Minimally doubled fermions and spontaneous chiral symmetry breaking

Rudina Osmanaj (Zeqirllari)
Department of Physics, University of Tirana

The 35th International Symposium on Lattice Field Theory, 18-24 June 2017
Palacio de Congresos, Granada, Spain

Lattice2017

Lattice2017

Outline

- Introduction
- BC fermions
- Dirac mode condensation
- Gauss – Lanczos quadrature
- Results
- Conclusions

Introduction

- Isospin breaking is a few percent:

  $\Rightarrow$ almost degenerate u and d quarks

- GMOR relation yields a few MeV quark masses:

  $\Rightarrow$ chiral symmetry broken explicitely by quark masses

- Why then a scalar should condense?
- Quark masses are small $\Rightarrow$ left and right movers are created at will
- QCD strings 

break chiral symmetry
- $\Rightarrow$ only 4 states condense instead of 8:

$$\pi^+ = u\bar{d}\ , \quad \pi^- = d\bar{u}\ , \quad \pi^0 = u\bar{u} - d\bar{d}\ , \quad \Sigma = u\bar{u} + d\bar{d}\ .$$

- $\Rightarrow$ pions and the chiral condensate: the blueprint of spontaneous ChSB

Introduction

- Fermions action discretization:
  - Doubling problem
  - If no doublers, chiral symmetry is broken

Nielsen – Ninomiya No – go theorem *(Nielsen, Ninomiya, 1981)*

"*A local, real, free fermion lattice action, having chiral and translational invariance, necessarily has fermion doubling* "

Introduction

- Doubling problem : obstacle to simulations

  Several ways to bypass No-Go theorem, but….

  - Wilson ➡ Broken chiral symmetry
  - DW or Overlap ➡ Numerical expensive (Non exact locality)
  - Staggered ➡ Rooting procedure (4 tastes)

➢ Another possibility : **Minimally doubled fermions**

i) 2 flavors ← 4 in Staggered

ii) Exact chiral: $U(1)_A \subset SU(2)$ ← Broken in Wilson

iii) Strict locality ← Not strict in DW or Overlap

Boriçi – Creutz action

Boriçi – Creutz fermionic action with the free Dirac operator (in the momentum space):

$$D(p) = \sum_{\mu} i\gamma_{\mu} \sin p_{\mu} + \sum_{\mu} i\gamma'_{\mu} \cos p_{\mu} - 2i\Gamma$$

This operator has zeros: $p_1 = (0,\ 0,\ 0,\ 0)$ dhe $p_2 = (\pi/2,\ \pi/2,\ \pi/2,\ \pi/2)$.

➢ There is always a special direction in euclidean space (given by the line that connects these two zeros: hypercubic diagonal)

⬇

➢Thus, these actions cannot maintain a full hypercubic symmetry *(P. F. Bedaque et al, 2008)*.

⬇

Hypercubic symmetry has to be restored
(Perturbative calculations *Capitani et al, 2010*)

$$D_{BC}(p) = \sum_{\mu} [i\gamma_{\mu} \sin p_{\mu} + i(\Gamma - \gamma_{\mu}) \cos p_{\mu}] + i(c_3 - 2)\Gamma$$

Banks-Casher relation

- Banks – Casher relation (*T. Banks, A. Casher, 1980*) provides a link between the chiral condensate and the spectral density $\rho(\lambda, m)$

$$\lim_{\lambda \to 0} \lim_{m \to 0} \lim_{V \to \infty} \rho(\lambda, m) = \frac{\Sigma}{\pi} \qquad \Sigma = -\lim_{m \to 0} \lim_{V \to \infty} \langle \bar{\psi}\psi \rangle$$

(If chiral symmetry is spontaneously broken by a non-zero value of the condensate the density of the quark modes in infinite volume does not vanish at the origin. A non-zero density conversely implies that the symmetry is broken)

- Instead of the spectral density, the average number ν(M,m) of eigenmodes of the Dirac operator with eigenvalues α ≤ M2 turns out to be a more convenient quantity to consider. Since

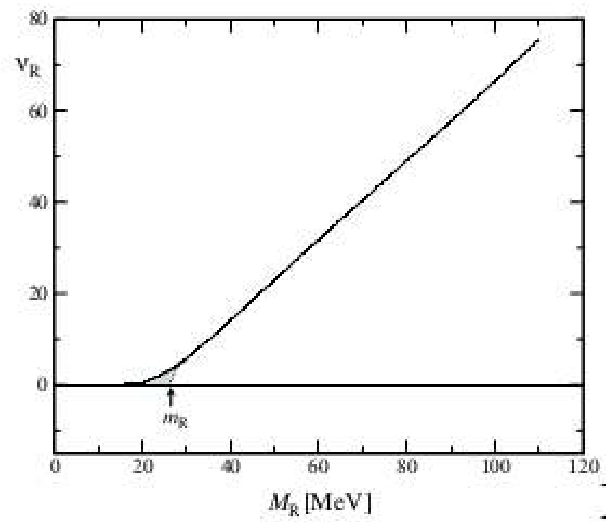
$$\nu(\Lambda) = V \int_{-\Lambda}^{\Lambda} d\lambda \rho(\lambda, m)$$

the mode number ultimately carries the same information as the spectral density.

The problem with Wilson fermions

- The number ν of eigenvalues as a function of a small cutoff:

$$\Lambda = \sqrt{M^2 - m^2}$$



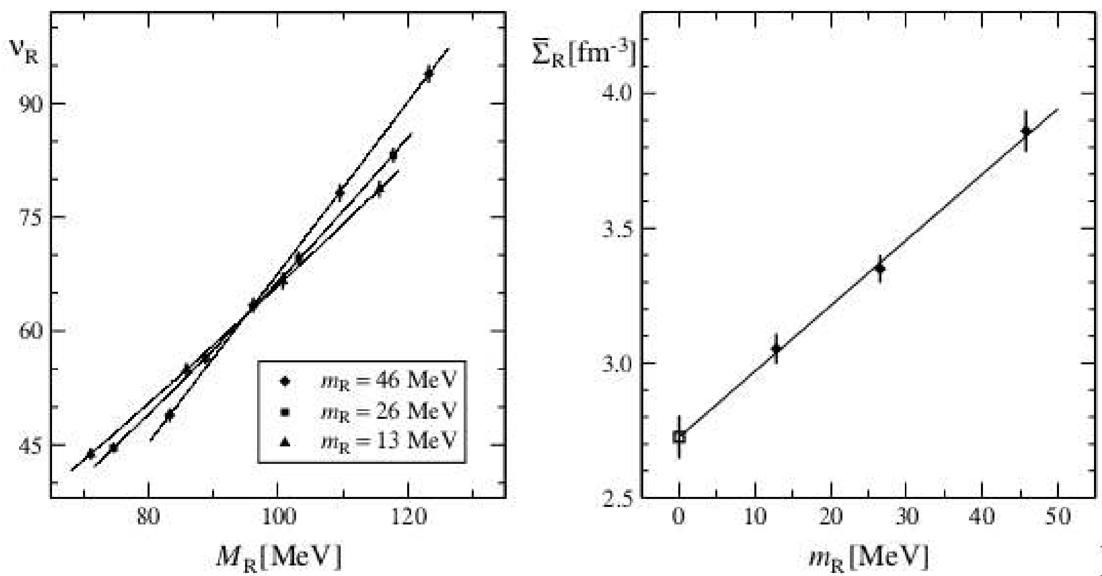
Lüscher 2008

- $\Rightarrow$ estimate the slope around $\Lambda \sim 100\text{MeV}$

Counting modes of the Wilson operator

- Count small modes $\Lambda = \sqrt{M^2 - m^2}$:

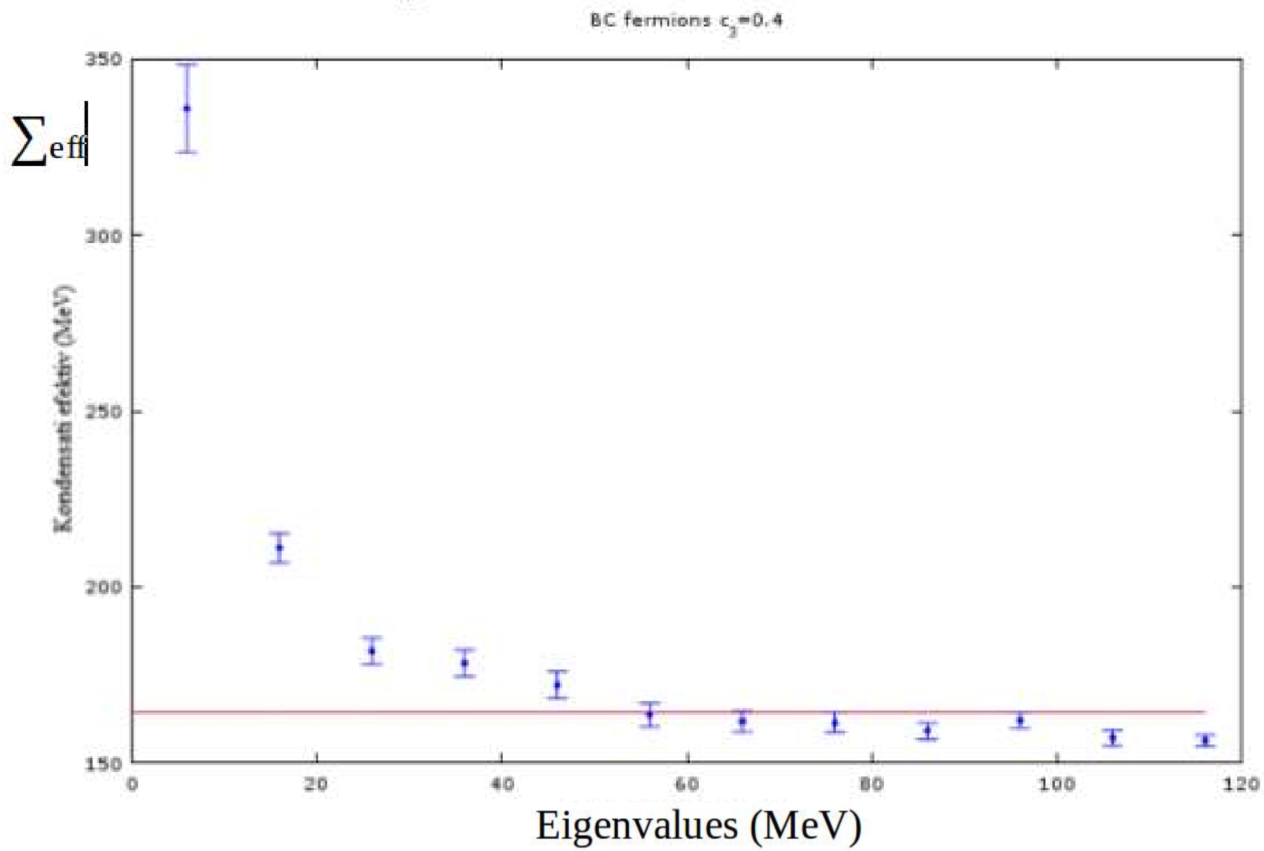
$$\nu(M,m) = 2\int_0^{\Lambda} \rho(\lambda)\, d\lambda \qquad \Leftrightarrow \qquad \Sigma_{\text{eff}} = \frac{\pi}{2}\sqrt{1 - \frac{m^2}{M^2}}\, \frac{\partial \nu(M,m)}{\partial M}$$



Lüscher 2008

- Count modes using Gauss-Lanczos quadrature:

$$\nu(\Lambda) = \sum_j w_j [1 + \mathrm{sgn}(\Lambda - \lambda_j)]/2 \, , \qquad \Sigma_{\mathrm{eff}} = \frac{\pi}{2} \frac{\nu(\Lambda)}{\Lambda}$$



Gauss-Lanczos quadrature

Let $A \in \mathbb{C}^{N\times N}$ be a hermitian matrix and $b \in \mathbb{R}^N$ a starting vector. Then the following algorithm computes the Gauss - Lanczos quadrature [17, 16]

Algorithm 1 Algorithm for the Gauss - Lanczos quadrature

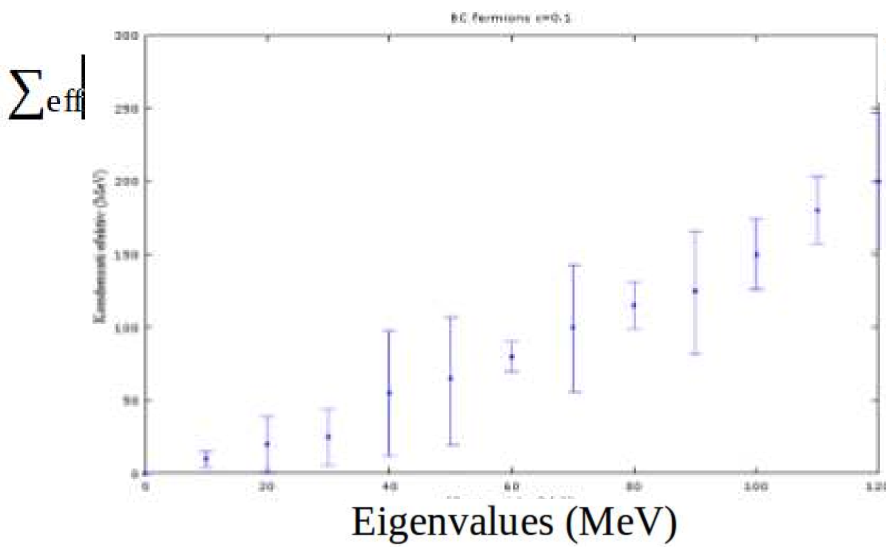
Compute α_i and β_i using Lanczos algorithm for $Ax = b$
Set $(T_n)_{i,i} = \alpha_i$, $(T_n)_{i+1,i} = (T_n)_{i,i+1} = \beta_i$ otherwise $(T_n)_{i,j} = 0$
Compute eigenvalues λ_i and eigenvectors v_i of T_n, where $i = 1....n$
Sort eigenvalues and eigenvectors in the increasing order of eigenvalues
Set k as the maximum index which correspond to the cut-off eigenvalue
Set θ_i to the positive square root of the original eigenvalues
Set z_i the first element of eigenvectors v_i where $i = 1....n$
Set $\omega_i = z_i^2$
Compute the mode number $\nu_k = \sum_{i=1}^{k} \omega_i$

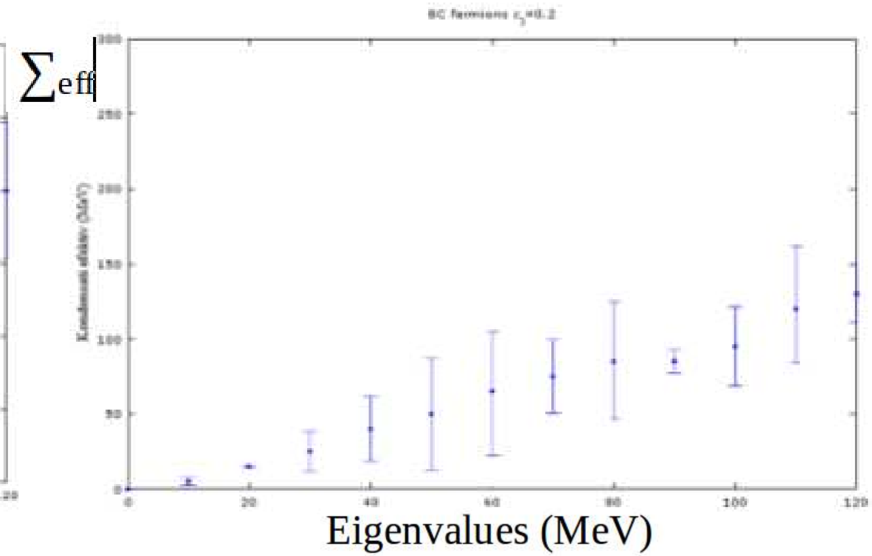
Details of simulations

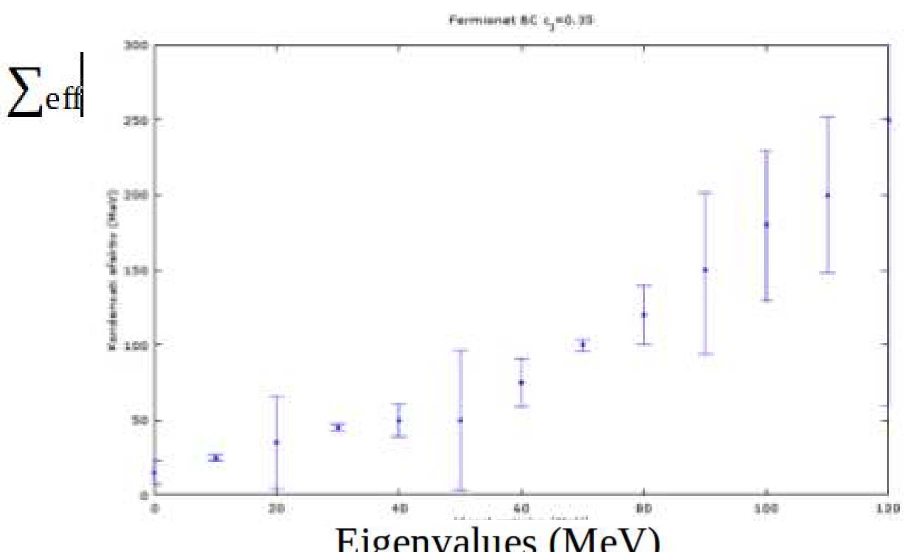
- Calculations of the effective chiral condensate.

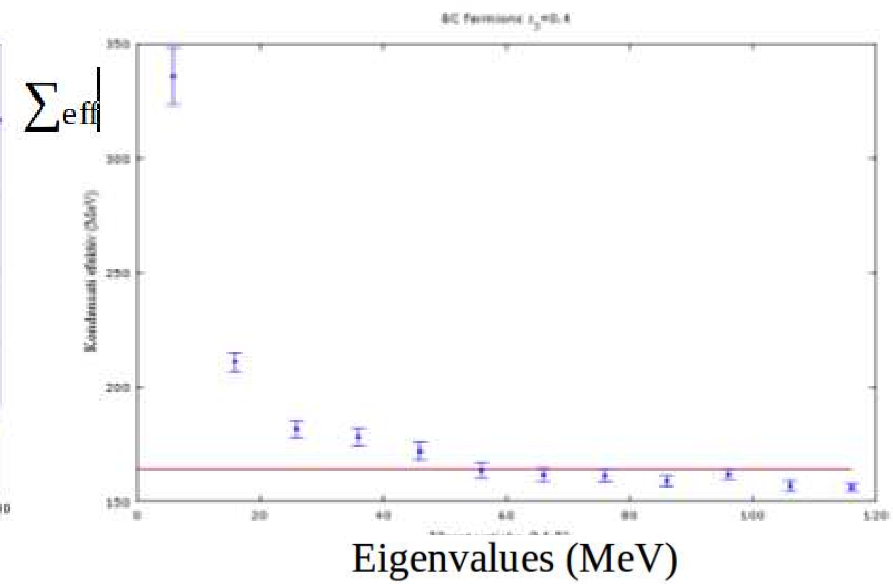
- Lattice 24^4
- Quenched approximation
- Wilson gauge action ($\beta = 6$)
- Boriçi – Creutz action
- Lanczos inverter
- Zero quark mass (BC fermions are chiral fermions)
- Seven different counterterms c3 (0.1, 0.2 0.35, 0.4, 0.6,0.8,1)

Results

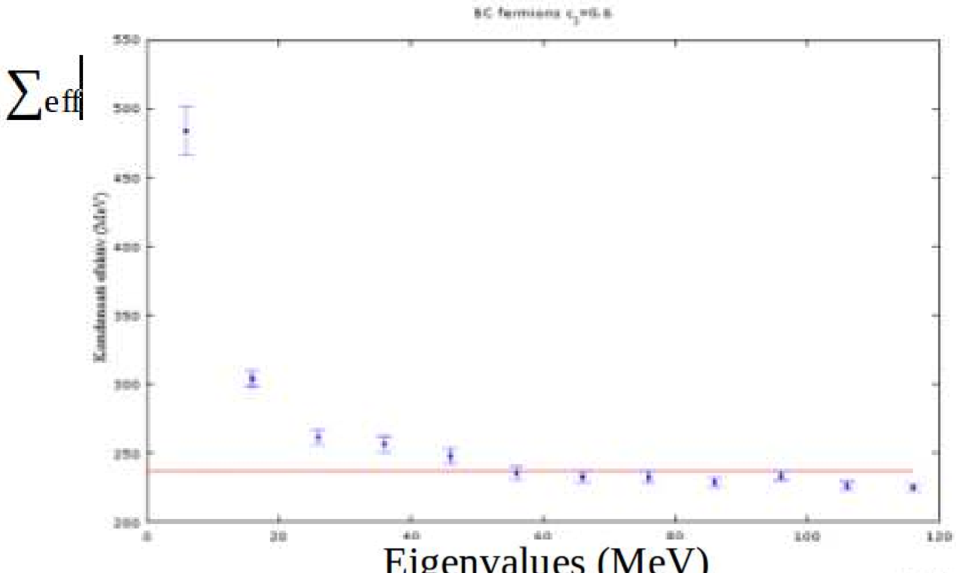

BC fermions c=0.1
∑eff
Eigenvalues (MeV)


BC fermions c_3=0.2
∑eff
Eigenvalues (MeV)

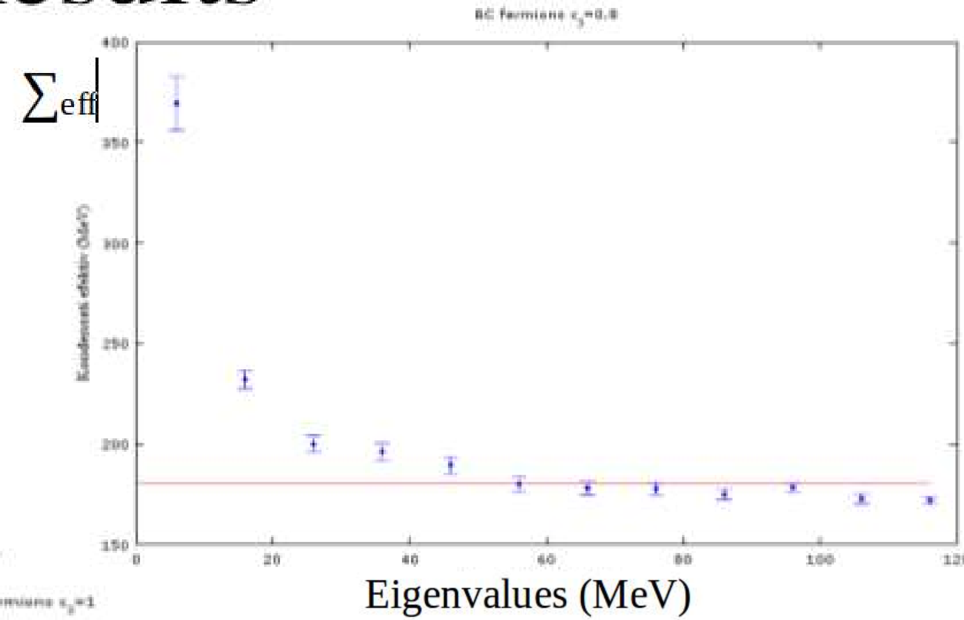

Fermionet BC c_3=0.35
∑eff
Eigenvalues (MeV)


BC fermions c_3=0.4
∑eff
Eigenvalues (MeV)

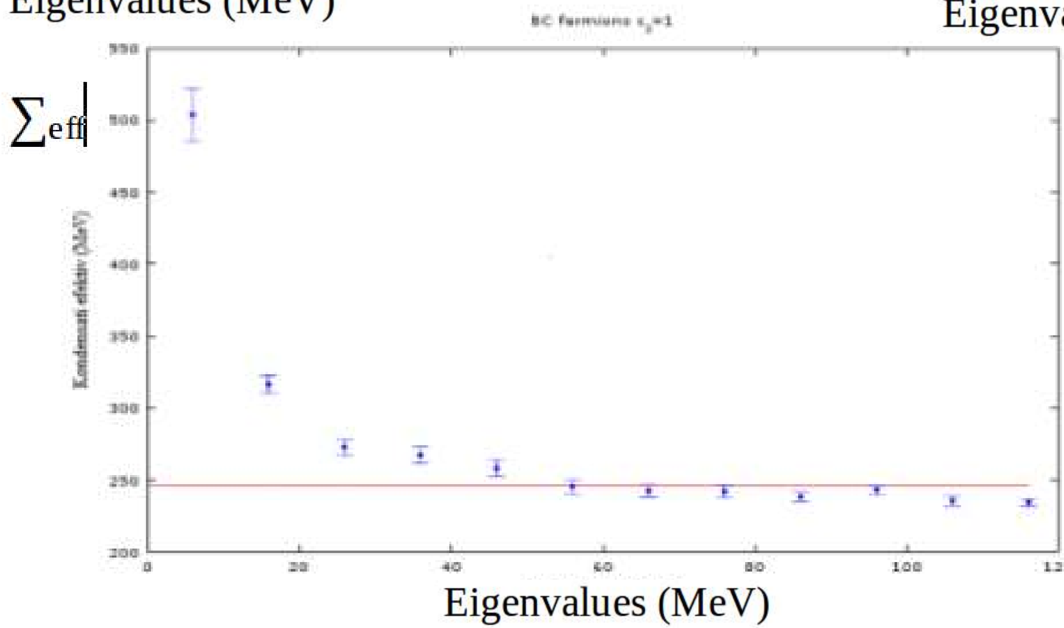
Results



BC fermions c_s=0.6
Kondensat efektiv (MeV)
∑eff
Eigenvalues (MeV)



BC fermions c_s=0.8
Kondensat efektiv (MeV)
∑eff
Eigenvalues (MeV)



BC fermions c_s=1
Kondensat efektiv (MeV)
∑eff
Eigenvalues (MeV)

Conclusions

Chiral symmetry and spontaneous chiral symmetry breaking are very imporant in QCD

Using MDF fermions (as chiral fermions) and Lanczos quadrature we can explore and understand the dynamical mechanism of SchSB in a very simple way.

The chiral condensate can be used as an order parameter for BC fermions, and help us to find the proper counterterms that restore partially the broken hypercubic symmetry.

This work aims to the use of this methodology for further detailed studies of minimally doubled fermions.