Dispersion Relation of Charmonia above T_c

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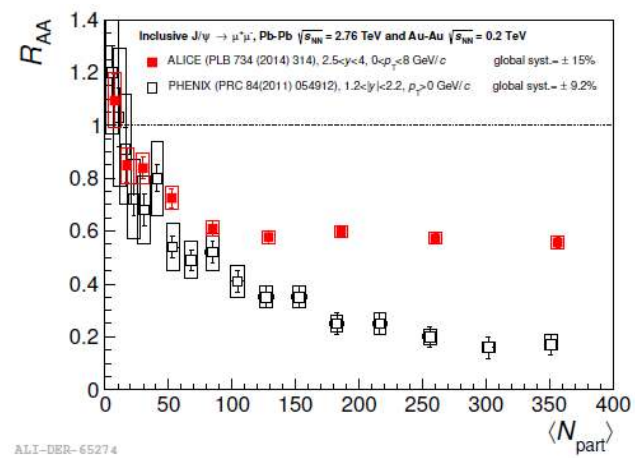
Ikeda, Asakawa, MK, PRD**95** (2017) 014504

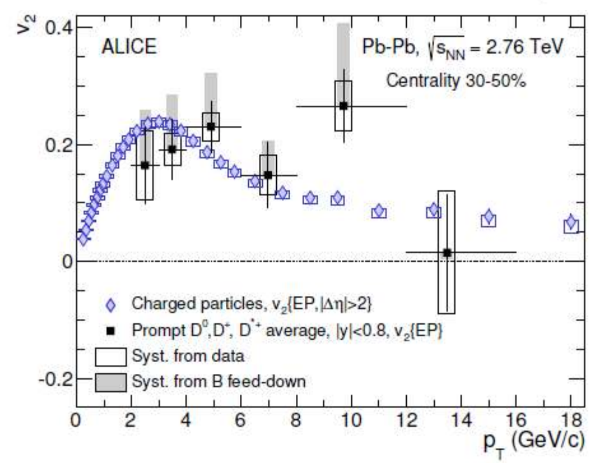
Charm Quarks in HIC

Impurity of QGP
= unique experimental probe

- production only in first stage
- small abundance

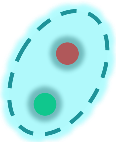
- J/p suppression
- transport property
- heavy-quark potential





Figs. from arXiv:1506.03981

Charmonia above Tc



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  - Melting temperature
  - Mass shift?

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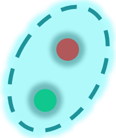
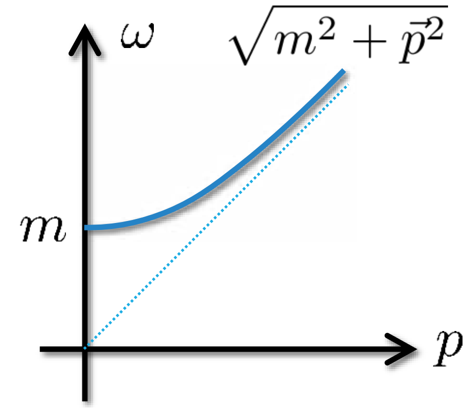
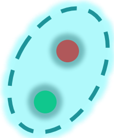


- Property of moving charmonia
  - Dispersion relation
  - residue
  - decay rate



In heavy-ion collisions, charmonia are typically moving!

Charmonia above Tc



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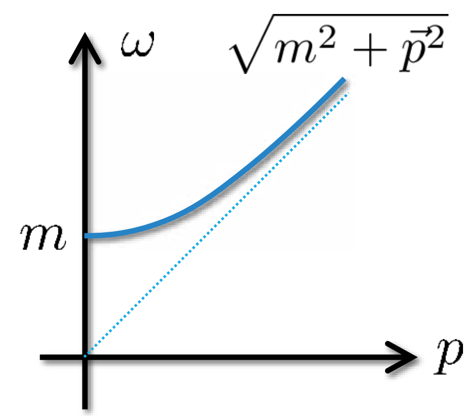
Let's study on the lattice



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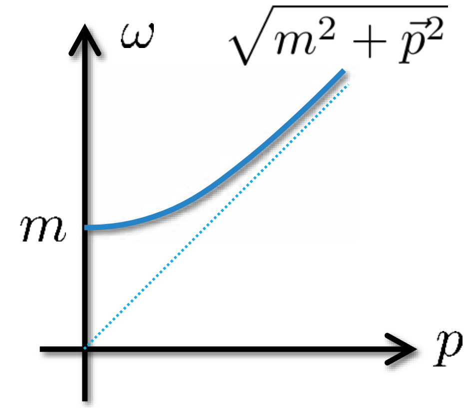
Nonzero-p Spectral Func.

In vacuum :Lorentz symmetry

- Tensor structure (V) $\rho_{\mu\nu}(\omega,\vec{p}) = \left(\frac{p_\mu p_\nu}{p^2} - g_{\mu\nu}\right)\rho_V(p)$
- Bound-state pole $\sim Z\delta(\omega^2 - E(\vec{p})^2) = \frac{Z}{2E(\vec{p})}\delta(\omega - 2E(\vec{p}))$
- Dispersion relation $E(\vec{p}) = \sqrt{m^2 + \vec{p}^2}$



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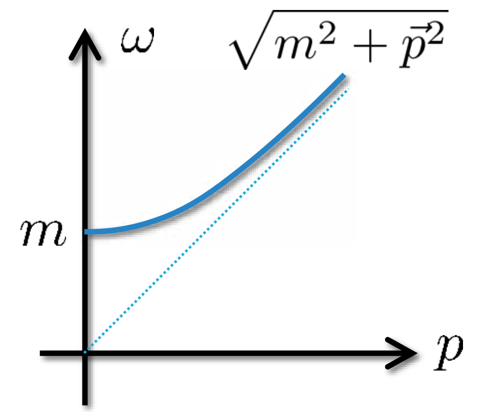
In medium

- Transverse and longitudinal splitting

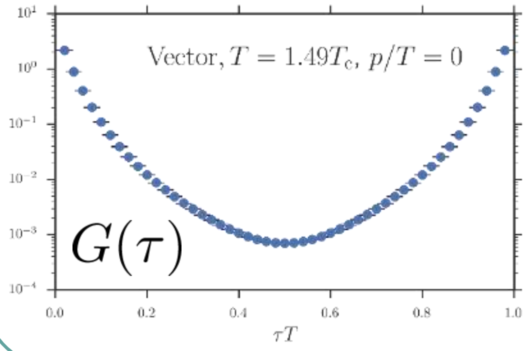
$$\rho_{\mu\nu}(\omega,\vec{p}) = \rho_{\mathrm{T}}(\omega,\vec{p})\Lambda_{\mathrm{T}} + \rho_{\mathrm{L}}(\omega,\vec{p})\Lambda_{\mathrm{L}}$$

- Dispersion relation can be modified
- Z no longer be a constant

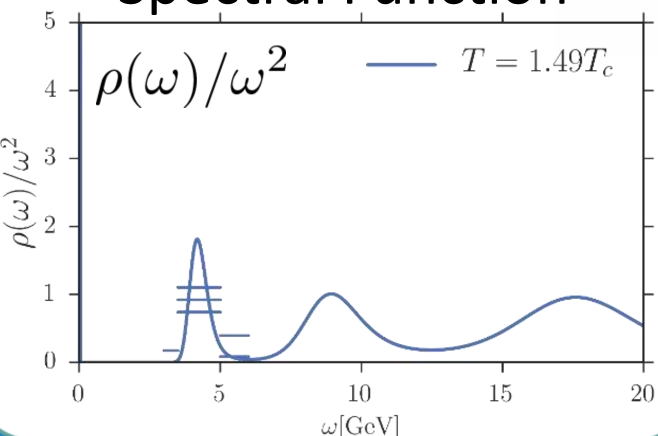


Maximum Entropy Method

Lattice data



Spectral Function



$$G(\tau) = \int_0^\infty d\omega \frac{\cosh(1/2T - \tau)\omega)}{\sinh(\omega/2T)} \rho(\omega)$$

"ill-posed problem"

Maximum Entropy Method

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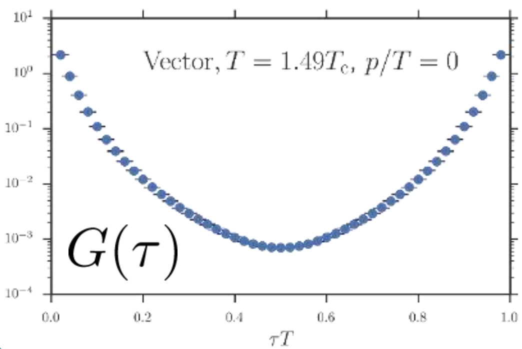


Bayes theorem

✖

Prior probability

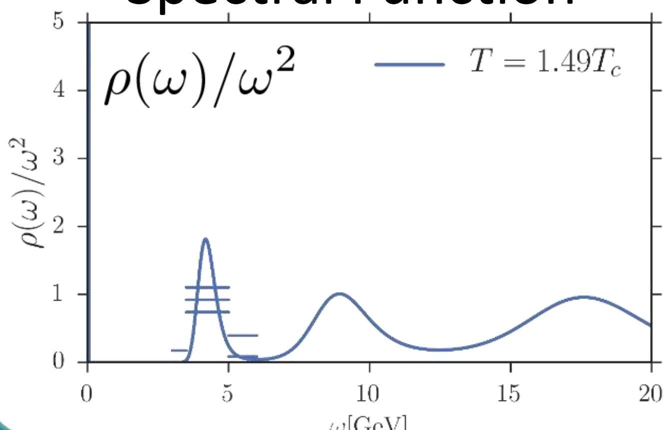
- Shannon-Jaynes entropy
- default model $m(\omega)$

=

Probability of $\rho(\omega)$

$P[\rho(\omega), \alpha]$

Spectral Function



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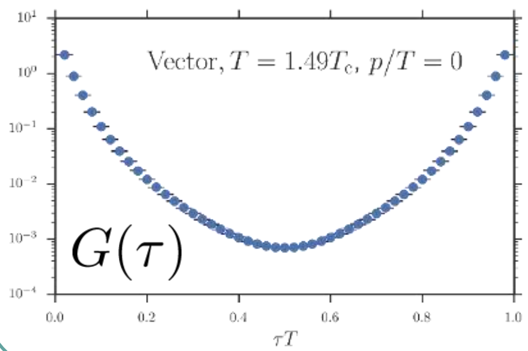

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Probability of $\rho(\omega)$

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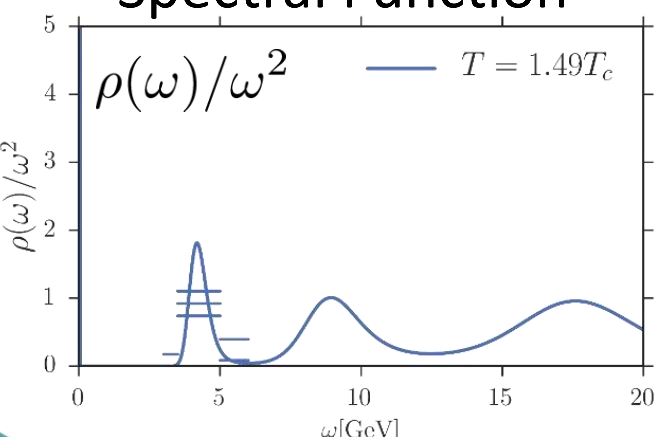
expectation value

$\langle \rho(\omega) \rangle_P$

$$\langle \mathcal{O} \rangle_P = \int d\alpha \int [d\rho] P[\rho, \alpha] \mathcal{O}$$

Spectral Function

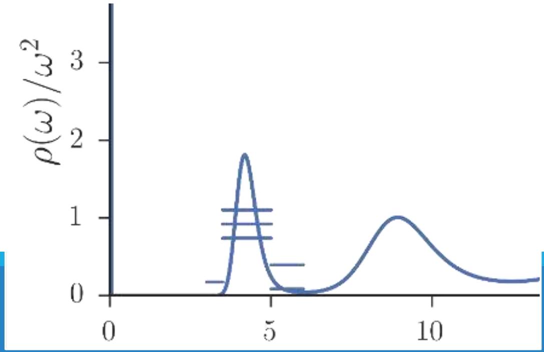


Error in MEM

MEM error = variance in P[ρ(ω),α] space

$$W = \int d\omega f(\omega)\rho(\omega)$$

- exp. val.: $\langle W \rangle_P = \int d\omega f(\omega) \langle \rho(\omega) \rangle_P$
- error: $\Delta W = \sqrt{(W - \langle W \rangle_P)^2}$



Lattice2017

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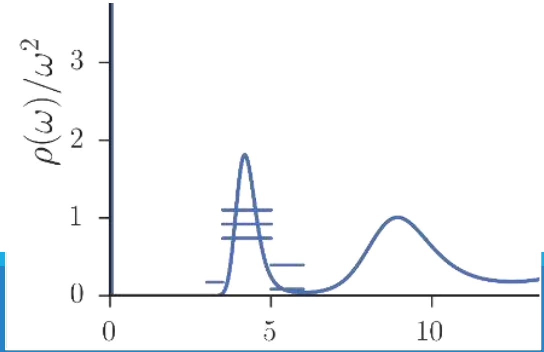
→ **typically huge**



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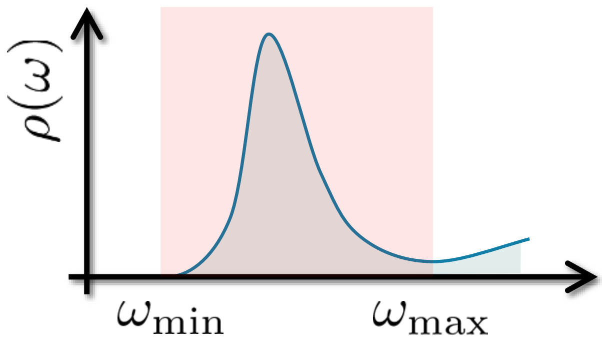
NOTE

- SPC obtained by MEM is just an image. No robust meaning.
- MEM error is more conservative than statistical one.

Defining Peak Position

Center of weight in a range $[\omega_{\rm min}:\omega_{\rm max}]$

$$\bar{E} = \frac{\int_{\omega_{\rm min}}^{\omega_{\rm max}} d\omega\omega(\rho(\omega)/\omega^2)}{\int_{\omega_{\rm min}}^{\omega_{\rm max}} d\omega\rho(\omega)/\omega^2}$$



- Represent peak position for a sufficiently sharp peak
- **Error analysis in MEM is possible!**
- $[\omega_{\rm min}, \omega_{\rm max}]$ dependence has to be checked

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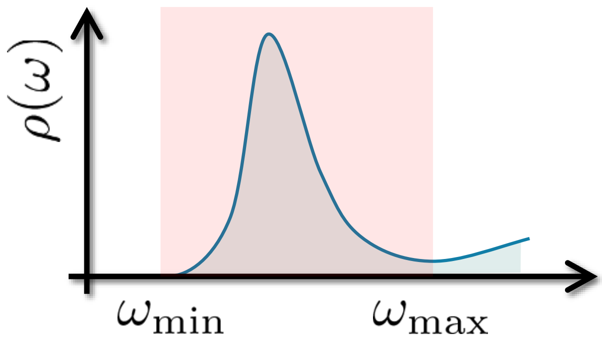

- Represent peak position for a sufficiently sharp peak
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- Residue: $\bar{Z} = \int_{\omega_1}^{\omega_2} d\omega 2\omega \rho(\omega)$

Lattice Setup

- quenched simulation
- Wilson fermion / gauge
- anisotropic lattice ($a_\sigma/a_\tau=4$)

$$\beta = 7.0,\ \gamma_F = 3.476,\ \kappa_\sigma = 0.8282$$
$$a_\sigma = 0.00975[\mathrm{fm}],\ a_\sigma/a_\tau = 4$$

Asakawa, Hatsuda, 2004

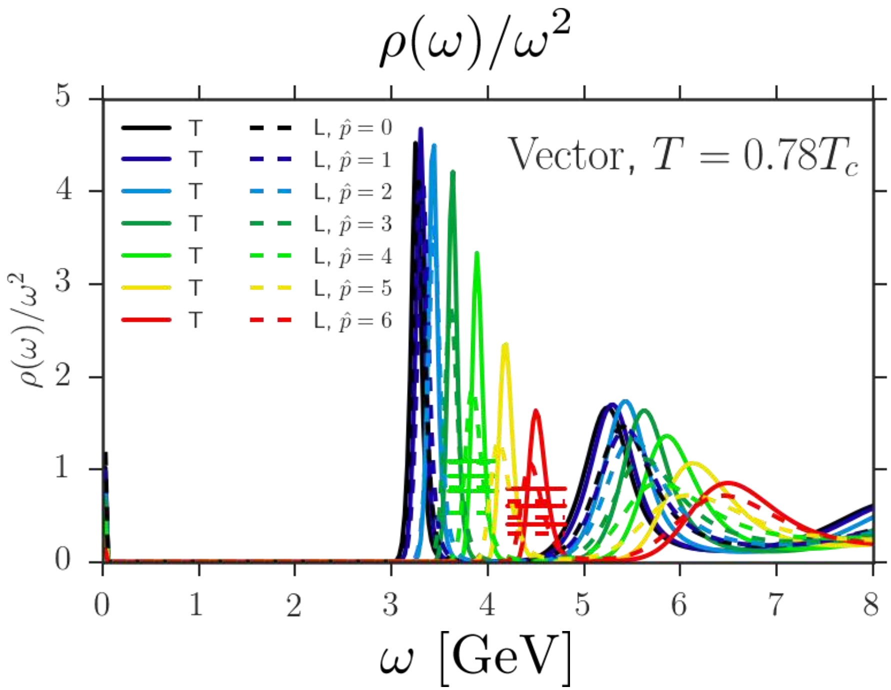
N_τ	T/T_c	N_σ	L_σ[fm]	N_{conf}
40	1.86	64	**2.5**	500x8
46	1.62	64	**2.5**	500x8
50	1.49	64	**2.5**	500x8
96	0.78	64	**2.5**	500x8

8 measurements on each conf.

BlueGene/Q@KEK
fermion part: Iroiro++

- Large spatial volume → high momentum resolution
- Large Nt / high statistics → high MEM precision

Spectral Func. @ T=0.78T



- ρ_T and ρ_L channels degenerate
- although correlators are different

$$\vec{p} = (p, 0, 0)$$

$$G_L = \frac{\omega^2 - p^2}{\omega^2} G_{11}$$

$$G_T = G_{22} = G_{33}$$

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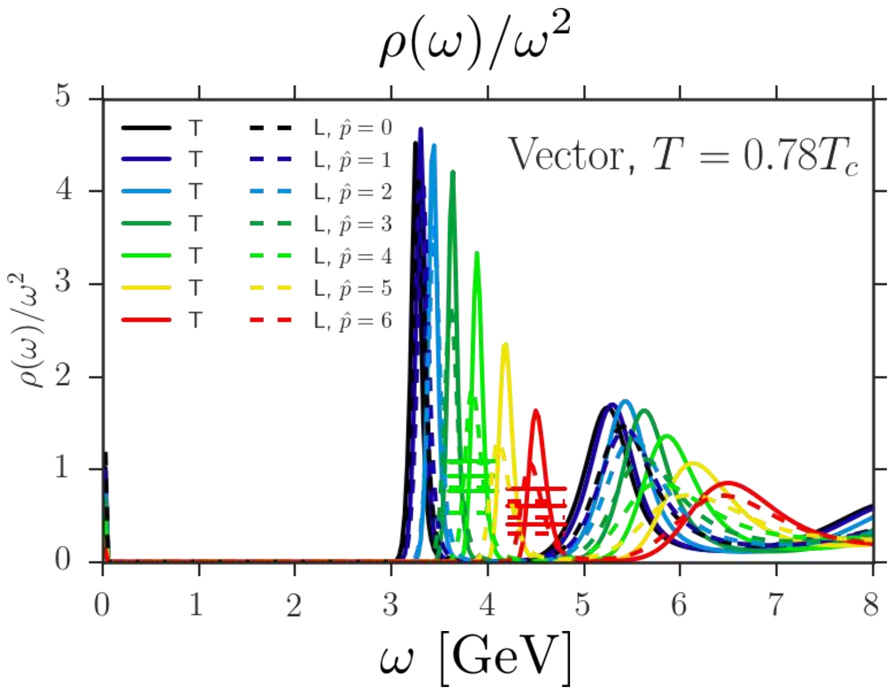

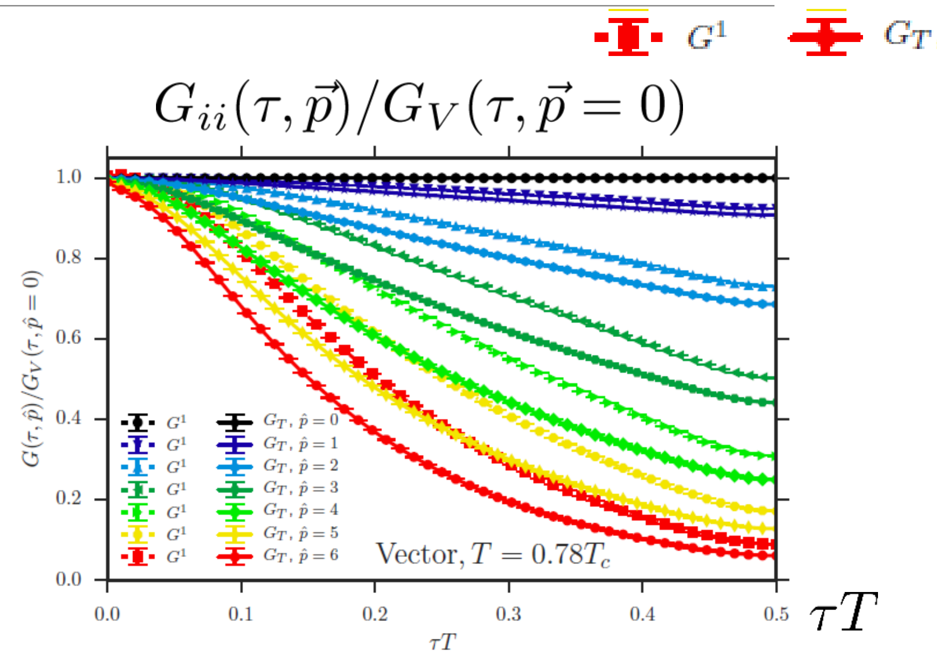

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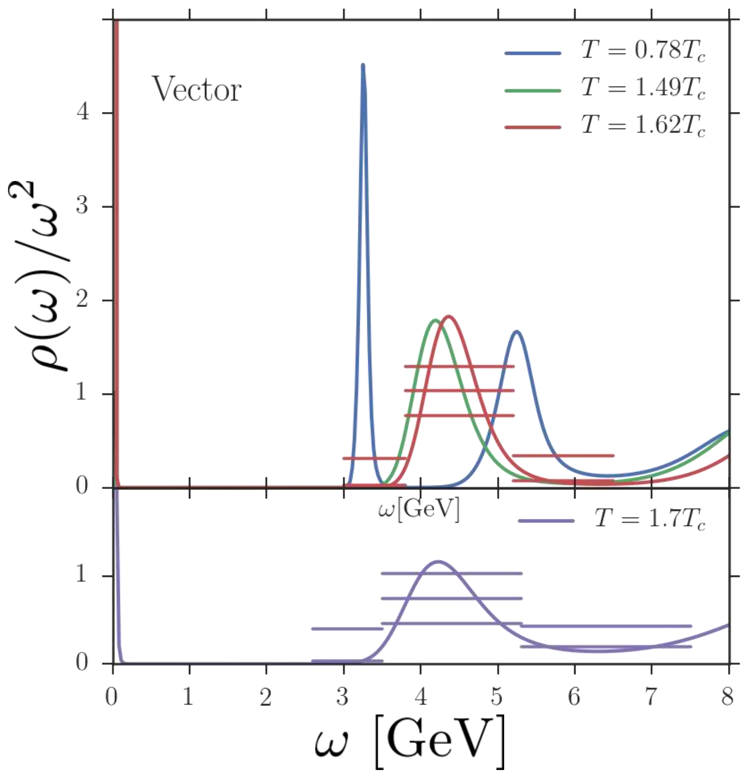
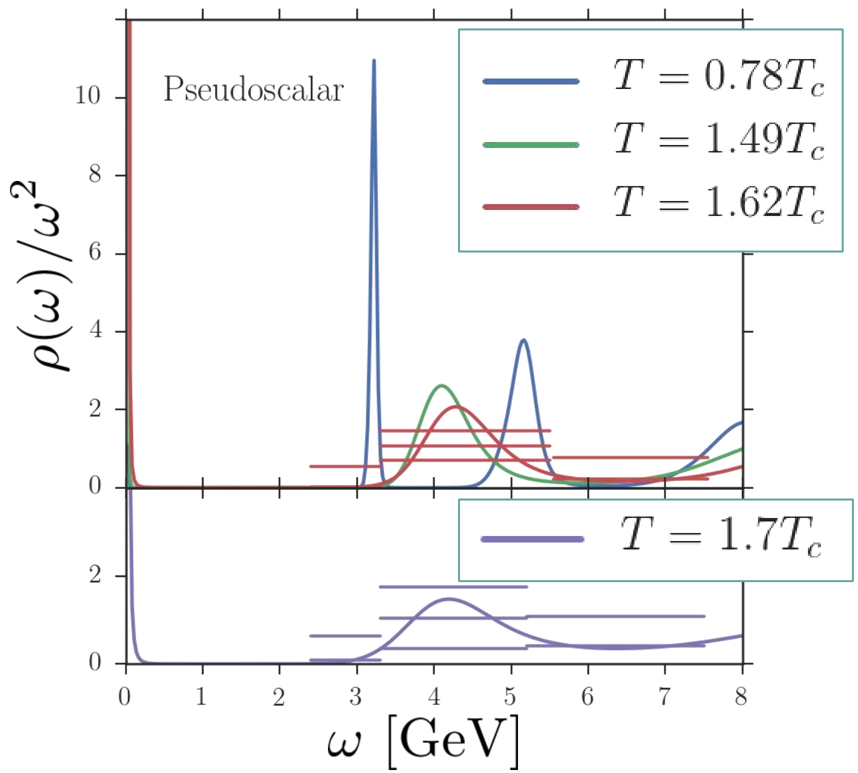
Spectral Function @ p=0

Vector channel

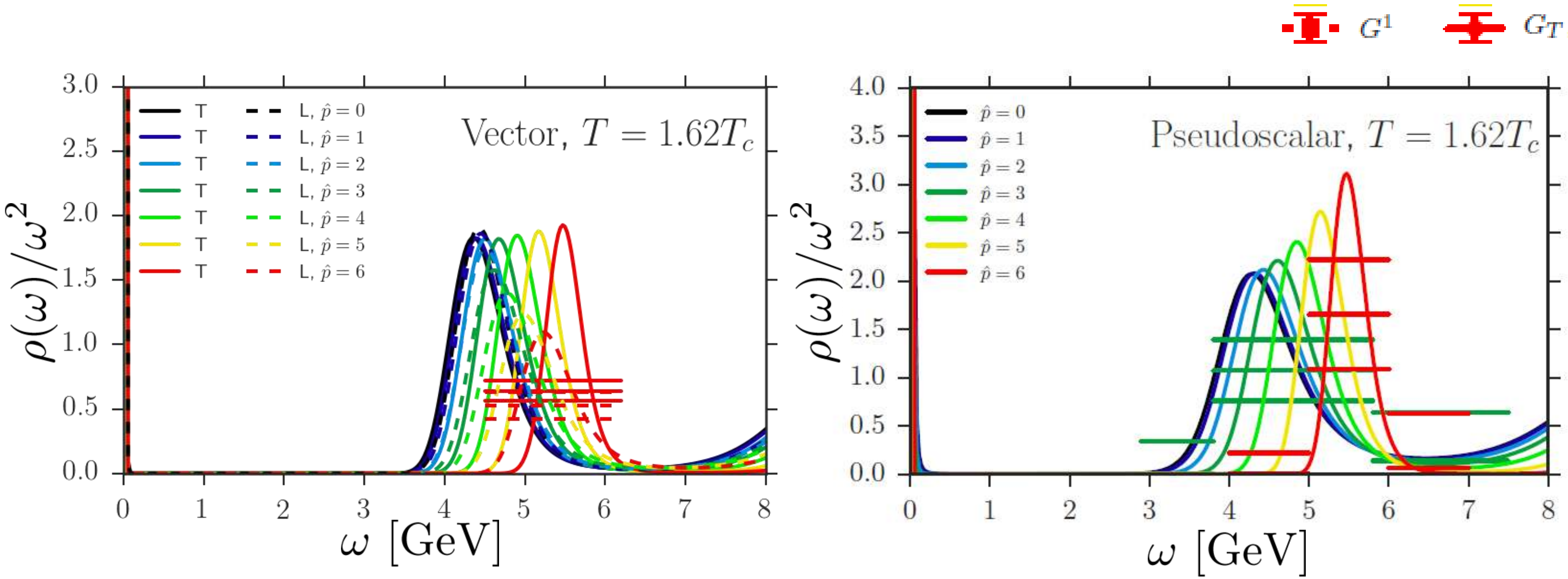


PS channel



- Bound state peaks seem to exist at T=1.62Tc.

Spectral Func. @ T=1.62T

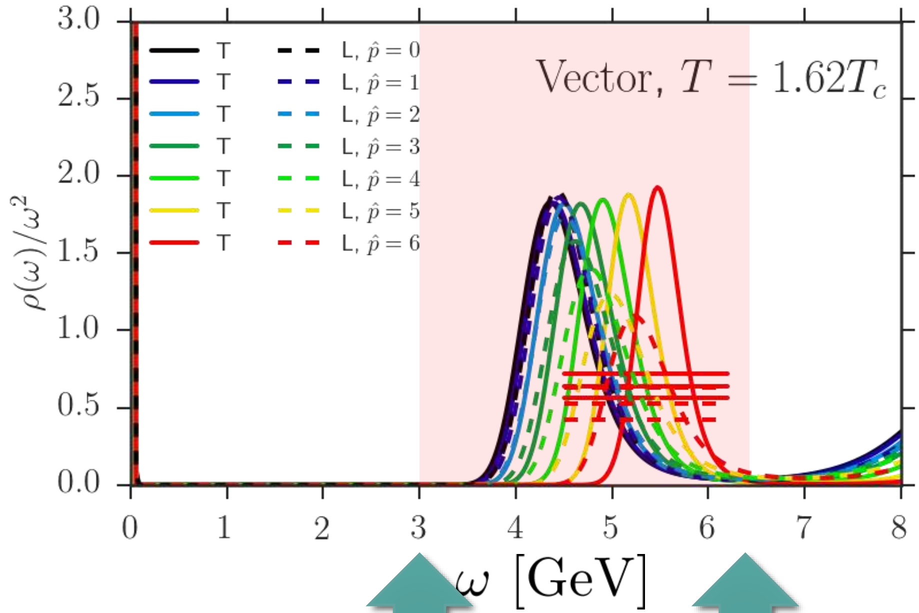


- ρ_T and ρ_L channels seem to degenerate.
- Peak exists for all momentum.

Lattice2017

Dispersion Relation

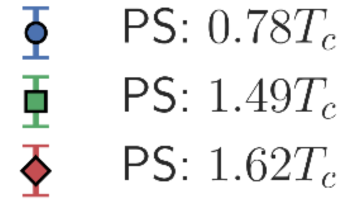
Energy interval [ω_{min}, ω_{max}] for disp. rel.

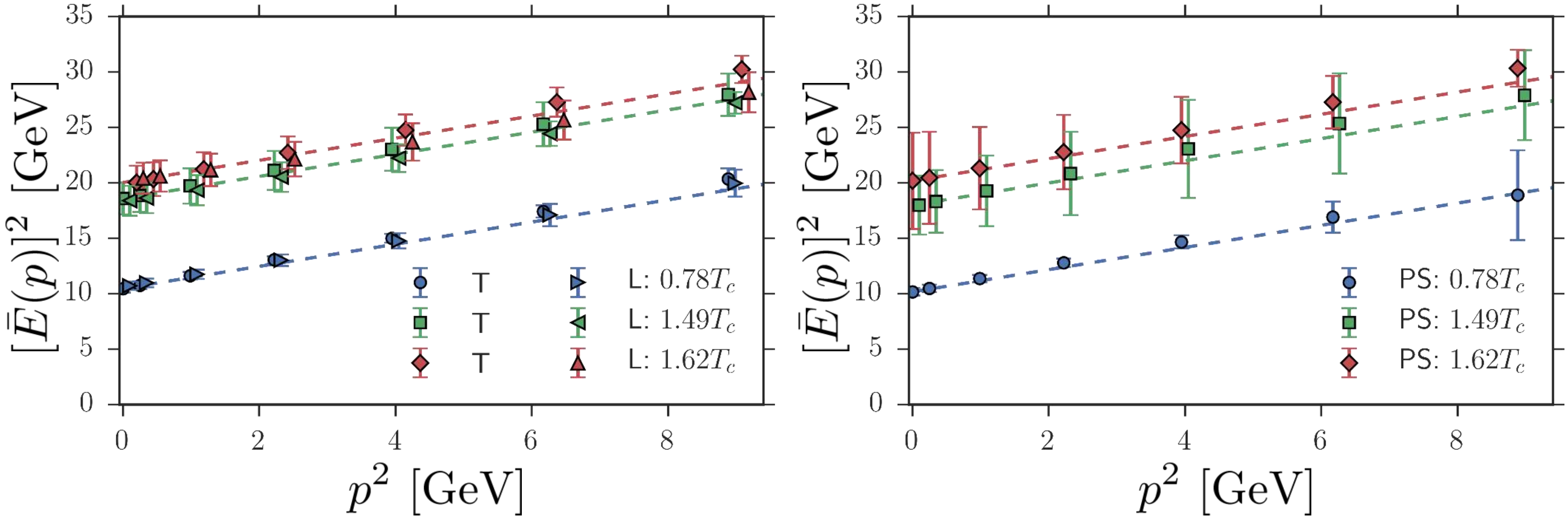


$\omega_{\rm min} = 3\text{GeV}$

$\omega_{\rm max}$:first minimum

Dispersion Relation





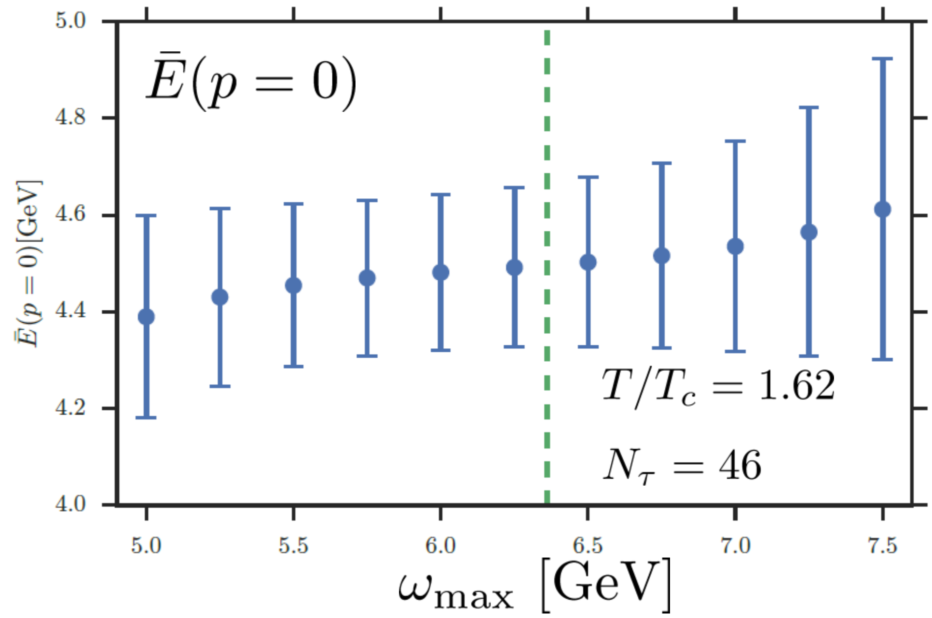
- Clear mass enhancement in medium.
- Dispersion relation is consistent with the Lorentz covariant form even at T=1.62Tc.

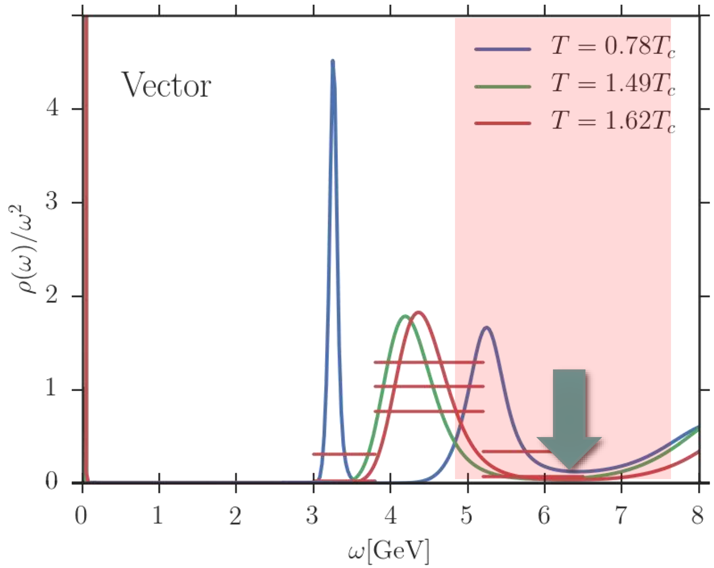
mass E(p=0)

T/T_c	0.78	1.49	1.62
J/ψ	3.24(6)	4.30(16)	4.47(16)
η_c	3.19(5)	4.24(31)	4.49(48)

[GeV]

Dependence on $[\omega_{\min} : \omega_{\max}]$





- $\omega_{\max}$ dependence is well suppressed.
- No $\omega_{\min}$ dependence for $\omega_{\min}$<3GeV.

Test: N_t Dependence

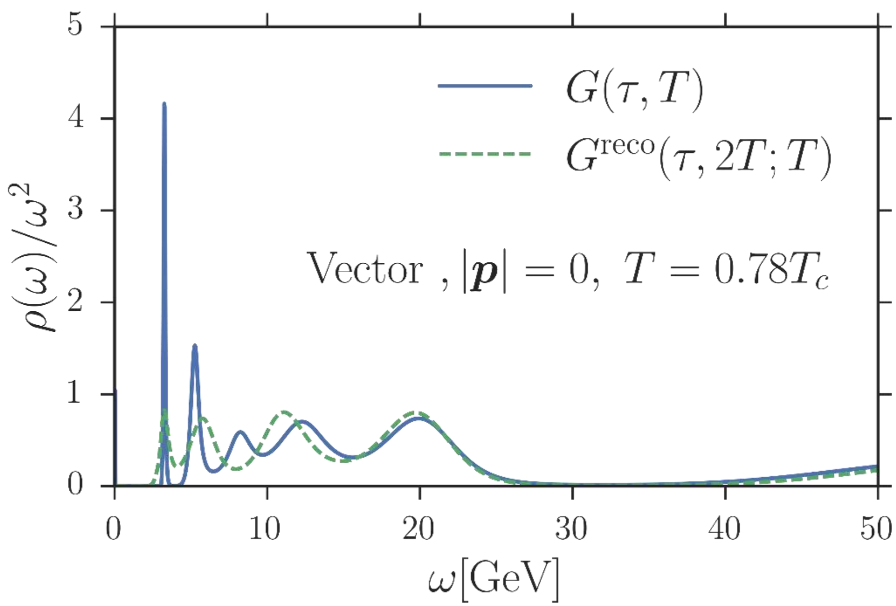
Special thanks to A. Rothkopf

correlator

$T = 0.78T_c,\ N_\tau = 96$

reconstructed cor.

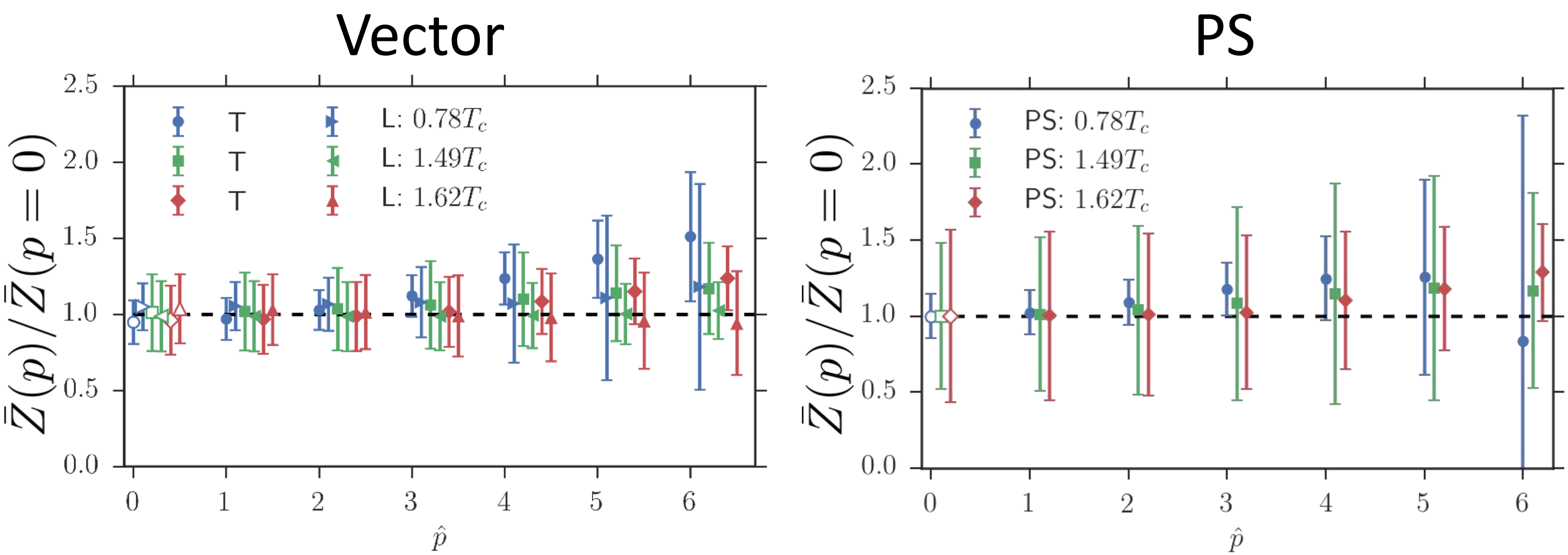
$T = 0.78T_c,\ N_\tau = 48$



- Peak position does not shift with the change of N_t

correlator	$G(\tau, 0, T)$	$G^{\rm rec}(\tau, 0, 2T; T)$
$\bar{m}$	3.24(6)	3.40(90)

Res due



- No p dependence of Z even for T=1.62Tc
- No T/L splitting in vector channel

Summary

- We analyzed the **peak positions in SPC with MEM error** by **defining** them in terms of the **center of weight**.

- Charmonia have **significant mass enhancement**.
- Dispersion relations are consistent with **Lorentz covariant form even at T=1.62Tc**.

Future Work

- much finer p resolution
- m_q dependence
- comparison with potential models and etc.