

Lattice2017

Equation of state in (2+1)-flavor QCD at physical point with improved Wilson fermion action using gradient flow



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with

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(WHOT-QCD Collaboration)

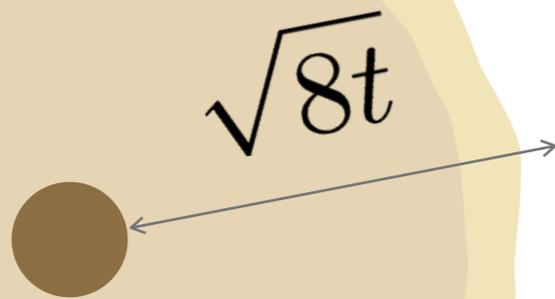
QCD thermodynamics with gradient flow

Gradient flow

Narayanan-Neuberger (2006), Lüscher (2009–)

Imaginary evolution of the system in terms of a fictitious "time" t preserving gauge sym. etc.:

(ex) pure gauge theory $\dot{B}_\mu = D_\nu G_{\nu\mu}, \quad B_\mu|_{t=0} = A_\mu \leftarrow$ original gauge field



We may view the flowed field B_μ as a smeared A_μ over a physical range of $\sqrt{(8t)}$.

It was shown that operators of flowed fields have no UV divergences nor short-distance singularities at $t > 0$. Lüscher-Weisz (2011)

GF provides us with a new physical (i.e. non-perturbative) renormalization scheme, which is directly calculable on the lattice in the $a \rightarrow 0$ limit.

This opened many possibilities to drastically simplify lattice evaluation of physical observables.

=> Using the finiteness of flowed observables, H. Suzuki proposed a new method to calculate the EMT, which is the generator of continuous coord. trans. and thus has not been simple to define/evaluate on the lattice.

QCD thermodynamics with gradient flow

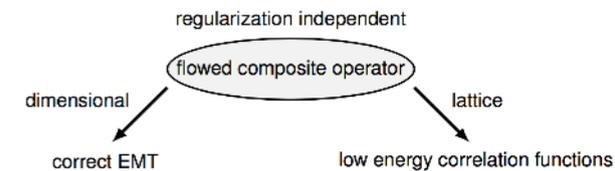
Energy-momentum tensor using GF

H.Suzuki (2013)

To avoid violation of Poincare inv. on the lattice,

1) Define EMT in a continuum scheme, using a W-T identity of Poincare inv., as usual.

2) When we flow this EMT to $t > 0$, because it is finite, it becomes directly calculable on the lattice in the $a \rightarrow 0$ limit.



By the GF evolution, however, higher- d operators can contaminate at $t > 0$.

3) Remove the unwanted contributions by another extrapolation of $t \rightarrow 0$.

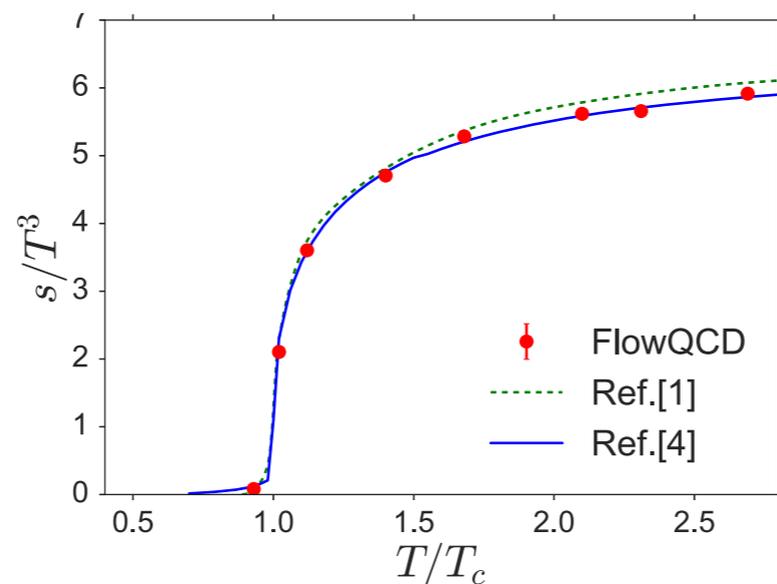
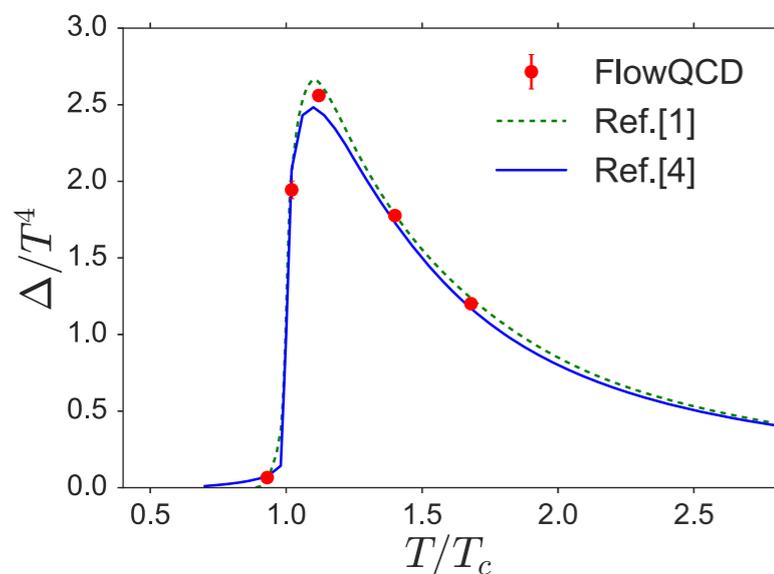
Can make this extrapol. smoother using a small- t oper. expansion by Lüscher:

The coeff's. c_i near the $t \rightarrow 0$ limit can be calculated by PT in AF theories.

$$\tilde{O}(t, x) \xrightarrow{t \rightarrow 0} \sum_i c_i(t) O_i^R(x)$$

Test in quenched QCD

FlowQCD, PRD94, 114512(2016); D90, 011501(2014) [E:D92,059902]



$$T_{\mu\nu}^R(x) = \lim_{t \rightarrow 0} \left\{ \frac{1}{\alpha_U(t)} U_{\mu\nu}(t, x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)} [E(t, x) - \langle E(t, x) \rangle_0] \right\}$$

$$\epsilon = -\langle T_{00} \rangle, \quad p = \frac{1}{3} \sum_i \langle T_{ii} \rangle$$

The EOS' from the conventional methods reproduced in $t \rightarrow 0$ and $a \rightarrow 0$.

(2+1)-flavor QCD thermodynamics with GF

Our project: Application to (2+1)-flavor QCD

GF with quarks : [Lüscher, JHEP 1304, 123 \(2013\)](#)

- * We can adopt pure gauge actions for GF,
- * at the price of a non-trivial field renormalization of quarks.

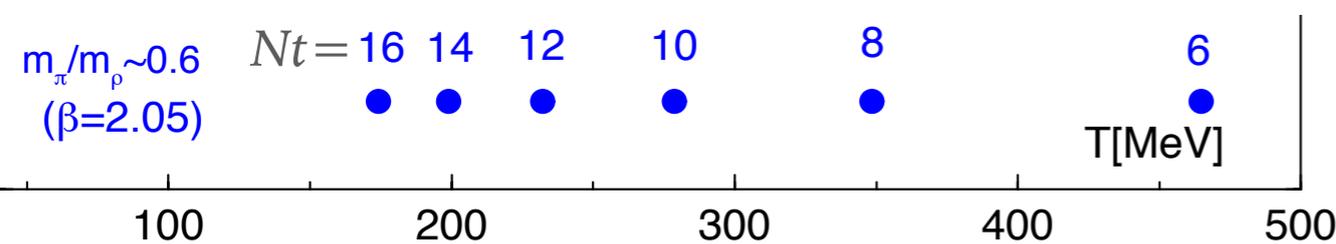
Full QCD EMT by GF : [Makino-Suzuki, PTEP 2014, 063B02 \(2014\)](#)

Chiral condensate by GF : [Hieda-Suzuki, Mod.Phys.Lett.A31, 1650214 \(2016\)](#)

Topological charge / susceptibility, etc. etc.

1st step:

- Heavy ud quarks with \approx physical s quark ($m_{PS}/m_V \approx 0.63$).
- Fine lattice ($a \approx 0.07\text{fm}$) with the fixed-scale approach.
- Compare with the results of the conventional methods.



* $N_f=2+1$ QCD, Iwasaki gauge + NP-clover

* CP-PACS+JLQCD's $T=0$ config. ($\beta=2.05, 28^3 \times 56, a \approx 0.07\text{fm}$)

* $T > 0$ by fixed-scale approach, ($32^3 \times Nt, Nt = 4, 6, \dots, 14, 16$):
 $T \approx 174--697\text{MeV}$

* EoS by T -integration method available (WHOT-QCD, PRD85)

* gauge meas. at every config., quark meas. every 10 config's.

Nf=2+1 QCD EMT by GF

Makino-Suzuki, PTEP 2014, 063B02 (2014)

EMT in full QCD

Operators on the lattice

$$\tilde{O}_{1\mu\nu}(t, x) \equiv G_{\mu\rho}^a(t, x)G_{\nu\rho}^a(t, x),$$

$$\tilde{O}_{2\mu\nu}(t, x) \equiv \delta_{\mu\nu}G_{\rho\sigma}^a(t, x)G_{\rho\sigma}^a(t, x),$$

$$\tilde{O}_{3\mu\nu}^f(t, x) \equiv \varphi_f(t)\bar{\chi}_f(t, x) \left(\gamma_\mu \overleftarrow{D}_\nu + \gamma_\nu \overleftarrow{D}_\mu \right) \chi_f(t, x),$$

$$\tilde{O}_{4\mu\nu}^f(t, x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t, x) \overleftarrow{D} \chi_f(t, x),$$

$$\tilde{O}_{5\mu\nu}^f(t, x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t, x)\chi_f(t, x),$$

Quark field renormalization

$$\varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \langle \bar{\chi}_f(t, x) \overleftarrow{D} \chi_f(t, x) \rangle_0}.$$

Physics extracted by $t \rightarrow 0$ extrapolation.

$$T_{\mu\nu}(x) = \lim_{t \rightarrow 0} \left\{ c_1(t) \left[\tilde{O}_{1\mu\nu}(t, x) - \frac{1}{4} \tilde{O}_{2\mu\nu}(t, x) \right] + c_2(t) \left[\tilde{O}_{2\mu\nu}(t, x) - \langle \tilde{O}_{2\mu\nu}(t, x) \rangle_0 \right] + c_3(t) \sum_{f=u,d,s} \left[\tilde{O}_{3\mu\nu}^f(t, x) - 2\tilde{O}_{4\mu\nu}^f(t, x) - \langle \tilde{O}_{3\mu\nu}^f(t, x) - 2\tilde{O}_{4\mu\nu}^f(t, x) \rangle_0 \right] + c_4(t) \sum_{f=u,d,s} \left[\tilde{O}_{4\mu\nu}^f(t, x) - \langle \tilde{O}_{4\mu\nu}^f(t, x) \rangle_0 \right] + \sum_{f=u,d,s} c_5^f(t) \left[\tilde{O}_{5\mu\nu}^f(t, x) - \langle \tilde{O}_{5\mu\nu}^f(t, x) \rangle_0 \right] \right\},$$

VEV-subtraction
($T = 0$ subtraction)

using coefficients by Makino-Suzuki evaluated in one-loop PT.

$$c_1(t) = \frac{1}{\bar{g}(1/\sqrt{8t})^2} - \frac{1}{(4\pi)^2} \left[9(\gamma - 2\ln 2) + \frac{19}{4} \right],$$

$$c_2(t) = \frac{1}{(4\pi)^2} \frac{33}{16},$$

$$c_3(t) = \frac{1}{4} \left\{ 1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2} \left[2 + \frac{4}{3} \ln(432) \right] \right\},$$

$$c_4(t) = \frac{1}{(4\pi)^2} \bar{g}(1/\sqrt{8t})^2,$$

$$c_5^f(t) = -\bar{m}_f(1/\sqrt{8t}) \left\{ 1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2} \left[4(\gamma - 2\ln 2) + \frac{14}{3} + \frac{4}{3} \ln(432) \right] \right\}$$

At $a \neq 0$, additional mixing with unwanted operators

$$T_{\mu\nu}(t, x, a) = T_{\mu\nu}(t, x) + A_{\mu\nu} \frac{a^2}{t} + \sum_f B_{f\mu\nu} (am_f)^2 + C_{\mu\nu} (aT)^2 + D_{\mu\nu} (a\Lambda_{\text{QCD}})^2 + a^2 S'_{\mu\nu}(x) + \mathcal{O}(a^4),$$

Note: lattice artifacts of NP-clover is $\mathcal{O}(a^2)$.

Singular terms at $t = 0$ due to mixing with $D=4$ ops.

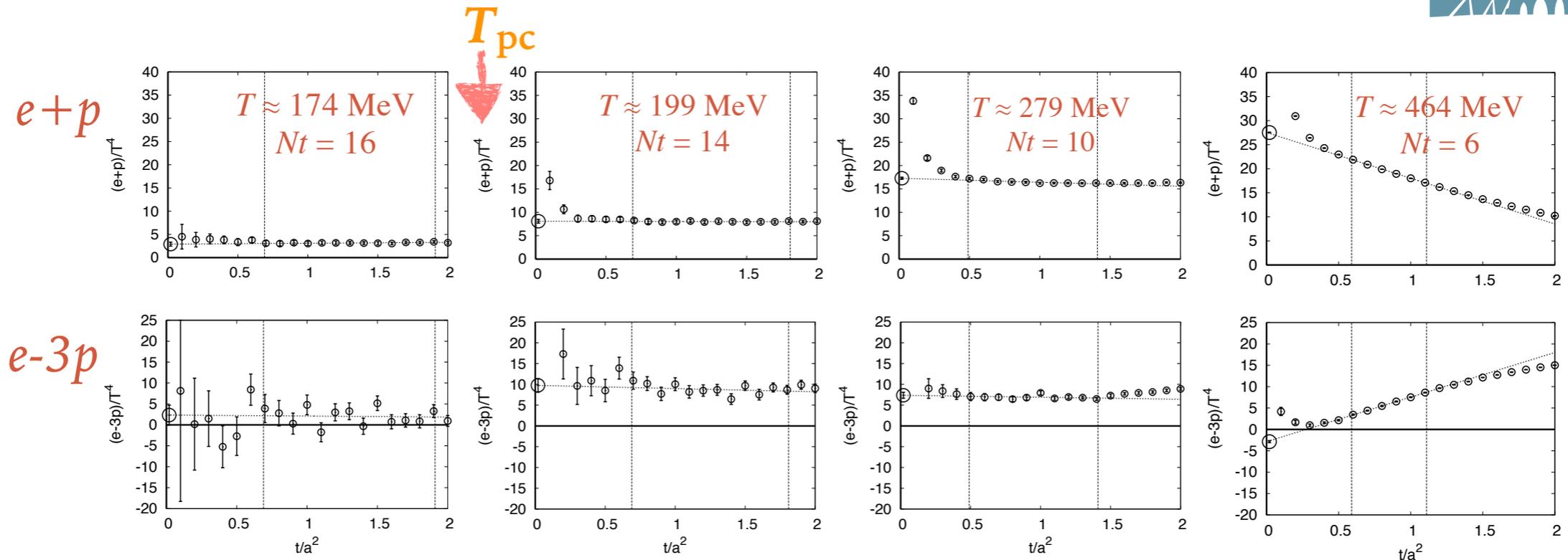
=> should be handled properly in the $t \rightarrow 0$ extrapolation.

Nf=2+1 heavy QCD EoS by GF

$$\epsilon = -\langle T_{00} \rangle, \quad p = \frac{1}{3} \sum_i \langle T_{ii} \rangle$$

1st step:

- Heavy ud quarks with \approx physical s quark ($m_{PS}/m_V \approx 0.63$), but on a fine lattice ($a \approx 0.07\text{fm}$) with the fixed-scale approach.
- Preliminary results presented at Lattice 2016, Southampton



- a^2/t -like behavior seen at $t \approx 0$.
- \exists meaningful range of t in which EMT is well linear. $\Leftarrow \sqrt{(8t/a^2)} \leq \min(Ns/2, Nt/2)$ to avoid oversmearing.
- a^2/t term looks negligible in the "linear windows" \Rightarrow **Linear extrapol.** using the windows.
- At $T \approx 697\text{ MeV}$ ($Nt=4$), no linear windows found.

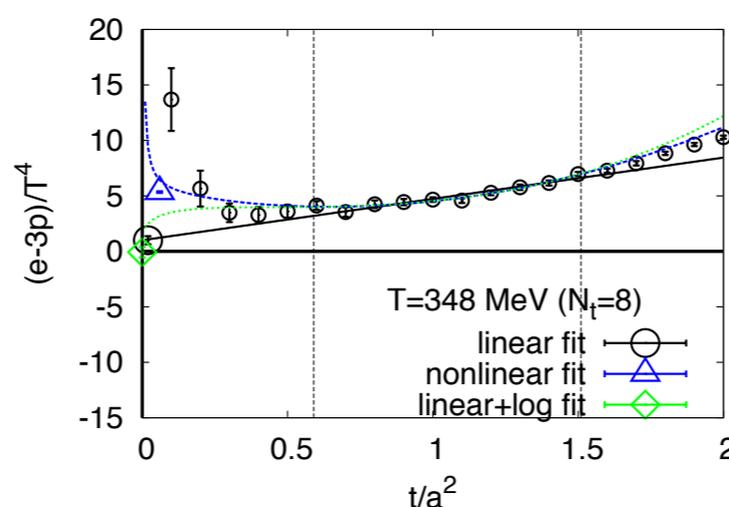
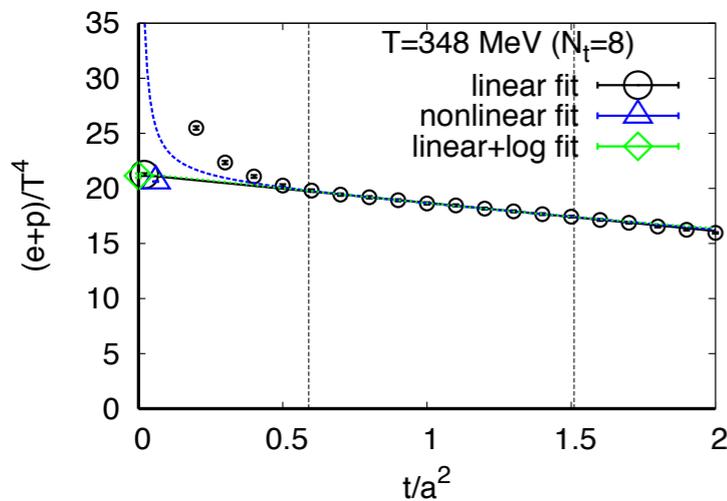
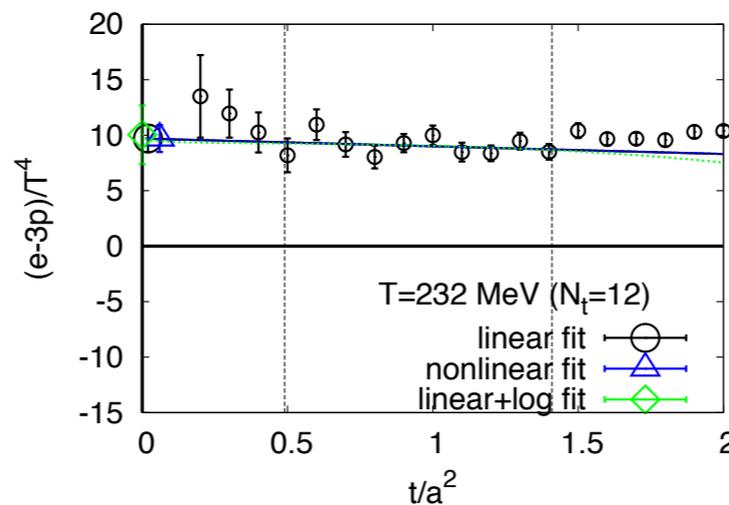
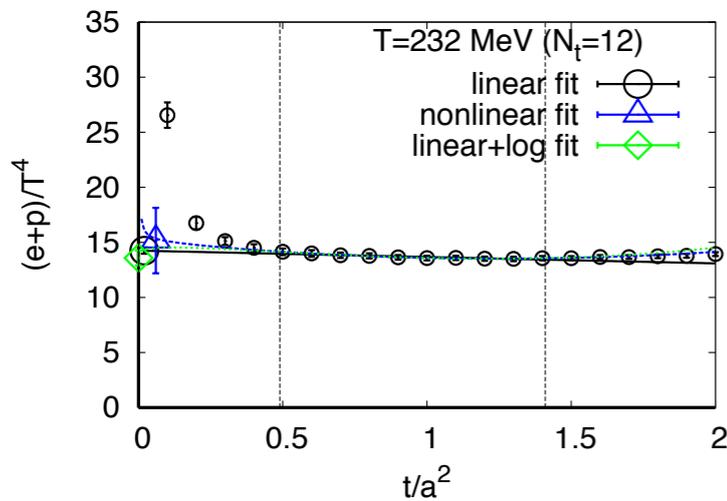
Nf=2+1 heavy QCD EoS by GF

1st step:

- After Lattice 2016, we made a series of additional analyses
 - to confirm the linear extrapolation procedure at $a > 0$
 - to estimate systematic error due to the fit ansatz

$\langle T_{\mu\nu}(t, a) \rangle = \langle T_{\mu\nu} \rangle + A_{\mu\nu} \frac{a^2}{t} + t S_{\mu\nu} + t^2 R_{\mu\nu}$ ➤ nonlinear fit, inspired from a^2/t as well as next-leading t corrections.

$\langle T_{\mu\nu}(t, a) \rangle = \langle T_{\mu\nu} \rangle + t S_{\mu\nu} + \frac{Q_{\mu\nu}}{\log^2(\sqrt{8t}/a)}$ ➤ linear+log fit, inspired from higher order PT corrections in the one-loop Suzuki coeff's. c_i .



☑ In most cases, all the fits are consistent with each other using the same window.

☑ Take the deviations as an estimate of systematic error due to the fit ansatz.

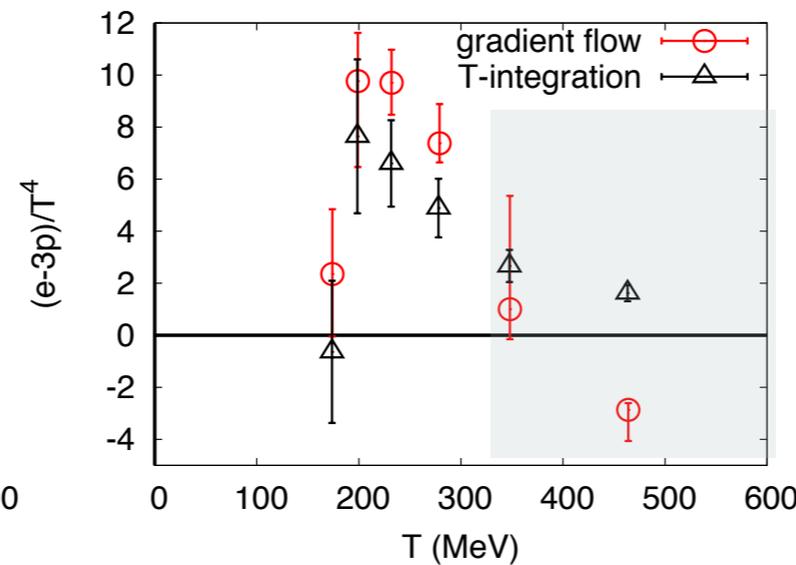
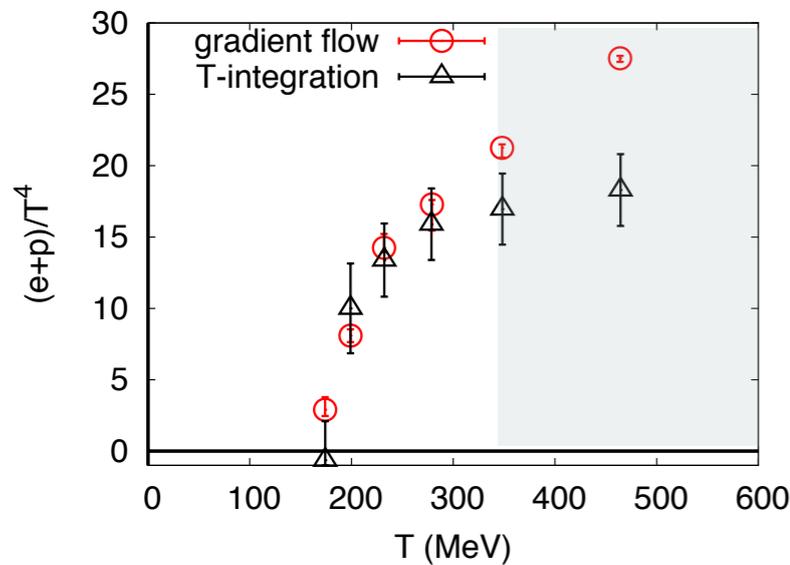
Nf=2+1 heavy QCD EoS / chiral transition by GF

1st step: Results

to be published in Phys.Rev.D

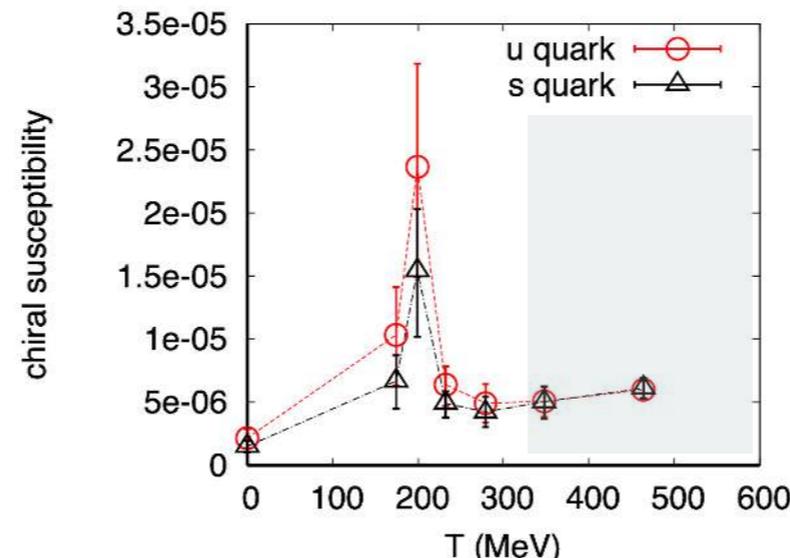
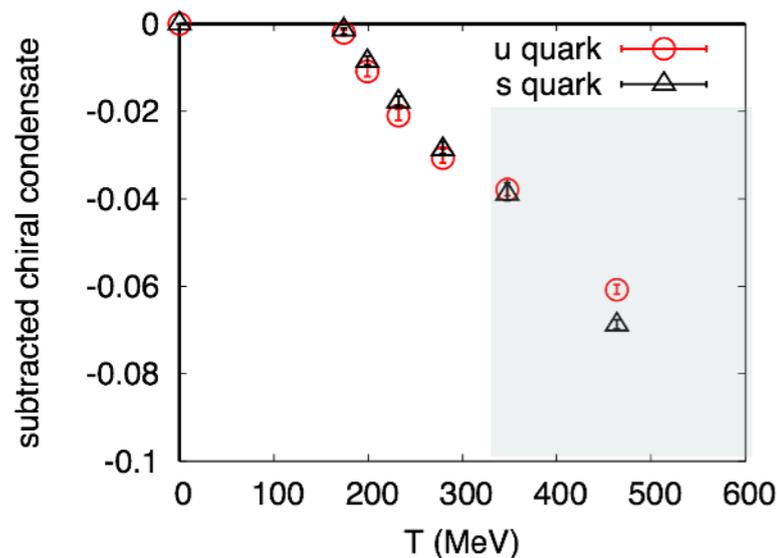
EoS

Errors include statistical as well as systematical ones due to fit ansatz etc.



- ✓ EoS by GF agrees with conventional method at $T \leq 300$ MeV ($Nt \geq 10$). Suggest $a \approx 0.07$ fm close to the cont. limit.
- ✓ Disagreement at $T \geq 350$ MeV due to $O((aT)^2 = 1/Nt^2)$ lattice artifact at $Nt \lesssim 8$.

Chiral cond. / disconnected susceptibility



$\overline{\text{MS}}$ scheme at $\mu = 2$ GeV

- ✓ Crossover suggested around $T_{pc} \approx 190$ MeV, consistent with previous study.
- ✓ Peak higher with decreasing m_q , as expected.
- ✓ Physically expected results even with Wilson-type quarks. GF method powerful to extract physical properties.

Nf=2+1 heavy QCD thermodynamics by GF

1st step: Topological charge / susceptibility

Phys.Rev.D95, 054502 (2017)

➤ Gluonic definition vs. fermionic definition

(a) gluonic definition Lüscher('10), Consonni-Engel-Giusti('15)

$$Q(t) = \int d^4x q(t, x) \quad q(t, x) = \frac{1}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} G_{\mu\nu}^a(t, x) G_{\rho\sigma}^a(t, x)$$

(b) fermionic definition Giusti-Rossi-Testa('04)

$$N_f^2 \langle Q^2 \rangle = m^2 (\langle P^0 P^0 \rangle - N_f \langle P^a P^a \rangle)$$

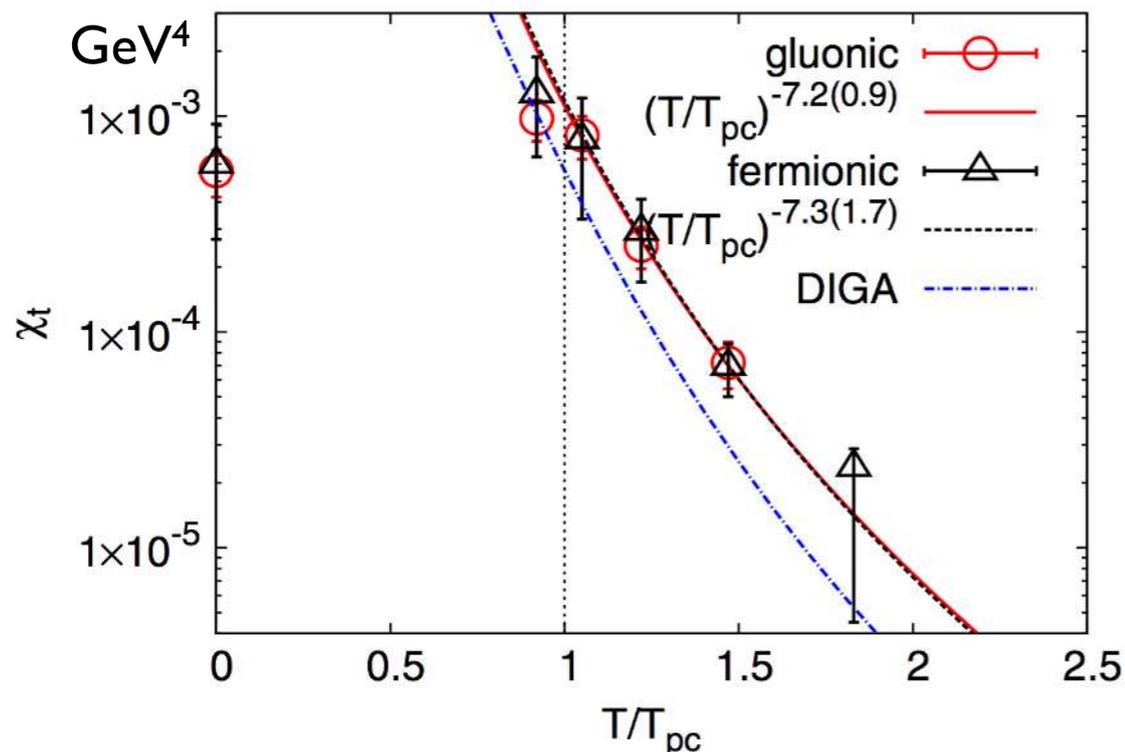
using chiral W-T identities

$$P^0 = \int d^4x \bar{\psi}(x) \gamma_5 \psi(x)$$

$$P^a = \int d^4x \bar{\psi}(x) T^a \gamma_5 \psi(x)$$

Equivalence shown with GW quarks, but large discrepancy found with non-chiral quarks.
E.g. Petreczky et al.(1606.03145): factor $\approx 2^4$ different χ at $Nt=12$ with HISQ.

➤ Topological susceptibility by GF



- ☑ Gluonic and fermionic definitions agree well with each other at $T/T_{pc} \leq 1.5$, even with Wilson-type quarks!
- ☑ Power-law behavior consistent with the dilute instant gas approximation (DIGA) which predicts the exponent to be -8 .

Next Steps

- Look fine, in spite of the use of Wilson-type quarks with explicit chiral violation!
- But, $m_\pi \sim 400$ MeV and $a \neq 0$ yet. A definite conclusion possible only after continuum extrapolation and at the physical point.
- Our good results suggests that our lattice is already close to the cont. limit, while the lattice artifact of $O((aT)^2 = 1/Nt^2)$ visible at $Nt \leq 8$.



● Application to other physical quantities

$T_{\mu\nu}$ correlation functions (towards specific heat, shear/bulk viscosities, ...)

=> Y. Taniguchi (Tuesday)

● Continuum extrapolation by adding a points

Available CP-PACS+JLQCD $T=0$ configurations have slightly different m_{PS}/m_V etc. To fine-tune on the same line of constant physics, we have decided to generate a new $T=0$ configuration near the CP-PACS+JLQCD simulation point. => on-going

● Physical point

$T=0$ configuration available from PACS-CS; $T>0$ configurations also in part available from WHOT-QCD's on-going project.



this talk

(2+1)-flavor **phys.pt.** QCD thermodynamics with GF

* $N_f=2+1$ QCD, Iwasaki gauge + NP-clover

* $T=0$ configs. of PACS-CS ($\beta=1.9$, $32^3 \times 64$, $a \approx 0.09\text{fm}$) [Phys.Rev.D79, 034503 (2009)] 80 configs. @ILDG/JLDG

Fine-tuned to the phys.pt. by reweighting. [Phys.Rev.D81, 074503 (2010)] using m_π , m_K , m_Ω inputs.

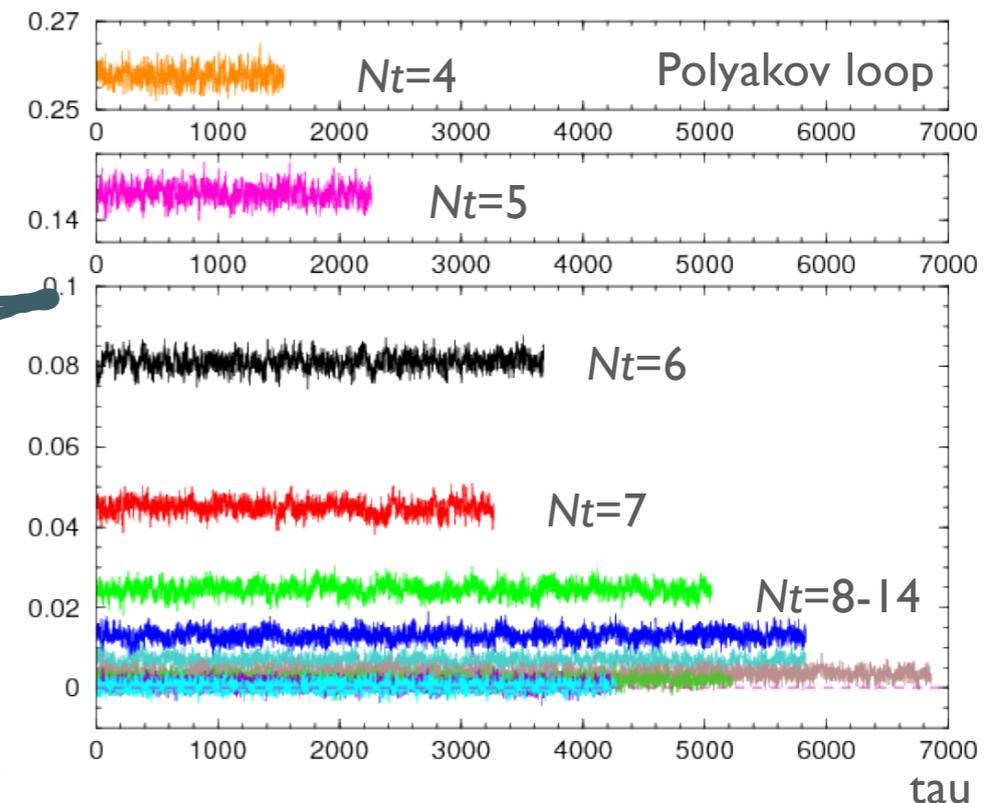
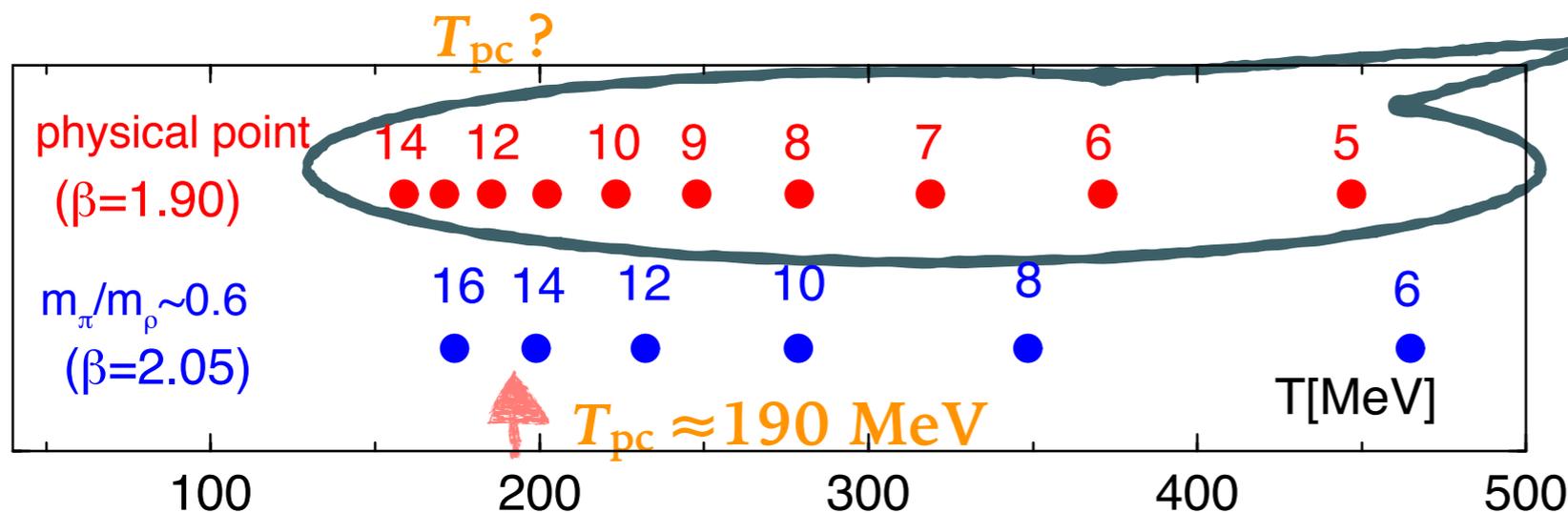
* $T>0$ by fixed-scale approach, ($32^3 \times N_t$, $N_t = 4, 5, \dots, 14$): $T \approx 157--549\text{MeV}$, on-going

Odd N_t too, to have a finer T -resolution.

Generated directly at the phys.pt. w/o reweighting.

$$\beta=1.9, K_{ud}=0.13779625, K_s=0.13663377$$

* Gauge meas. every 5 tau, quark meas. every 50 tau.



- Where is T_{pc} for physical m_q ? Expect $T_{pc}^{\text{phys}} < 190\text{ MeV}$.
- Lattice slightly coarser than the heavy case: $a \approx 0.09\text{fm} > 0.07\text{fm}$.
- May have the lattice artifact of $O((aT)^2 = 1/N_t^2)$ at $N_t < 8$.

But we have some configurations. Let us try!

GF with reweighting

* $T=0$ configs. of PACS-CS ($\beta=1.9$, $32^3 \times 64$, $a \approx 0.09\text{fm}$)

Fine-tuned to the phys.pt. by **reweighting**.

Simulation: $\beta=1.9$, $K_{ud}=0.137785$, $K_s=0.136600$

=> Phys.pt.: $\beta=1.9$, $K_{ud}=0.13779625$, $K_s=0.13663377$

$$\langle \mathcal{O}[U](\kappa_{ud}^*, \kappa_s^*) \rangle_{(\kappa_{ud}^*, \kappa_s^*)} = \frac{\langle \mathcal{O}[U](\kappa_{ud}^*, \kappa_s^*) R_{ud}[U] R_s[U] \rangle_{(\kappa_{ud}, \kappa_s)}}{\langle R_{ud}[U] R_s[U] \rangle_{(\kappa_{ud}, \kappa_s)}}$$

$$R_{ud}[U] = \left| \det \left[\frac{D_{\kappa_{ud}^*}[U]}{D_{\kappa_{ud}}[U]} \right] \right|^2$$

$$R_s[U] = \det \left[\frac{D_{\kappa_s^*}[U]}{D_{\kappa_s}[U]} \right]$$

Reweighting factors provided for each of 80 configs.

Because GF is introduced to define renormalized observables, we compute flowed fields/observables on each config. as usual, and average them over configurations using the provided reweighting factors,

i.e., we do not treat the reweighting factors as a part of observables.

□ In this talk, errors from the reweighting factors not estimated yet.

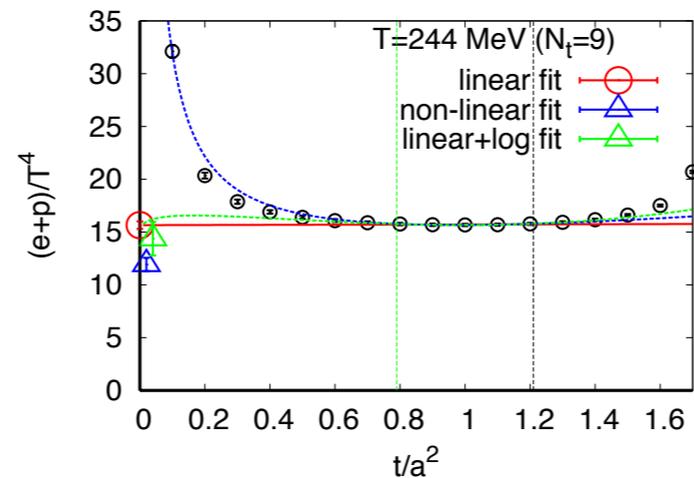
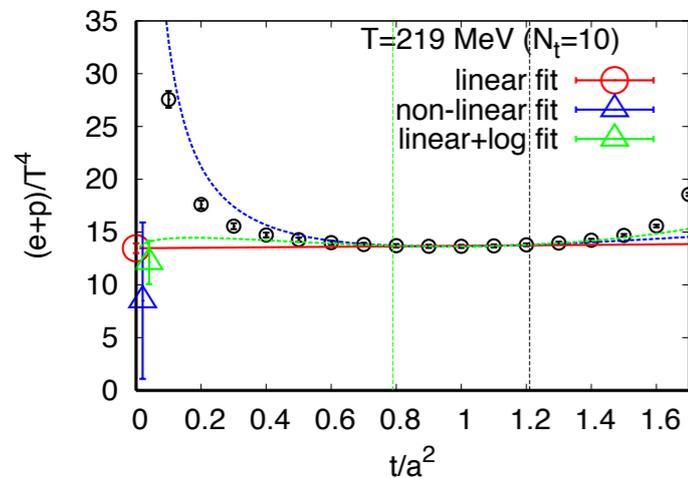
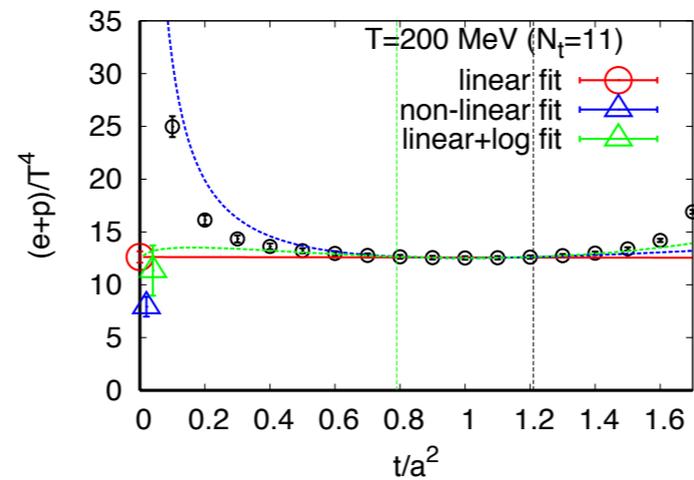
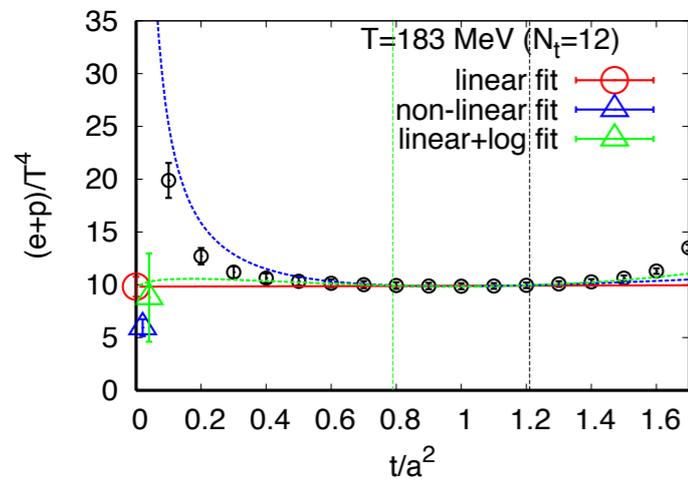
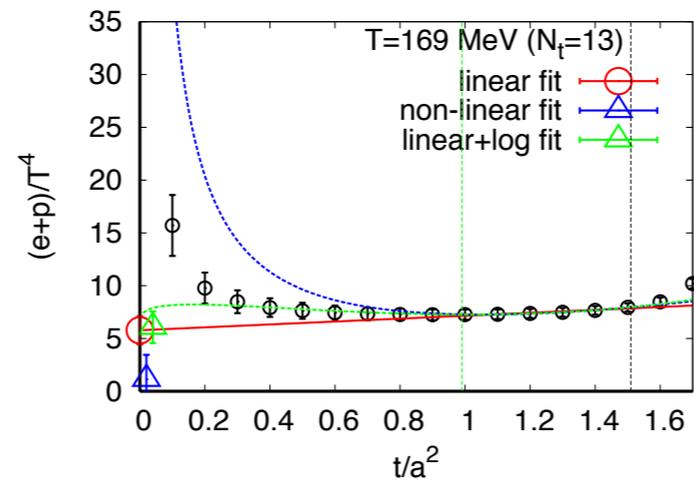
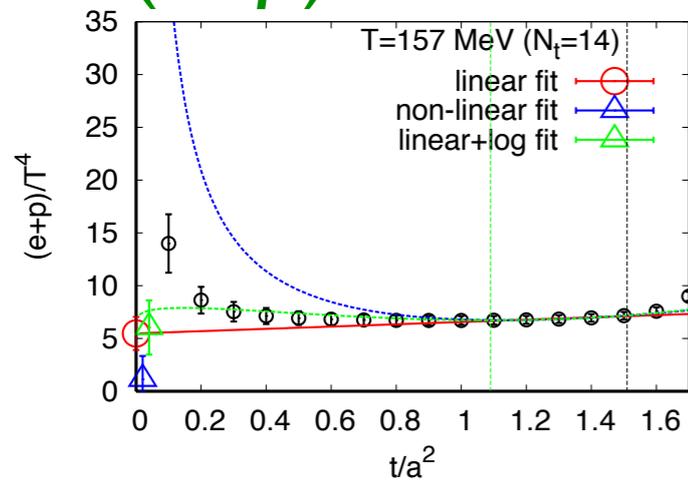
* $T>0$ by fixed-scale approach ($32^3 \times Nt$, $Nt = 4, 5, \dots, 13, 14$)

Generated directly at the phys.pt. w/o reweighting.

(2+1)-flavor **phys.pt.** QCD EoS with GF

Results of 1st trial fits: *Preliminary*

➤ $(e+p)/T^4$



■ **linear** $\langle T_{\mu\nu}(t, a) \rangle = \langle T_{\mu\nu} \rangle + t S_{\mu\nu} + O(a^2, t^2)$

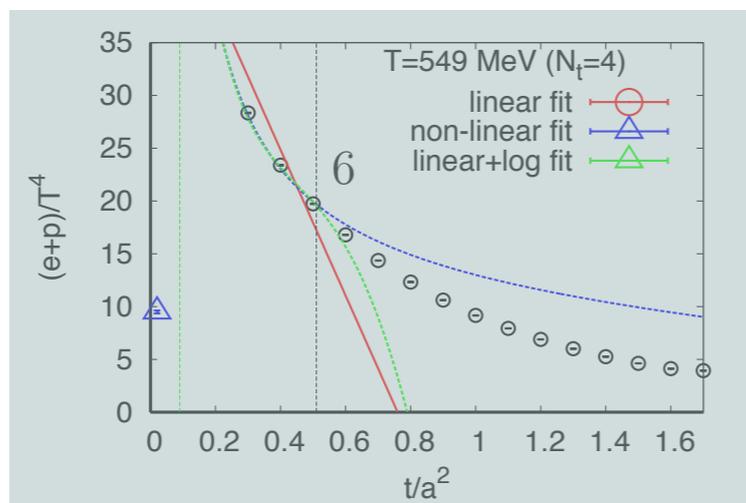
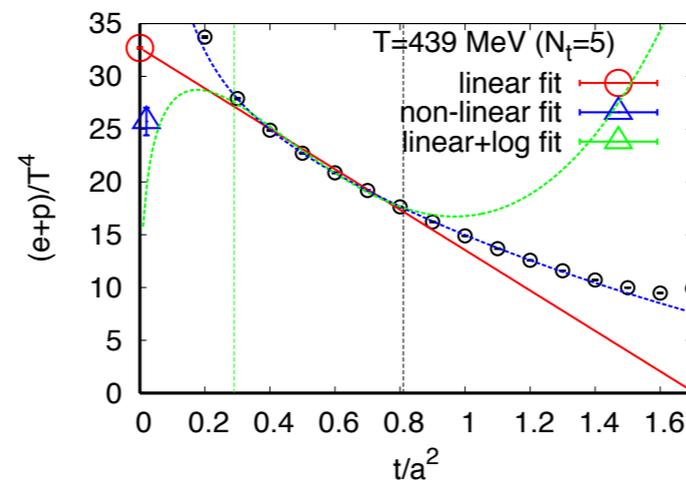
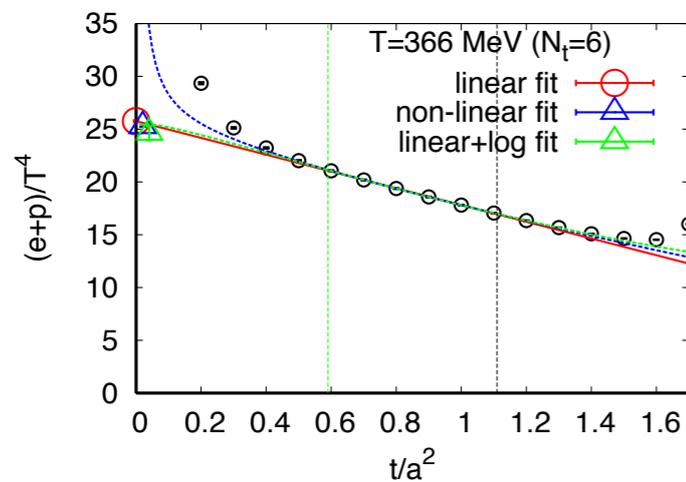
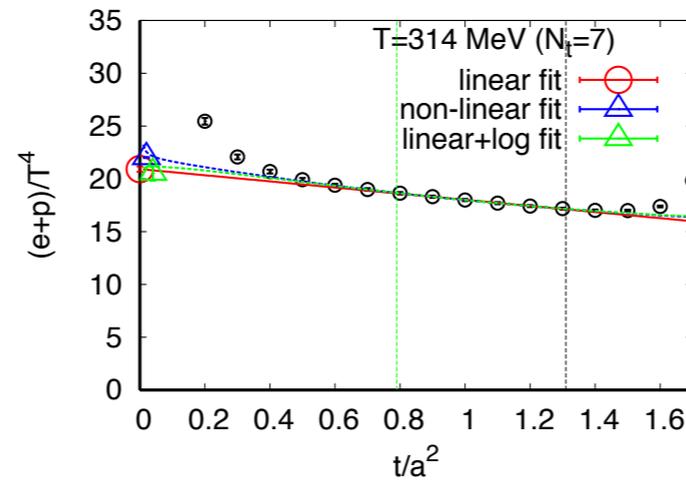
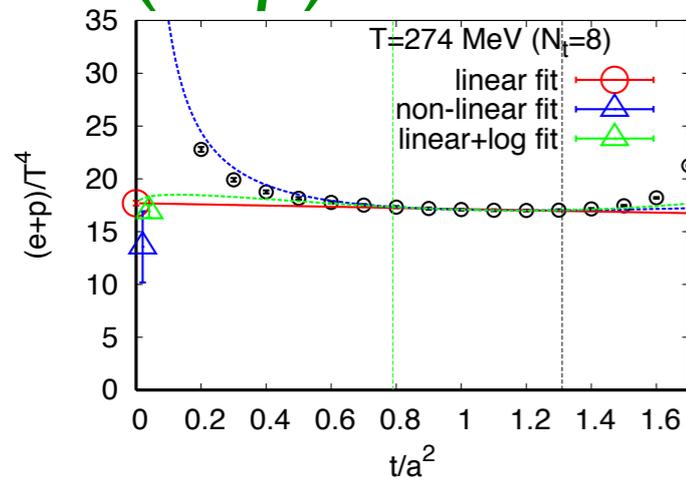
■ **nonlinear** $\langle T_{\mu\nu}(t, a) \rangle = \langle T_{\mu\nu} \rangle + A_{\mu\nu} \frac{a^2}{t} + t S_{\mu\nu} + t^2 R_{\mu\nu}$

■ **linear+log** $\langle T_{\mu\nu}(t, a) \rangle = \langle T_{\mu\nu} \rangle + t S_{\mu\nu} + \frac{Q_{\mu\nu}}{\log^2(\sqrt{8t}/a)}$

(2+1)-flavor **phys.pt.** QCD EoS with GF

Preliminary

➤ $(e+p)/T^4$



- linear $\langle T_{\mu\nu}(t, a) \rangle = \langle T_{\mu\nu} \rangle + t S_{\mu\nu} + O(a^2, t^2)$
- nonlinear $\langle T_{\mu\nu}(t, a) \rangle = \langle T_{\mu\nu} \rangle + A_{\mu\nu} \frac{a^2}{t} + t S_{\mu\nu} + t^2 R_{\mu\nu}$
- linear+log $\langle T_{\mu\nu}(t, a) \rangle = \langle T_{\mu\nu} \rangle + t S_{\mu\nu} + \frac{Q_{\mu\nu}}{\log^2(\sqrt{8t}/a)}$

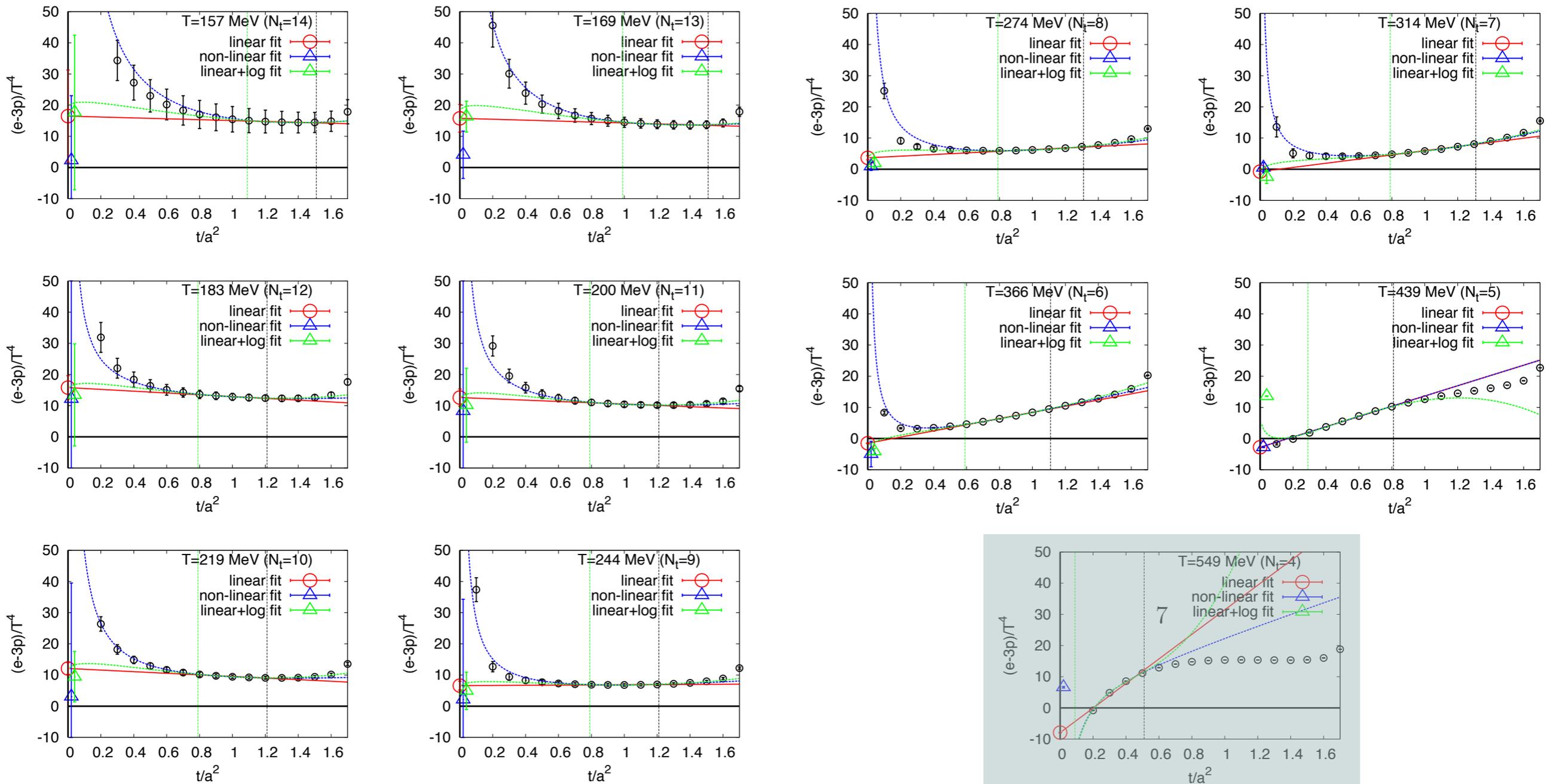
- ☑ Linear windows narrower than the heavy QCD case.
 <= coarser lattice and/or smaller m_q ?
- ☑ Take the difference among fits as an estimate of the systematic error due to the fit ansatz.
- ☑ No linear windows at $Nt=4$ ($T \approx 549$ MeV). like the heavy case.
 <= $t/a^2 \leq t_{1/2} = [\min(Ns/2, Nt/2)]^2 / \sqrt{8}$ to avoid overlapped smearing.

(2+1)-flavor **phys.pt.** QCD EoS with GF

Preliminary

➤ $(e-3p)/T^4$

- linear $\langle T_{\mu\nu}(t, a) \rangle = \langle T_{\mu\nu} \rangle + t S_{\mu\nu} + O(a^2, t^2)$
- nonlinear $\langle T_{\mu\nu}(t, a) \rangle = \langle T_{\mu\nu} \rangle + A_{\mu\nu} \frac{a^2}{t} + t S_{\mu\nu} + t^2 R_{\mu\nu}$
- linear+log $\langle T_{\mu\nu}(t, a) \rangle = \langle T_{\mu\nu} \rangle + t S_{\mu\nu} + \frac{Q_{\mu\nu}}{\log^2(\sqrt{8t}/a)}$



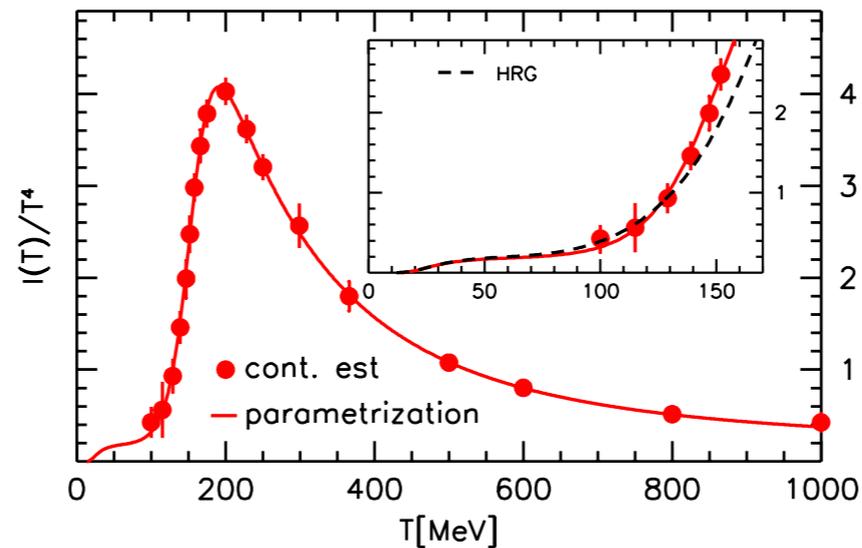
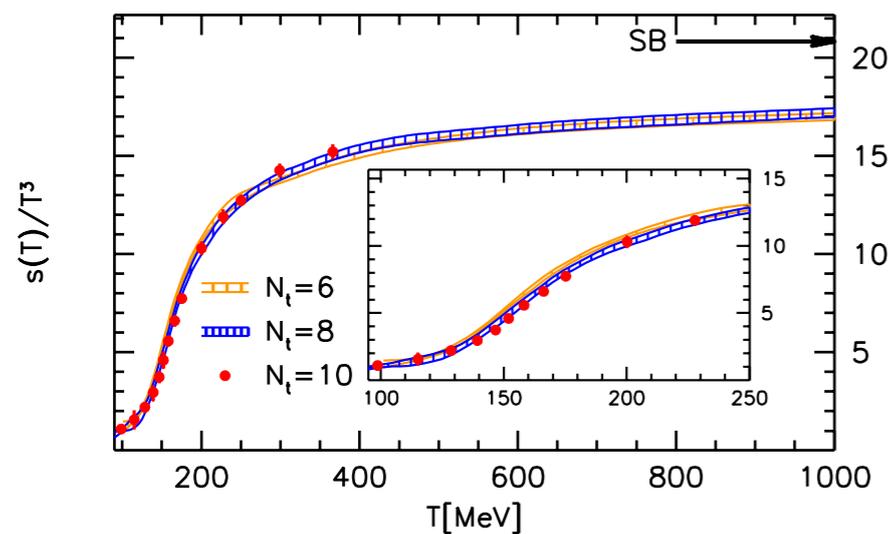
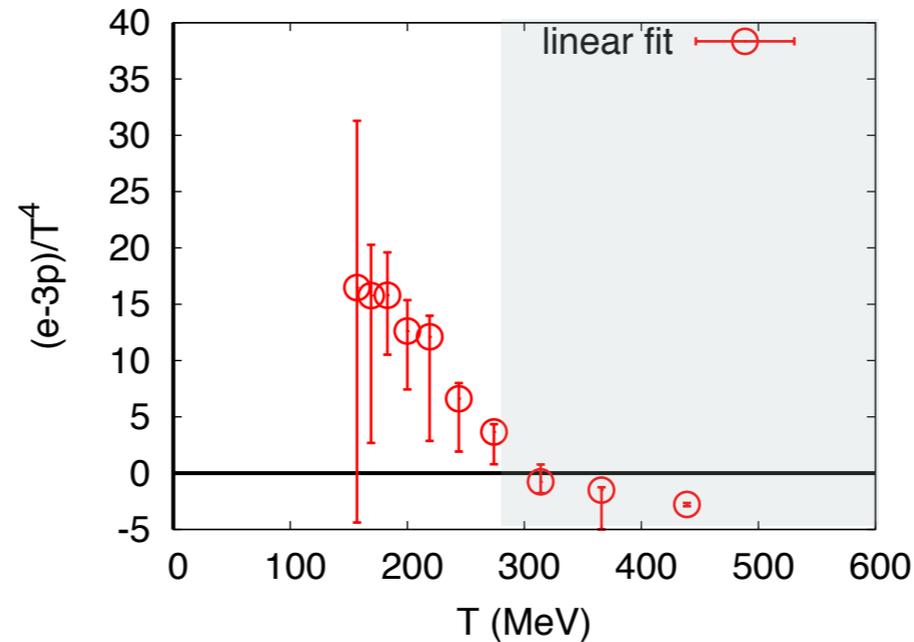
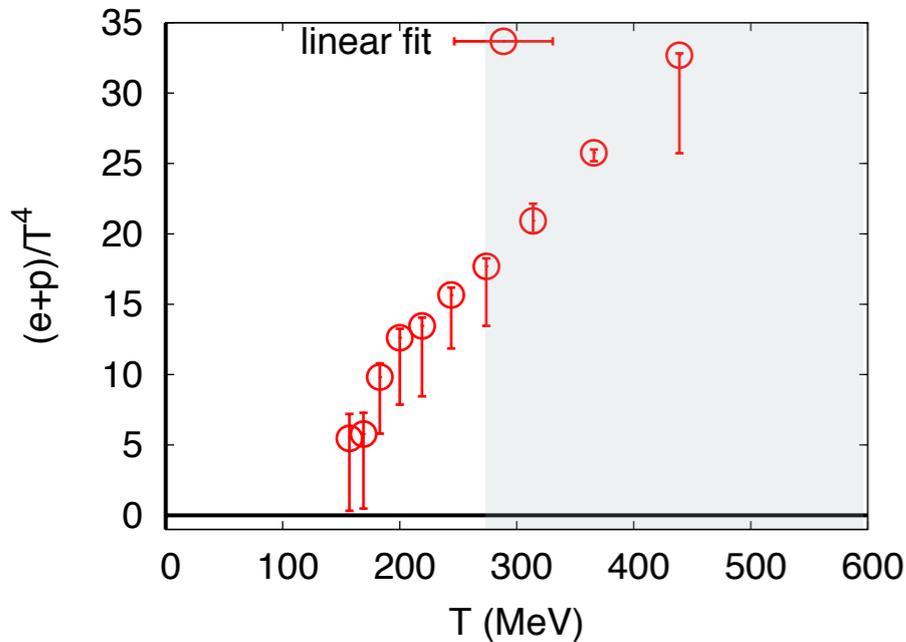
(2+1)-flavor **phys.pt.** QCD EoS with GF

Preliminary

➤ EoS

Central from the linear fit.

Errors include static as well as systematical one due to fit ansatz.



* Experience with the heavy case suggests that $T > 274$ MeV may be contaminated by the $O((aT)^2 = 1/Nt^2)$ lattice artifact at $Nt < 8$.

* Results of a conventional method on the same configurations not available yet.

* Borsany et al., JHEP 1011, 077 (2010), stout.

* HISQ/asqtad agree.

- ❑ Definite comparison possible only after continuum extrapolation.
- ❑ More statistics? VEV-subtraction with reweighting??

Chiral Condensate by GF

Hieda-Suzuki, Mod.Phys.Lett.A31, 1650214 (2016)

From axial W-T identity

$$\{\bar{\psi}_f \psi_f\}^{(0)}(t, x) = \left\{ 1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2} \left[4(\gamma - 2 \ln 2) + 8 + \frac{4}{3} \ln(432) \right] \right\} \times \frac{\bar{m}_f(1/\sqrt{8t})}{m_f} [\varphi_f(t) \bar{\chi}_f(t, x) \chi_f(t, x)]$$

At $m_f > 0$, chiral cond. in usual lattice simulation can have m_f/a^2 singularity.

With GF, such divergence is prohibited by the finiteness of flowed operators, but m_f/t can appear, instead.

In fact, to the lowest order of PT, we do encounter such m_f/t term.

$$\sum_{f, f'=u, d, s} \sqrt{\varphi_f(t)} \sqrt{\varphi_{f'}(t)} \bar{\chi}_f(t, x) \{\{t^A, M\}, t^B\}_{ff'} \chi_{f'}(t, x)$$

$$\stackrel{t \rightarrow 0}{\sim} \left[-\frac{12}{(4\pi)^2} \sum_{f=u, d, s} \left(\{\{t^A, M\}, t^B\} M \left\{ \frac{1}{2t} + M^2 [\gamma + \ln(2M^2 t)] + \mathcal{O}(t) \right\} \right)_{ff} + \mathcal{O}(g^2) \right] \mathbb{1}$$

$$+ [1 + \mathcal{O}(g^2)] \bar{\psi}(x) \{\{t^A, M\}, t^B\} \psi(x) + \mathcal{O}(t).$$

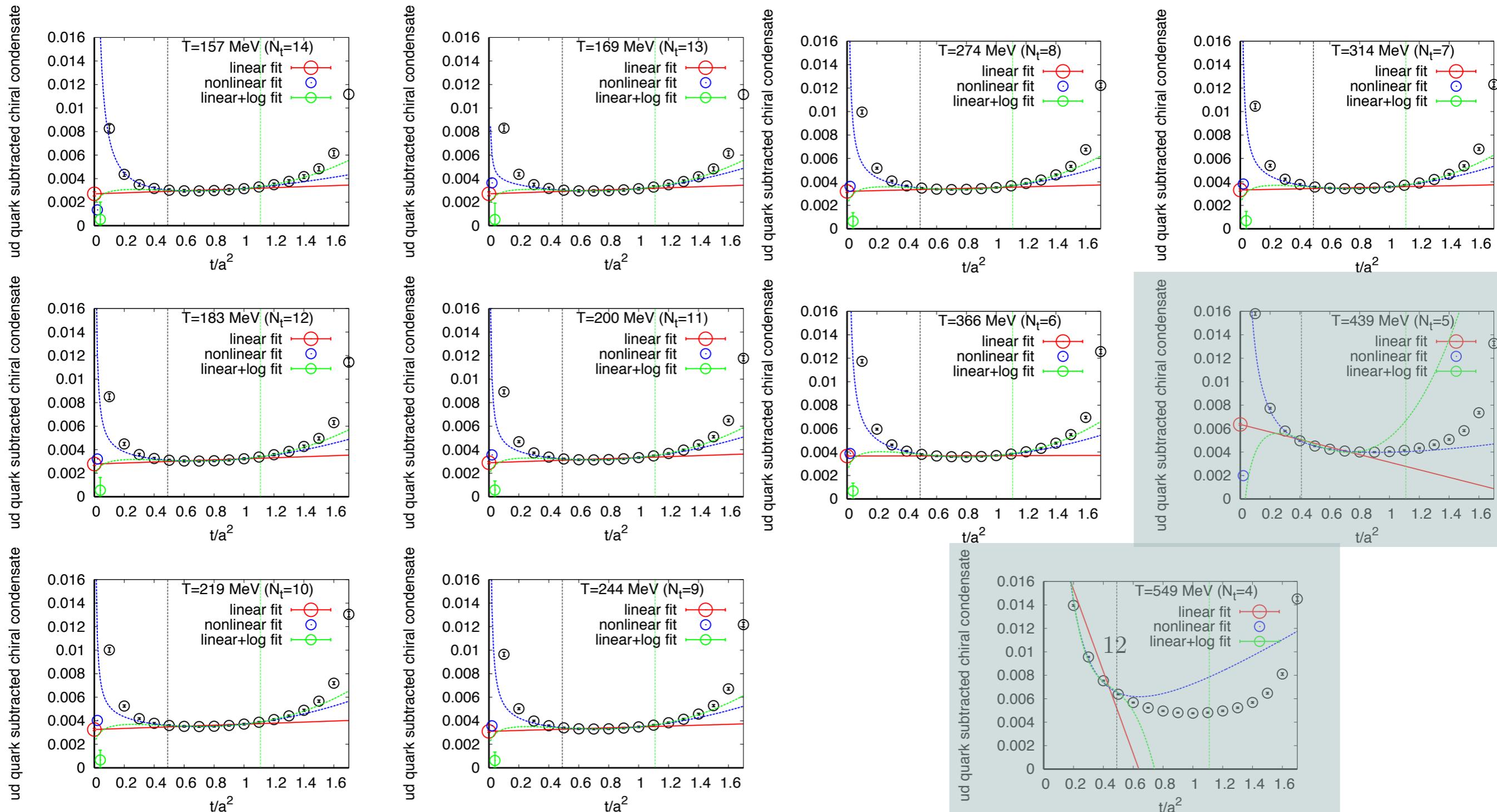
To remove this obstacle in the $t \rightarrow 0$ extrapolation, Hieda-Suzuki suggests a **VEV-subtraction**.

$$\{\bar{\psi}_f \psi_f\}(x) = \lim_{t \rightarrow 0} \left\{ 1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2} \left[4(\gamma - 2 \ln 2) + 8 + \frac{4}{3} \ln(432) \right] \right\} \times \frac{\bar{m}_f(1/\sqrt{8t})}{m_f} [\varphi_f(t) \bar{\chi}_f(t, x) \chi_f(t, x) - \text{VEV}].$$

(2+1)-flavor **phys.pt.** QCD chiral cond. with GF

Preliminary

► ud-quark chiral cond. w/ VEV-subtraction

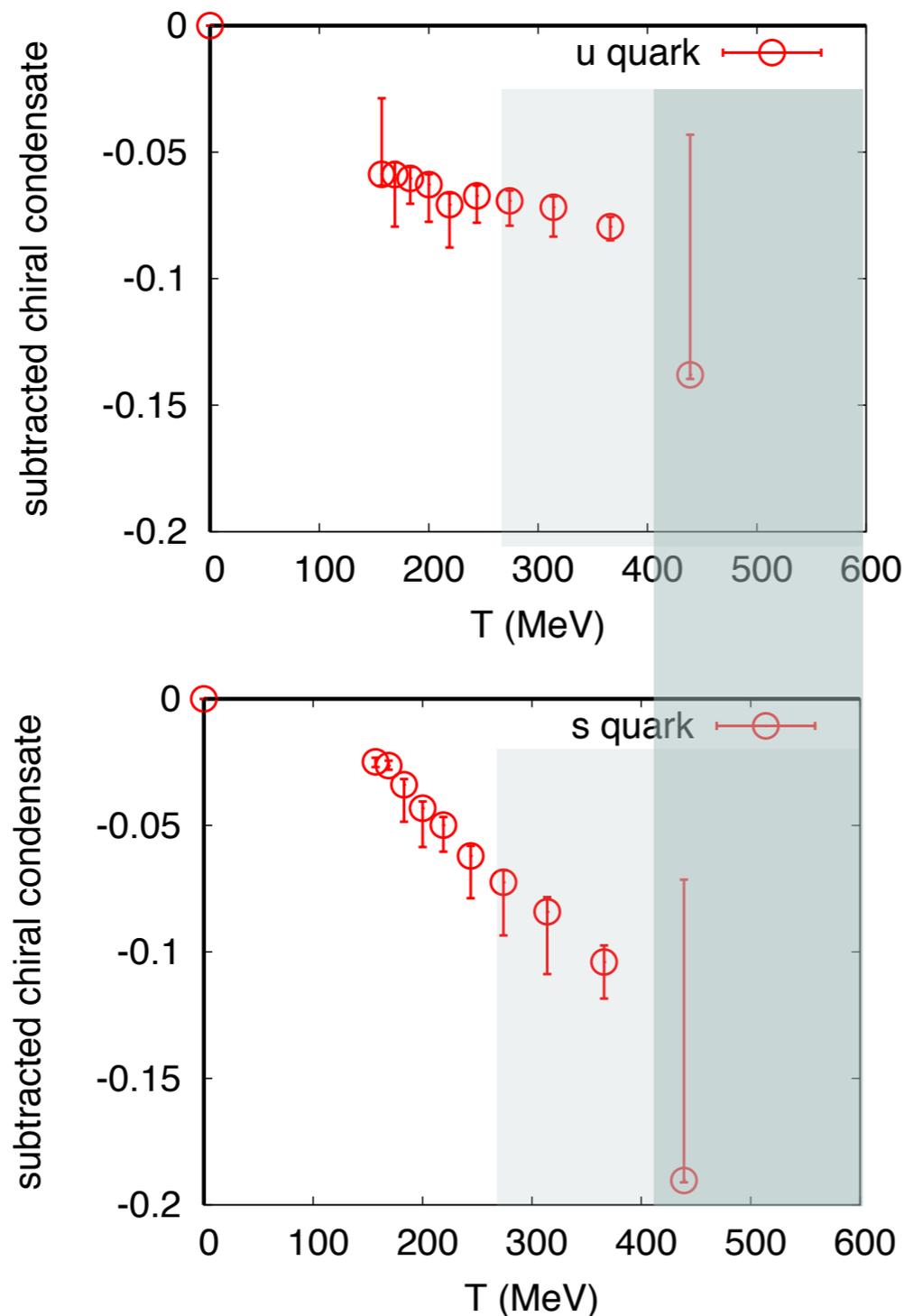


Some higher-order fits are not converged yet.

(2+1)-flavor **phys.pt.** QCD chiral cond. with GF

Preliminary

➤ chiral conds. w/ VEV-subtraction



- * Larger difference between ud and s quarks than the heavy case.
- * Sharper crossover/transition with lighter ud quarks.
- * Experience with the heavy case suggests that $T > 274$ MeV may be contaminated by the $O((aT)^2 = 1/Nt^2)$ lattice artifact at $Nt < 8$.

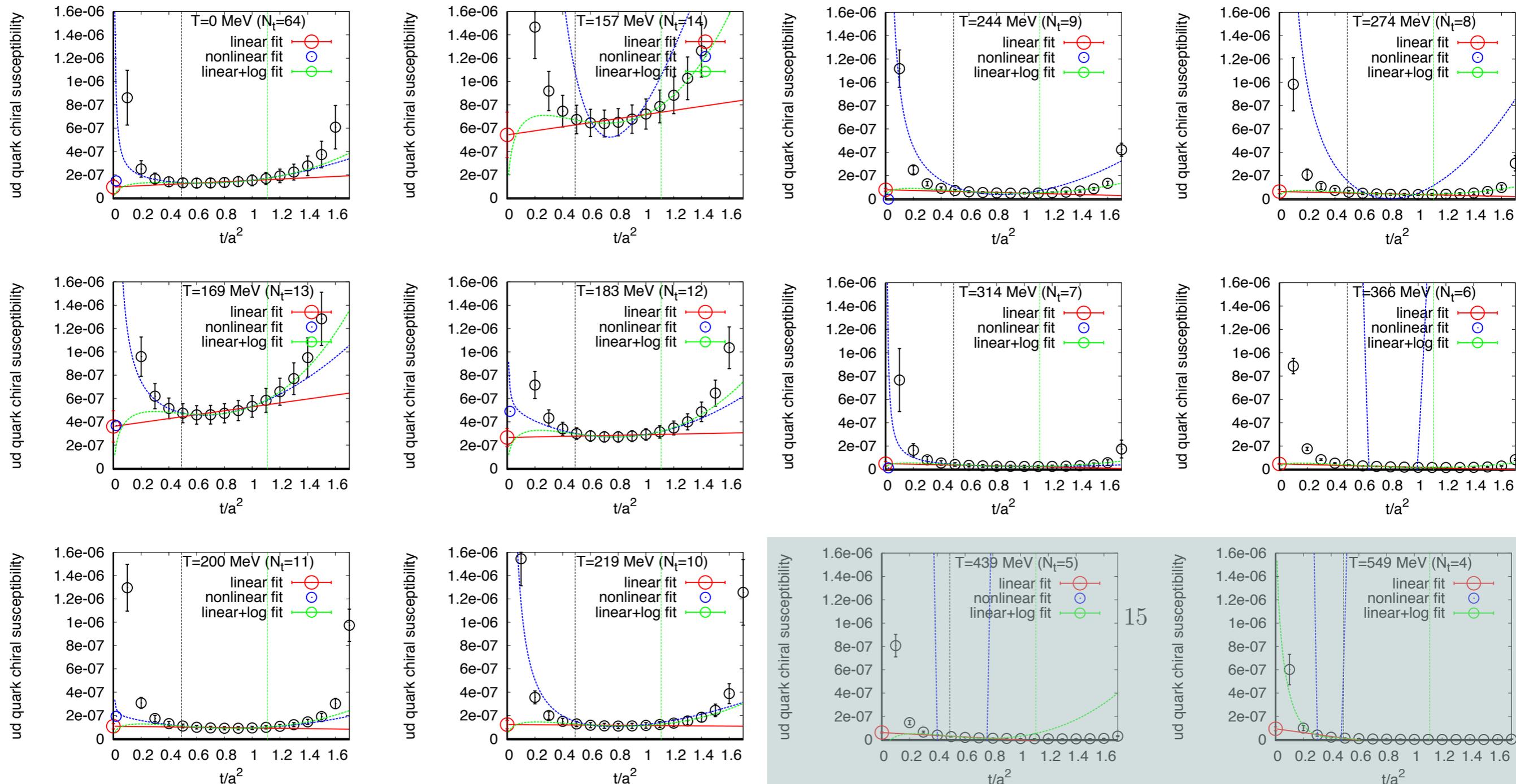
Some higher-order fits are not converged yet.

(2+1)-flavor **phys.pt.** QCD chiral suscept. with GF

Preliminary

➤ **disconnected ud chiral susceptibility** $\chi_{ff}^{\text{disc.}} = \left\langle \left[\frac{1}{N_T} \sum_x \{\bar{\psi}_f \psi_f\}(x) \right]^2 \right\rangle_{\text{disconnected}} - \left[\left\langle \frac{1}{N_T} \sum_x \{\bar{\psi}_f \psi_f\}(x) \right\rangle \right]^2$

Note: no VEV-subtraction needed in the susceptibility.



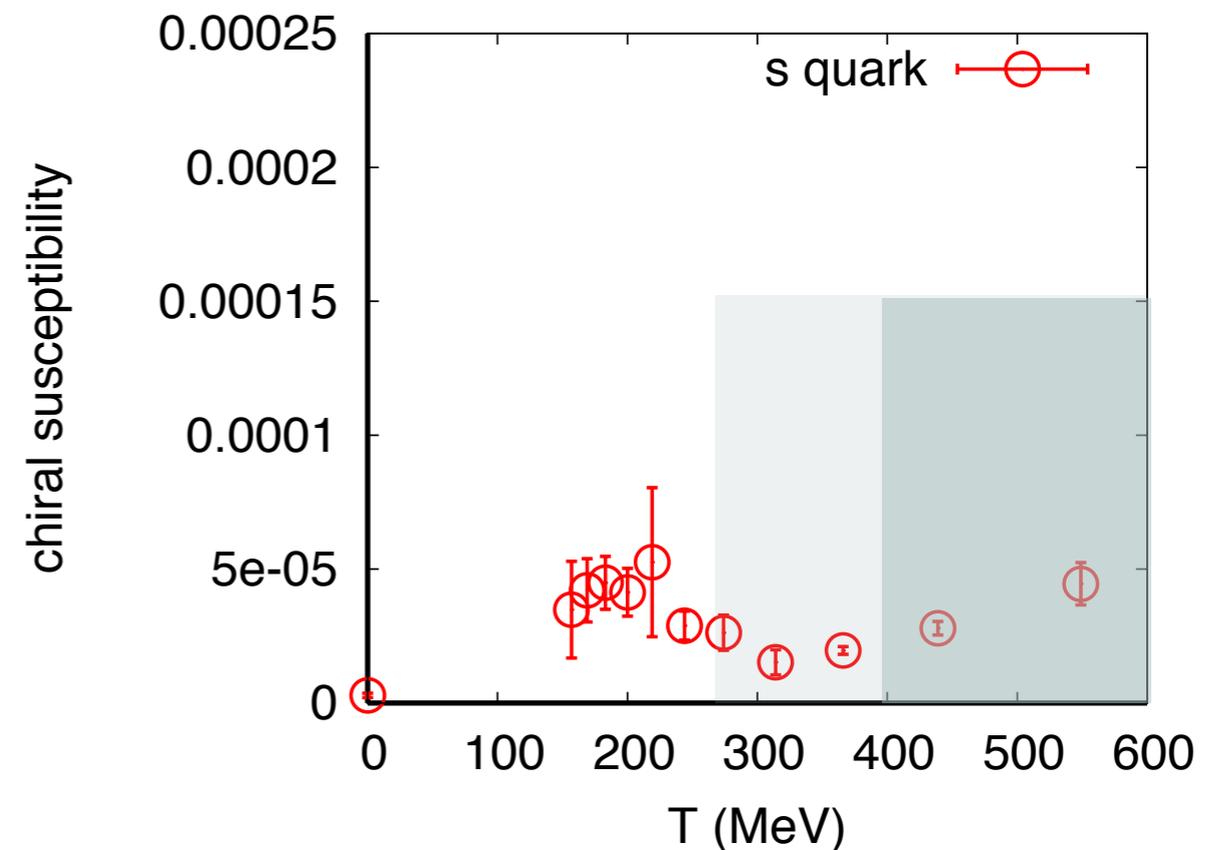
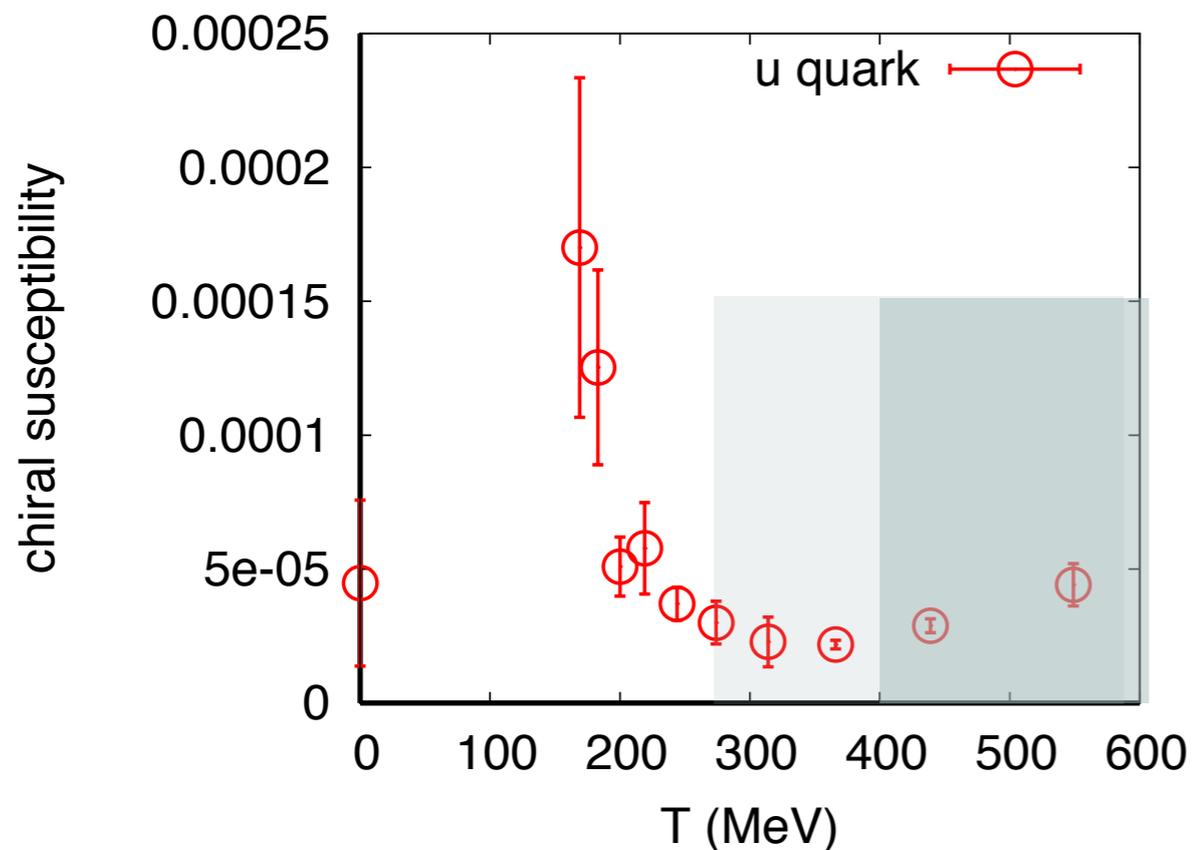
Some higher-order fits are not converged yet.

(2+1)-flavor **phys.pt.** QCD chiral transition with GF

Preliminary

➤ **disconnected chiral susceptibilities** (very preliminary)

Errors are statistical only,
because nonlinear/linear+log fits sometimes failed to converge yet.



➤ $T_{pc} \lesssim 169 \text{ MeV}$ (around 157 ? or lower?)
need lower T + more statics + more trials of the fits

* Experience with the heavy case suggests that $T > 274 \text{ MeV}$ may be contaminated by the $O((aT)^2 = 1/Nt^2)$ lattice artifact at $Nt < 8$.

Summary

(2+1)-flavor **heavy** QCD thermodynamics with GF Phys.Rev.D95, 054502 (2017), and to be published.

- Heavy ud ($m_{PS}/m_V \approx 0.63$), fine lattice ($a \approx 0.07\text{fm}$), $32^3 \times Nt$ ($Nt=4,6,\dots,16$): $T \approx 174\text{-}697\text{MeV}$
- ☑ EoS consistent with conventional method.
- ☑ Chiral suscept. shows peak at expected $T_{pc} \sim 190\text{MeV}$ even with Wilson-type quark.
- ☑ $a \approx 0.07\text{fm}$ seems to be close to the continuum limit, but $O((aT)^2)$ lattice artifacts at $Nt \lesssim 8$.
- ☑ The GF method works well.

(2+1)-flavor **phys.pt.** QCD thermodynamics with GF on-going

- Physical point, slightly coarser lattice ($a \approx 0.09\text{fm}$), $32^3 \times Nt$ ($Nt=4,5,\dots,14$): $T \approx 157\text{-}549\text{MeV}$

Preliminary results suggest

- ☑ Similar to the heavy case. The method seems to work.
However, ...
- ☑ Windows for linear fit narrower. \Leftarrow Coarser lattice and/or lighter quarks?
- ☑ $T_{pc}^{\text{phys}} \lesssim 169\text{ MeV}$ (around 157 or lower?)
- ☐ Definite conclusions possible only after continuum extrapolation.
- ☐ Need more work on the fits and the VEV subtraction procedure with reweighting.
- ☐ Need more statistics at this light m_q . ($T=0$ also).
- ☐ Need lower T (larger Nt) too.
- ☐ Errors due to the reweighting factor not estimated yet.