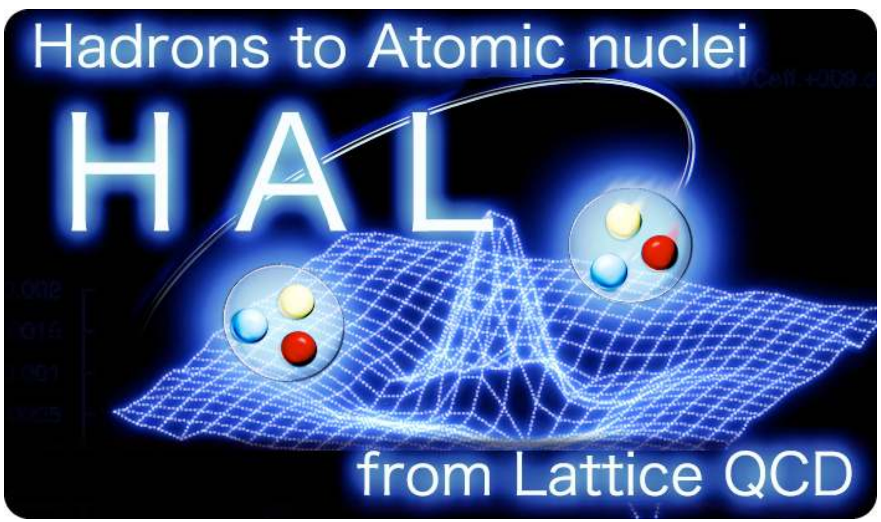
Baryon interactions from lattice QCD with physical masses
-- S=-3 sector: XiSigma & XiLambda-XiSigma --

Noriyoshi Ishii for HAL QCD Coll.



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Univ. Birjand: F.Etminan
RIKEN: T.Doi, T.Hatsuda, T.Iritani, S.Gongyo, T.M.Doi
Nihon Univ.: T.Inoue
YITP, Kyoto Univ.: S.Aoki, T.Miyamoto, K.Sasaki, D.Kawai

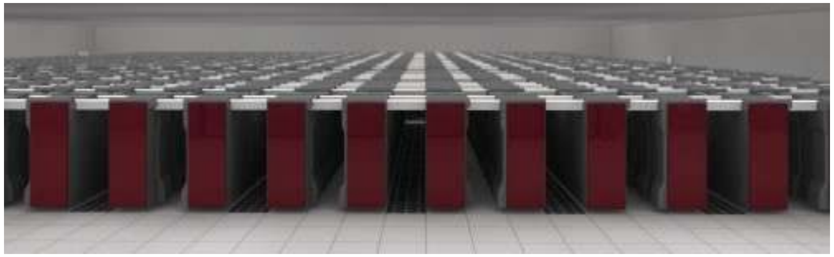
Background

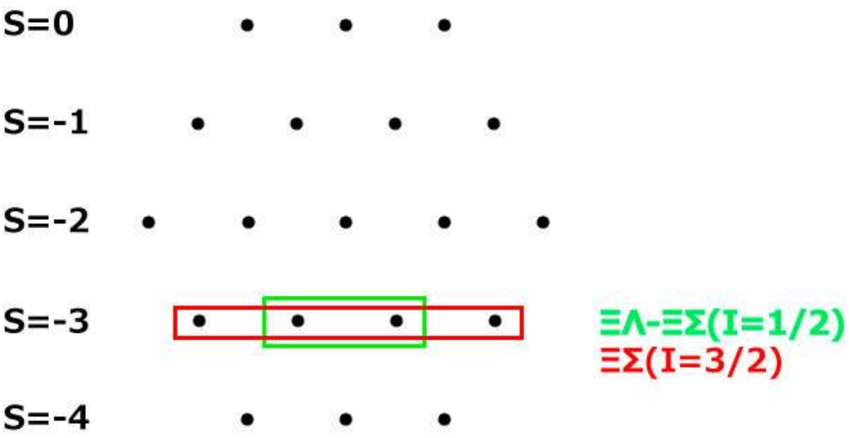
- Experimental determination of hyperon interaction is one of the most important subjects at J-PARC.
- Main interest is S=-1 and -2 sectors.
  (Exp. becomes more difficult
  as the number of strange quark increases.)
- For LQCD, the calculation becomes easier as the number of strange quarks increases.
  (due to the reduction of statistical noise)
- In this talk, we use "physical point" gauge configs. to calculate

  hyperon interaction in S=-3 sector

  with increased statistics.

J-PARC
Exploration of multi-strangeness world





Setup

In this talk, we use

- "physical point" gauge configs. on 96^4 lattice generated by K computer (AICS)

[K.-I.Ishikawa eta l., [PACS Coll] PoS LATTICE2015(2016)075]

  - 1/a = 2.3 GeV, L= 8.1 fm
  - 400 gauge configs.
  - 48(source points) * 4(rotation)
  - bin size = 20 (=100 HMC traj.)

- hadron masses:

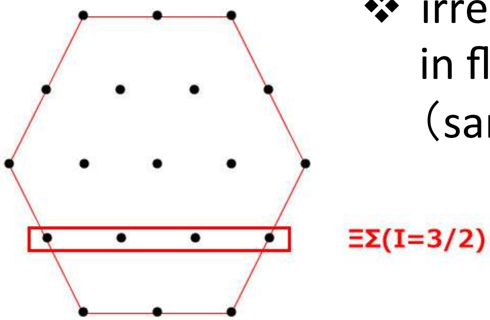
m(pion) = 146 MeV
m(K) = 525 MeV

m(N) = 962 MeV
m(Lambda) = 1139 MeV
m(Sigma) = 1222 MeV
m(Xi) = 1356 MeV

two-baryon sector (S=-3)

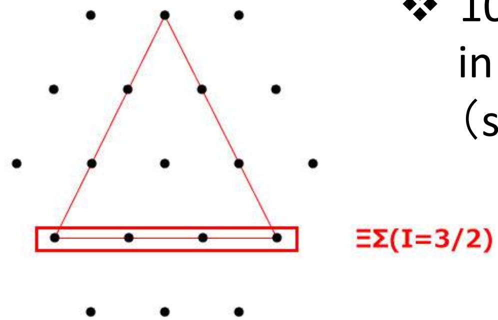
◆ I=3/2 (XiSigma single channel)

☐ total spin singlet



❖ irrep. 27
in flavor SU(3) limit
(same as dineutron)

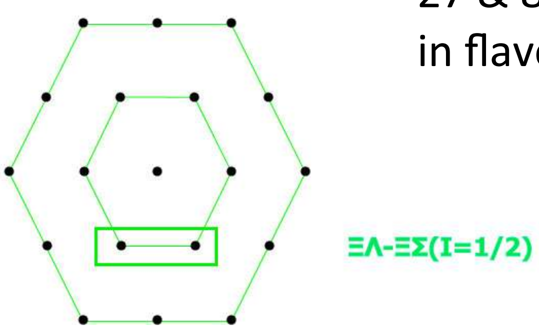
☐ total spin triplet



❖ 10* irrep.
in flavor SU(3) limit
(same as deutron)

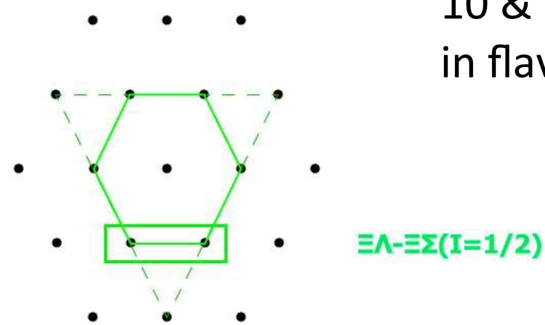
◆ I=1/2 (XiLambda-XiSigma coupled channel)

☐ total spin singlet



❖ mixture of
27 & 8s irreps.
in flavor SU(3) limit

☐ total spin triplet



❖ mixture of
10 & 8a irreps.
in flavor SU(3) limit

XiSigma (I=3/2) single channel

Time-dep. Schrödinger-like eq.（for unequal mass system）

◆ **R-correlator**

$$R(\vec{x}-\vec{y},t) \equiv e^{(m_\Xi+m_\Sigma)t}\left\langle 0 \left| T\left[\Xi(\vec{x},t)\Sigma(\vec{y},t)\cdot\overline{\Xi\Sigma}(t=0)\right]\right| 0\right\rangle$$

$$= \sum_n \psi_{k_n}(\vec{x}-\vec{y})\cdot\exp\left(-(E_n - m_\Xi - m_\Sigma)t\right)\cdot a_n$$

where $\psi_{k_n}(\vec{x}-\vec{y}) \equiv \left\langle 0 \left| \Xi(\vec{x})\Sigma(\vec{y}) \right| n\right\rangle$

◆ An identity for two-particle energy in C.M. frame

$$k^2E^2 = \frac{1}{4}\left(E^2-(m_\Xi+m_\Sigma)^2\right)\left(E^2-(m_\Xi-m_\Sigma)^2\right)$$

where $E \equiv \sqrt{m_\Xi^2+k^2}+\sqrt{m_\Sigma^2+k^2}$

◆ Schrödinger eq. satisfied by E-indep. HAL QCD potential

$$\left(\frac{\nabla^2}{2\mu}-\frac{k_n^2}{2\mu}\right)\psi_{k_n}(\vec{r}) = \int d^3r'\, V(\vec{r},\vec{r}\,')\psi_{k_n}(\vec{r}\,')$$

with $\mu \equiv \dfrac{m_\Xi m_\Sigma}{m_\Xi+m_\Sigma}$

➔ R-correlator satisfies **time-dependent Schrödinger-like eq.**

$$\left(\frac{\nabla^2}{2\mu}D_t^2+\frac{1}{8\mu}\left(D_t^2-(m_\Xi+m_\Sigma)^2\right)\left(D_t^2-(m_\Xi-m_\Sigma)^2\right)\right)R(\vec{r},t) = \int d^3r'\, V(\vec{r},\vec{r}\,')D_t^2R(\vec{r}\,',t)$$

$$D_t \equiv \partial_t - m_\Xi - m_\Sigma$$

This eq. enables us to obtain the potential without relying on the ground state saturation

Time-dep. Schrödinger-like eq. (non-rela. approx.)

Numerical evaluation of 4th derivative is unstable

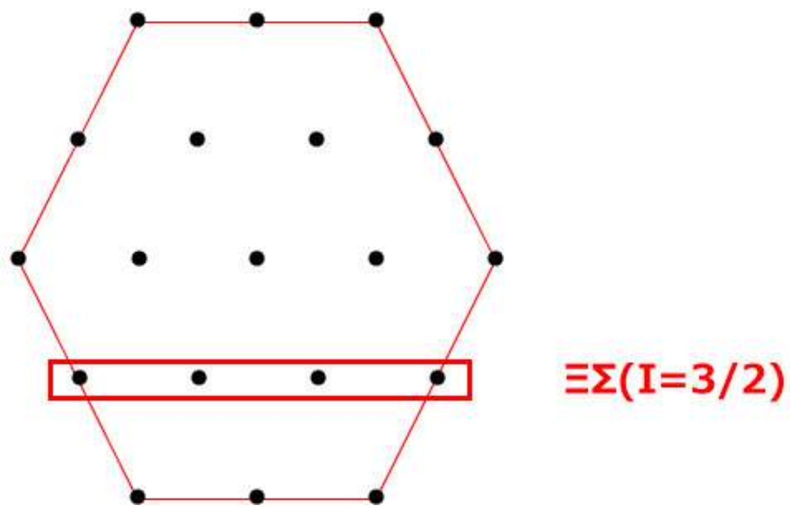
➔ we use non-relativistic approx. version

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r}, t) \simeq \int d^3 r' V(\vec{r}, \vec{r}') R(\vec{r}', t)$$

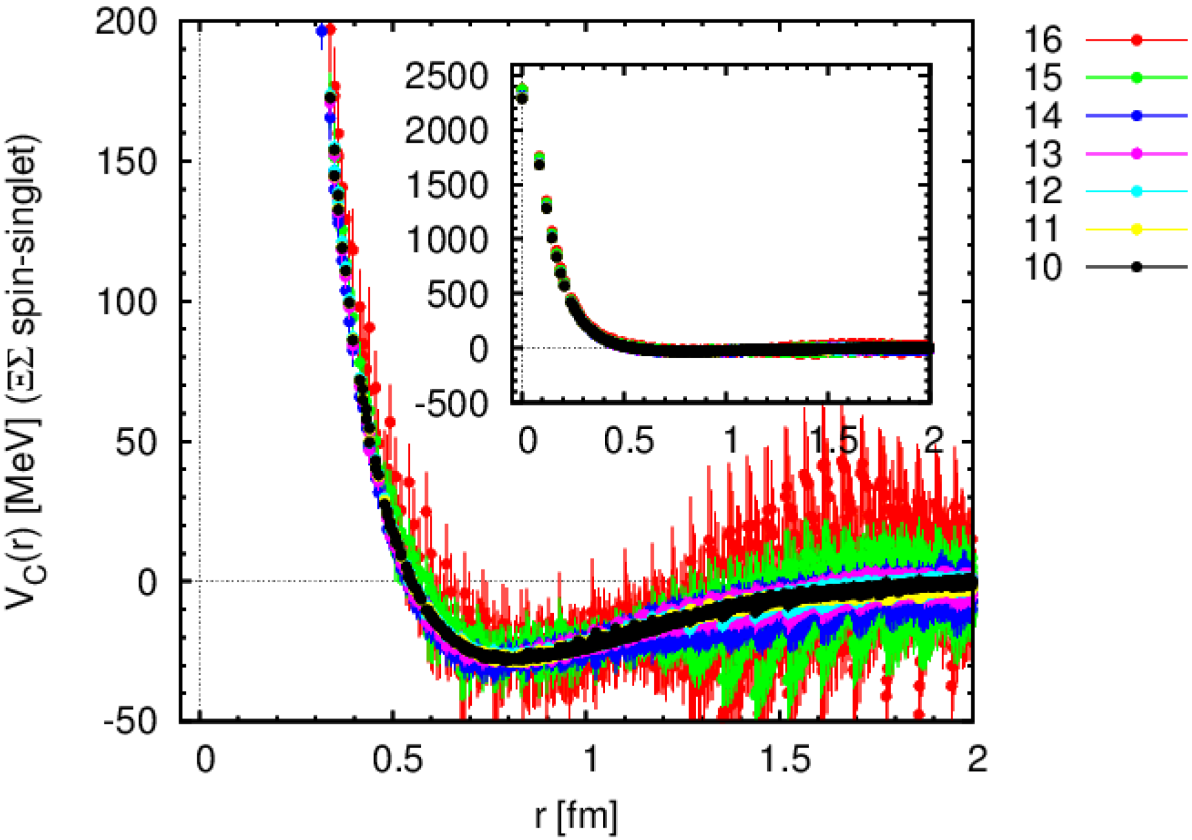
XiSigma (I=3/2) spin singlet

◆Total spin singlet

◆ 27 irrep. in flavor SU(3) limit
(same as dineutron)



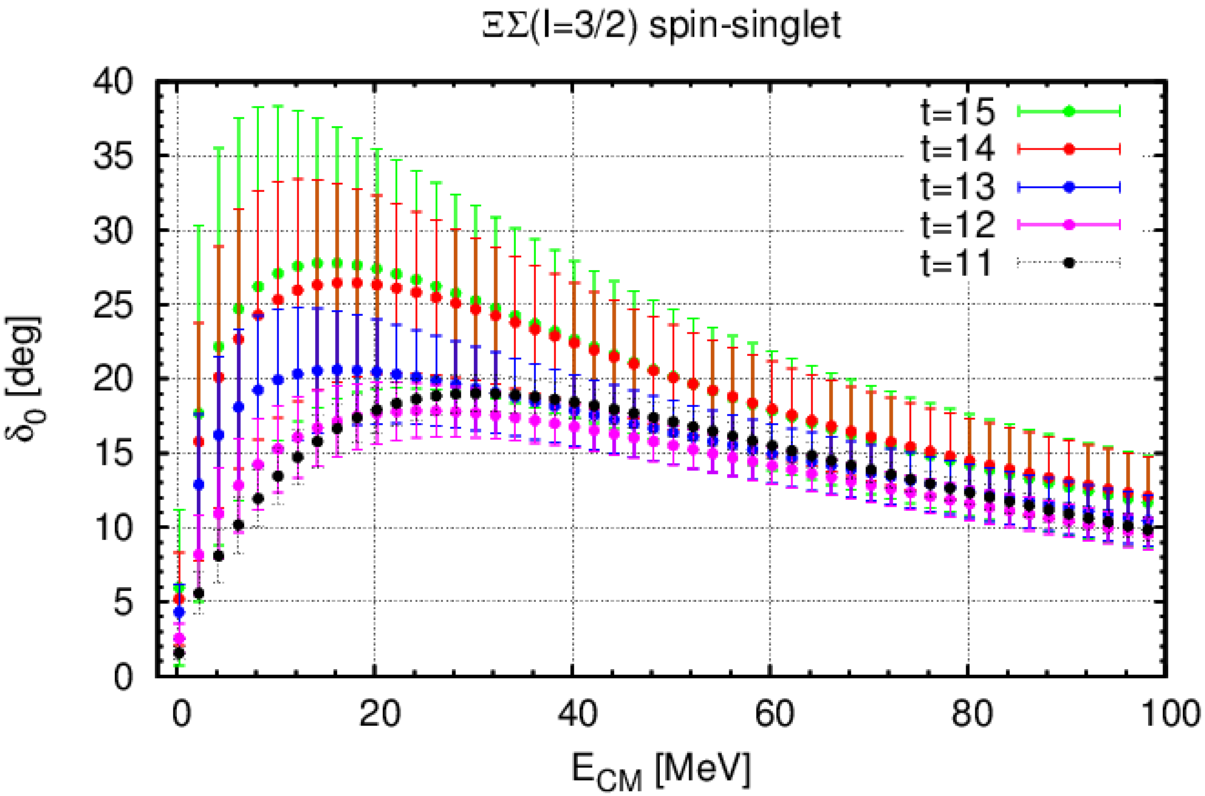
XiSigma(I=3/2, spin singlet)



- 1/a ∼2333 MeV, L ∼8.1 fm
- m_{pion} ∼146 MeV
- 400 gauge confs. are used.
- Binsize = 20 (= 100 HMC traj.)
- 48 source points * 4 rotations
- Point sink and wall source

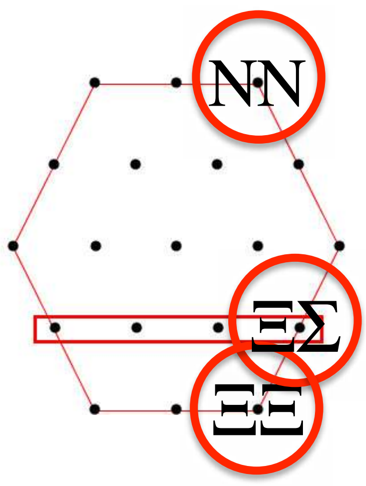
- Qualitative behavior is similar to dineutron channel (NN).
  - repulsive core at short distance surrounded by attraction

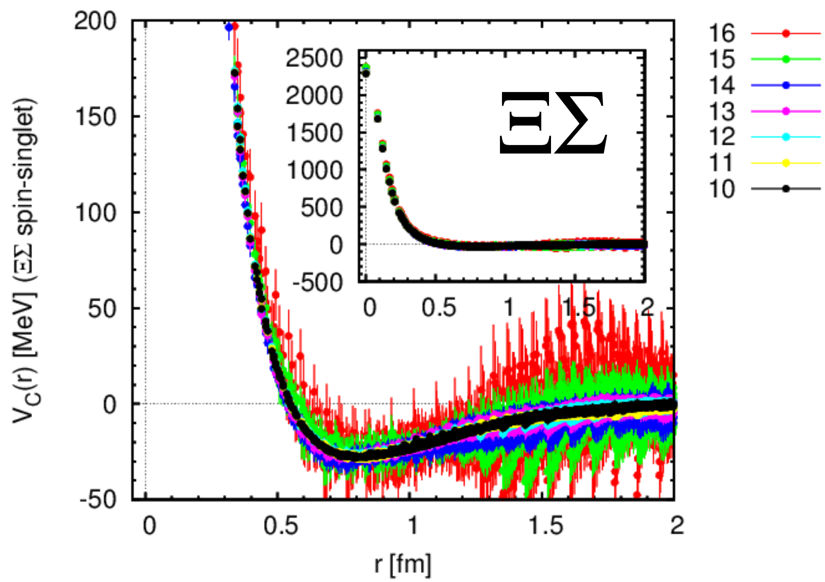
XiSigma(I=3/2, spin singlet) scattering phase shift



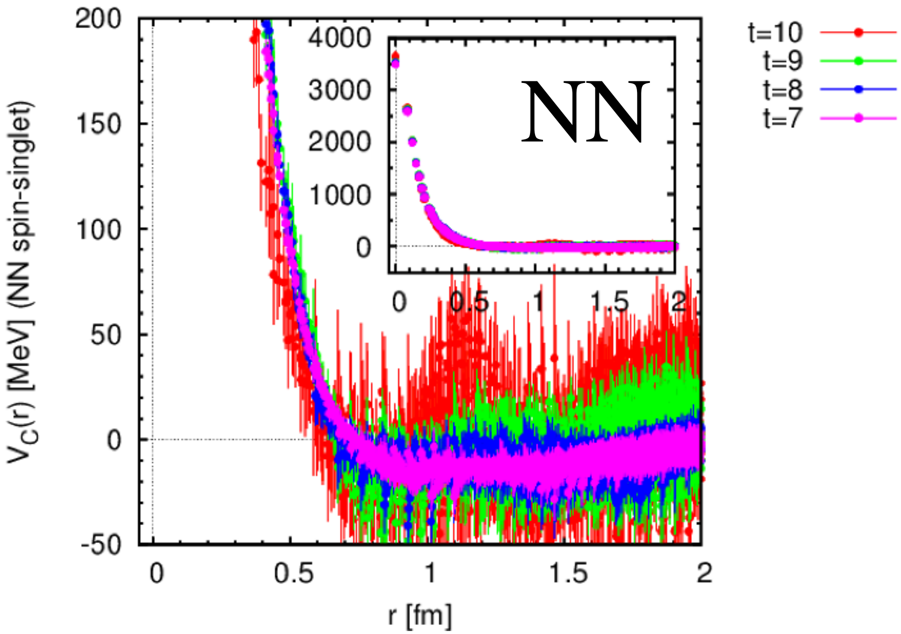
- Qualitative behavior is similar to dineutron channel
  - attractive
  - no bound state

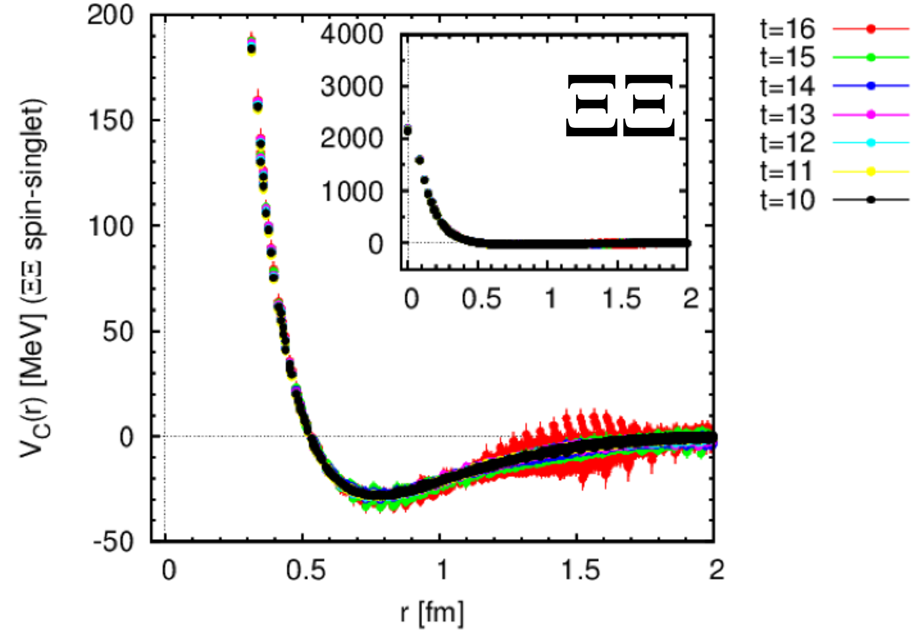
flavor SU(3) breaking: other states in irrep. 27





- repulsive core:
  NN is huge
- attraction:
  unclear due to the large stat. noise of NN

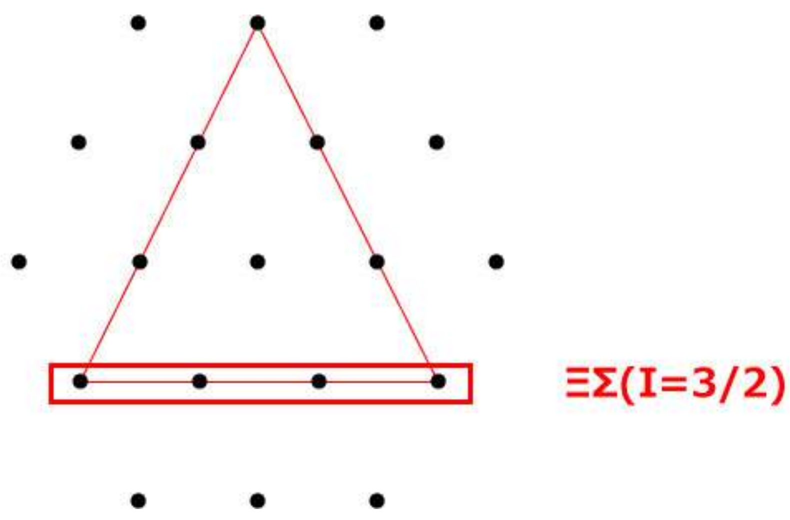




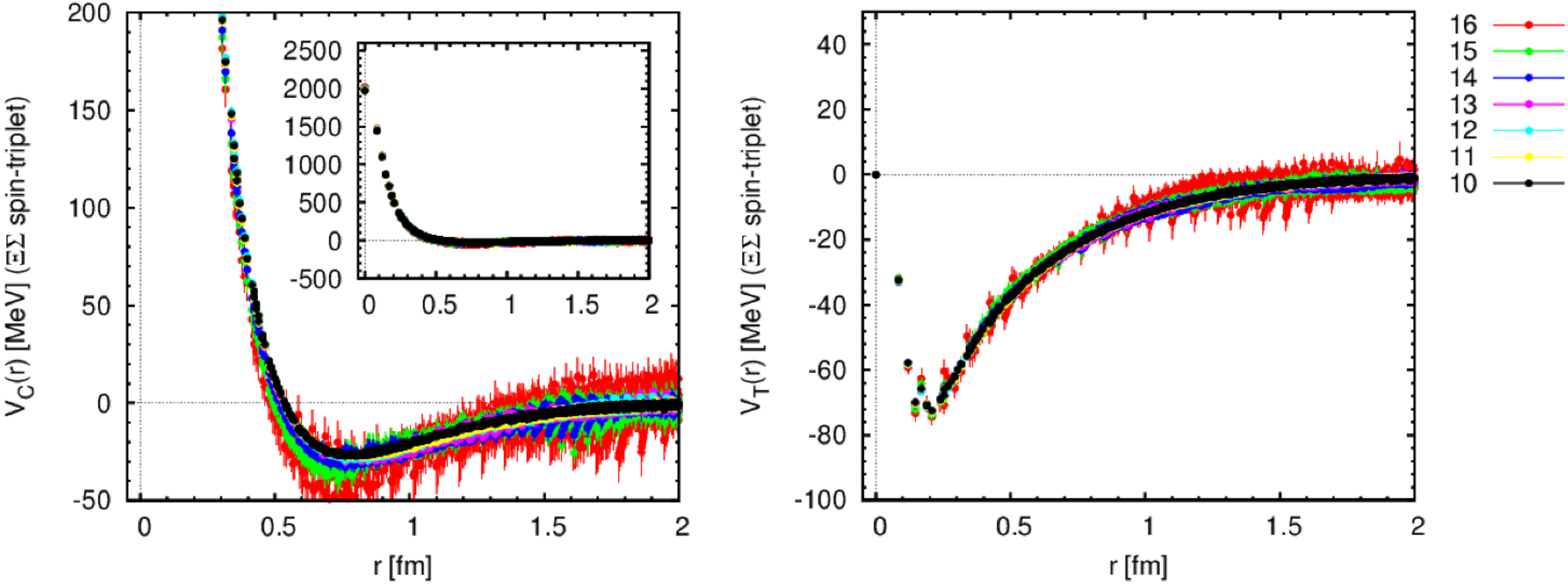
XiSigma (I=3/2) spin triplet

◆Total spin triplet

◆Irrep. **10*** in flavor SU(3) limit
(same as deuteron channel)



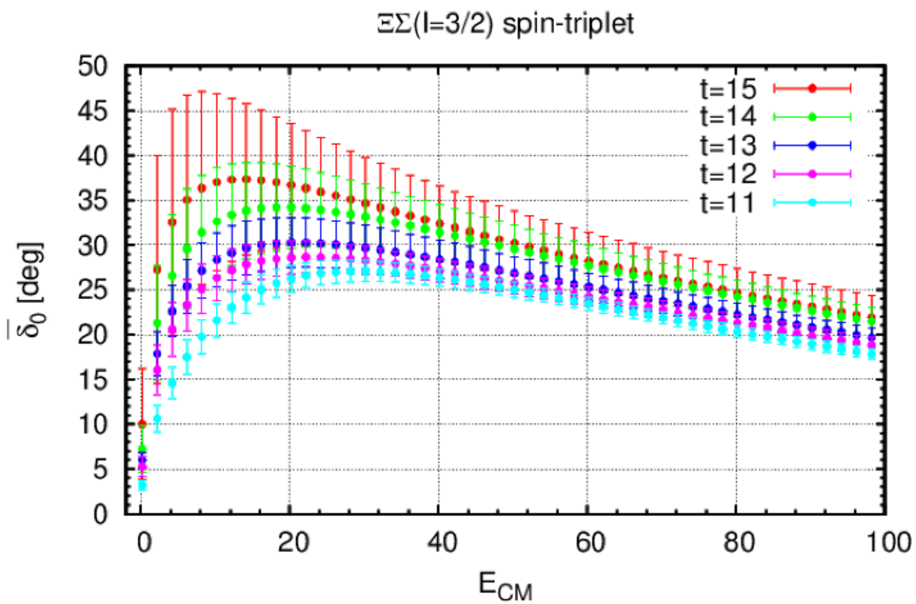
XiSigma (I=3/2, spin triplet)

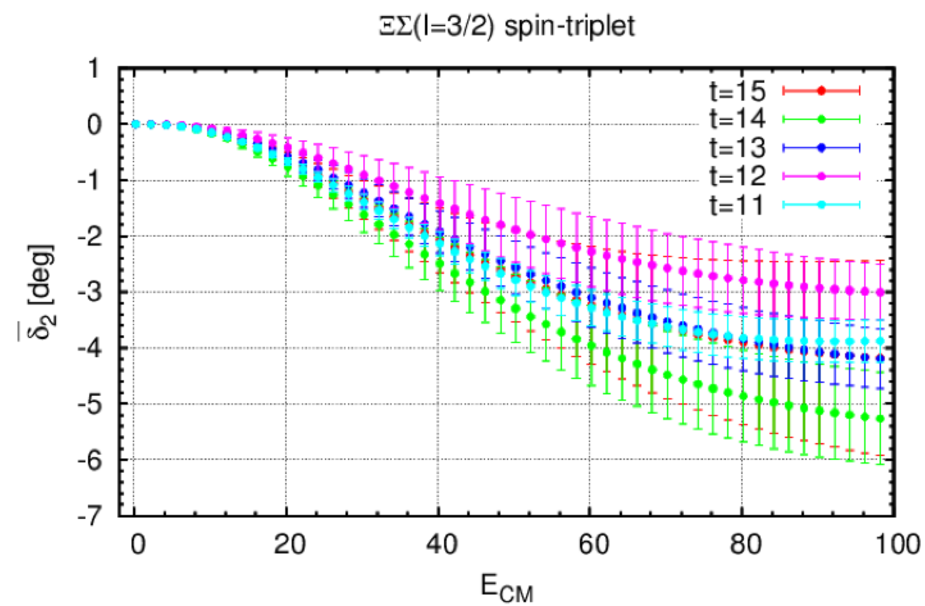


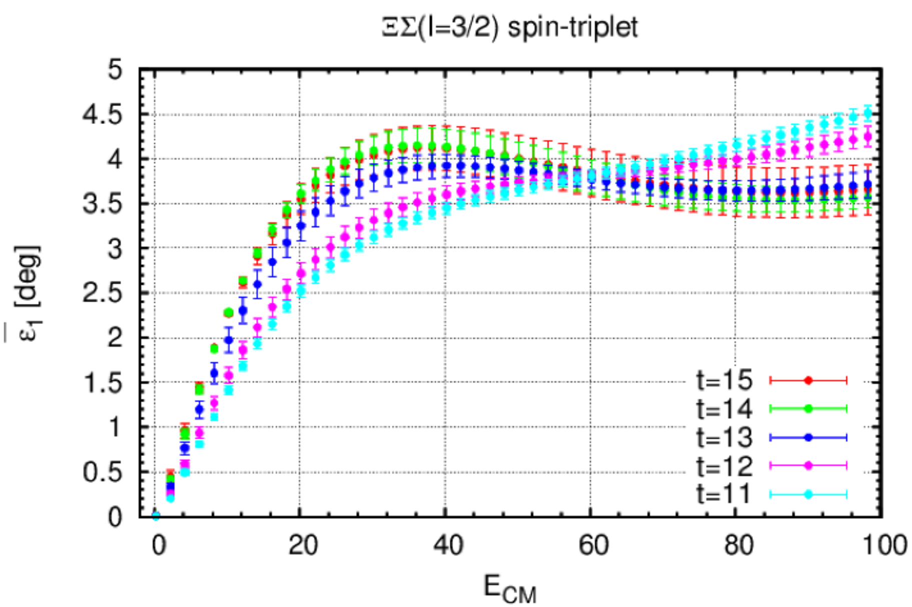
◆ Qualitative behaviors are similar to the deuteron channel for both central and tensor potentials

- ◆ 1/a ～2333 MeV, L ～8.1 fm
- ◆ m_{pion}～146 MeV
- ◆ 400 gauge confs. are used.
- ◆ Binsize = 20 (= 100 HMC traj.)
- ◆ 48 source points * 4 rotations
- ◆ Point sink and wall source

XiSigma (I=3/2, spin triplet) scattering phase shift



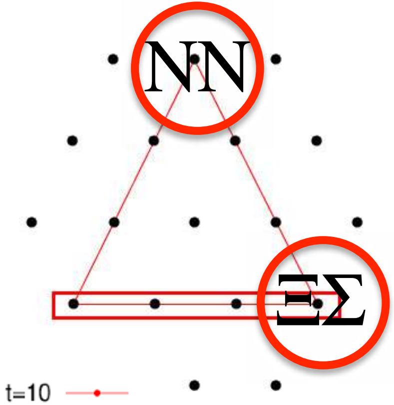


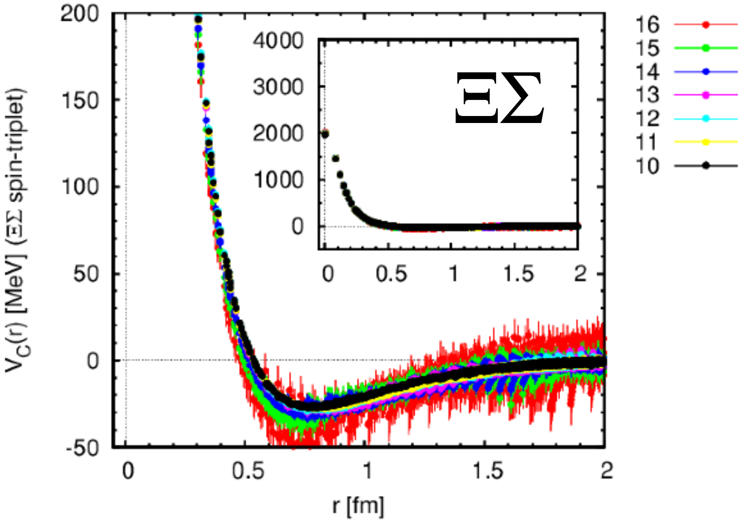


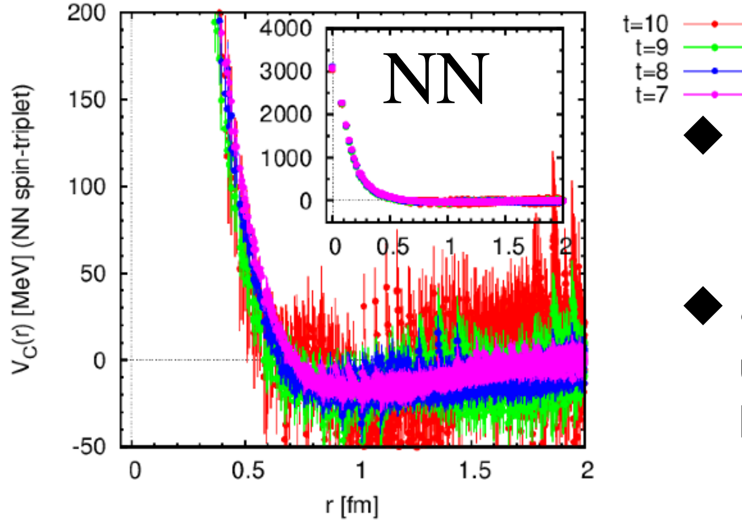
◆Qualitative behaviors are similar to deuteron channel (except for the absense of a bound state)

- ☐ attractive
- ☐ no bound state (strength are weaker than the deuteron channel)

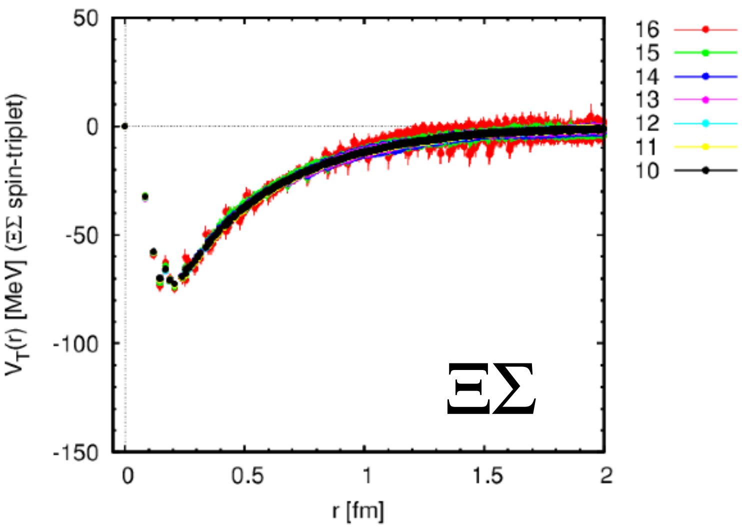
Flavor SU(3) breaking: other states in irrep. 10*

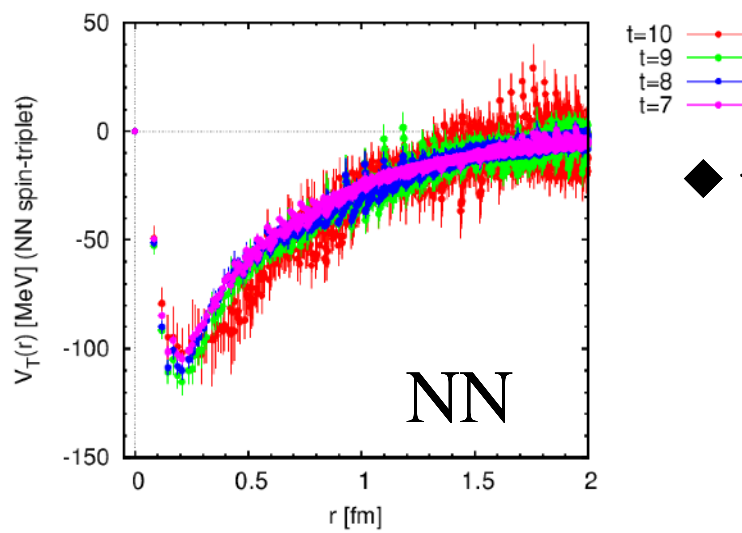






- repulsive core: NN is huge
- attraction: unclear due to the large stat. noise of NN.





- tensor potential: NN is quite strong.

XiLambda-XiSigma coupled channel (I=1/2)

Coupled ch. generalization of time-dep. Schrödinger-like eq.

◆ **coupled channel generalization** of the **time-dep. Schrödinger-like eq.**

$$\begin{bmatrix} \mathcal{D}_{\Xi\Lambda} R_{\Xi\Lambda}(\vec{r},t;\mathcal{J}) \\ \mathcal{D}_{\Xi\Sigma} R_{\Xi\Sigma}(\vec{r},t;\mathcal{J}) \end{bmatrix} = \int d^3r' \begin{bmatrix} V_{\Xi\Lambda;\Xi\Lambda}(\vec{r},\vec{r}') & \zeta^{t} V_{\Xi\Lambda;\Xi\Sigma}(\vec{r},\vec{r}') \\ \zeta^{-t} V_{\Xi\Sigma;\Xi\Lambda}(\vec{r},\vec{r}') & V_{\Xi\Sigma;\Xi\Sigma}(\vec{r},\vec{r}') \end{bmatrix} \cdot \begin{bmatrix} D^2_{t;\Xi\Lambda} R_{\Xi\Lambda}(\vec{r}',t;\mathcal{J}) \\ D^2_{t;\Xi\Sigma} R_{\Xi\Sigma}(\vec{r}',t;\mathcal{J}) \end{bmatrix}$$

where (quite complicated)

$$R_{\Xi\Lambda}(\vec{x}-\vec{y},t;\mathcal{J}) \equiv Z_{\Xi}^{-1/2} Z_{\Lambda}^{-1/2} e^{(m_\Xi+m_\Lambda)t} \left\langle 0 \middle| T\left[\Xi(\vec{x},t)\Lambda(\vec{y},t)\cdot\mathcal{J}(t=0)\right] \middle| 0 \right\rangle$$

$$R_{\Xi\Sigma}(\vec{x}-\vec{y},t;\mathcal{J}) \equiv Z_{\Xi}^{-1/2} Z_{\Sigma}^{-1/2} e^{(m_\Xi+m_\Sigma)t} \left\langle 0 \middle| T\left[\Xi(\vec{x},t)\Sigma(\vec{y},t)\cdot\mathcal{J}(t=0)\right] \middle| 0 \right\rangle$$

$$\zeta \equiv \exp\left(m_\Sigma - m_\Lambda\right)$$

$$\mathcal{D}_{\Xi\Lambda} \equiv \frac{\nabla^2}{2\mu_{\Xi\Lambda}} D^2_{t;\Xi\Lambda} + \frac{1}{8\mu_{\Xi\Lambda}}\left(D^2_{t;\Xi\Lambda} - (m_\Xi+m_\Lambda)^2\right)\left(D^2_{t;\Xi\Lambda} - (m_\Xi-m_\Lambda)^2\right)$$

$$\mathcal{D}_{\Xi\Sigma} \equiv \frac{\nabla^2}{2\mu_{\Xi\Sigma}} D^2_{t;\Xi\Sigma} + \frac{1}{8\mu_{\Xi\Sigma}}\left(D^2_{t;\Xi\Sigma} - (m_\Xi+m_\Sigma)^2\right)\left(D^2_{t;\Xi\Sigma} - (m_\Xi-m_\Sigma)^2\right)$$

$$D_{t;\Xi\Lambda} \equiv \partial_t - m_\Xi - m_\Lambda$$

$$D_{t;\Xi\Sigma} \equiv \partial_t - m_\Xi - m_\Sigma$$

$$\mu_{\Xi\Lambda} \equiv \frac{1}{1/m_\Xi + 1/m_\Lambda}$$

$$\mu_{\Xi\Sigma} \equiv \frac{1}{1/m_\Xi + 1/m_\Sigma}$$

This eq. enables us to obtain the E-indep. HAL QCD potential without relying on single state saturation

However

Numerical evaluation of 4th time derivative is unstable

➔ We use non-relativistic approx. version:

$$\left(\frac{\nabla^2}{2\mu_{\Xi\Lambda}} - \frac{\partial}{\partial t}\right) R_{\Xi\Lambda}(\vec{r},t;\mathcal{J}) \simeq V_{\Xi\Lambda;\Xi\Lambda}(\vec{r}) R_{\Xi\Lambda}(\vec{r}',t;\mathcal{J}) + e^{+(m_\Sigma - m_\Lambda)t} V_{\Xi\Lambda;\Xi\Sigma}(\vec{r}) R_{\Xi\Sigma}(\vec{r}',t;\mathcal{J})$$

$$\left(\frac{\nabla^2}{2\mu_{\Xi\Sigma}} - \frac{\partial}{\partial t}\right) R_{\Xi\Sigma}(\vec{r},t;\mathcal{J}) \simeq e^{-(m_\Sigma - m_\Lambda)t} V_{\Xi\Sigma;\Xi\Lambda}(\vec{r}) R_{\Xi\Lambda}(\vec{r}',t;\mathcal{J}) + V_{\Xi\Sigma;\Xi\Sigma}(\vec{r}) R_{\Xi\Sigma}(\vec{r}',t;\mathcal{J})$$

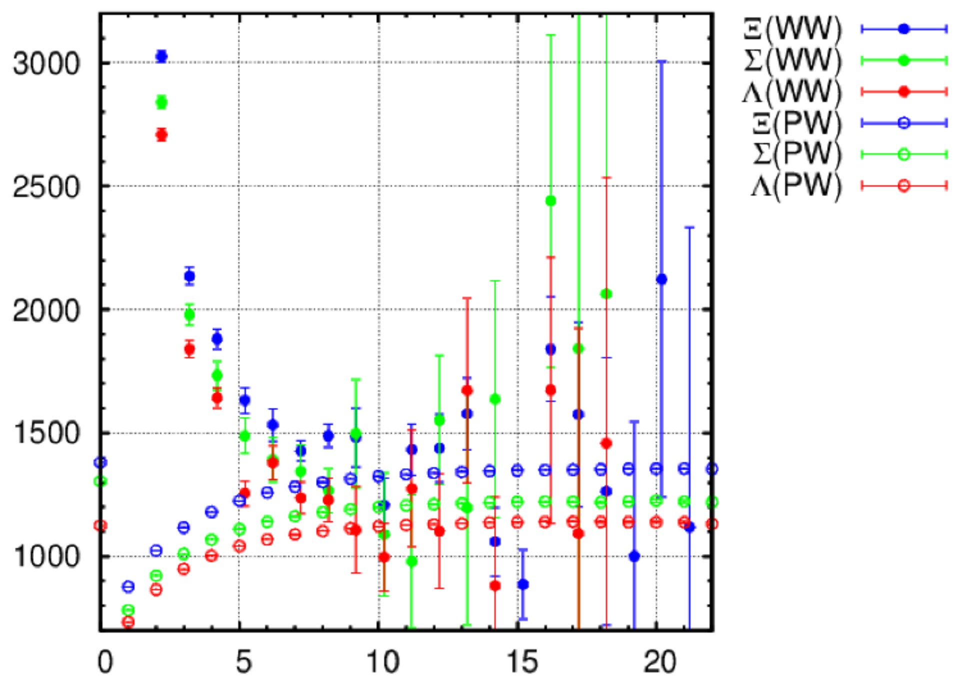
Z factors of composite baryon fields

$$\psi(x) \to Z^{1/2}\psi_{\text{out}}(x)$$

- To define R-corr, Z factors for Lambda and Sigma are needed.

- 2pt. corr. of point-wall and wall-wall are used to obtain Z-factors.

effective mass plot



- Simultaneous fit of wall-wall data with pont-wall data.

- The results depend on Z factors though the combination

$$\sqrt{Z_\Lambda / Z_\Sigma}$$

- Wall-wall data is quite noisy.
  → Fit-range is hard to identify.
  - fit range = 10-15

$$\sqrt{Z_\Lambda / Z_\Sigma} = 1.01(3)$$

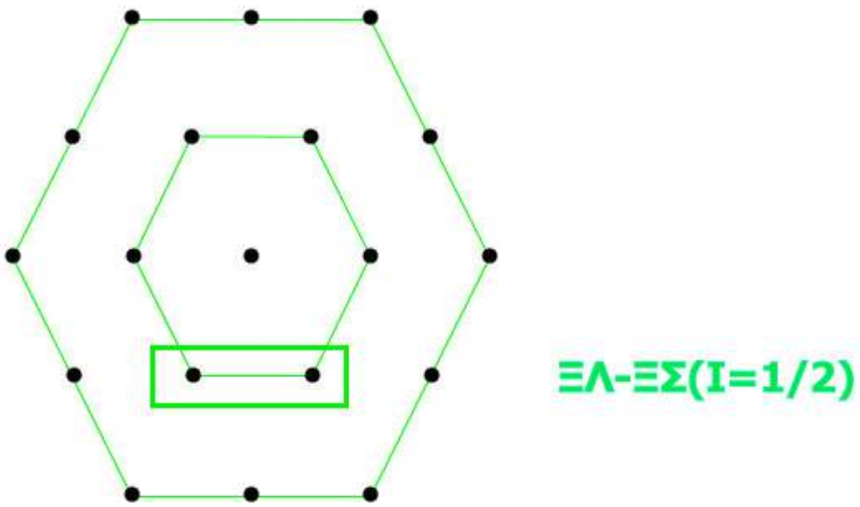
  - fit range = 15-20

$$\sqrt{Z_\Lambda / Z_\Sigma} = 1.02(6)$$

XiLambda-XiSigma coupled channel: (I=1/2) spin singlet

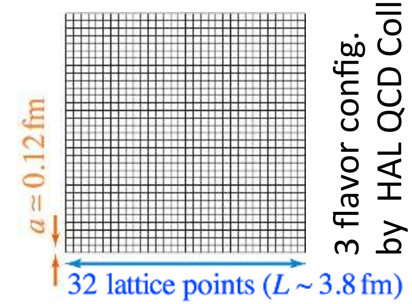
◆spin singlet

◆flavor SU(3) limit :
mixture of of **27** & **8s** irreps.
(27 irrep. contains NN)



XiLambda-XiSigma coupled channel (I=1/2)

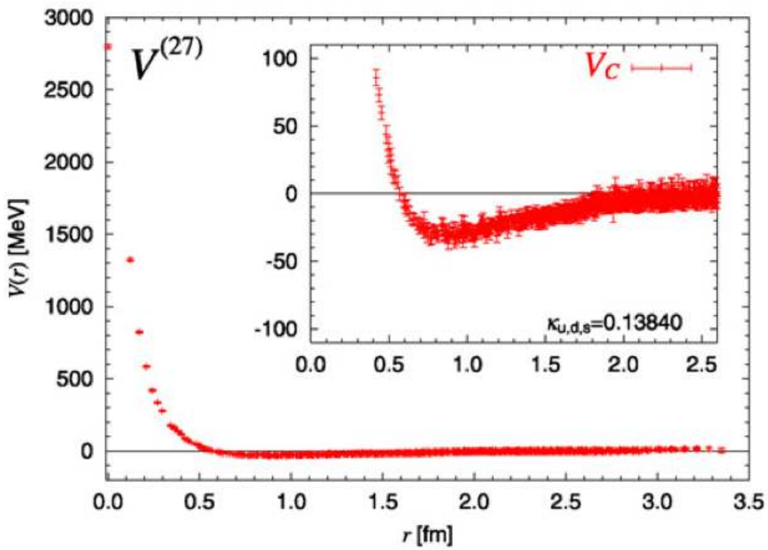
For qualitative understanding, we use



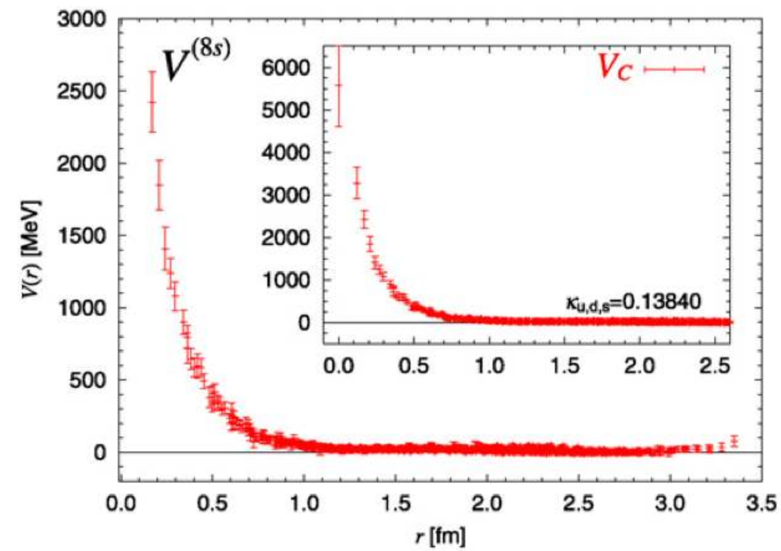
3 flavor config.
by HAL QCD Coll.
m(PS) = 469 MeV
m(B) = 1161 MeV

Potentials in the flavor SU(3) limit
(T.Inoue et al., NPA881(2012)28)

27 irrep.



8S irrep.



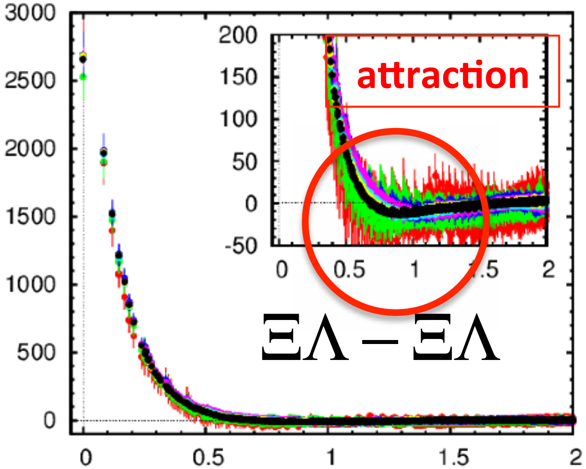
XiLambda-XiSigma potentials are linear combinations of these two as

$$\begin{pmatrix} V_{\Xi\Lambda;\Xi\Lambda} & V_{\Xi\Lambda;\Xi\Sigma} \\ V_{\Xi\Sigma;\Xi\Lambda} & V_{\Xi\Sigma;\Xi\Sigma} \end{pmatrix} = \begin{pmatrix} \frac{9}{10}V^{(27)} + \frac{1}{10}V^{(8s)} & -\frac{3}{10}V^{(27)} + \frac{3}{10}V^{(8s)} \\ -\frac{3}{10}V^{(27)} + \frac{3}{10}V^{(8s)} & \frac{1}{10}V^{(27)} + \frac{9}{10}V^{(8s)} \end{pmatrix}$$

XiLambda-XiSigma coupled channel(I=1/2) spin singlet

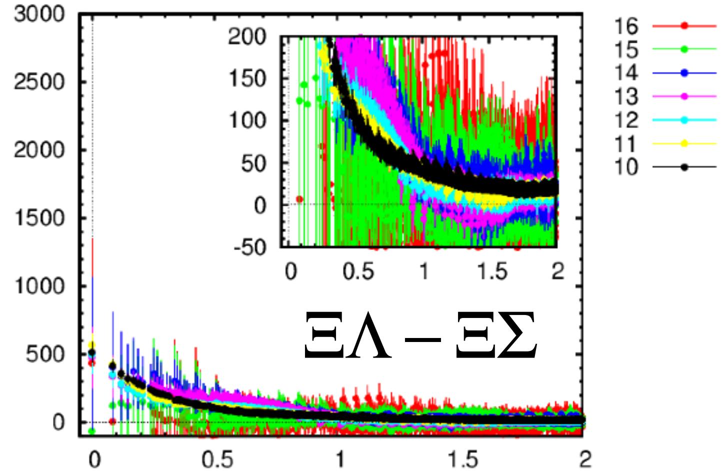
"phys. pt." potential

$$V_{\Xi\Lambda;\Xi\Lambda} \leftrightarrow \frac{9}{10}V^{(27)} + \frac{1}{10}V^{(8S)}$$

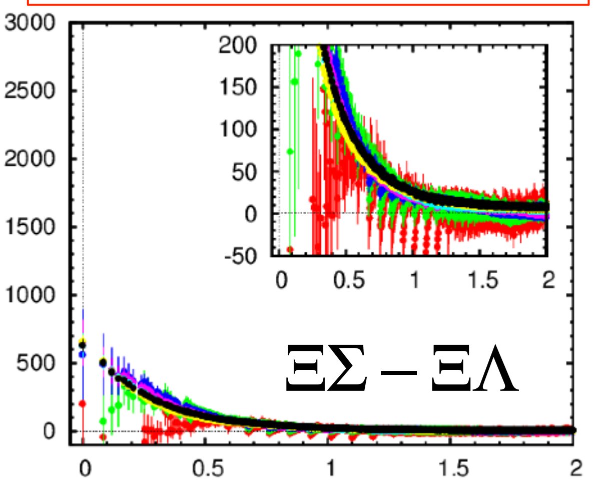


"phys. pt." potential

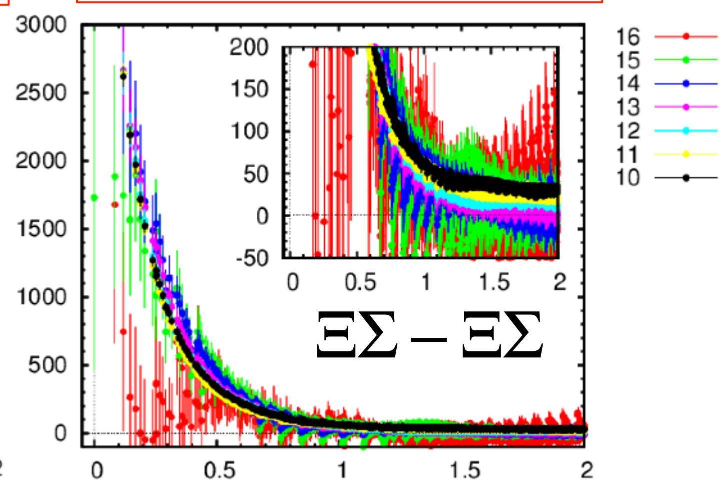
$$V_{\Xi\Lambda;\Xi\Sigma} \leftrightarrow -\frac{3}{10}V^{(27)} + \frac{3}{10}V^{(8S)}$$



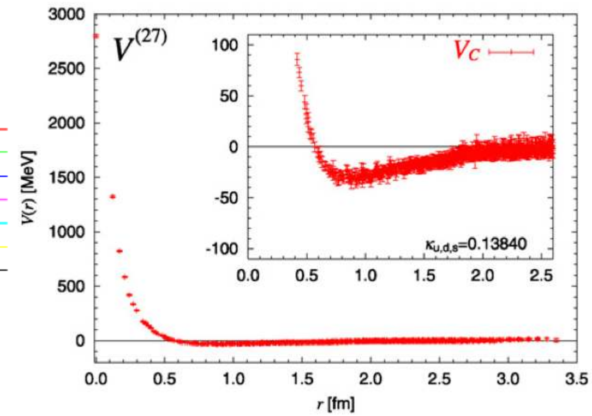
$$V_{\Xi\Sigma;\Xi\Lambda} \leftrightarrow -\frac{3}{10}V^{(27)} + \frac{3}{10}V^{(8S)}$$



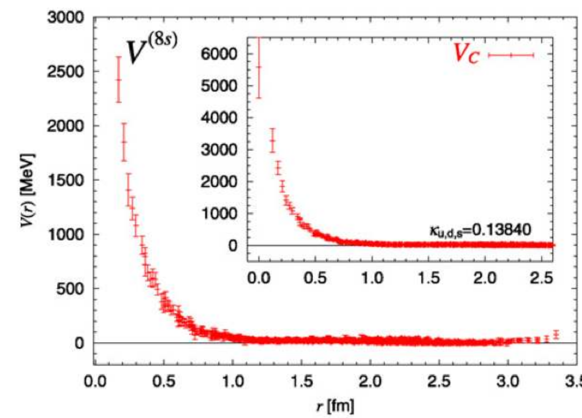
$$V_{\Xi\Sigma;\Xi\Sigma} \leftrightarrow \frac{1}{10}V^{(27)} + \frac{9}{10}V^{(8S)}$$



27 irrep. in flavor SU(3) limit



8S irrep. in flavor SU(3) limit



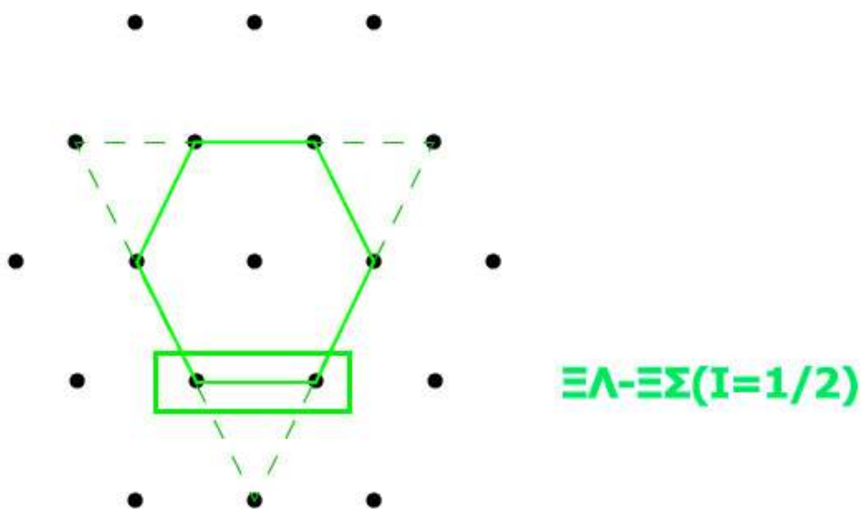
Once 8S is involved

- ☐ stat. noise becomes large
- ☐ t-dependence appears (convergence of long distance behavior is slow)

XiLambda-XiSigma coupled channel (I=1/2) spin triplet

◆spin triplet

◆Flavor SU(3) limit :
Linear combination of **10** & **8a** irreps.



XiLambda-XiSigma coupled channel(I=1/2, spin triplet)

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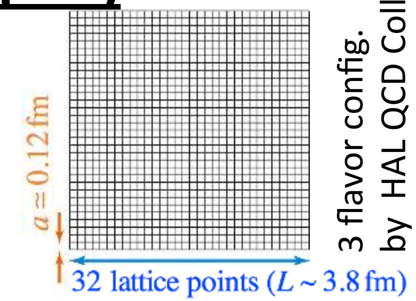
Potentials in the flavor SU(3) limit
(T.Inoue et al., NPA881(2012)28)



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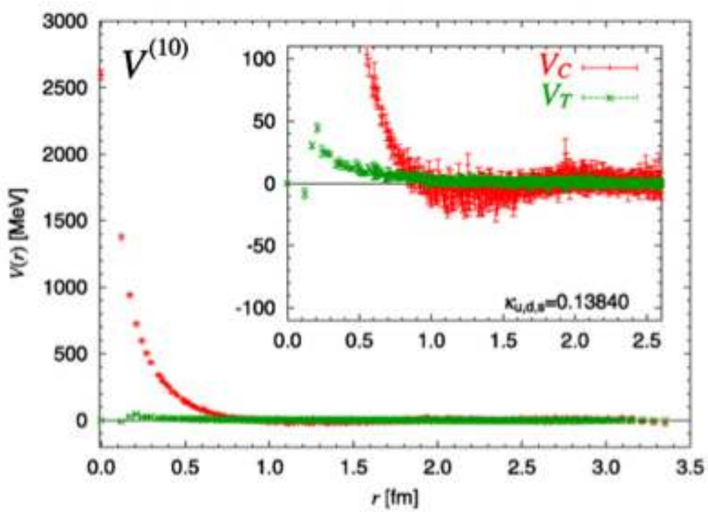
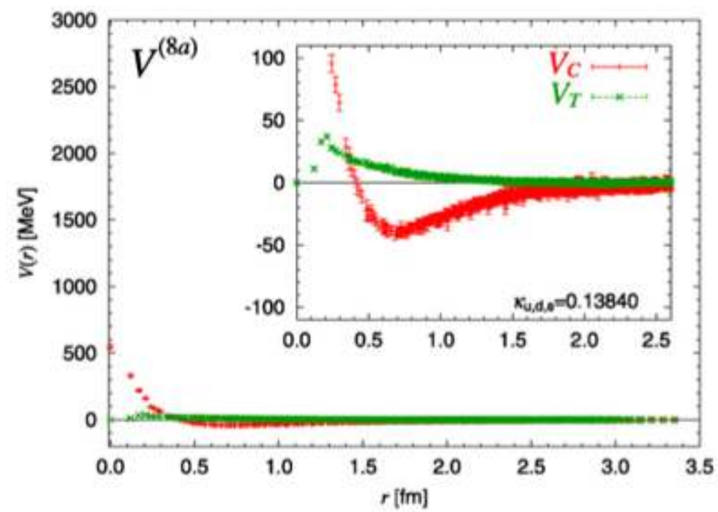
10 irrep.



8A irrep.



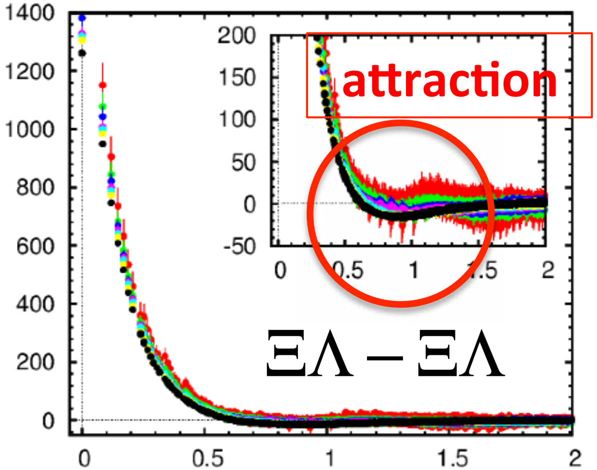
XiLambda-XiSigma potentials are linear combinations of these two as

$$\begin{pmatrix} V_{\Xi\Lambda,\Xi\Lambda} & V_{\Xi\Lambda,\Xi\Sigma} \\ V_{\Xi\Sigma,\Xi\Lambda} & V_{\Xi\Sigma,\Xi\Sigma} \end{pmatrix} = \begin{pmatrix} \frac{V_{10}+V_{8A}}{2} & \frac{V_{10}-V_{8A}}{2} \\ \frac{V_{10}-V_{8A}}{2} & \frac{V_{10}+V_{8A}}{2} \end{pmatrix}$$

XiLambda-XiSigma coupled channel (I=1/2, spin triplet) central potential

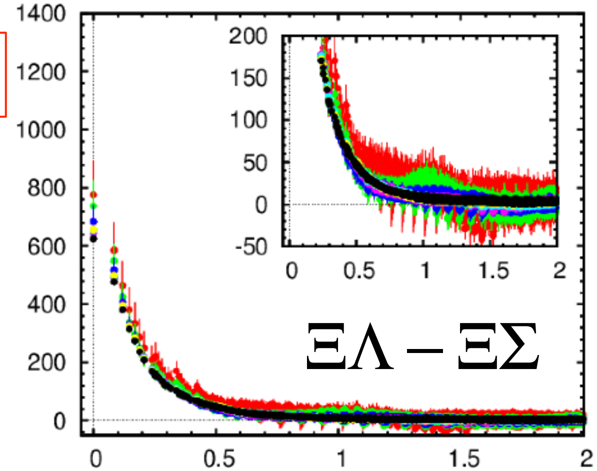
"phys. pt." potential

$$V_{\Xi\Lambda;\Xi\Lambda} \leftrightarrow \frac{1}{2}V^{(10)} + \frac{1}{2}V^{(8a)}$$

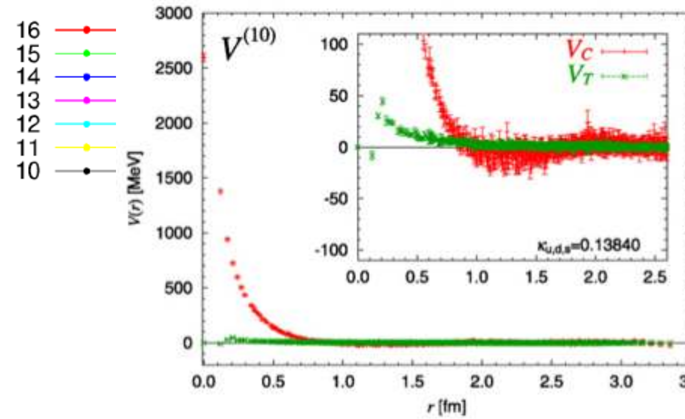


"phys. pt." potential

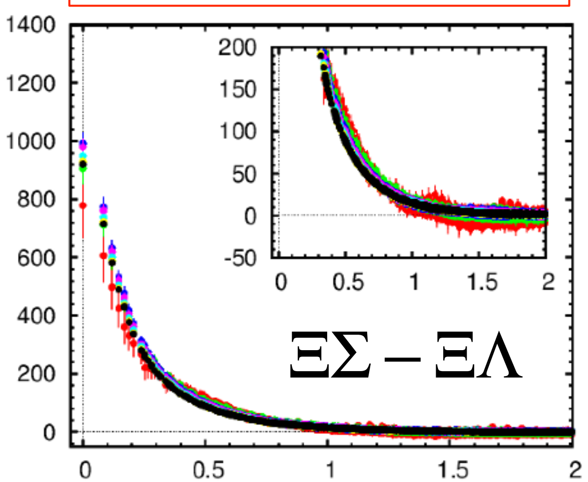
$$V_{\Xi\Lambda;\Xi\Sigma} \leftrightarrow \frac{1}{2}V^{(10)} - \frac{1}{2}V^{(8a)}$$



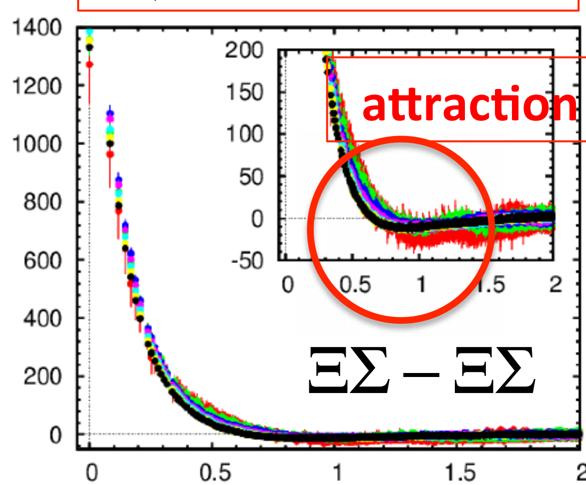
10 irrep. in flavor SU(3) limit



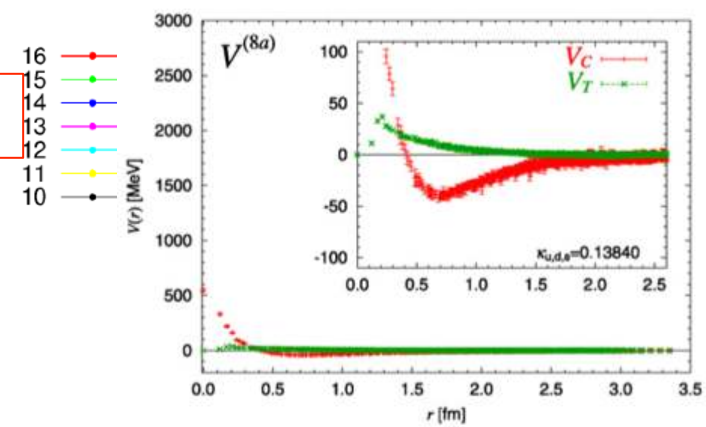
$$V_{\Xi\Lambda;\Xi\Lambda} \leftrightarrow \frac{1}{2}V^{(10)} - \frac{1}{2}V^{(8a)}$$



$$V_{\Xi\Lambda;\Xi\Lambda} \leftrightarrow \frac{1}{2}V^{(10)} + \frac{1}{2}V^{(8a)}$$



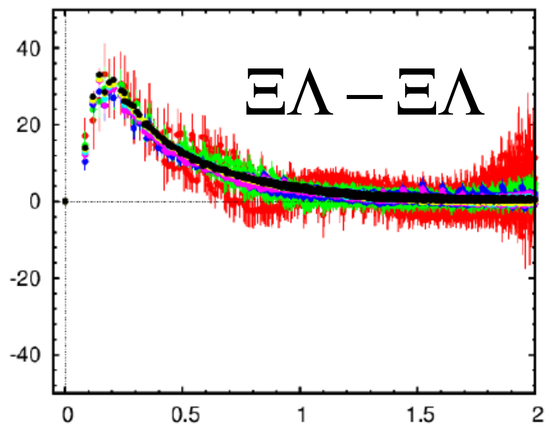
8A irrep. in flavor SU(3) limit



XiLambda-XiSigma coupled channel(I=1/2, spin triplet) tensor potential

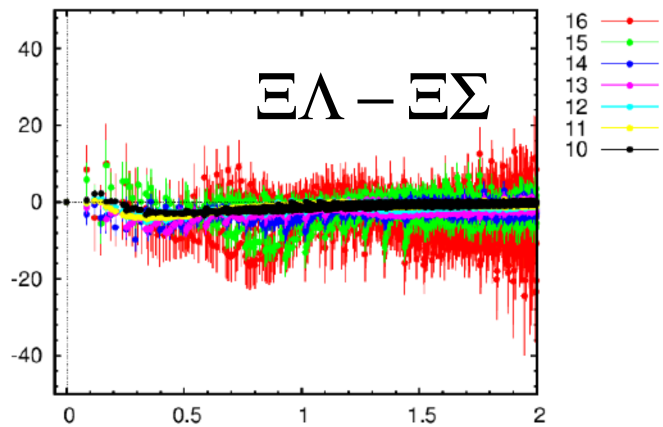
"phys. pt." potential

$$V_{\Xi\Lambda;\Xi\Lambda} \leftrightarrow \frac{1}{2}V^{(10)} + \frac{1}{2}V^{(8a)}$$

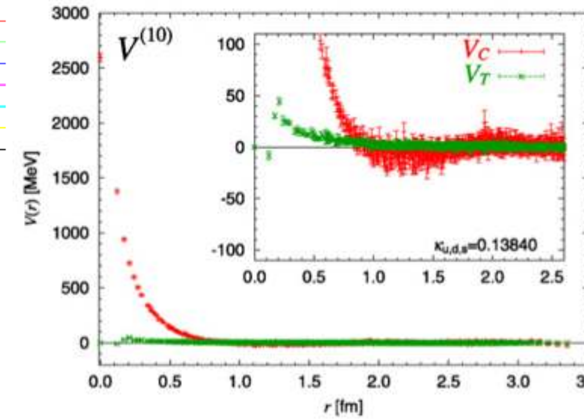


"phys. pt." potential

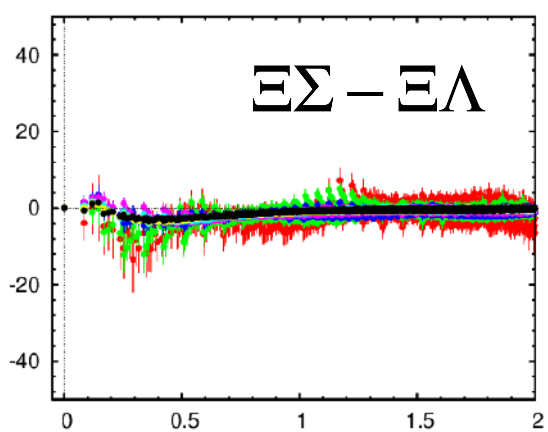
$$V_{\Xi\Lambda;\Xi\Sigma} \leftrightarrow \frac{1}{2}V^{(10)} - \frac{1}{2}V^{(8a)}$$



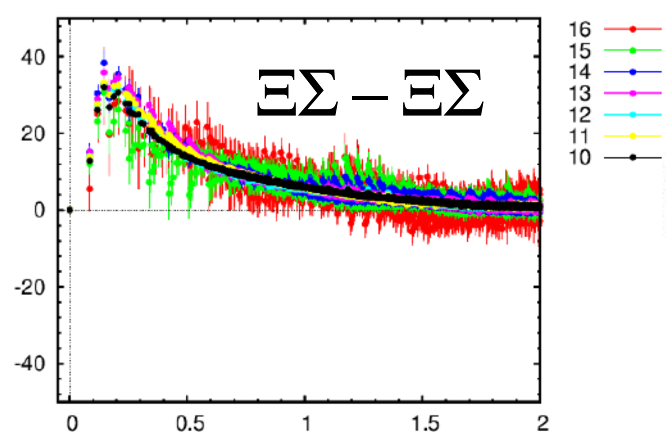
10 irrep. in flavor SU(3) limit



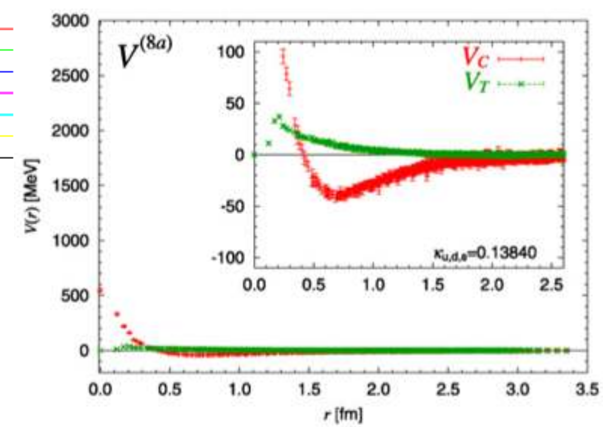
$$V_{\Xi\Lambda;\Xi\Lambda} \leftrightarrow \frac{1}{2}V^{(10)} - \frac{1}{2}V^{(8a)}$$



$$V_{\Xi\Lambda;\Xi\Lambda} \leftrightarrow \frac{1}{2}V^{(10)} + \frac{1}{2}V^{(8a)}$$



8A irrep. in flavor SU(3) limit



Comment

- In the derivation of the time-dependent Schrodinger-like eq.

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \simeq V(\vec{r},\vec{r}\,') R(\vec{r}\,',t)$$

R-corr. is defined as

$$R(\vec{r},t) \equiv C_{BB}(\vec{r},t) \Big/ e^{-2m_B t}$$

$$C_{BB}(\vec{x}-\vec{y},t) \equiv \left\langle 0 \middle| T\left[B(\vec{x},t)B(\vec{y},t) \cdot \overline{BB(t=0)} \right] \middle| 0 \right\rangle$$

- In an actual calc., we replace it by

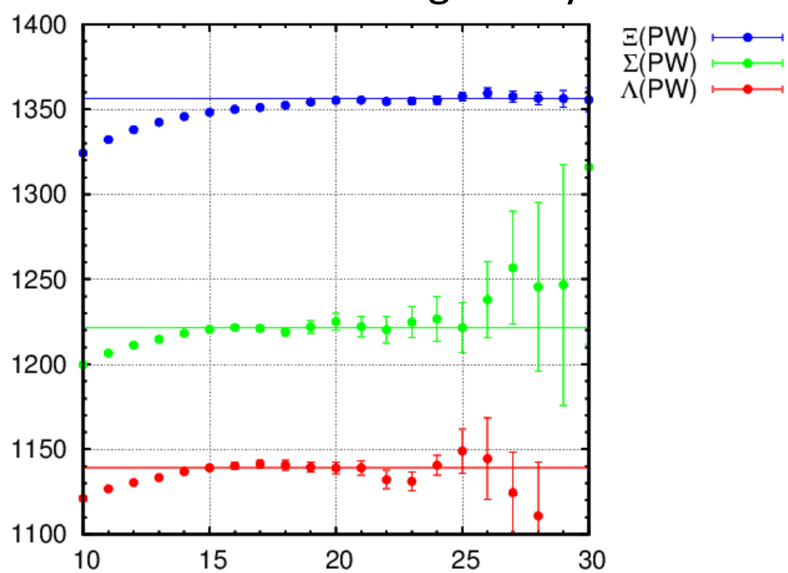
$$R(\vec{r},t) \equiv C_{BB}(\vec{r},t) \Big/ C_B(t)^2$$

with $C_B(t) \equiv \left\langle 0 \middle| T\left[B(t)B(0) \right] \middle| 0 \right\rangle$

(to reduce the statistical noise)

➔ Ground state saturation of single baryon corr. is required

single baryon effective mass plot



- The region we used for the potentials
  t = 10-16
  (due to statistical reason)
- Plateaux t～15
  (Lambda and Sigma)
- Xi is not in the plateau region at t ～15. (～10 MeV smaller)

Summary

- "physical point" LQCD results (m(pion)=146 MeV) for hyperon-potentials and scattering phase shift in the S=-3 sector.
  - Potentials
    - ✓ XiSigma(I=3/2) : spin singlet and triplet
    - ✓ XiLambda-XiSigma(I=1/2) : spin singlet and triplet
  - scattering phase shift
    - ✓ XiSigma(I=3/2): spin singlet and triplet
  - Qualitative behaviors are understood by considering the flavor SU(3) limit

- Todo
  - Improvement of statistics

    Now : 400 conf x 4 rot x 48 src. pts.

    ⇩ x 2

    Future : 400 conf x 4 rot x 96 src. pts.

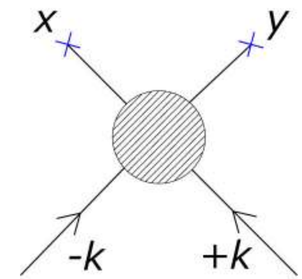
  - full time-dependent Schrödinger-like eq.

backup

HALQCD method

[Aoki,Hatsuda,Ishii,PTP123(2010)89]

◆Nambu-Bethe-Salpeter (NBS) wave func.



$$\langle 0 | T[N(x)N(y)] | N(+k)N(-k), in \rangle$$

◆**Relation with S-matrix** (by LSZ reduction formula)

$$\langle N(p_1)N(p_2), out | N(+k)N(-k), in \rangle_{\text{connected}}$$

$$= \left(iZ_N^{-1/2}\right)^2 \int d^4x_1 d^4x_2\, e^{ip_1x_1}\left(\Box_1 + m_N^2\right) e^{ip_2x_2}\left(\Box_2 + m_N^2\right) \langle 0 | T[N(x_1)N(x_2)] | N(+k)N(-k), in \rangle$$

◆**equal-time NBS wave func.**

$$\psi_k(\vec{x}-\vec{y}) \equiv Z_N^{-1} \langle 0 | N(\vec{x},0)N(\vec{y},0) | N(+k)N(-k), in \rangle$$

$$\simeq e^{i\delta(k)} \frac{\sin\left(kr + \delta(k)\right)}{kr} + \cdots \quad \text{as } r \equiv |\vec{x}-\vec{y}| \to \text{large}$$

(for S-wave)

- ❖ **Exactly the same func. form as non-rela. Q.M. scat. wave func.**
- ❖ **We demand that the potential reproduces the NBS wave.**
  → Potential is faithful to the scattering phase shift.

$$\left(k^2 / m_N - H_0\right)\psi_k(\vec{r}) = \int d^3r' \, U(\vec{r},\vec{r}')\psi_k(\vec{r}')$$

HALQCD method

SKIP

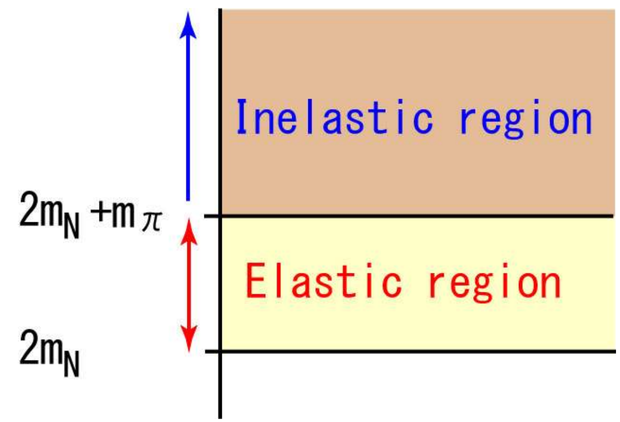
◆Proof of existence of E-indep. U(r,r')

◆Assumption:

Linear indep. of NBS wave func's for E < Eth.

➔ Dual basis exists

$$\int d^3r\, \tilde{\psi}_{\vec{k}'}(\vec{r})\psi_{\vec{k}}(\vec{r}) = (2\pi)^3\delta^3(\vec{k}'-\vec{k})$$



◆Proof:

$$K_{\vec{k}}(\vec{r}) \equiv \left(k^2/m_N - H_0\right)\psi_{\vec{k}}(\vec{r})$$

$$K_{\vec{k}}(\vec{r}) = \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r}) \int d^3r'\, \tilde{\psi}_{\vec{k}'}(\vec{r})\psi_{\vec{k}}(\vec{r})$$

$$= \int d^3r' \left\{ \int \frac{d^3k}{(2\pi)^3} K_{\vec{k}'}(\vec{r})\tilde{\psi}_{\vec{k}'}(\vec{r}') \right\} \psi_{\vec{k}}(\vec{r}')$$

$$\left(k^2/m_N - H_0\right)\psi_{\vec{k}}(\vec{r}) = \int d^3r'\, U(\vec{r},\vec{r}')\psi_{\vec{k}}(\vec{r}')$$

$$U(\vec{r},\vec{r}') \equiv \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r})\tilde{\psi}_{\vec{k}'}(\vec{r})$$

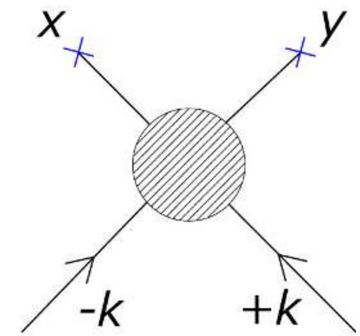
U(r,r') does not depend on E because of the integration of k'.

Potential is determined without relying on the ground state saturation

◆Def: **R-correlator**

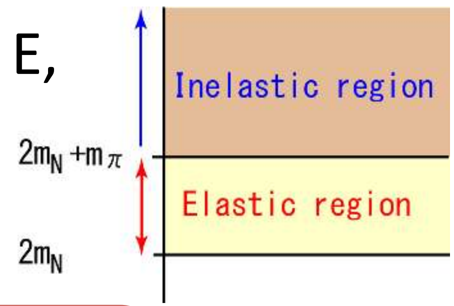
[Ishii et al.,PLB712(2012)437]

$$R(\vec{x}-\vec{y},t) \equiv e^{2mt} \left\langle 0 \middle| T\left[B(\vec{x},t)B(\vec{y},t)\cdot \overline{BB}(t=0)\right] \middle| 0 \right\rangle$$
$$= \sum_n \psi_{k_n}(\vec{x}-\vec{y})\cdot \exp\left(-(E_n-2m)t\right)\cdot a_n$$



◆Using the fact that our potential does not depend on E, we determine the potential from the time-evolution

➔ **time-dep. Schroedinger-like eq.**



$$\left(-H_0 + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r' V(\vec{r},\vec{r}') R(\vec{r}',t)$$

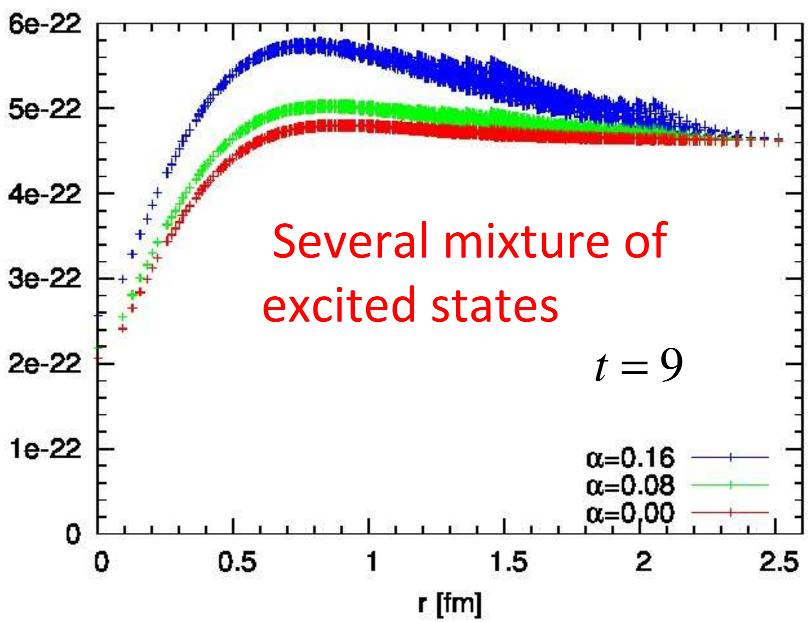
◆ **We need Elastic saturation.**
（We do not need **Ground saturation**）

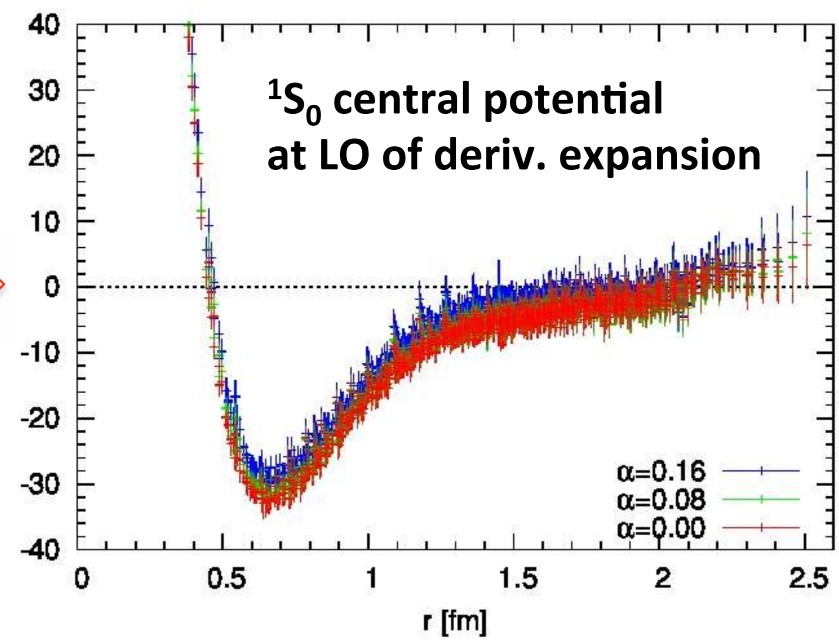
◆ **Elastic saturation** is easier than **ground state saturation**.

Example: Potential is determined uniquely even in the presence of excited state contamination.

$$\langle 0 | T[N(\vec{x},t)N(\vec{y},t)\cdot \overline{NN}(t=0;\alpha)] | 0\rangle = \sum_n \psi_n(\vec{x}-\vec{y})\cdot a_n(\alpha)\cdot \exp(-E_n t)$$

$$V_C(\vec{x}) = -\frac{H_0 R(t,\vec{x})}{R(t,\vec{x})} - \frac{(\partial/\partial t)R(t,\vec{x})}{R(t,\vec{x})} + \frac{1}{4m_N}\frac{(\partial/\partial t)^2 R(t,\vec{x})}{R(t,\vec{x})}$$





Good agreement !
➔ Our method works !

source function:

$$f(x,y,z) = 1 + \alpha\left(\cos(2\pi x/L) + \cos(2\pi y/L) + \cos(2\pi z/L)\right)$$