"Calculation of B_K with Wilson fermion using gradient flow"

- This is unfinished report towards B_K
- I report on the calculation of m_{PCAC}, f_π using Gradient flow

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Back ground

- $\boldsymbol{B_K}$ gives QCD contribution to the Kaon mixing phenomenon

$$B_K = \left(\frac{8}{3} f_K^2 m_K^2\right)^{-1} \langle \overline{K_0} | \hat{O}^{\Delta s=2} | K_0 \rangle, \qquad \hat{O}^{\Delta s=2} = \{\bar{s}(\gamma^L)_\mu d\}\{\bar{s}(\gamma^L)_\mu d\}$$

- Contamination from the breaking of the chiral symmetry gives as an undesirable contribution

$$\hat{O}^{\Delta s=2(R)} = Z\{\hat{O}^{\Delta s=2} + \Delta_2 \hat{O}_{VV-AA} + \Delta_3 \hat{O}_{SS-PP} + \Delta_4 \hat{O}_{SS+PP} + \Delta_5 \hat{O}_{TT}\}$$

- Hard to subtruct them

$$\Delta_i \sim \mathcal{O}(10^{-2}), \langle \hat{O}_{\Gamma\Gamma} \rangle \gtrsim \left\langle \hat{O}^{\Delta s=2} \right\rangle$$

D. Becirevic, V. Gim ´ enez, V. Lubicz, G. Martinelli, M. Papinutto and J. Reyes, hep-lat/0401033.

- Gradient flow supports to measure observables related with the chiral symmetry ?

(WHOT-QCD, Phys. Rev. D 95, 054502 (2017))

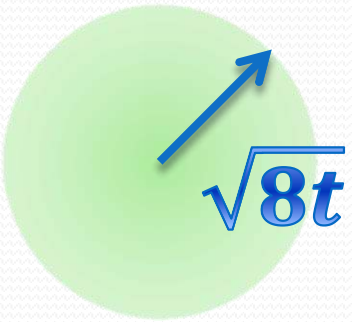
Gradient flow

- diffusion equation in 5dim (fictitious time t)

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x) + D_\mu \partial_\nu B_\nu(t,x)$$
$$\partial_t \chi(t,x) = D^2 \chi(t,x) - \partial_\mu B_\mu(t,x)$$
$$\partial_t \bar{\chi}(t,x) = \bar{\chi}(t,x) \overleftarrow{D}^2 + \partial_\mu B_\mu(t,x)$$
$$B_\mu(t=0) = A_\mu, \chi(t=0) = \psi, \bar{\chi}(t=0) = \bar{\chi}$$

$$\chi(t,x) \sim \int dy\, e^{-\frac{(x-y)^2}{4t}} \psi(y)$$

remove UV divergence
definition of EM tensor
scale setting
renormalization of composite op.
definition of topological saceptibility
...



Why gradient flow?

- Measurement of topological susceptibility

  Y. Taniguchi, K. Kanaya, H. Suzuki, T. Umeda, Phys Rev D.95.054502

- Gluonic definition

$$Q(t) = \frac{1}{64\pi^2} \int d^4x \, \epsilon_{\mu\nu\rho\sigma} G_{\mu\nu}{}^a(t,x) G_{\rho\sigma}{}^a(t,x)$$

- Fermionic definition (from chiral WT id.)

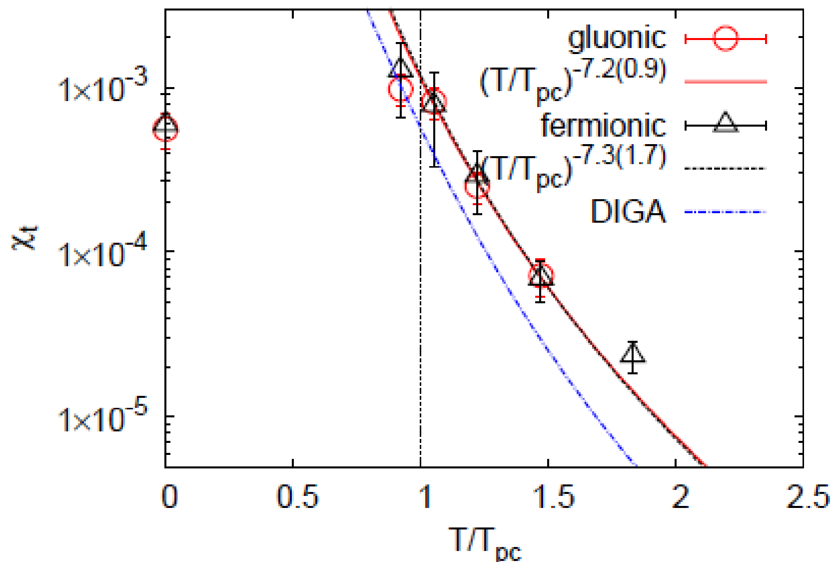
$$\langle Q^2 \rangle = \frac{m^2}{n_f^2} \left(\langle P^0 P^0 \rangle - n_f \langle P^a P^a \rangle \right)$$

$$P^a = \int d^4x \ \bar{\psi}(x) T^a \gamma_5 \psi(x)$$

Why gradient flow?

- Measurement of topological susceptibility

Y. Taniguchi, K. Kanaya, H. Suzuki, T. Umeda, Phys Rev D.95.054502



- We can take the continuum limit without bothering the UV divergence.
- Gradient flow and B_K have good chemistry ?

$$\hat{O}^{\Delta s=2} + \Delta_2 \hat{O}_{VV-AA} + \Delta_3 \hat{O}_{SS-PP} + \Delta_4 \hat{O}_{SS+PP} + \Delta_5 \hat{O}_{TT}$$

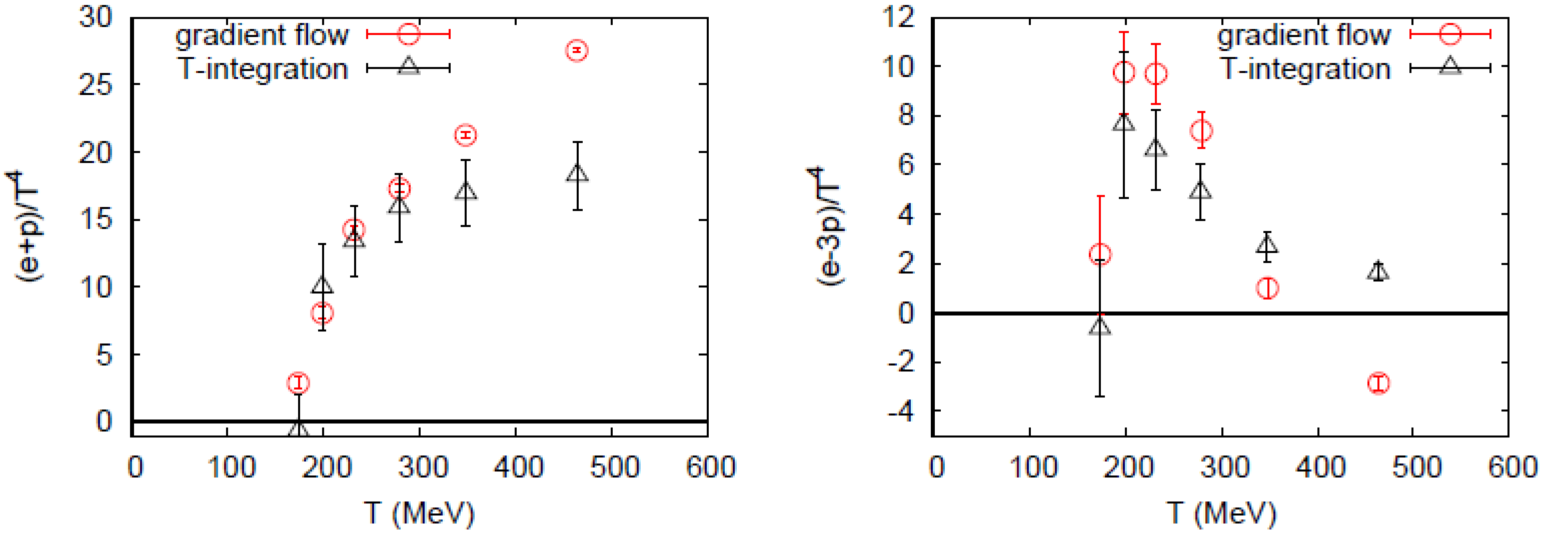
Road to B_K

Our goal is $B_K = \left(\frac{8}{3} f_K^2 m_K^2\right)^{-1} \langle \overline{K_0} | \hat{O}^{\Delta s=2} | K_0 \rangle$

3pt. function

Many success for 1pt. Function

- Yang-Mills flow and Fermion flow
- ex. EM tensor on the lattice



(WHOT-QCD, Lattice2016)

Road to B_K

Our goal is $B_K = \left(\frac{8}{3} f_K^2 m_K^2\right)^{-1} \underline{\langle \overline{K_0} | \hat{O}^{\Delta s=2} | K_0 \rangle}$

3pt. function

Many success for 1pt. Function

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The next step is 2pt. Function!

- Target : PCAC mass, pion mass and decay constant
- from : $\langle \boldsymbol{A_\mu(t,0,\vec{x})\pi(t,0,\vec{y})} \rangle, \langle \boldsymbol{\pi(t,0,\vec{x})\pi(t,0,\vec{y})} \rangle$
- towards denominator of B_K (f_K and m_K)

Calculation of 3pt. Function is in preparation

Strategy of calculation

Ⅰ. $\lim_{a\to 0}\langle A_\mu(t,0,\vec{x})\pi(t,0,\vec{y})\rangle$ at flow time $t \neq 0$

→ non-perturbatively renormalized in GF scheme

Ⅱ. $C_A(t)C_\pi(t)\langle A_\mu(t,0,\vec{x})\pi(t,0,\vec{y})\rangle$

transfer from GF scheme to $\overline{MS}$ scheme

matching factor $C_\Gamma(t)$ have been calculated

K. Hieda and H. Suzuki, arXiv:1606.04193 [hep-lat].

Ⅲ. $\lim_{t\to 0} C_A(t)C_\pi(t)\langle A_\mu(t,0,\vec{x})\pi(t,0,\vec{y})\rangle$

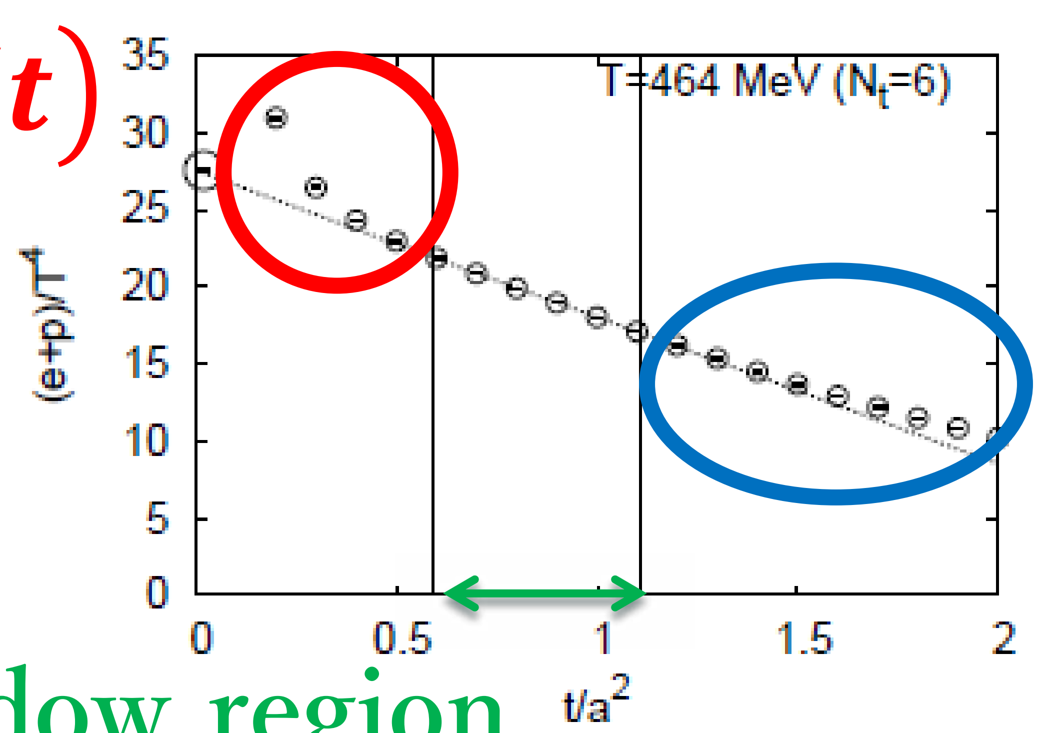
screen off "$t \times (\text{dim6 op.})$" term

To calculate the matching factor perturbatively

Some techniques

(WHOT-QCD, Lattice2016)

$\mathcal{O}(a^2/t)$



$\mathcal{O}(t^2)$

Window region

Window region works well to extract the physics

Some techniques

- Propagator at non-zero flow time is nontrivial

$$\langle \chi(t)\, \bar{\chi}(t) \rangle \neq (D(t) + m)^{-1}$$

We have to flow the fermion field back to flow time 0

$$\chi(t,x) = \sum_y K(t,x;0,y)\psi(y)$$

standard flow : $0 \to t$

$$\bar{\chi}(t,x) = \sum_y \bar{\psi}(y)\, K^{\dagger}(t,x;0.y)$$

adjoin flow : $t \to 0$

Lüscher, JHEP 1304, 123 (2013)

Lattice setup

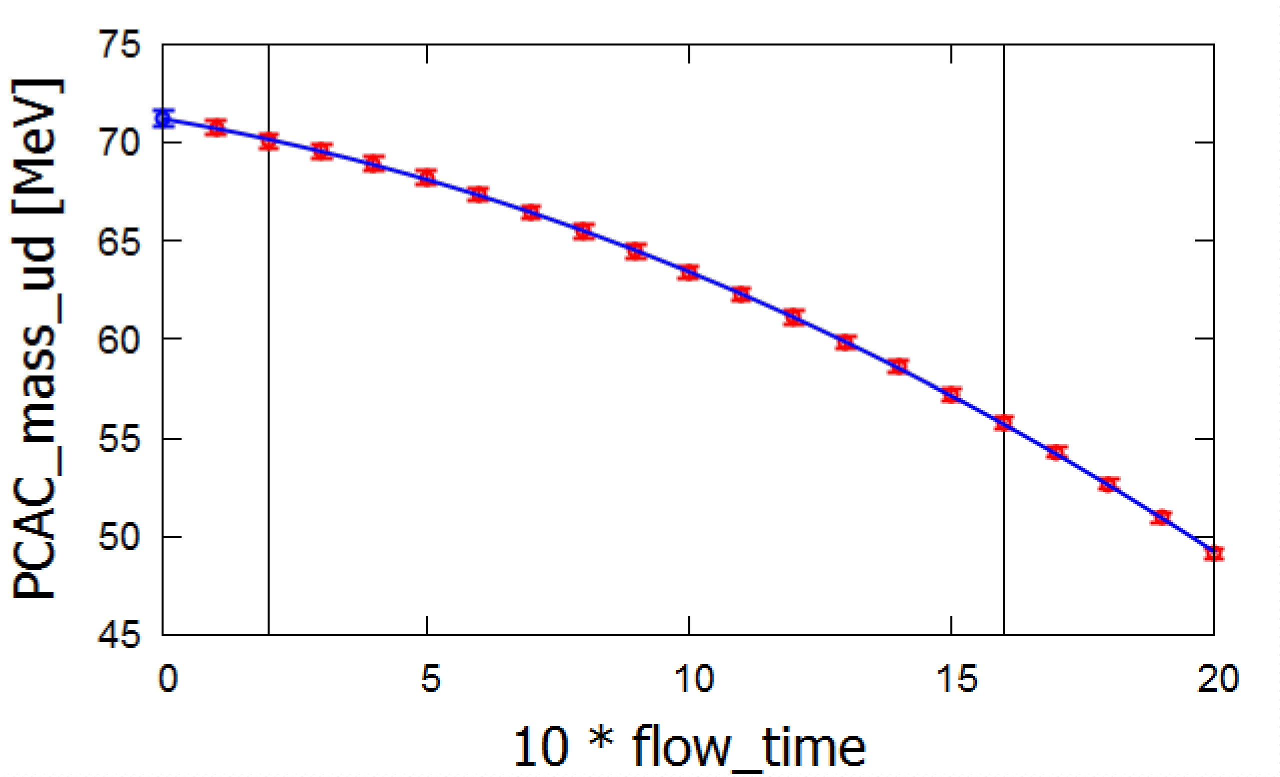
- NP $\mathcal{O}(a)$ improved Wilson fermion
- Iwasaki gauge action
- $2 + 1$ flavors
- Lattice size : $\mathbf{28^3 \times 56}$
- $\boldsymbol{\beta = 2.05 : a = 0.0701(29)}$ fm

  $\boldsymbol{(1/a \simeq 2.79\ GeV)}$
- Heavy ud quark masses,

  $\boldsymbol{m_\pi / m_\rho \simeq 0.63}$
- Almost physical s quark

  $\boldsymbol{m_{\eta_{ss}} / m_\phi \simeq 0.74}$

PCAC mass



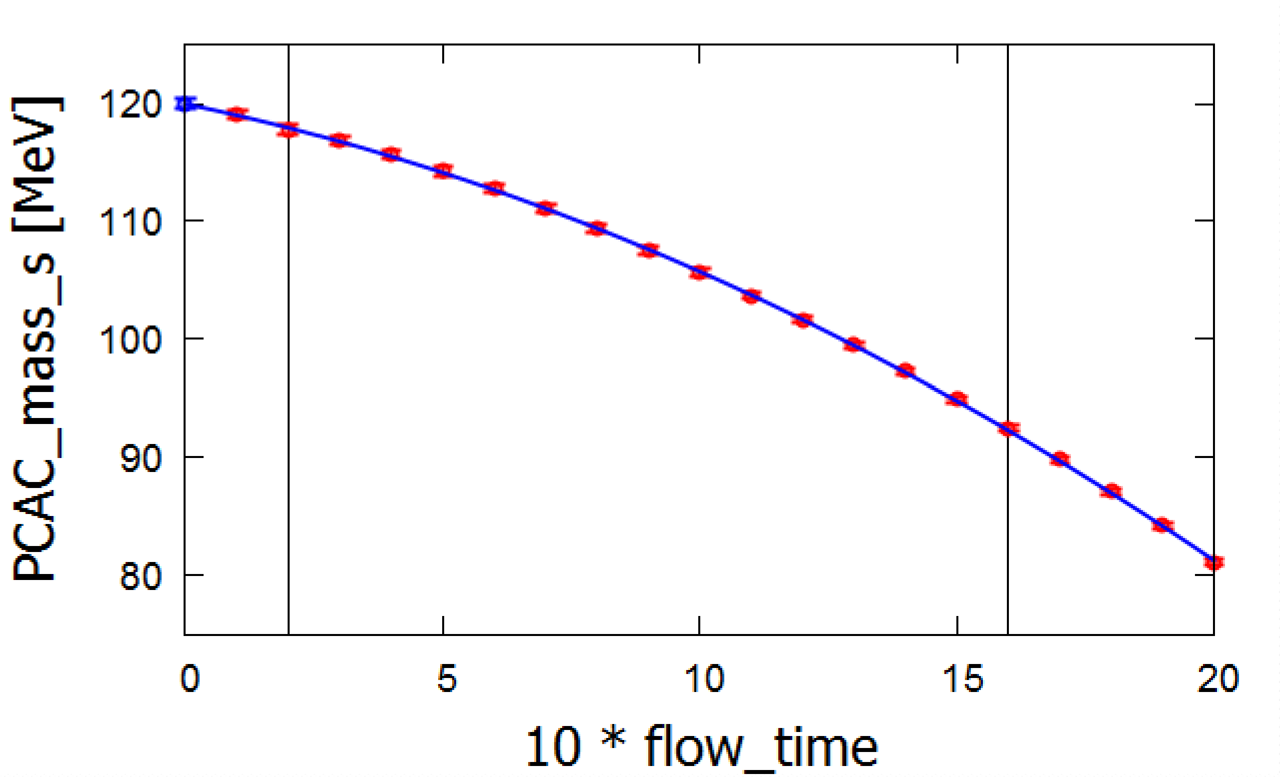
$m_{ud} = 71.210 \pm 0.391$ MeV

non-perturbatively

renormalized in $\overline{\mathrm{MS}}$ at 2GeV

We cannot find the window region → fit with second order poly.

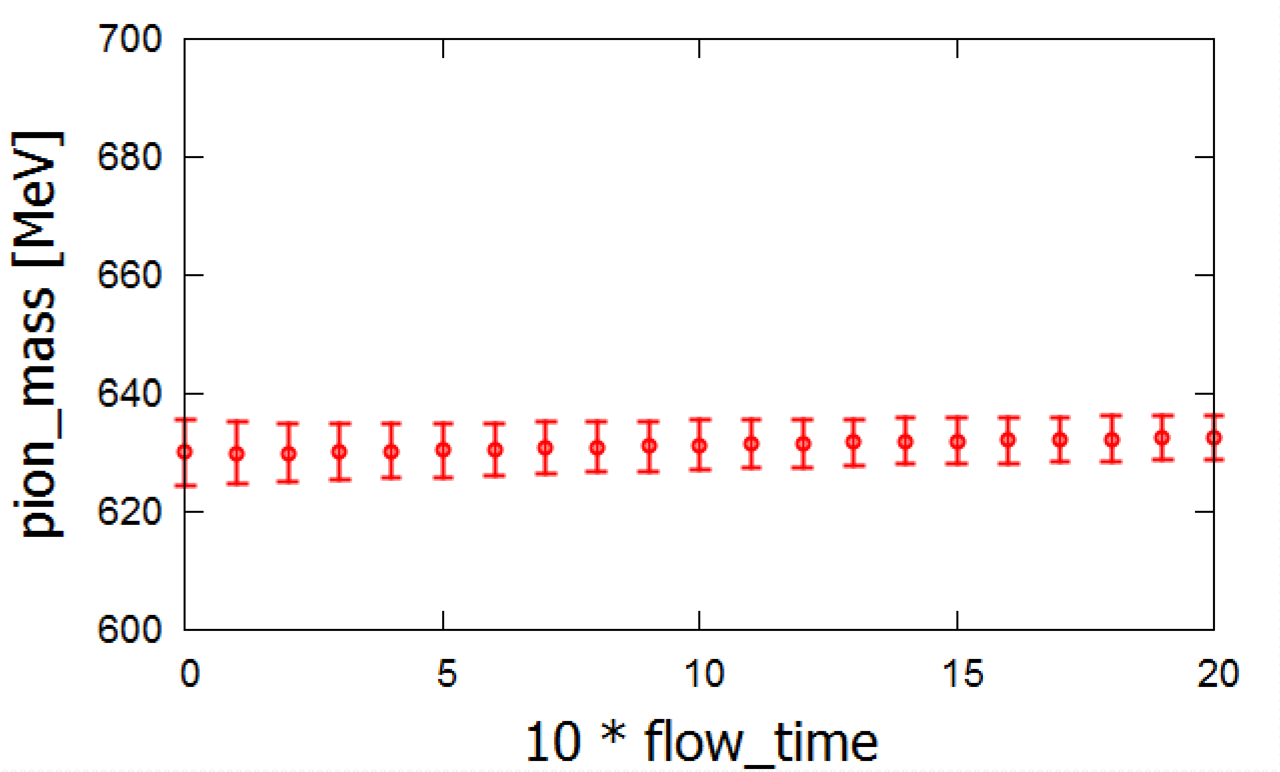
PCAC mass



$m_s = 119.96 \pm 0.412$ MeV

non-perturbatively
renormalized in $\overline{\mathrm{MS}}$ at 2GeV

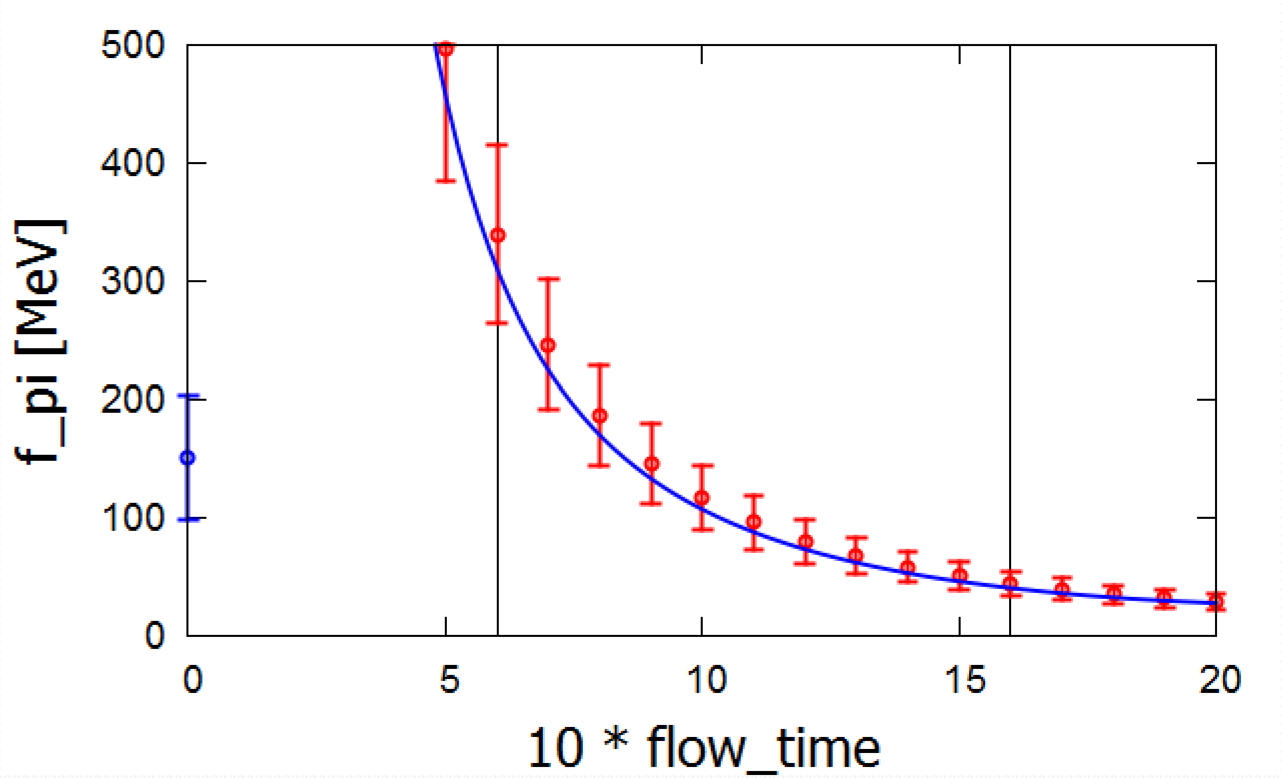
Pion mass



Flow eq. does not effect to the pion mass

$$\langle \chi(t)\, \bar{\chi}(t) \rangle \neq (D(t) + m)^{-1}$$

Pion decay constant



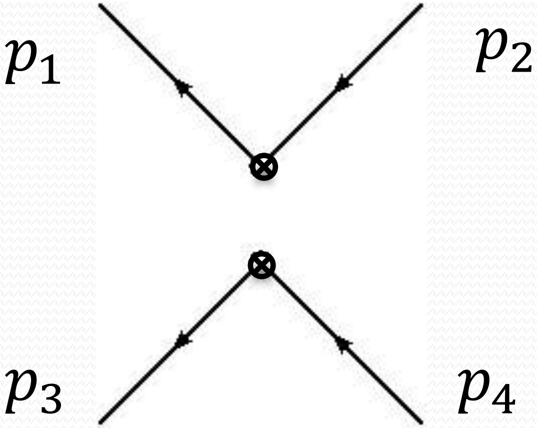
$$f_\pi = 149.96 \pm 52.62 \text{ MeV}$$

$$a^2/t \; is \; dominant \rightarrow \text{fit in } A\frac{1}{t^2} + B\frac{1}{t} + C + Dt + Et^2$$

Progress of B_K

- Calculation of $\langle \overline{K_0} | \hat{O}^{\Delta s=2} | K_0 \rangle$ is in preparation

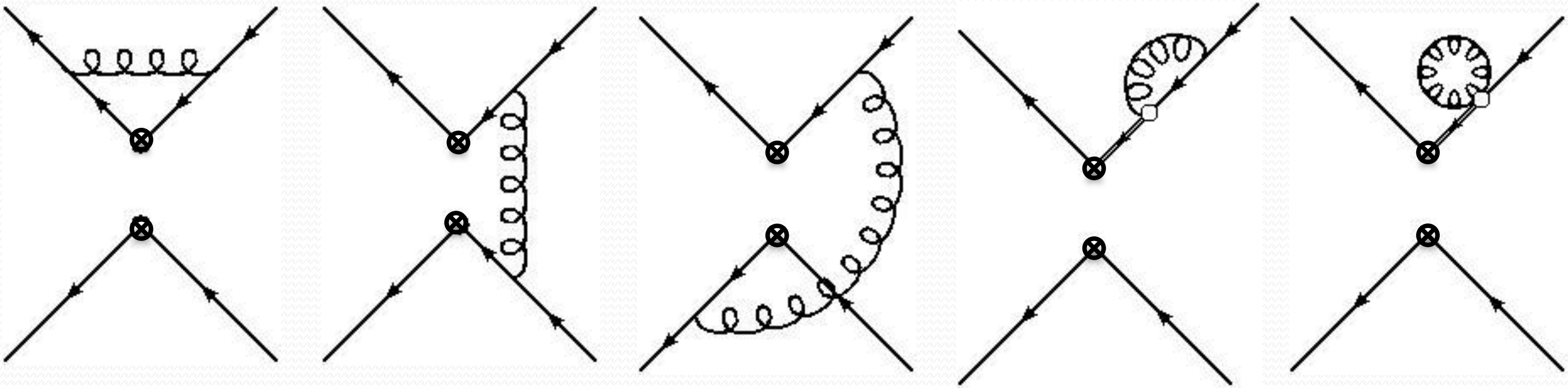
- We calculated the matching factor



$$O^{\pm} = \{\bar{s}_1(\gamma^L)_\mu d_2\}\{\bar{s}_3(\gamma^L)_\mu d_4\} \pm \{\bar{s}_1(\gamma^L)_\mu d_4\}\{\bar{s}_3(\gamma^L)_\mu d_2\}\Big|_t$$

- 1st order perturbation
- set $\boldsymbol{p_i} = \boldsymbol{0}$ for simplicity

Matching factor of 4-fermi op.



 means Heat kernel $K_t(x) = \int_p e^{ip\cdot x} e^{-tp^2}$

$$Z^{GF\to\overline{MS}}(\pm)$$

$$= 1 + \frac{g^2}{(4\pi)^2}\left\{-3\,\frac{\pm N_c - 1}{N_c}\left(\log\left(8\mu^2 t\right) + 1 + \gamma - \log(4)\right) + \frac{N_c^2 \mp 6N_c + 5}{2N_c} + 2C_F \log(432)\right\}$$

Summary and outlook

- We calculate 2 pt. function as a step towards the calculation of B_K
  - Specifically, PCAC mass, pion mass and decay constant
  - They are non-perturbatively renormalized in $\overline{MS}$ at 2GeV
- We have to work more about fitting
  - We did not find the window region → non linear fit
  - fitting with $1/t$ term makes error too large
- We calculate the matching factor of $O^{\pm}$
- Calculation of B_K is still on its way!