The thermal photon rate from dynamical lattice QCD – part 1

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What is the thermal photon rate and why is it interesting?

The differential photon production rate is, to $O(\alpha_{\rm em})$,

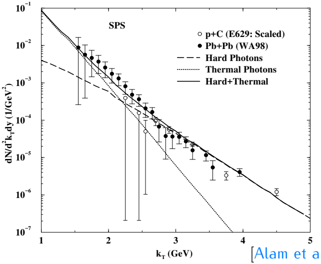
$$k\frac{d\Gamma_\gamma}{d^3k} = \left(\sum_f Q_f^2\right)\frac{\alpha_{\rm em}}{4\pi^2}\, n_B(\omega=k)\,\rho^\mu{}_\mu(\omega=k,k),$$

[McLerran & Toimela (1985)]

where $\rho^\mu{}_\mu(\omega,k)$, the vector channel spectral function, and $n_B(\omega)$, the Bose-Einstein distribution, depend implicitly on the temperature, T.

In heavy-ion collisions, photons are produced thermally and in initial hard partonic reactions.

The temperature of the initial equilibrated thermal medium can be inferred from the thermal contribution to the photon spectra.



[Alam et al. (2003)]

By modelling the evolution of the medium, estimates for the photon spectra can be obtained from the photon rate.

What is known about the photon rate?

The thermal photon rate has been estimated in weakly-coupled QCD and other theories at weak and strong coupling.

[Arnold et al. (2001); Huot et al. (2006)]

Strongly-interacting media require a non-perturbative cross-check of weak-coupling approaches.

[Ghiglieri et al. (2016)]

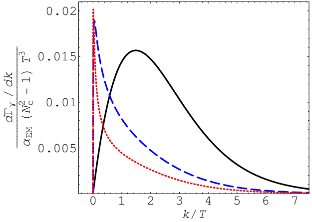


Figure: Strong-coupling (solid black) and weakly-coupled (dashed) $\mathcal{N} = 4$ SYM

[Huot et al. (2006)]

Due to the conservation of the electric charge, hydrodynamics predicts the $k \to 0$ behaviour of $\rho^{\mu}{}_{\mu}(\omega, k)$,

$$\frac{\rho^{\mu}{}_{\mu}(\omega, k)}{\omega} \longrightarrow \frac{4\chi_s D k^2}{\omega^2 + (Dk^2)^2} \quad \text{as} \quad \omega, k \to 0$$

Outline

UV-finite spectral function

Lattice set-up with $N_\mathrm{f} = 2$ O(a)-improved Wilson fermions

- tree-level improvement
- continuum limit

Photon rate from the UV-finite spectral function

- A linear method – Backus-Gilbert method
- Constraint from a model spectral function

Results

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Real-time phenomena from Euclidean correlators

Photon emission is a real-time phenomenon, which, like scattering or transport, is challenging for lattice QCD due to the analytic continuation required. [Cuniberti et al. (2001)]

In frequency-space, the Euclidean correlation has the spectral representation

$$\tilde{G}_{\mathrm{E},AB}(\omega_n,\boldsymbol{k}) \equiv \int_0^{\beta} \mathrm{d}\tau\, \mathrm{e}^{i\omega_n\tau} \int \mathrm{d}^3x\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\boldsymbol{x}}\, \mathrm{Tr}\left\{\mathrm{e}^{-\beta\hat{H}}\hat{A}(\tau,\boldsymbol{x})\hat{B}(0)\right\} = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, \frac{\rho_{AB}(\omega,\boldsymbol{k})}{\omega - i\omega_n}$$

or, in coordinate space, if $G_{\mathrm{E},AB}(\tau,\boldsymbol{k}) = G_{\mathrm{E},BA}(\tau,\boldsymbol{k})$,

$$G_{\mathrm{E},AB}(\tau,\boldsymbol{k}) = \int_0^{\infty} \frac{\mathrm{d}\omega}{2\pi}\, \rho_{AB}(\omega,\boldsymbol{k})\, \frac{\cosh(\omega(\beta/2-\tau))}{\sinh(\beta\omega/2)}$$

which is numerically ill-posed for ρ_{AB} when $G_{\mathrm{E},AB}$ is known only on a finite set,

$$\underbrace{\begin{bmatrix} \cdot \\ \cdot \end{bmatrix}}_{G_\mathrm{E}} = \underbrace{\begin{bmatrix} \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \end{bmatrix}}_{K} \underbrace{\begin{bmatrix} \vdots \\ \vdots \\ \vdots \end{bmatrix}}_{\rho}$$

especially if the spectral function is quickly varying, e.g. a diverging background

A UV-finite spectral density for the photon rate

We introduce the spectral function

$$\rho_\lambda(\omega,k) \equiv \left(\delta^{ij} - \frac{k^i k^j}{k^2}\right)\rho^{ij}(\omega,k) - \frac{\lambda}{k^2}\left(k^2\rho^{00}(\omega,k) - k^i k^j \rho^{ij}(\omega,k)\right)$$

which is equal to $\rho^\mu{}_\mu(\omega,k)$ for $\lambda = 1$ and furthermore, due to

$$\omega^2\rho^{00}(\omega,k) - k^i k^j \rho^{ij}(\omega,k) = 0,$$

is independent of λ on the light-cone, $\omega = k$.

$\rho_\lambda(\omega,k)$ with $\lambda = 1$ has a ω^2-divergence but $\lambda = -2$ is UV-finite.

In the following, we study the $\lambda = -2$ choice.

Note that the photon rate vanishes at tree-level.

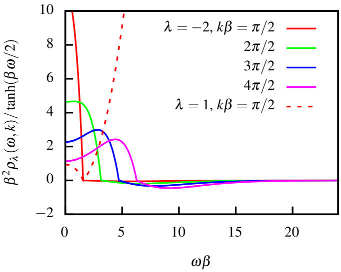
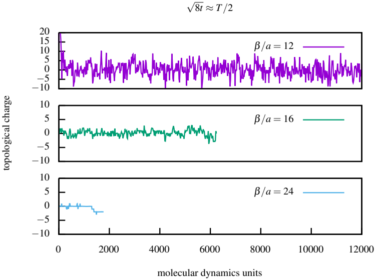


Figure: Free massless spectral function [Laine (2013)]

Lattice set-up with $N_f = 2$ O(a)-improved Wilson fermions

T (MeV)	T/T_c	β_{LAT}	β/a	L/a	$m_{\overline{MS}(2\,GeV)}$ (MeV)	N_{meas}
250	1.2	5.3	12	48	12	8256
"	"	5.5	16	64	"	4880
"	"	5.83	24	96	"	1680
500	2.4	6.04	16	64	"	8064

- enables continuum limit at $T = 250$ MeV



- further investigation of autocorrelation of topological charge required

Continuum limit 1/3

There are four independent discretizations of the $\lambda = -2$ isovector vector correlator

$$G^{\lambda=-2}(\tau, \boldsymbol{k}) = \left(\delta^{ij} - \frac{3k^i k^j}{k^2}\right) G^{ij}(\tau, \boldsymbol{k}) + 2G^{00}(\tau, \boldsymbol{k})$$

where $G^{\mu\nu}(\tau, \boldsymbol{k}) = \int \mathrm{d}^3 x e^{-\mathrm{i}\boldsymbol{k}\boldsymbol{x}} \langle V^{\mu}(\tau, \boldsymbol{x}) V^{\nu}(0) \rangle$ using both the local or exactly-conserved lattice vector current

In the local-conserved case, there are two discretizations possible by defining the local current on the link, or the conserved current on the site

$$G^{ij}(\tau + a/2, \boldsymbol{k}) = \frac{1}{2}\left(G^{ij}(\tau, \boldsymbol{k}) + G^{ij}(\tau + a, \boldsymbol{k})\right)$$
$$G^{00}(\tau, \boldsymbol{k}) = \frac{1}{2}\left(G^{00}(\tau - a/2, \boldsymbol{k}) + G^{00}(\tau + a/2, \boldsymbol{k})\right)$$

Project to all spatial momenta, on and off-axis, with $k\beta \leq 2\pi$

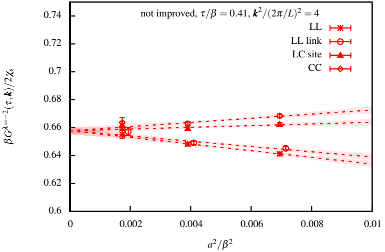
Continuum limit 2/3

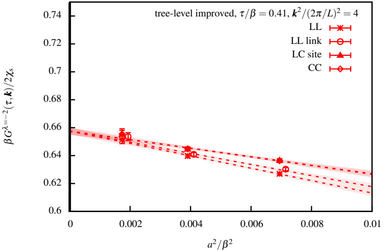
In the chirally-symmetric phase, the matrix-elements of the O(a)-improvement counterterms are suppressed, so we perform a continuum limit in a^2/β^2

Instead we perform tree-level improvement, defined via

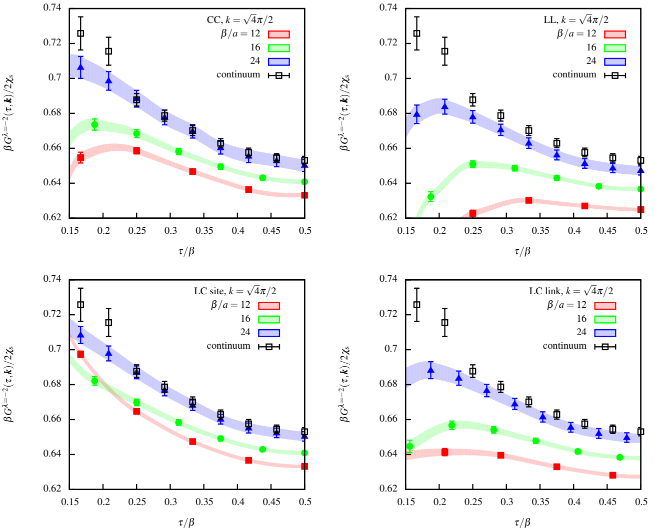
$$G^{\lambda=-2}(\tau,\boldsymbol{k}) \to \frac{G^{\lambda=-2}_{\text{cont.t.l.}}(\tau,\boldsymbol{k})}{G^{\lambda=-2}_{\text{lat.t.l}}(\tau,\boldsymbol{k})} G^{\lambda=-2}(\tau,\boldsymbol{k})$$

A piecewise spline interpolation is used before taking the combined continuum limit of the four discretizations of $\beta G^{\lambda=-2}(\tau,\boldsymbol{k})/\chi_s$



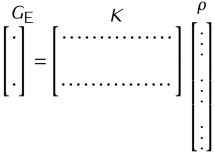


Continuum limit 3/3



Backus-Gilbert method 1/3

The finite-dimensional analogue of the inverse problem is ill-posed if $N_\omega \gg N_\tau = \beta/a$



The Backus-Gilbert (BG) method is a linear method, which constructs an approximate inverse map, $K^{\text{“}-1\text{”}}(\bar\omega,\tau)$, which localizes the kernel in frequency space,

$$G_\mathrm{E}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} K(\tau,\omega)\rho(\omega) \quad \xrightarrow{K^{\text{“}-1\text{”}}} \quad \hat\rho_\mathrm{BG}(\bar\omega) \equiv \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \underbrace{K^{\text{“}-1\text{”}}(\bar\omega,\tau)K(\tau,\omega)}_{\hat\delta(\bar\omega,\omega)}\rho(\omega)$$

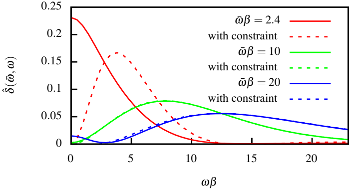
The linear map is uniquely defined by optimizing a quadratic form,

$$K^{\text{“}-1\text{”}}(\bar\omega,\tau) = \underset{K^{\text{“}-1\text{”}}}{\operatorname{argmin}} \left\{ \alpha K^{\text{“}-1\text{”}}(\bar\omega,\tau)\mathrm{cov}(\tau,\sigma)K^{\text{“}-1\text{”}}(\bar\omega,\sigma) + (1-\alpha)\int_0^\infty \mathrm{d}\omega\, \hat\delta(\bar\omega,\omega)^2(\bar\omega-\omega)^2 \right\},$$

subject to a linear constraint.

There is no maximum likelihood interpretation of $\rho_\mathrm{BG}(\bar\omega)$!

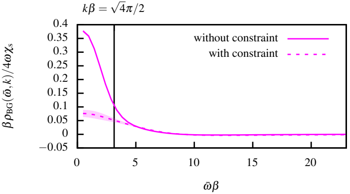
Backus-Gilbert method 2/3



← resolution function $\hat{\delta}(\bar{\omega},\omega)$

acts like a smearing kernel

a linear constraint $\hat{\delta}(\bar{\omega} = 0,\omega) = 0$ removes contributions from $\rho(\omega = 0, k)$

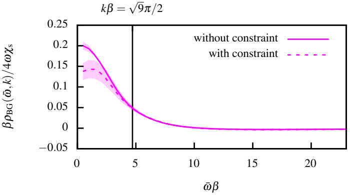


← spectral function $\rho_{\mathrm{BG}}(\bar{\omega},k)$

at $k\beta \gtrsim \pi$, the photon rate is consistent with or without the constraint

estimate the effective diffusion constant

$$D_{\mathrm{eff}}(k,\beta) = \frac{\rho_{\mathrm{BG}}(k,k)}{4k\chi_s}$$

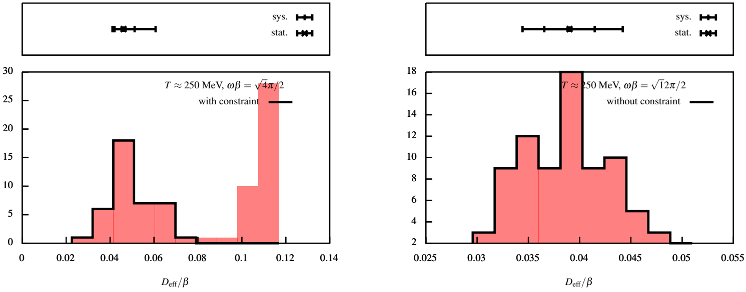


[Ghiglieri et al. (2016)]

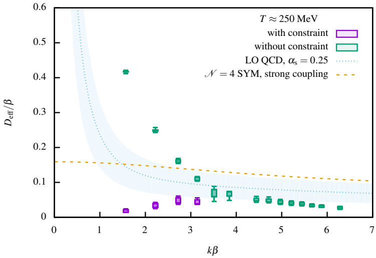
Backus-Gilbert method 3/3

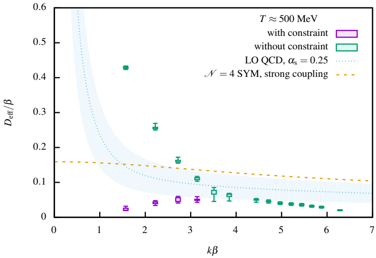
Estimate a systematic uncertainty by repeating with many variations.

variation	values
τ_{min}/β	$\{0.1,\ldots,0.25\}$
extra constraint	{yes, no}
α	$\{10^{-2},\cdots,10^{-4}\}$
tree-level improved	{yes, no}
discretization (@ $T = 500$ MeV)	{LL, LC site, LC link, CC}



Preliminary results from the BG method





Results display virtually no visible temperature effects

Inverse problem appears to be controlled when $k\beta > \pi$

Improved momentum resolution using on- and off-axis momenta

Summary part 1

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Derivation of a sum rule for $\rho \equiv \rho_{\lambda=-2}$

i. In vacuum, Lorentz invariance and transversity of $G^{\mu\nu}(\omega,\boldsymbol{k})$

$$G(\omega_n,k) = \left(\delta^{ij} - \frac{3k^i k^j}{k^2}\right)\rho^{ij}(\omega,k) + 2\rho^{00}(\omega,k) \longrightarrow 0$$

ii. The UV finite correlation admits an OPE $\Rightarrow \rho(\omega,k) \sim \frac{\langle\mathcal{O}_4\rangle}{\omega^2} + \cdots$ as $\omega \to \infty$
However, in order to satisfy, $\rho(\omega,k) \to 0$ as $k \to 0$ and $\omega > 0$ due to charge conservation,

$$\rho(\omega,k) \sim \frac{k^2\langle\mathcal{O}_4\rangle}{\omega^4} \qquad (\dagger)$$

iii. As

$$\tilde{G}(\omega_n,k) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi}\,\omega\,\frac{\rho(\omega,k)}{\omega^2+\omega_n^2}$$

$$\xrightarrow{\omega_n\to\infty} \frac{1}{\pi\omega_n^2}\int_0^\infty \mathrm{d}\omega\,\omega\,\rho(\omega,\mathrm{k})$$

But the Euclidean correlator is the analytic continuation of $\rho(\omega,k)$, so via eqn. $(\dagger)$

$$\int_0^\infty \mathrm{d}\omega\,\omega\,\rho(\omega,\mathrm{k}) \sim \frac{\mathrm{k}^2\langle\mathcal{O}_4\rangle}{\omega_\mathrm{n}^2} \longrightarrow 0 \quad \text{as} \quad \omega_\mathrm{n} \longrightarrow \infty$$

Derivation of a sum rule for $\rho \equiv \rho_{\lambda=-2}$ short version

i. Lorentz invariance and transversity $\Rightarrow \tilde{G}_E(\omega_n, k) = 0$ in vacuum and UV finite at $T > 0$

ii. UV finite correlation admits an OPE $\tilde{G}_E(\omega_n, k) \sim \frac{\langle \mathcal{O}_4 \rangle}{\omega_n^2}$

Furthermore, charge conservation demands $\tilde{G}_E(\omega_n, k) \to 0$ as $k \to 0$ and $\omega > 0$, so

$$\tilde{G}_E(\omega_n, k) \sim \frac{k^2 \langle \mathcal{O}_4 \rangle}{\omega_n^4} \tag{†}$$

iii. Matching the large ω_n-behaviour

$$\tilde{G}_E(\omega_n, k) = \int_0^\infty \frac{d\omega}{\pi} \omega \frac{\rho(\omega, k)}{\omega^2 + \omega_n^2}$$

$$\xrightarrow{\omega_n \to \infty} \frac{1}{\pi \omega_n^2} \int_0^\infty d\omega\, \omega\, \rho(\omega, \mathrm{k}) \sim \frac{\mathrm{k}^2 \langle \mathcal{O}_4 \rangle}{\omega_\mathrm{n}^4} \quad \text{using (†)}$$

results in the sum rule $\int_0^\infty d\omega\, \omega \rho(\omega, \mathrm{k}) = 0$