New methods for B decay constants and form factors from lattice NRQCD

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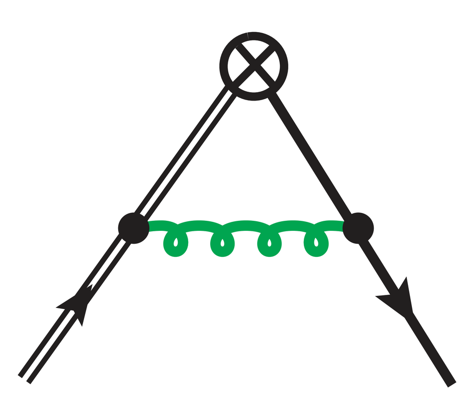
Abstract

We determine the normalisation of scalar and pseudo scalar current operators made from NonRelativistic QCD b quarks and HISQ light quarks through O($\alpha_s\Lambda/m_b$). We use matrix elements of these operators to extract B meson decay constants and form factors and compare to those obtained using the standard vector and axial vector operators. We work on MILC second-generation 2+1+1 gluon field configurations including those with physical light quarks in the sea. This provides a test of systematic uncertainties in these calculations and we find agreement between the results to the few % level of uncertainty quoted.

Normalisation of Lattice NRQCD current operators



Continuum current operators can be systematically expressed as a non-relativistic expansion in terms of lattice NRQCD-HISQ current operators multiplied by coefficients which can be calculated in perturbation theory as a power series in α_s. The coefficients depend on the bare NRQCD quark mass and are calculated by matching a heavy quark to light quark scattering process induced by the continuum current and by the combination of NRQCD-HISQ currents. A key Feynman diagram is the one shown above.

In [1] (based on perturbative calculations from [2]) the following expansion of the temporal axial current was used:

$$A_0 = (1+\alpha_s z_0^{A_0})\times$$
$$(J^{(0)}_{A_0,lat} + (1+\alpha_s z_1^{A_0})J^{(1)}_{A_0,lat} + \alpha_s z_2^{A_0} J^{(2)}_{A_0,lat})$$

with

$$J^{(0)}_{A_0,lat} = \overline{q}(x)\gamma_5\gamma_0 Q(x)$$
$$J^{(1)}_{A_0,lat} = -\frac{1}{2m_b}\overline{q}(x)\gamma_5\gamma_0\gamma\cdot\nabla Q(x)$$
$$J^{(2)}_{A_0,lat} = -\frac{1}{2m_b}\overline{q}(x)\gamma\cdot\nabla\gamma_0\gamma_5\gamma_0 Q(x)$$

where the two derivatives acting in opposite directions in $J^{(1)}$ and $J^{(2)}$. By determining the matrix elements of these three currents between the vacuum and a B meson in a lattice QCD calculation, [1] was able to determine the B meson decay constant using:

$$\langle 0|A_0|B(\vec{p}=0)\rangle = f_B m_B$$

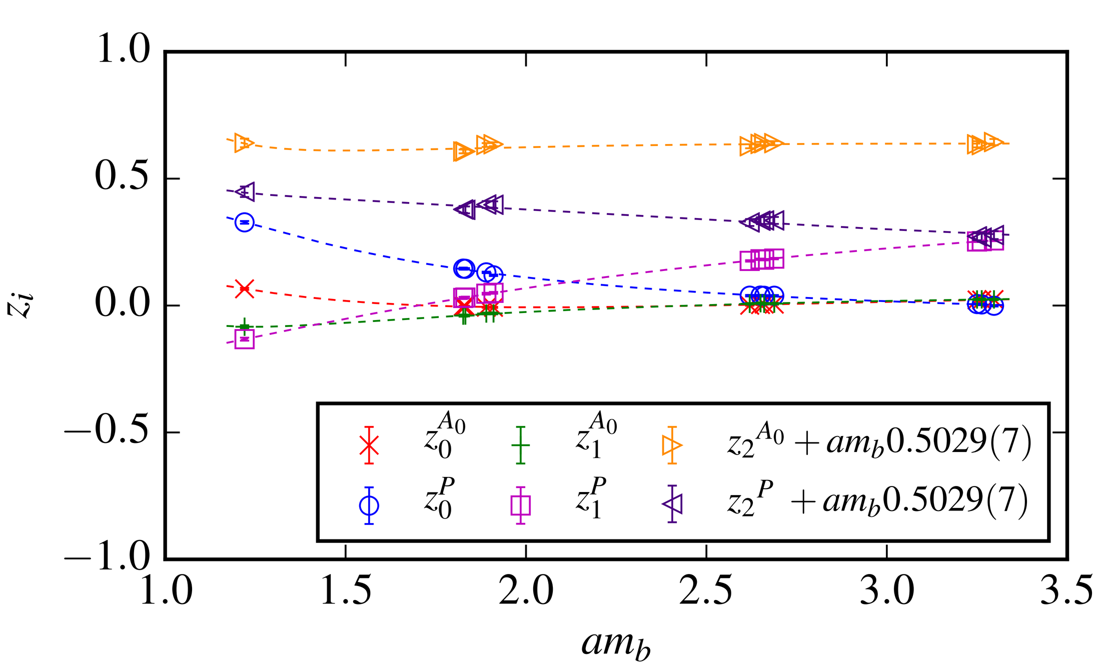
Here we use the alternative continuum expression:

$$m_b\left(1+\frac{m_l}{m_b}\right)\langle 0|P|B(\vec{p}=0)\rangle = f_B m_B^2$$

The continuum pseudo scalar density, P, can be expressed as an expansion in terms of NRQCD-HISQ currents. Because $\gamma_0 Q = Q$, the NRQCD-HISQ currents are in fact the same ones that appear in the expansion of A_0. We find

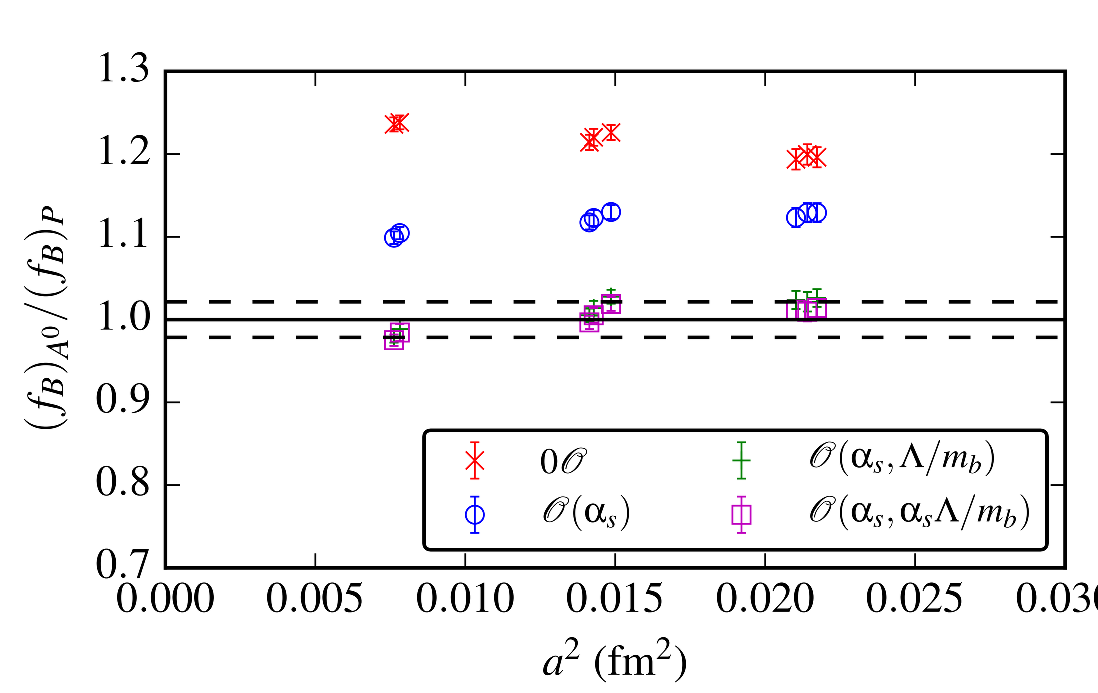
$$m_b^{\overline{MS}} P = m_b(1+\alpha_s z_0^P)\times$$
$$(J^{(0)}_{A_0,lat} - (1+\alpha_s z_1^P)J^{(1)}_{A_0,lat} + \alpha_s z_2^P J^{(2)}_{A_0,lat})$$

through O($\alpha_s\Lambda/m_b$). The z coefficients are related to those for A_0. The plot below shows the different z coefficients as a function of the lattice NRQCD b quark mass. Exactly the same coefficients pertain to the scalar operator because of the chiral symmetry of the HISQ action.



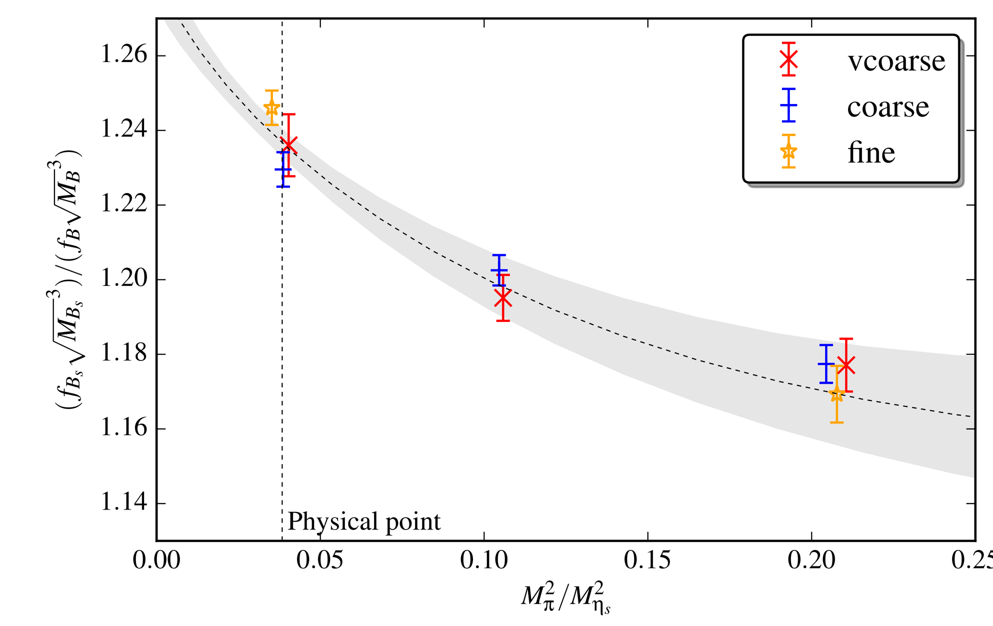
Results

Using matrix elements from [1] we can compare the determination of B meson decay constants from A_0 and P. The figure below shows the comparison for f_B as we add successively higher orders in the expansion of the current. Zeroth order includes only $J^{(0)}$ at tree-level, $O(\alpha_s)$ includes the appropriate z_0 etc. We take the experimental B meson mass for m_B.

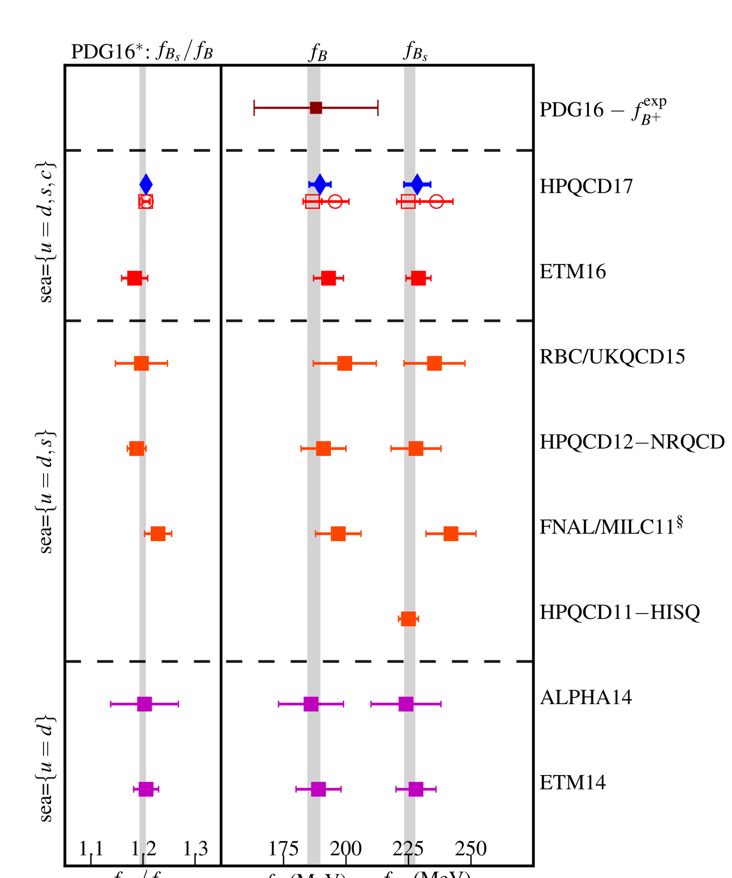


The dashed black lines give the relatively uncertainty in f_B quoted in [1]. We see that, as we improve the approximation to the continuum current, the ratio of the two determinations of the decay constant gets closer to 1. The best result, to the order we are working, is within the quoted uncertainty.

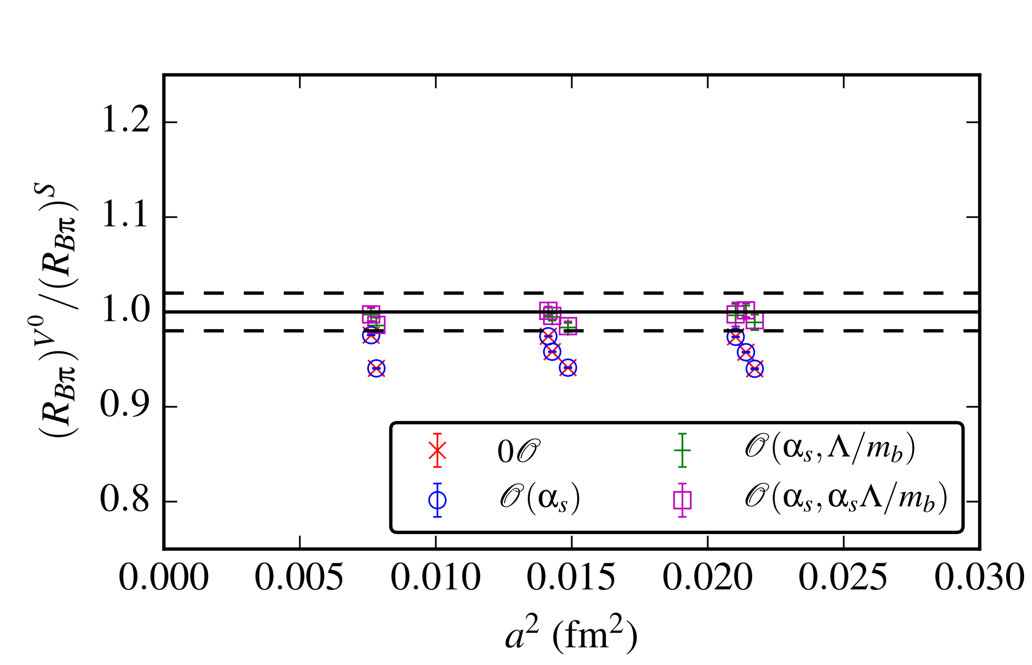
From our results at multiple values of the lattice spacing and sea quark masses we can determine physical values for f_B and f_{B_s} using the pseudo scalar density. The plot below shows results, using fit functions from [1], that allow determination of f_{B_s}/f_B.



Here we give a summary plot of lattice QCD determinations of f_B (corresponding to m_u=m_d) and f_{B_s}. HPQCD results are: red open squares for A_0, red open circles for P, and a blue diamond for the average (allowing for correlations through the input data).



A similar analysis for the temporal vector and scalar currents gives similar plots for the comparison of the scalar form factor for B to π decay at zero recoil [3]. Below we show how the same result for the ratio $R_{B\pi}$ between f_0 and f_B/f_π is obtained with the two methods. In future this will enable us to determine f_+ and f_0 from temporal vector and scalar currents, avoiding the use of the spatial vector NRQCD-HISQ current which has significant correction terms away from zero recoil.



[1] R. Dowdall et al, HPQCD, 1302.2644.
[2] C. Monahan et al, HPQCD, 1211.6966.
[3] B. Colquhoun et al, HPQCD, 1510.07446.

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