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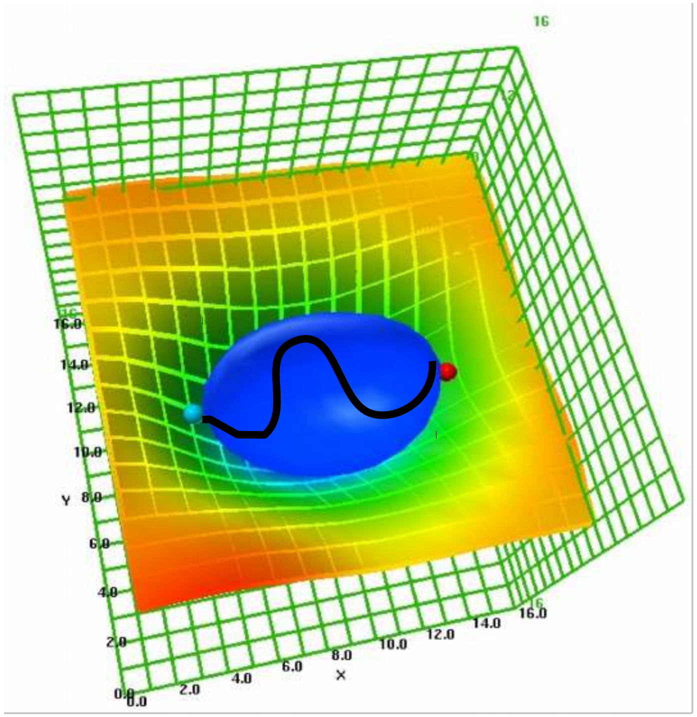
Stiff-Self-Interacting String at high temperature SU(3) Yang-Mills theory.



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Talk Outline

–- Effective bosonic string model of confinement.

-- Casimir Energy in String models.

-- Monte-Carlo simulations of the quark-antiquark potential.

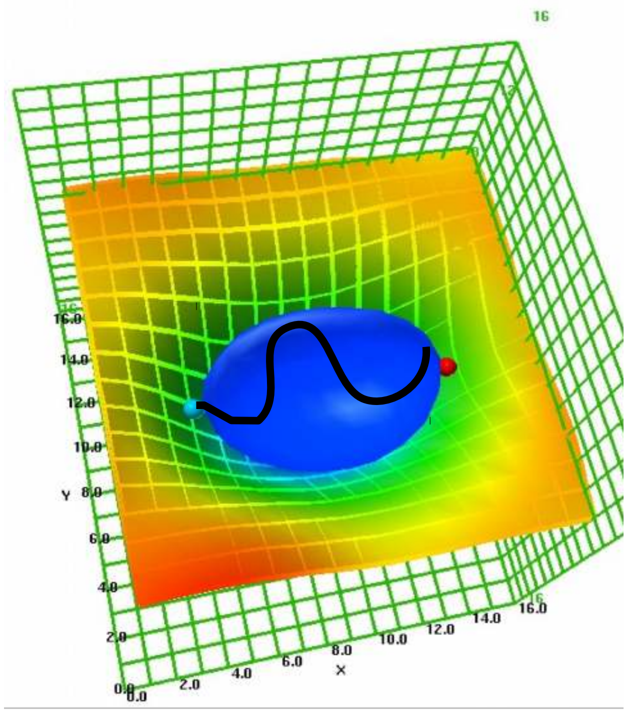
--- Width of the Numbu-Goto and Polyakov-Kleinert Strings.

-- Numerical results and fit analysis.

String picture of confinement

The origin of the linearly rise has been identified to be due to the formation of a thin *string-like* colour-electric flux tube between the quark colour sources.

The quantum fluctuations of the string result in sub-leading correction to the QQ potential known as the Lüscher term.
Simulations of many confining gauge models have tested and verified the existence of this term and the subsequent growth properties of the tube.



M. Luscher and P. Weisz, JHEP0207 049 (2002).

M. Caselle, M. Pepe, and A. Rago, JHEP 10, 5 (2004).

M. Caselle, F. Gliozzi, U. Magnea, and S. Vinti, Nucl.Phys. B460, 397 (1996).

M. Caselle, F. Gliozzi, U. Magnea, S. Vinti, Nucl. Phys. B460, 397 (1996).

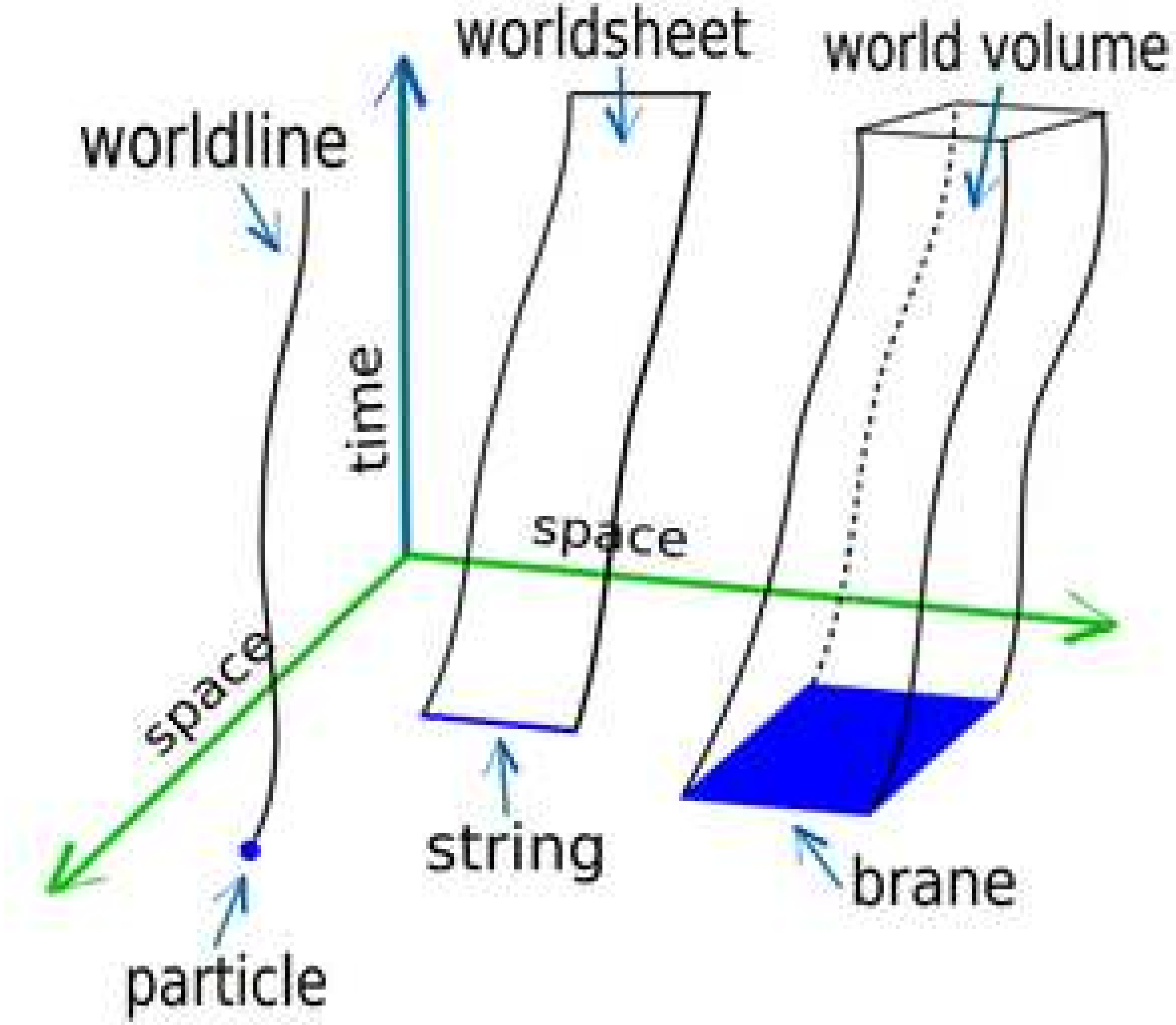
F. Gliozzi, M. Pepe, U.J. Wiese, Phys.Rev.Lett. 104, 232001 (2010).

C. Bonati, Phys. Lett. B 703 (2011) 376.

A. Allais and M. Caselle, J. High Energy Phys. 01 (2009)073.

A. S. Bakry, et al, Phys. Rev. D 82, 094503 (2010).

A. S. Bakry, D. B. Leinweber, and A. G. Williams, Phys. Rev. D 85, 034504 (2012).



- string world-sheet.

QCD string signatures

A- Zero Temperature T=0

* **Luscher- term.**

M. Luscher, K. Symanzik, and P. Weisz, Nucl. Phys. B173, 365 (1980).

* **Log growth for the flux tube width.**

M. Luscher, G. Munster, and P. Weisz, Nucl. Phys. B180,1(1981).

B- Near the Deconfinment point Tc

* **Logarithmic term.**

P. de Forcrand, G. Schierholz, H. Schneider, and M. Teper, Phys. Lett. 160B, 137 (1985).
M. Gao, Phys. Rev. D 40, 2708 (1989).

* **Linear growth at large distances.**

A. Allais and M. Caselle, J. High Energy Phys. 01 (2009)073.

Effective bosonic string model of confinement.

- Nambu-Goto model
- The action of the Nambu-Goto model is proportional to the area of the string world-sheet

$$S[X] = \sigma \int_0^L d\nu \int_0^R du \sqrt{1 + (\partial_\nu X)^2 + (\partial_u X)^2}$$

- Where σ is the string tension which appears as a parameter of the effective model, the effective string world-sheet associated with a two-point Polyakov loop correlation function obeys periodic b.c. in the compactified direction and Dirichlet b.c. along the interquark axis direction:

$$X^i(\tau + L, \varsigma) = X^i(\tau, \varsigma)$$
$$X^i(\tau, 0) = X^i(\tau, R) = 0$$

- The Nambu-Goto action is the most simple form of string actions proportional to area of the world-sheet.

$$S_{NG}[X] = \sigma \int d\zeta_1 \int d\zeta_2 \sqrt{g},$$

- The Lüscher and Weiss (LW) effective action up to four-derivative term read

$$S_{LW}[X] = \sigma A + \frac{\sigma}{2} \int d\zeta_1 \int d\zeta_2 [(\frac{\partial X}{\partial \zeta_\alpha} \cdot \frac{\partial X}{\partial \zeta_\alpha}) + \sigma \int d\zeta_1 \int d\zeta_2 [\kappa_2 (\frac{\partial X}{\partial \zeta_\alpha} \cdot \frac{\partial X}{\partial \zeta_\alpha})^2 + \kappa_3 (\frac{\partial X}{\partial \zeta_\alpha} \cdot \frac{\partial X}{\partial \zeta_\beta})^2] + S_b$$

- The vector $X_\mu(\zeta_1, \zeta_2)$ maps the region $C \subset R_2$ into R_4 and couplings κ_1, κ_2 are effective low-energy parameters.
- The parameters are not arbitrary and are fixed by Lorentz-Invariance

$$(D-2)\kappa_2 + \kappa_3 = \left(\frac{D-4}{2\sigma}\right).$$

M. Luscher and P. Weisz, JHEP 07, 014 (2004), hep-th/0406205.

- A simple generalization of the Nambu-Goto string has been proposed by Polyakov and Kleinert to stabilize the NG action.

- The action of the bosonic (Polyakov) string with the extrinsic curvature term reads

$$S_{PK} = \frac{\sigma}{2}\int d^2\zeta\sqrt{g}g^{\alpha\beta}\partial_\alpha X.\partial_\beta X + \frac{1}{2\alpha}\int d^2z\sqrt{g}\Delta X.\Delta X$$

- The extrinsic-curvature term favors smooth string configurations over those that are sharply curved, the rigidity of the string makes it smoother and was first investigated in the context of fluid membranes

H. Kleinert, Physics Letters B, 335(1986), ISSN 0370-2693.
G. German and H. Kleinert, Phys. Rev. D40, 1108 (1989).
H. Kleinert and A. Chervyakov (1996), hep-th/9601030.
A. Polyakov, Nuclear Physics B 268, 406 (1986).

- Additional terms consistent with Poincare and parity invariance may contribute to (LW) action for couplings of dimension $[\text{Length}]^2$.

- One geometrically-invariant term would appears as a contribution from the extrinsic curvature

$$S_K = \alpha \int d^2\zeta \sqrt{g} K^2$$

with the extrinsic curvature K defined as

$$K = \Delta(g) \partial_\alpha [\sqrt{g} g^{\alpha\beta} \partial_\beta]$$

- delta is Laplace operator.

- The partition function of the NG model in the physical gauge is functional integrals over all the world sheet configurations swept by the string

$$Z(R,T) = \int_{C} [DX] \, \exp(-S(X))$$

- The Casimir potential is extracted from the string partition function as

$$V(R,T) = -\frac{1}{T} \log(Z(R,T))$$

- Employing ζ function regularization scheme, the path integrals yield the quantum corrections to the Casimir energy at the next to leading order. This results in a temperature-dependent decrease in the string tension given by

$$\sigma(T) = \sigma_0 - \frac{\pi}{6} T^2 - \frac{\pi^2}{72\sigma_0} T^4 + O(T^6)$$

Employing zeta function regularization scheme, the path integral yields the leading order contribution to the potential

$$V_{\ell o}(R,T) = \sigma R + (D-2)T \, \ln \eta \, (i\tau) + \mu(T)$$

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1-q^n); \quad q = e^{\frac{-\pi L_T}{R}}.$$

Dedekind eta

Deitz and Filk extracted the first model-dependent corrections to the Casimir effect from the explicit calculation of the two-loop approximation using zeta regularization scheme as

$$V_{n\ell o}(R,T) = \sigma R + 2T \, \ln \eta \, (i\tau) - T \ln \left(1 - \frac{(D-2)\pi^2 T}{1152 \sigma_o R^3} \left[2E_4(\tau) + (D-4)E_2^2(\tau) \right] \right)$$

$$E_{2k}(\tau) = 1 + (-1)^k \frac{4k}{B_k} \sum_{n=1}^{\infty} \frac{n^{2k-1} q^n}{1-q^n}.$$

Eisenstein series

K. Dietz and T. Filk, Phys. Rev. D 27, 2944 (1983).

Extrinsic Curvature/Rigid/Stiff string

The potential of rigid string at finite temperature is given by

$$V^{\ell o}_{Stiff}(R,T) = \sigma R + T\ \ln\eta\,(i\tau) + T\sum_{n=0}^{\infty}\ln(1 - e^{-2R\sqrt{\Omega_n^2+\omega_0^2}}) + ..$$

$$V^{n\ell o}_{Stiff}(R,T) = V^{\ell o}_{Stiff}(R,T) - T\ln\left(1 - \frac{(D-2)\pi^2 T}{1152\sigma_o R^3}\left[2E_4(\tau) + (D-4)E_2^2(\tau)\right]\right),$$

Stiffness effects for QCD flux tube may become noticeable at high temperature near the deconfinement point.

We discuss stiffness effects in conjunction with the leading and next to leading approximations to NG action separately.

V. V. Nesterenko and I. G. Pirozhenko, J. Math. Phys. 38, 6265 (1997), hep-th/9703097.

Boundary terms

The boundary term describes the interaction of the effective string with the Polyakov loops at the fixed ends of the string and is given by

$$S_b = \int d\zeta_0 [b_1 \partial_2 X_i . \partial_2 X^i + b_2 \partial_2 \partial_1 X_i . \partial_2 \partial_1 X^i + ...].$$

Consistency with the open-closed string duality implies a vanishing value of the first boundary coupling b_1=0.

The leading order corrections due to second boundary terms with the coupling b_2 appears at higher order than the four derivative term in the bulk

For Dirichlet boundary condition, the contribution of the boundary terms to the static potential is

$$V_b = (d-2) b_2 \frac{-\pi^3 L_T}{60 R^4}$$

O. Aharony and M. Field, JHEP 01, 065 (2011), 1008.2636.

Simulation Set-up

The gauge configurations were generated using the standard Wilson gauge action.

The two lattices employed are of spatial size Nx=36, and temporal extents of Nt=,10, 8. beta=6.0, lattice spacing a=0.1 fm.

Bins of 6 measurements separated by 100 sweeps of updates. Each bin of measurments is taken following a 2000 of updating sweeps.   500 bin corresponding to 10,000 measurement at each temperature.

Quark-antiquark potential

The lattice data of the quark-antiquark potential are extracted from the two point Polyakov correlator

$$\mathcal{P}_{2\mathrm{Q}} = \int d[U]\, P(0)\, P^{\dagger}(R) \exp(-S_w),$$
$$= \exp(-V(R,T)/T).$$

The polyakov loop correlator is evaluated after averaging the time links.

The temporal links are integrated out directly by evaluating the equivalent contour integral.

$$\bar{U}_t = \frac{\int dU U\, e^{-Tr(Q\,U^{\dagger}+U\,Q^{\dagger})}}{\int dU e^{-Tr(Q\,U^{\dagger}+U\,Q^{\dagger})}}.$$

G. Parisi, R. Petronzio, and F. Rapuano, Phys. Lett. B128, 418 (1983).

P. de Forcrand and C. Roiesnel, Phys. Lett. B 151, 77 (1985).

NG Potential at Leading Order (Free String Approximation)

$$V_{\ell o}(R,T) = \sigma R + (D-2)T\,\ln\eta\,(i\tau) + \mu(T)$$

The numerical data for the quark-antiquark potential match the free leading-order NG string.

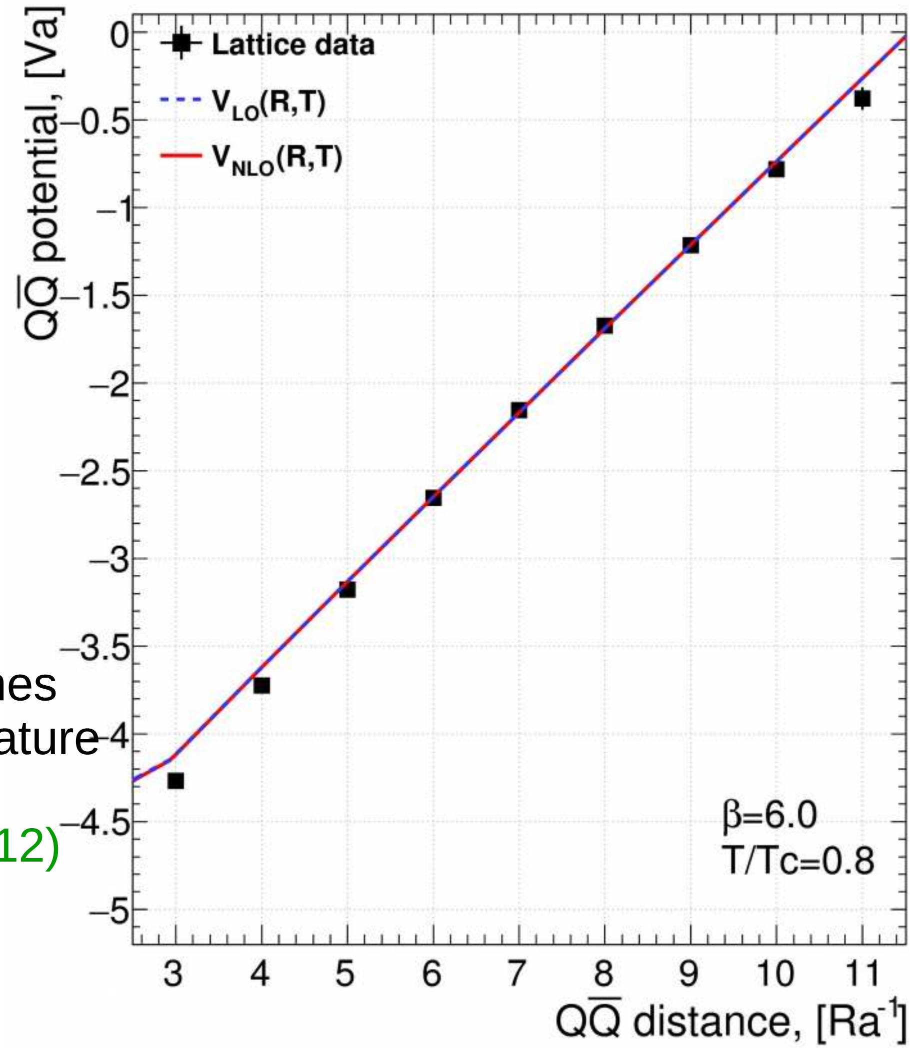
The inclusion of NLO self-interacting terms return good fits as well

Approximately the same value of the string tension is retrieved for fit domains involving short and large separation distances. This points to
1- NLO terms of NG action are relevant to QCD strings.

2-The minor role of the higher order modes at the end of the QCD plateau T/Tc=0.8.

Other indications on the pale out of thermal effects comes from the flat string tension plateau region at this temperature

N. Cardoso and P. Bicudo, Phys. Rev. D 85,077501 (2012)



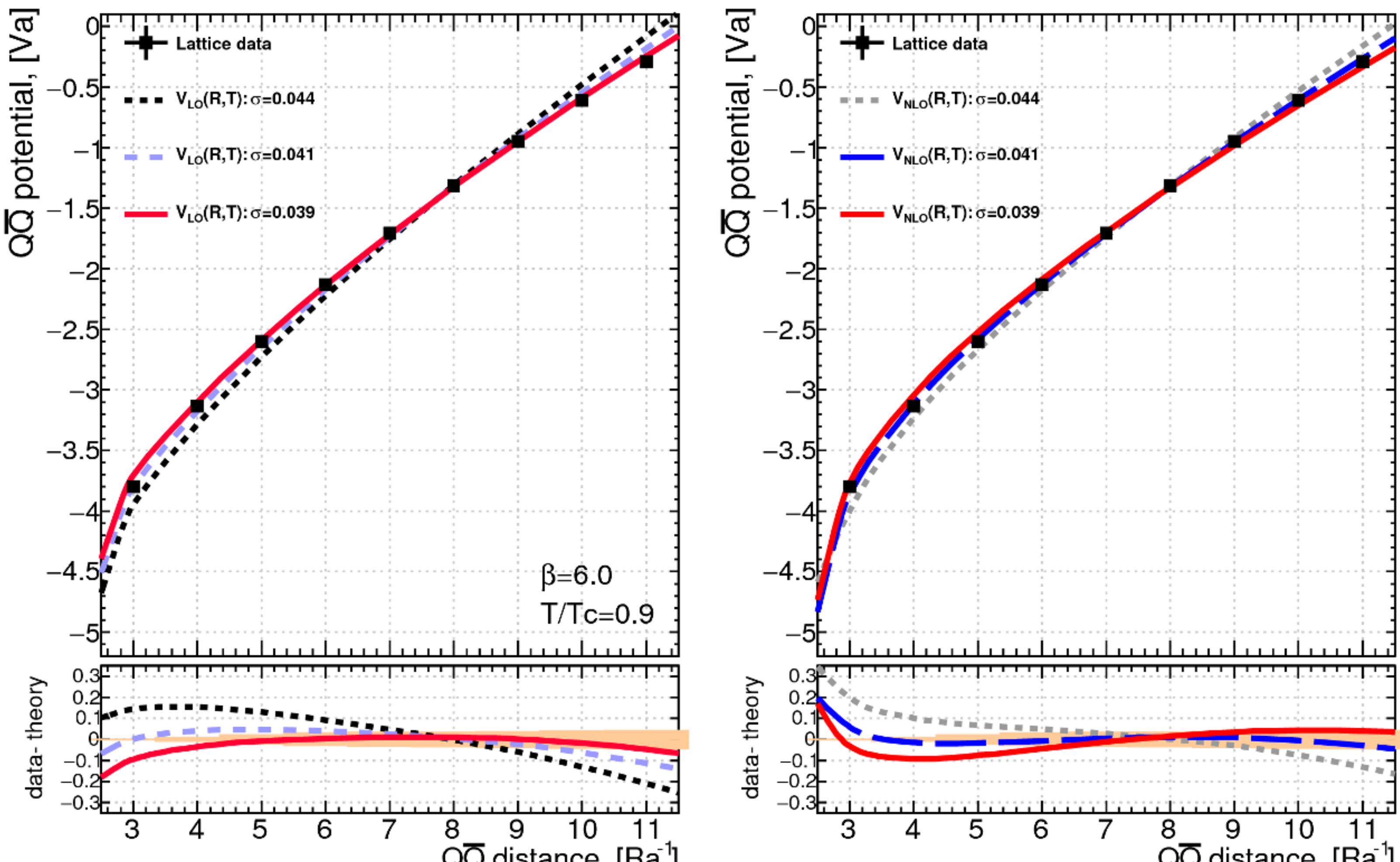
And the recent Monte-Carlo measurements which reproduces the same value of string tension 0.044 at zero temperature.
Miho Koma (Nihon U.) 2017. Phys.Rev. D95 (2017) no.9, 094513.

Fits of NG action at higher temperature T/Tc=0.9

$$V_{\ell o}(R,T) = \sigma R + (D-2)T\,\ln\eta\,(i\tau) + \mu(T)$$

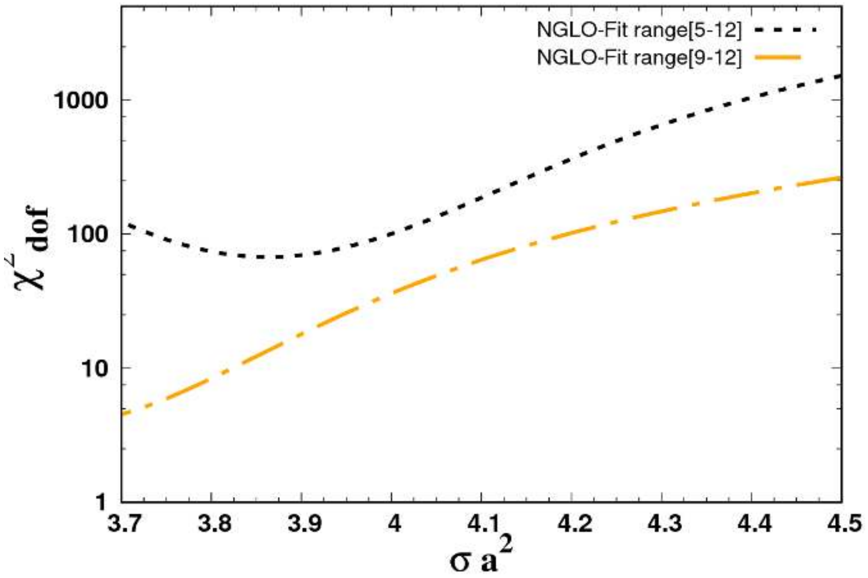
$$V_{n\ell o}(R,T) = \sigma R + 2T\,\ln\eta\,(i\tau) - T\ln\left(1 - \frac{(D-2)\pi^2 T}{1152\sigma_o R^3}\left[2E_4(\tau) + (D-4)E_2^2(\tau)\right]\right)$$

The leading order approximation returns a minimal of Chi squared at string tension 0.039. However, the values of residuals are outstandingly higher than the corresponding values considering the next-to-leading approximation.

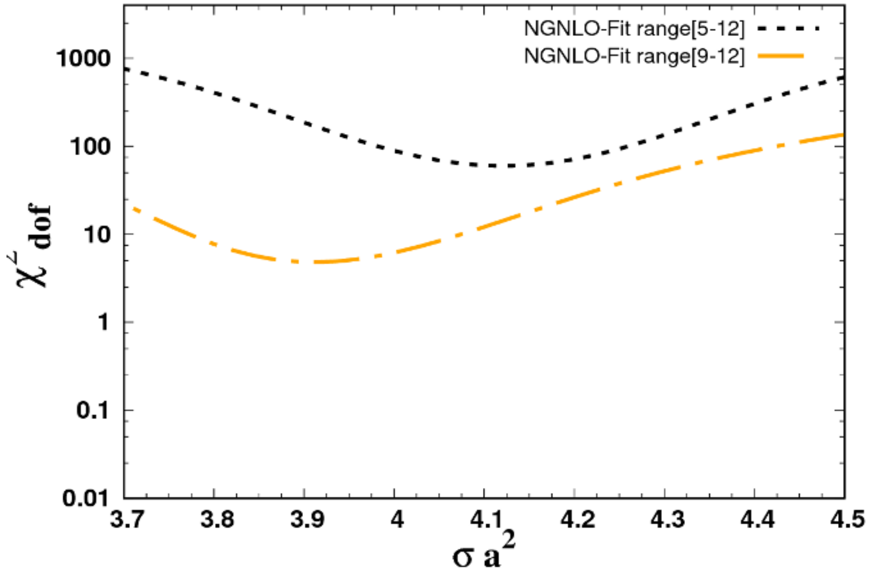


Nambu-Goto action

The returned Chi squared per degree of freedom from the fits of leading and next leading approximations of NG action to the quark-aniquark potential data.



Free String



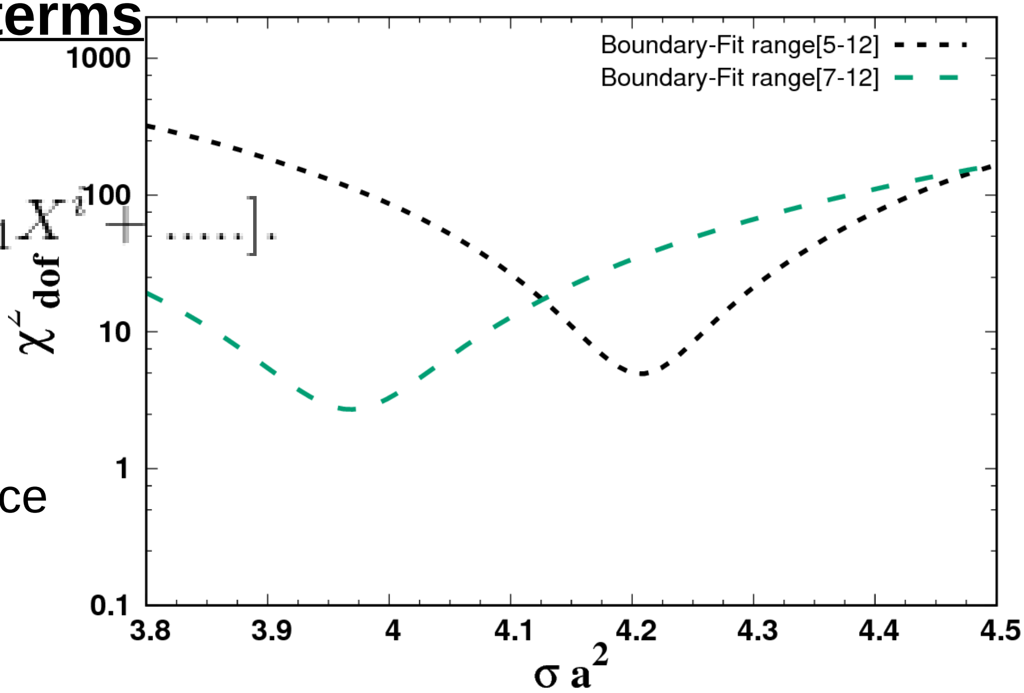
Self-Interacting String

Fit ranges covering small source separation [0.9,1.2] fm reduce the returned value of Chi squared for both Free and Self-Interacting string.

The increase in the value of the string tension at Chi minimal is the same for both approximations.

The consideration of the two-loop approximation at this temperatures does not provide a correct value for the free-parameter $\sigma\, a^{2}$ interpreted as the zero temperature string tension.

Luscher-Weiss action with boundary terms



$$S_b = \int d\zeta_0 [b_1 \partial_2 X_i . \partial_2 X^i + b_2 \partial_2 \partial_1 X_i . \partial_2 \partial_1 X^i + ...].$$

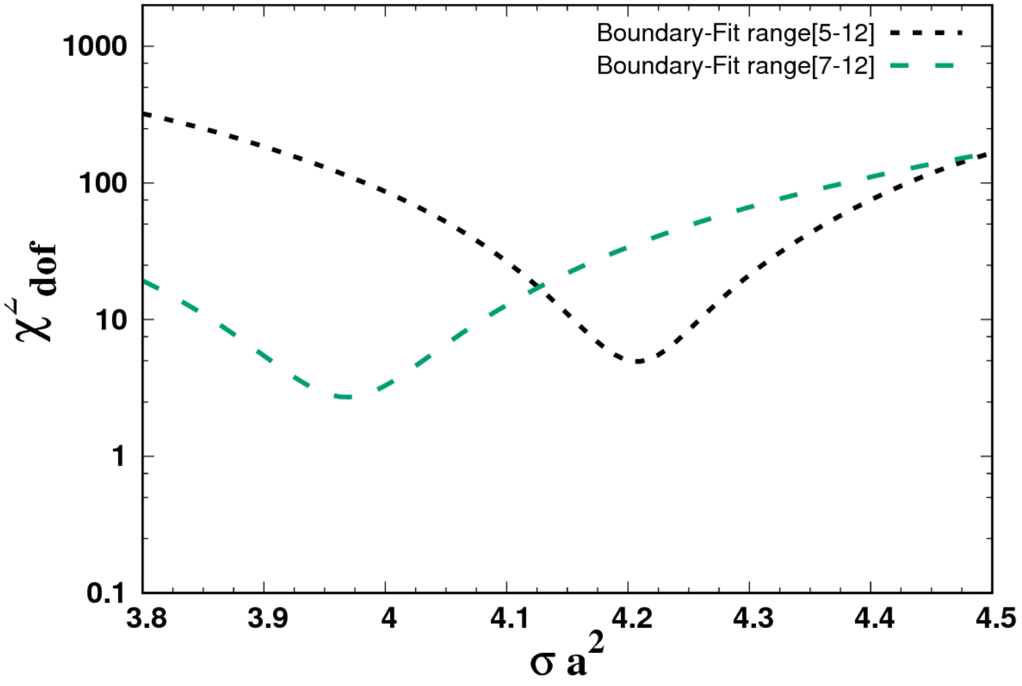
Effects such as the interaction of the string with the boundaries may explain the discrepancy in the NG string description for intermediate distance and could retrieve of the correct behavior of string tension versus the temperature.

The contribution to the Casmir energy due to the leading nonvanshing term Sb in Luscher-Wiess action is given by

$$V_{Q\bar{Q}} = V_{n\ell o} + V_b.$$

$$V_{n\ell o}(R,T) = \sigma R + 2T \, \ln \eta \, (i\tau) - T \ln \left(1 - \frac{(D-2)\pi^2 T}{1152 \sigma_o R^3} \left[2E_4(\tau) + (D-4) E_2^2(\tau) \right] \right)$$

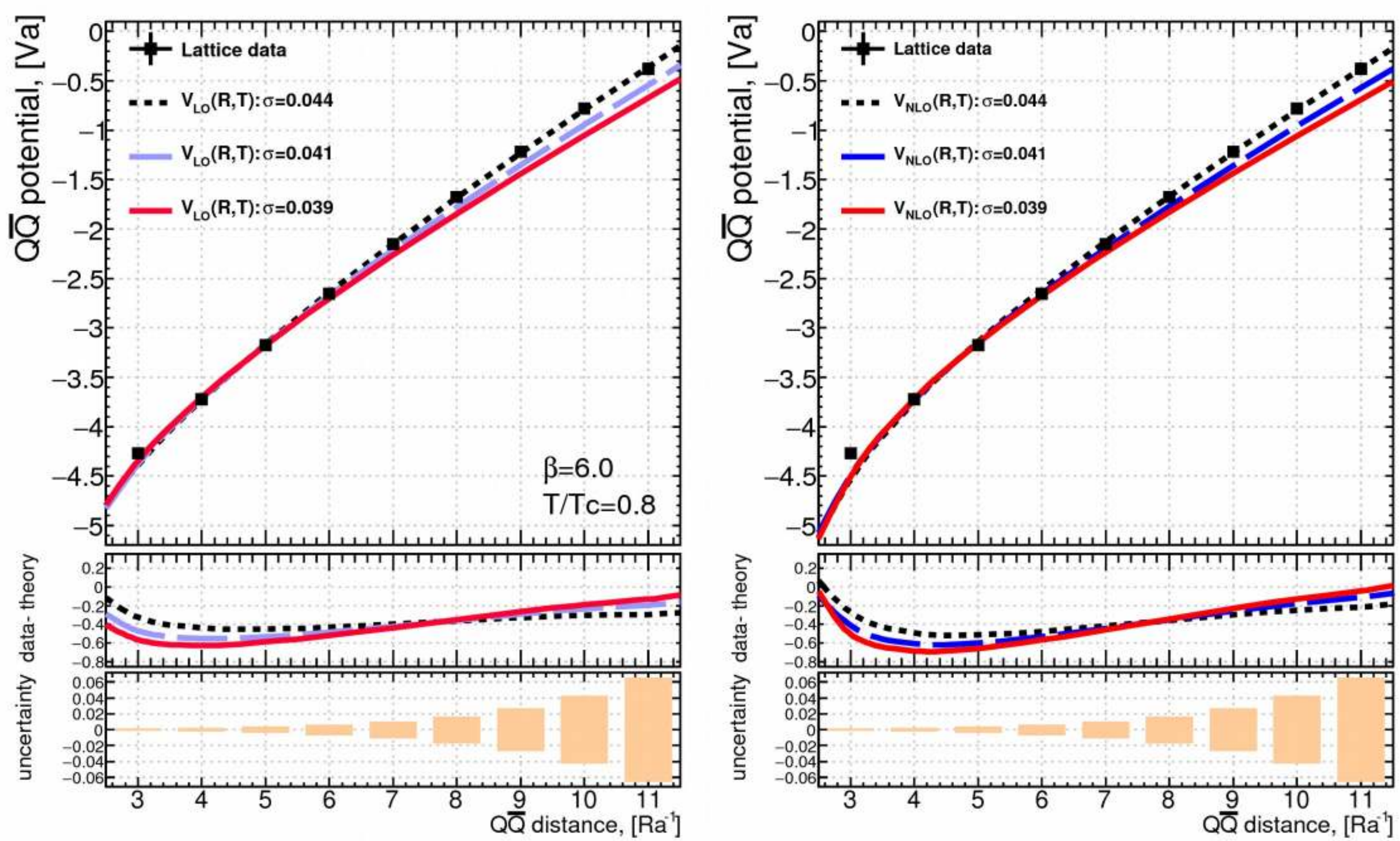
$$V_b = (d-2) b_2 \frac{-\pi^3 L_T}{60 R^4}$$



The values of χ^2_{dof} show improvements by the inclusion of the boundary terms Nevertheless these values are still high.

Despite of the reductions in the values of χ^2_{dof}, the consideration of the leading boundary term does not significantly alter the value of string tension for the returned minimal . We have almost the same values of the string tension as considering Self-interacting NG potential without boundary term.

Stiff/Rigid string



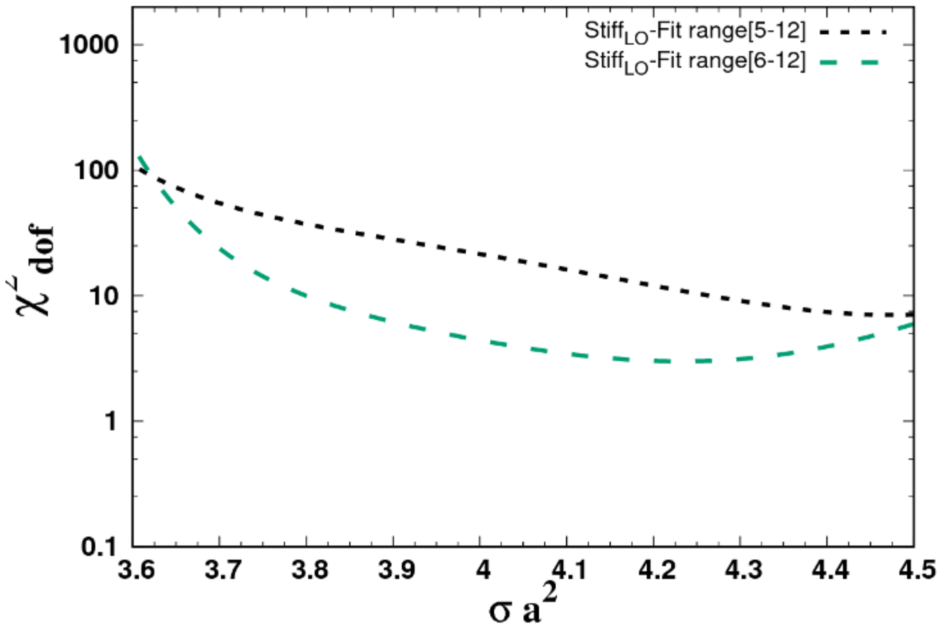
At the temperature T/T_c=0.8 the fits are returning low values of chi^2 with almost stable value for the rigidity parameter in the extrinsic curvature term in Polyakov-Kleinert action.

We measure the same value of the string tension. This is consistent with the fact that at relatively low temperatures the string's smooth fluctuations are dominant.

At the temperature T/T_c=0.9 close to the deconfinement point. We found significant improvement in the fit behavior when considering both the leading and the next to leading order approximation together with the stiffness or rigidity of the flux tube.

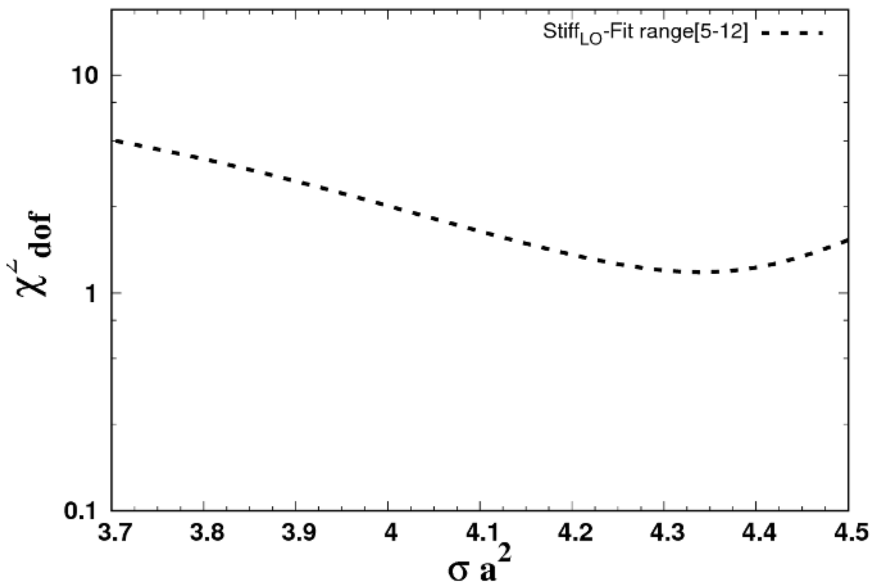
Stiff/Rigid string

The returned residual per degree of freedom from the fits of leading and next leading approximations of NG action to the quark-aniquark potential data.



Stiff free-string $V_{Q\bar{Q}} = V^{\ell o}_{stiff}$

$$V_{Q\bar{Q}} = V^{n\ell o}_{stiff} + V_b$$



Stiff Self-Interacting string

Comparison between the fit behavior and NG string with boundary terms show improvements in the fits retrieved by considering stiffness effects of the stringlike flux tube is not artifact of additional parameters

- **<u>Width of Nambu-Goto string</u>**

- The bosonic string model yields a prediction for the thickness of the fluctuating strings. The width of the junction itself can be calculated by taking the expectation value

$$W^2(\xi;\tau) = \langle\, X^2(\xi;\tau)\,\rangle$$
$$= \frac{\int_C [D\,X]\, X^2 \exp(-S_{NG}[X])}{\int_C [D\,X] \exp(-S_{NG}[X])}$$

- The Nambu-Goto effective action up next to leading order expansion is given by

$$S_{NG}[X] = S_{LO} + S_{NLO}$$

where S_{LO} is given by

$$S_{LO} = \sigma A + \frac{\sigma}{2}\int d\zeta_1 \int d\zeta_2 [(\frac{\partial X}{\partial \zeta_\alpha} \cdot \frac{\partial X}{\partial \zeta_\alpha})$$

and $S_{N\,LO}$ is

$$S_{NLO} = \sigma \int d\zeta_1 \int d\zeta_2 [\kappa_2 (\frac{\partial X}{\partial \zeta_\alpha} \cdot \frac{\partial X}{\partial \zeta_\alpha})^2 + \kappa_3 (\frac{\partial X}{\partial \zeta_\alpha} \cdot \frac{\partial X}{\partial \zeta_\beta})^2]$$

- Lüscher, Münster and Weisz have shown long-ago that the leading order approximation of the NG string entails a logarithmic divergence of the mean square width of the string at the middle plane in the zero temperature limit

$$W^2 \sim \frac{1}{\pi\sigma} \log(\frac{R}{R_0})$$

where R_0 is an ultra-violet scale. In D dimension and for cylindrical boundary conditions, the mean-square width would read

$$W^2_{\ell o}(\xi, \tau) = \frac{D-2}{2\pi\sigma} \log\left(\frac{R}{R_0(\xi)}\right) + \frac{D-2}{2\pi\sigma} \log\left|\frac{\theta_2(\pi\,\xi/R;\tau)}{\theta_1'(0;\tau)}\right|$$

A. Allais and M. Caselle, JHEP 01, 073 (2009), 0812.0284.

- F. Gliozzi. M. Pepe and Wiese computed analytically the width of the string at next-to-leading order

$$W^2(\xi) = W^2_{\ell o}(\xi) + W^2_{n\ell o}(\xi)$$

with the next to leading term given by

$$W^2_{n\ell o}(\xi) = \frac{\pi}{12\sigma R^2}\left[E_2(i\tau) - 4E_2(2i\tau)\right]\left(W^2_{lo}(\xi) - \frac{D-2}{4\pi\sigma}\right)$$
$$+ \frac{(D-2)\pi}{12\sigma^2 R^2}\left\{\tau\left(q_2\frac{d}{dq_2} - \frac{D-2}{12}E_2(i\tau)\right)\left[E_2(2i\tau) - E_2(i\tau)\right] - \frac{D-2}{8\pi}E_2(i\tau)\right\}$$

F. Gliozzi, M. Pepe, and U.-J. Wiese, Phys.Rev.Lett. 104, 232001 (2010), 1002.4888.

Generalization of the Nambu-Goto action:Width of Plyakov-Kleinert String

- The smooth configurations of quantum fluctuations swept in the Euclidean space-time by the Nambu-Goto string are favored by adding a new term

- The term is proportional to the geometrical second fundamental form of The world sheet or the so-called extrinsic curvature/rigidity/stiffness of the world-sheet.

- The squared width of the string is defined as the second moment of the field.

$$W^2(\xi;\tau) = \frac{\int_C [D\,X]\, X^2 \exp(-S_{NG}[X])}{\int_C [D\,X] \exp(-S_{NG}[X])}$$

We investigate the extrinsic curvature term contribution to the mean-square width of the free and self interacting string approximation

$$W^2_{free}(x) = W^2(x)_{LO} + W^2_{ext}.$$
$$W^2_{inter}(x) = W^2(x)_{LO} + W^2(x)_{NLO} + W^2_{ext}$$

$$\langle (X(x,t)^2) S_{ext} \rangle =$$
$$(D-2) \lim_{\epsilon,\epsilon' \to 0} \int_0^R dx' \int_0^{L_T} dt' \left(G(x,t;x',t')\, \partial_\mu \partial_\mu \partial_{\mu'} \partial_{\mu'} G(x,t;x',t') + \right.$$
$$\left. \partial_\mu \partial_\mu G(x,t;x',t')\, \partial_{\mu'} \partial_{\mu'} G(x,t;x',t') \right)$$

The term involving the extrinsic string action becomes The contribution mean-square width of the generalized smooth string is given by

$$W^2_{ext} = \langle X(x,t)^2 S_{ext} \rangle =$$
$$4RL_T(D-2) \left(\frac{2}{\pi\sigma} \right)^2 \left(\frac{\pi^2}{4R^2} \right)^2 \left(-\frac{1}{24} \left(E_2(q) \right) \right)^2$$

In the following, We report the fit behavior to the numerical lattice data.

- Here S[X] is the effective string action given by

$$S[X] = S_2[X] + S_4[X] + S_{ext}[X]$$

- Expanding around the free-string action

$$W^2(x,t) = W^2_{LO}(x,t) - < X(x,t)^2 (S_{NLO} + S_{ext}) >_0 + 2\alpha < (\partial_\mu X(x,t))^2 >_0$$
$$+\alpha^2 < (\partial_\mu \partial_\mu X(x,t))^2 >_0 - \beta r \alpha^2 \int dt\, dx\, dt'\, dx' < \partial_\mu \partial_\mu X(x,t) \partial_{\mu'} \partial_{\mu'} X(x',t') >_0$$

- In the following, we calculate the width of the rigid string up to one loop order.

$$S_{ext} = \frac{1}{2\alpha_o} \int_0^{L_T} d\tau \int_0^R dx [(\partial_x \partial_x X)^2 + (\partial_t \partial_t X)^2]$$

- Expanding around the square width of the free action

$$W^2(x) = W^2(x)_{LO} + W^2(x)_{NLO} + \langle (X(x,t)^2) S_{ext} \rangle$$

Define Green function $G(x, t; x_0, t_0) = \langle X(x, t)X(x_0, t_0) \rangle$ as the two point propagator. then the last term in, representing the perturbation with respect to the instantaneous string, would read in terms of the corresponding Green functions as

$$
\begin{aligned}
&\left\langle (X(x,t)^2) S_{ext} \right\rangle = \\
&(D-2) \lim_{\epsilon,\epsilon' \to 0} \int_0^R dx' \int_0^{L_T} dt' \left(G(x,t;x',t')\ \partial_\mu \partial_\mu \partial_{\mu'} \partial_{\mu'} G(x,t;x',t') + \right. \\
&\left. \partial_\mu \partial_\mu G(x,t;x',t')\ \partial_{\mu'} \partial_{\mu'} G(x,t;x',t') \right)
\end{aligned}
$$

The term involving the extrinsic string action becomes The contribution mean-square width of the generalized smooth string is given by

$$
\begin{aligned}
&W^2_{ext} = \left\langle X(x,t)^2 S_{ext} \right\rangle = \\
&4RL_T(D-2) \left(\frac{2}{\pi\sigma} \right)^2 \left(\frac{\pi^2}{4R^2} \right)^2 \left(-\frac{1}{24} \left(E_2(q) \right) \right)^2
\end{aligned}
$$

A. Bakry, X. Chen, M. Deliyergiyev, A. Galal, A. Khalaf, Will and P.M. Zhang, in preparation
Width of stiff strings at two loop order.

Simulation Set-up

The gauge configurations were generated using the standard Wilson gauge action.

The two lattices employed are of spatial size Nx=36, and temporal extents of Nt=,10, 8. beta=6.0, lattice spacing a=0.1 fm.

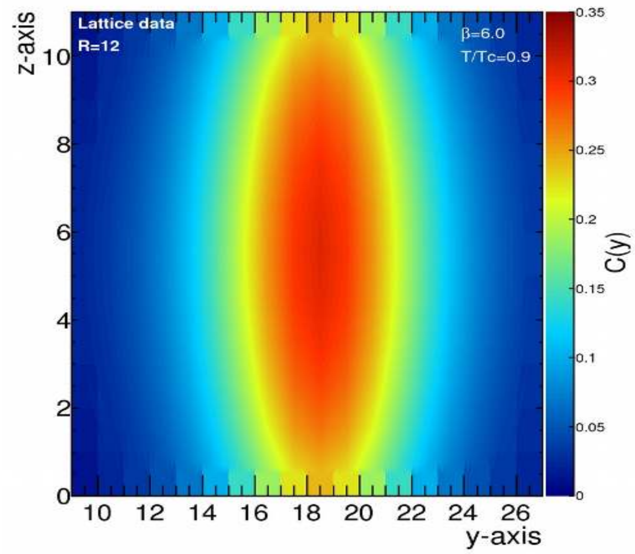
Bins of 6 measurements separated by 100 sweeps of updates. Each bin of measurements is taken following a 2000 of updating sweeps.   500 bin corresponding to 10,000 measurement at each temperature.

To eliminate statistical fluctuations, uncompromising the physical observables are left intact, 20 sweeps of UV filtering using an over-improved algorithm have been applied on all gauge configurations.

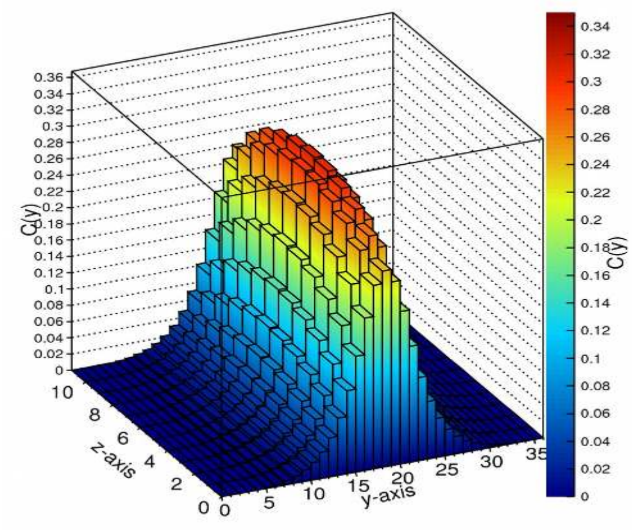
C. Morningstar and M. Peardon, Phys. Rev. D 69,054501 (2004).

Action Density Measurements

A scalar field characterizing the action density distribution in the Polyakov vacuum or in the presence of color sources can be defined as



$$\mathcal{C}(\vec{\rho};\vec{r}_1,\vec{r}_2)=\frac{\langle\,\mathcal{P}_{2Q}(\vec{r}_1,\vec{r}_2)\,S(\vec{\rho})\,\rangle}{\langle\,\mathcal{P}_{2Q}(\vec{r}_1,\vec{r}_2)\,\rangle\,\langle S(\vec{\rho})\,\rangle},$$



with the vector rho referring to the spatial position of the energy probe with respect to some origin.

The brackets < ...... > stands for averaging over gauge configurations and lattice symmetries.

The measurements are taken at a fixed color source's separations R are repeated at each point of the three-dimensional torus and time slice then averaged.

Cluster decomposition of the operators leads to C = 1 away from the quarks.

Width of the action density

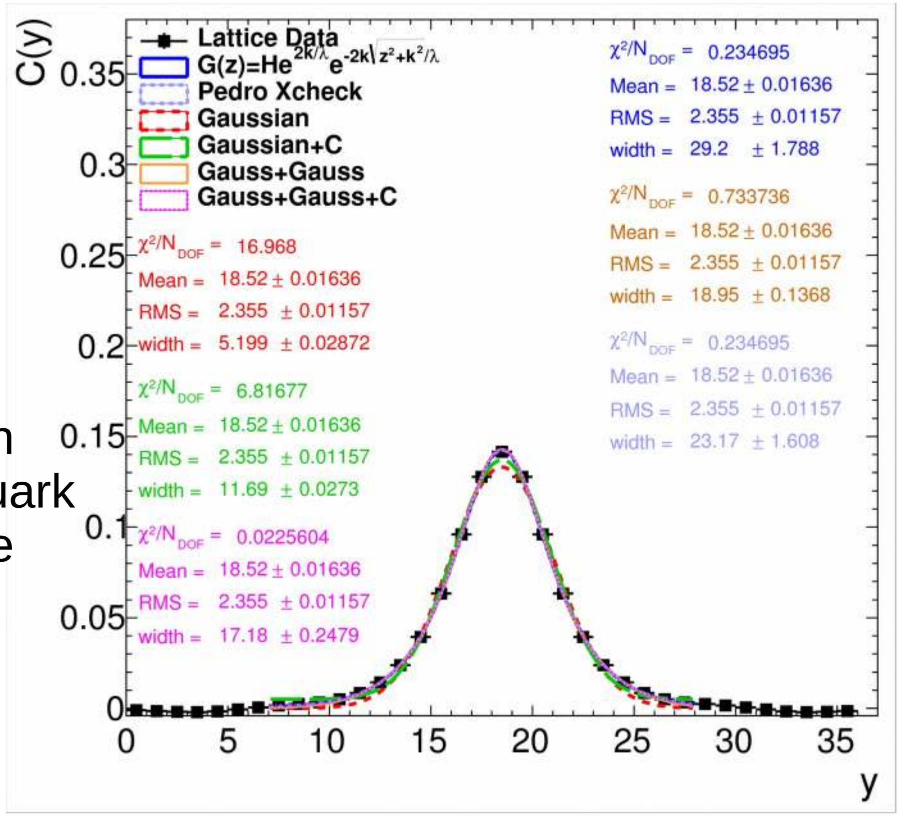
We examine several fit functions to measure the width of the action density profile.

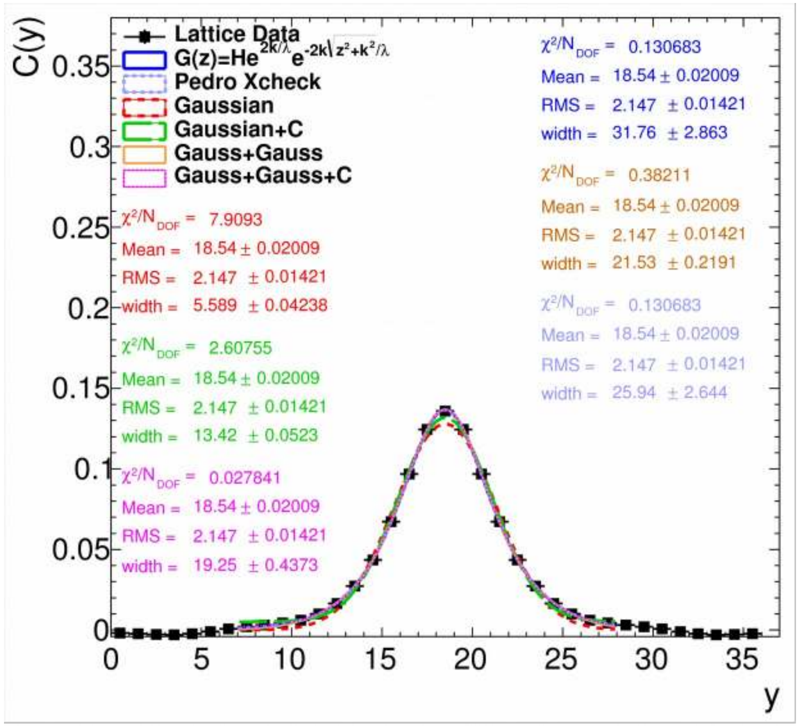
To estimate the mean-square width of the gluonic action density a long the planes transverse to the quark-antiquark axis, we choose a double Gaussian function of the same amplitude, A, and mean value mu=0.

$$G(r,\theta;z) = A(e^{-r^2/\sigma_1^2} + e^{-r^2/\sigma_2^2}) + \kappa$$

In the above form the constraint sigma1=sigma2 corresponds to the standard Gaussian distribution.

The fits of the double Gaussian form return stable values for the mean-square width and good Chi squared at the intermediate distances.

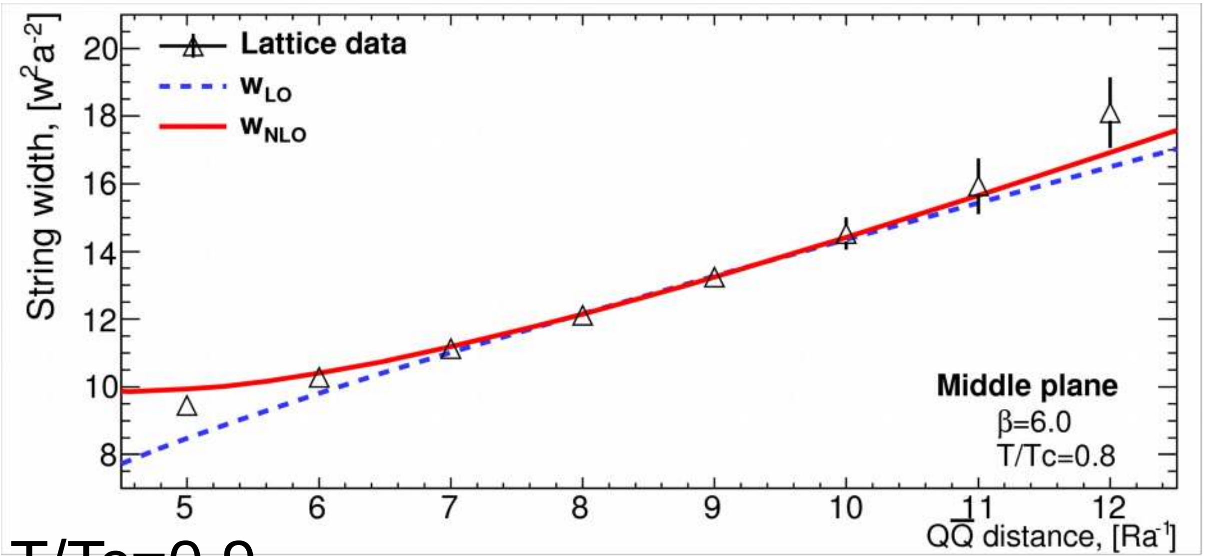




Width in the middle plane at T/Tc=0.8

Thermal effects fade out
near the end of QCD
plateau region T/Tc=0.8

Free string and Self-Interacting NG
string show good fit beahviour
with subtle differences.



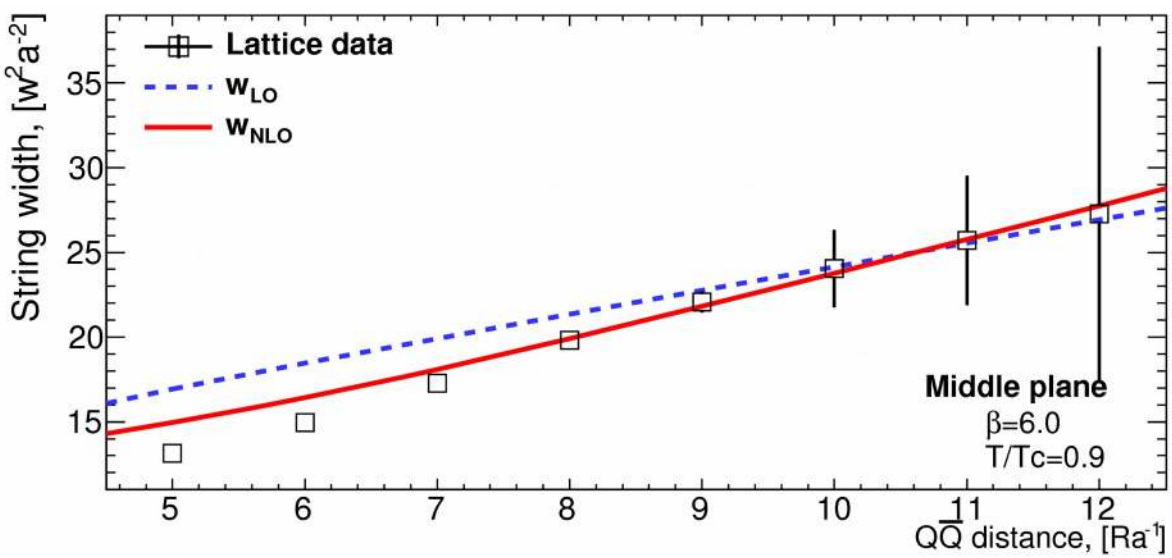
Width in the middle plane at T/Tc=0.9

Temperature effects are pronounced

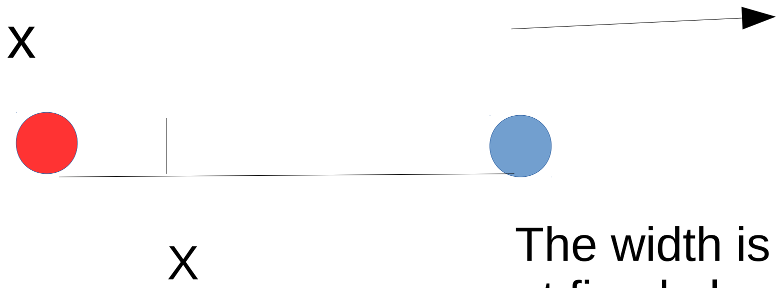
Free LO-NG significantly deviates
from the lattice data for intermediate
source separation.

Improvements with respect to NLO
NG string.

Nevertheless, the fits return large
residuals "Chi" for intermediate distances

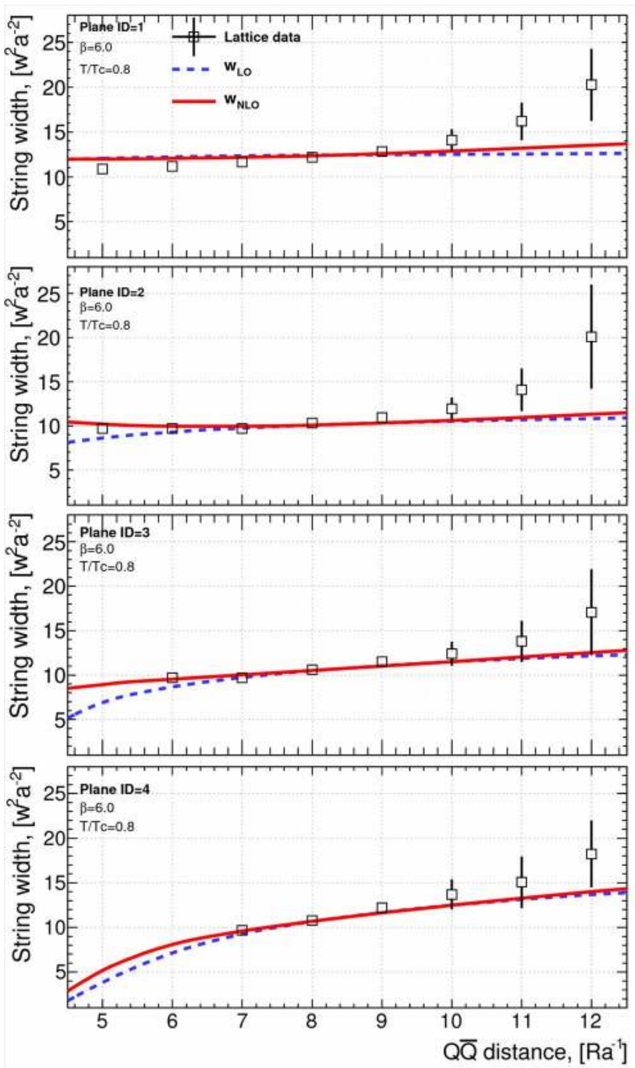


Mean-square width at different planes x

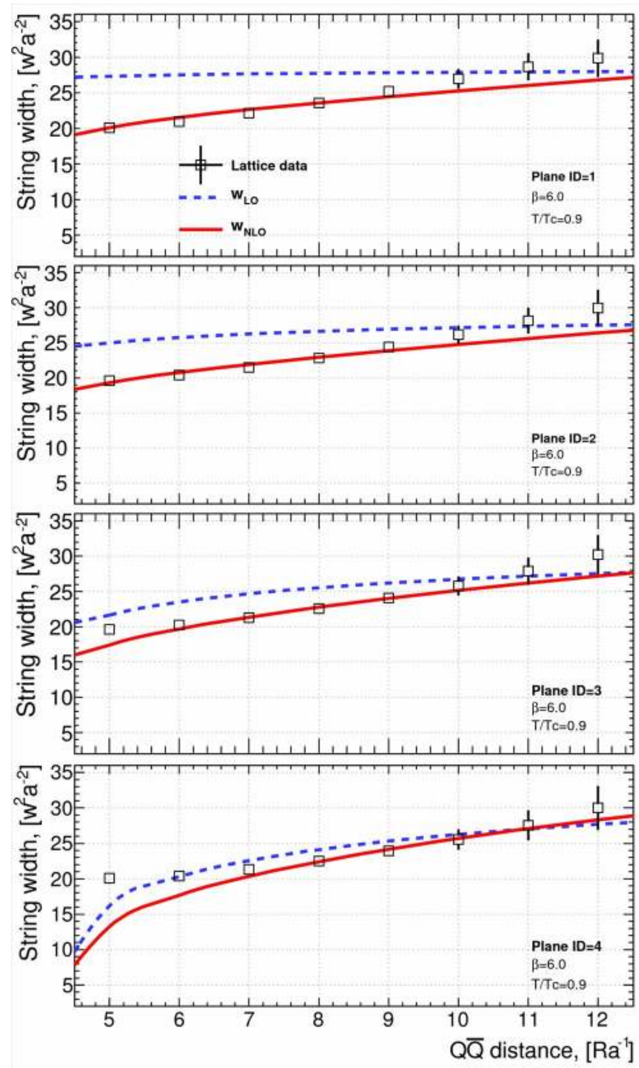


Leading and next to leading order NG string

The width is measured at fixed plane as we pull one quark apart.

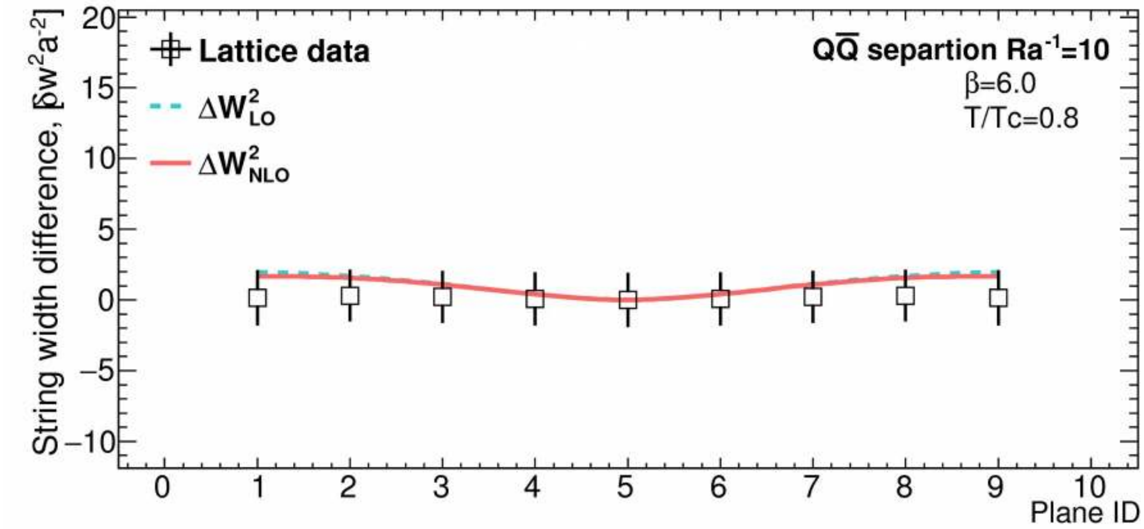


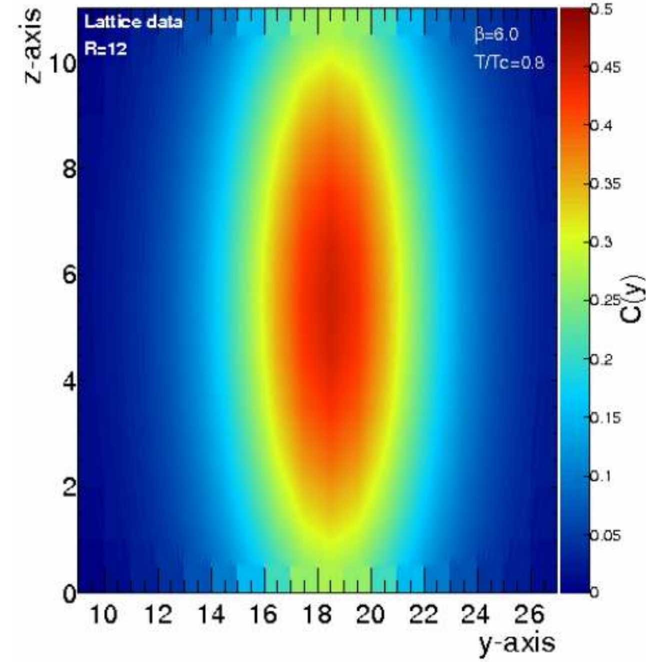
T/Tc=0.8



T/Tc=0.9

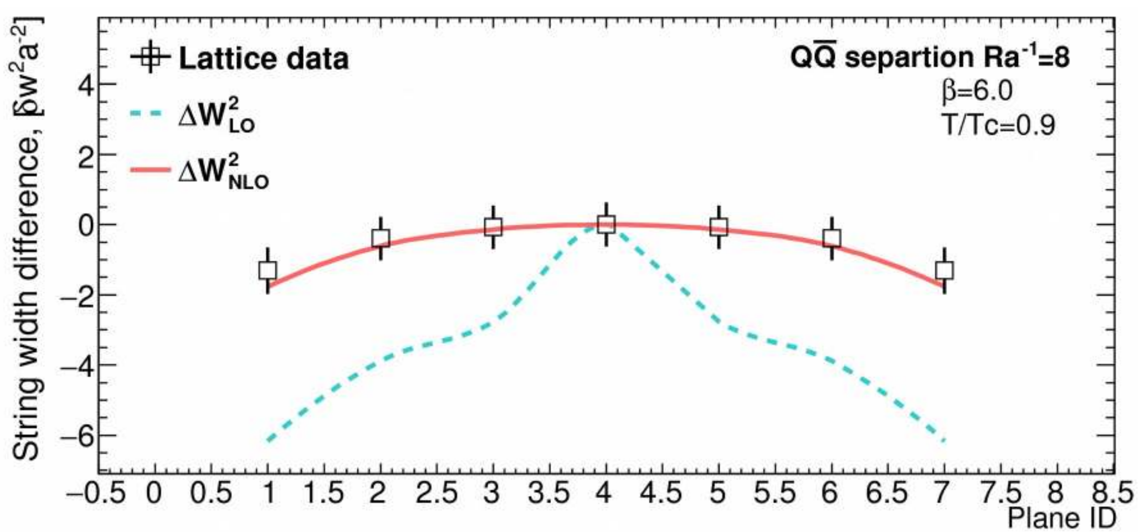
Curvature of the flux-tube

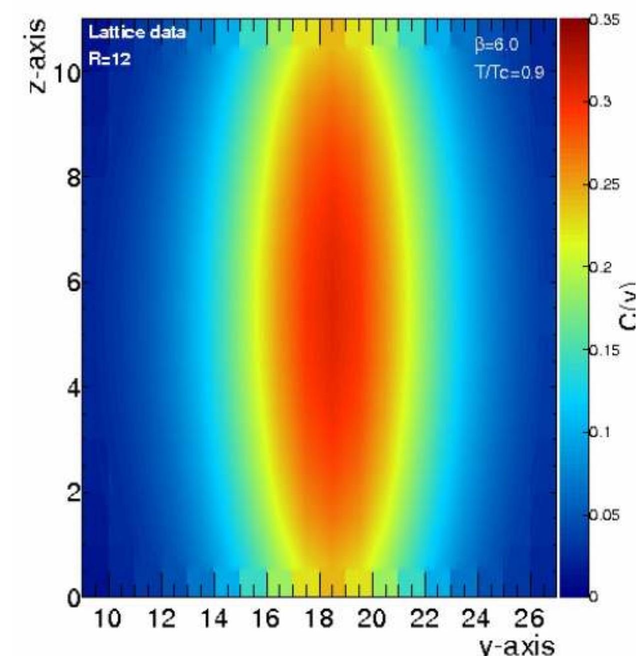




Action density T/Tc=0.8

Thermal effects diminish at T/Tc=0.8. The action density
Exhibits with a constant width profile.

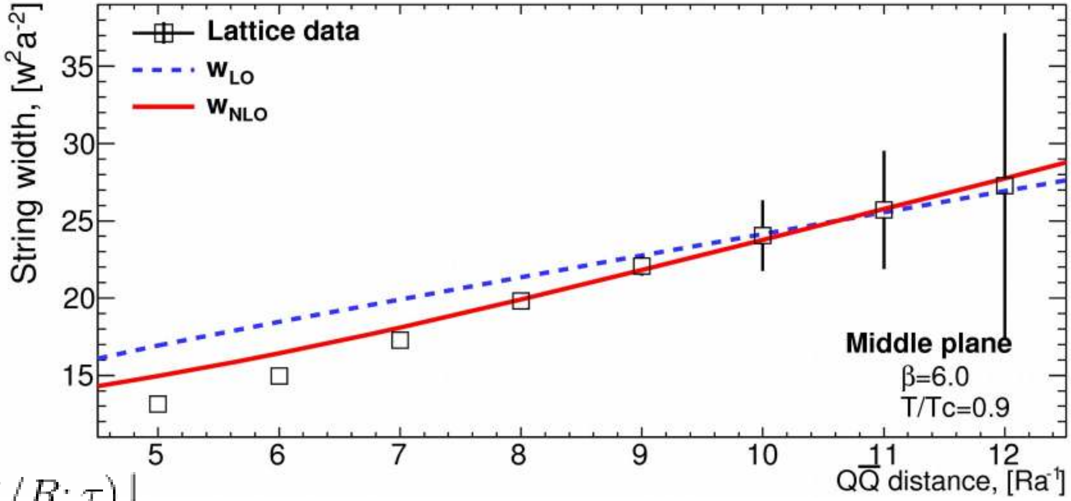




Near the deconfinement point T/Tc=0.9 the action density profile unveils
a curvature which is consistent with string-self interacting picture.

Action density T/Tc=0.9

Nambu-Goto versus Stiff/Rigid string

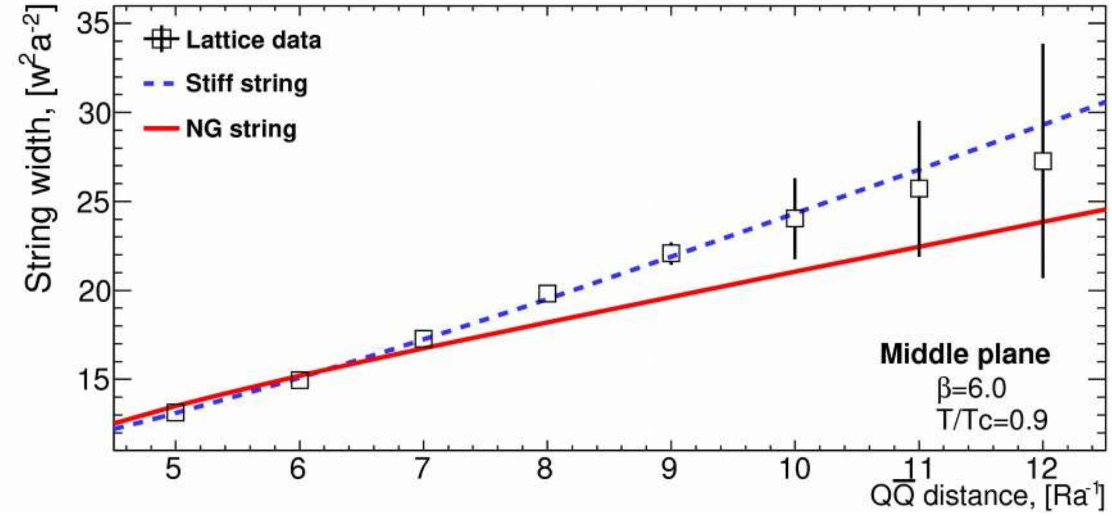


$$W^2_{free}(x) = W^2(x)_{LO} + W^2_{ext}.$$
$$W^2_{inter}(x) = W^2(x)_{LO} + W^2(x)_{NLO} + W^2_{ext}$$

$$W^2_{\ell o}(\xi,\tau) = \frac{D-2}{2\pi\sigma}\log\left(\frac{R}{R_0(\xi)}\right) + \frac{D-2}{2\pi\sigma}\log\left|\frac{\theta_2(\pi\,\xi/R;\tau)}{\theta_1'(0;\tau)}\right|$$ Nambu-Goto string

$$W^2_{n\ell o}(\xi) = \frac{\pi}{12\sigma R^2}\left[E_2(i\tau) - 4E_2(2i\tau)\right]\left(W^2_{lo}(\xi) - \frac{D-2}{4\pi\sigma}\right)$$
$$+\frac{(D-2)\pi}{12\sigma^2 R^2}\left\{\tau\left(q_2\frac{d}{dq_2} - \frac{D-2}{12}E_2(i\tau)\right)\left[E_2(2i\tau) - E_2(i\tau)\right] - \frac{D-2}{8\pi}E_2(i\tau)\right\}$$

$$W^2_{ext} = \langle X(x,t)^2 S_{ext}\rangle =$$
$$4RL_T(D-2)\left(\frac{2}{\pi\sigma}\right)^2\left(\frac{\pi^2}{4R^2}\right)^2\left(-\frac{1}{24}\left(E_2(q)\right)\right)^2$$



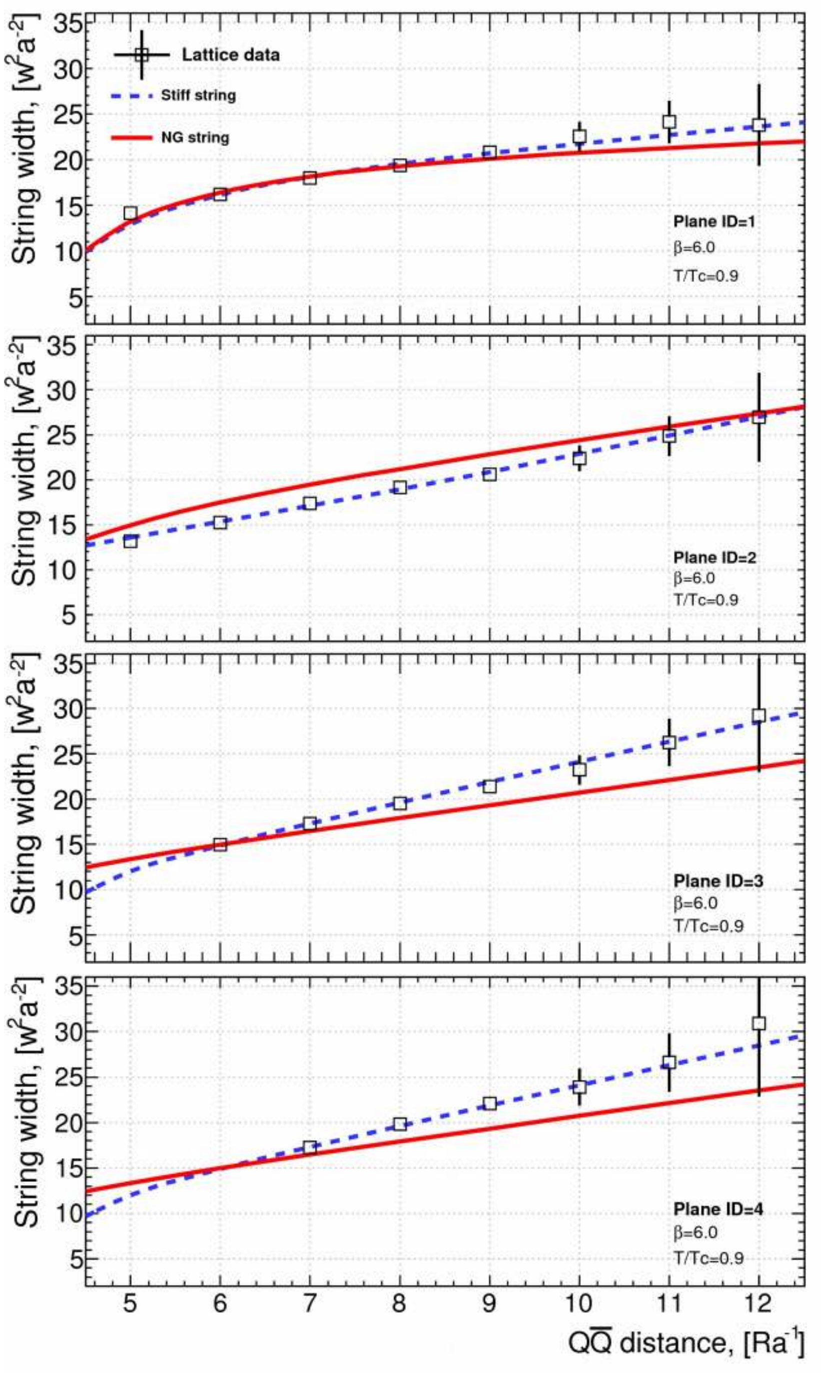
Stiff string

Compares fits of Stiff-string to the NG String at x plane

Fits are considered on an interval corresponding to the whole intermediate color source separation [5,12]

Significant reductions in the values of χ^{2} to acceptable values in the intermediate and long separation distances are returned.

This is in favor of the parameterization behavior with respect to the stiff string together with the next to leading approximation against considering only the NG approximation.



SUMMARY AND CONCLUSION

- We discussed stiffness effects of bosonic strings in the vicinity of QCD phase transition point. The geometrical extrinsic curvatures or stiffness effects of the string have been considered within the framework of both the universal free Nambu-Goto string in addition to the correction to the action expanded up to to the fourth derivative term.

- We derive a formalism to calculate the mean square width of the stiff string at high temperature. The theoretical predictions laid down by the free and self-interacting picture are then set into a comparison with the corresponding SU(3) Yang-Mills lattice data in four space-time dimension.

- Near the end of the QCD plateau region at temperatures $T/T_c = 0.8$. We found subtle changes in the fit behavior of the mean-square width of the NG string for both the free and self-interacting stiff strings pictures to the lattice data with small differences in χ^2 observed in the intermediate distances and the asymptotic large distances regions.

- We conclude that the same as zero and low temperatures, $T/T_c = 0.8$ thermal effects fade out and higher order loops and the rigidity of the string has very small effects for source sparations commencing from $R = 0.5$ fm.

- The NG effective description does not accurately describe the $Q\bar{Q}$ potential and the energy density profile just close to the deconfinement point which manifests as a deviation from the standard value of the string tension and the width of the quantum fluctuations of the effective string in the intermediate separation region.

- In this investigation, we found that the geometrical smoothness of the string fluctuations with the use of the additional Polyakov-Kleinert extrinsic curvature term to the NG action can be proposed to successfully account for deviations from the string fine structure and Yang-Mills data at high temperature.

- This introduces strong incentives to scrutinize the relevant properties of the energy density profile in other framework such as the London penetration length in superconductivity models or the UV filtering of the gauge field to understand the stiffness physics of the QCD-flux-tube on a more profoundly.

A. Bakry, X. Chen, M. Deliyergiyev, A. Galal, S. Xu, and P.M. Zhang.
String models near QCD deconfinement point. In preparation.