Electromagnetic corrections to the hadronic vacuum polarization of the photon within $\mathrm{QED_L}$ and $\mathrm{QED_M}$

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CP³ Origins

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Plan of the talk

Motivations

Well known 3σ tension in $(g-2)_\mu$

Future experiments will shrink the error!

$\sigma\left(e^+ e^- \rightarrow \text{Had}\right)$-method still the most accurate

(includes all SM contributions)

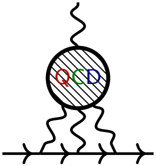
Exp. data with space-like kin. allow for direct comparison with Lattice

[Carloni Calame et al. Phys. Lett. B746:325–329, 2015]

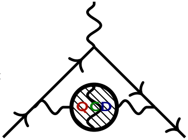
$3\sigma \simeq 4\%$ on a_μ^{HLO}

QED corrections $\approx 1\%$

$a_\mu^{\mathrm{HLbL}} =$



$= \mathrm{O}\left(e^7\right) =$



$= a_\mu^{\mathrm{HLO}}(\alpha)$

In this talk we explore EM corrections to the HVP

Can we pin them down? (lattice error $a_\mu^{\mathrm{HLO}} \approx 5\%$)

IR regularizations of choice

We do not discuss all the (many) issues with QED here

[Portelli PoS KAON 13 (2013) 023] [Patella PoS LATTICE 2016 (2017)]

$\mathrm{QED_L}$

[Borsanyi et al. Science 347 (2015) 1452]

- Easy implementation
- Non-local constraint
- Power-like finite vol. corr.
- Renorm. issues for op. $d > 4$
- Convincing spectrum results
- Non-comm. $L \to \infty \leftrightarrow a \to 0$

$\mathrm{QED_M}$

[Endres et al. Phys. Rev. Lett. 117 (2016) no.7, 072002]

- Easy implementation
- Local formulation
- Exp. suppressed finite vol. corr.
- m_γ "too small"
  - t-dep. in eff. masses
  - $\underline{p}$ stiffness in eff. en.
- Non-comm. $m_\gamma \to 0 \leftrightarrow L \to \infty$

Non-comm. of limits in $\mathrm{QED_M}$ resembles p and ϵ regimes in χ-PT

In χ-PT: $m\Sigma V$ vs 1 In $\mathrm{QED_M}$: ? vs 1

Remark: Massive photons form Bose-Einstein condensate

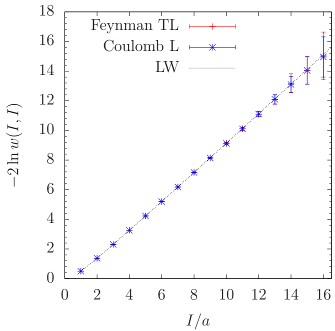
Wilson loops in pure QED: $V = 32^4$

$$w_{\mu\nu}(I,I) = \exp\left(2e^2Q^2\left[C_\mu(I,0) - C_\nu(I,I\hat{\nu})\right]\right), \quad C_\mu(I,x) = ID(x) + \sum_{\tau=1}^{I-1}(I-\tau)D(x+\tau\hat{\mu})$$

$D(x)$ is the infinite lattice massless/massive scalar propagator in coordinate space

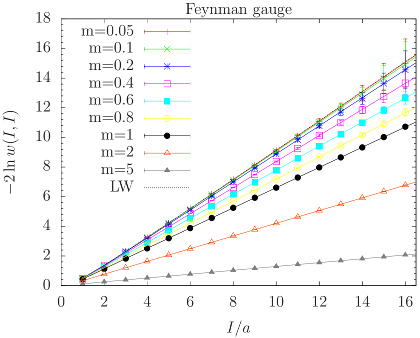
Lüscher-Weisz method

[Luscher and Weisz, Nucl. Phys. B 445 (1995) 429]



Borasoy-Krebs method

[Borasoy and Krebs Phys. Rev. D 72 (2005) 056003]



QCD ensembles

Goal: QED corrections to a_μ^{HLO} in QCD+qQED framework

Dynamical QCD cnfs generated by CLS with $N_f = 2$ degenerate flavors of non-perturbatively O(a) improved Wilson fermions

[Capitani et al. Phys. Rev. D 92 (2015) no.5, 054511]

$$\beta = 5.2,\ c_{sw} = 2.01715,\ \kappa_c = 0.1360546,\ a[\mathrm{fm}] = 0.079(3)(2),\ L/a = 32$$

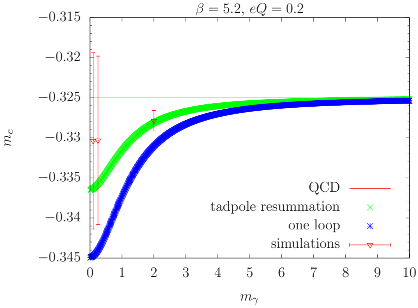
Run	κ	am_π	$m_\pi L$	m_π[MeV]
A3	0.13580	.1893(6)	6.0	473
A4	0.13590	.1459(6)	4.7	364
A5	0.13594	.1265(8)	4.0	316

QED inclusion shifts the critical mass!

Remark: 1% Net effect on m_c translates in O(100%) change in m_q (for $m_0 \simeq m_c^{\mathrm{QCD}}$)
Important for m_π and therefore HVP!

QCD+qQED ensembles

Inclusion of qQED with $\alpha = 1/137$ and physical charges $Q = 2/3, -1/3$



Run	am_γ	$am_{\pi^0=u\bar{u}}$	$am_{\pi^0=d\bar{d}}$	$am_{\pi^\pm}$
A3	0	.2549(9)	.2071(9)	.2330(9)
	0.1	.2556(7)	.2074(8)	.2337(8)
	0.25	.2553(7)	.2072(8)	.2331(8)
A4	0	.2240(8)	.1691(9)	.1994(9)
	0.1	.2252(9)	.1699(9)	.2005(9)
	0.25	.2246(8)	.1700(10)	.1998(9)
A5	0	.2105(7)	.1526(9)	.1849(8)
	0.1	.2114(7)	.1528(9)	.1856(8)
	0.25	.2111(7)	.1531(9)	.1852(8)

Pion masses going from 380 MeV to 640 MeV

Notice: m_c EM shift in A5 gives $m_{\pi^0=u\bar{u}}^{Q(C+E)D} \simeq 2m_\pi^{QCD}$

Notice: Matching between ensembles $m_{\pi^\pm}^{Q(C+E)D}(\mathrm{A5}) \simeq m_\pi^{QCD}(\mathrm{A3})$

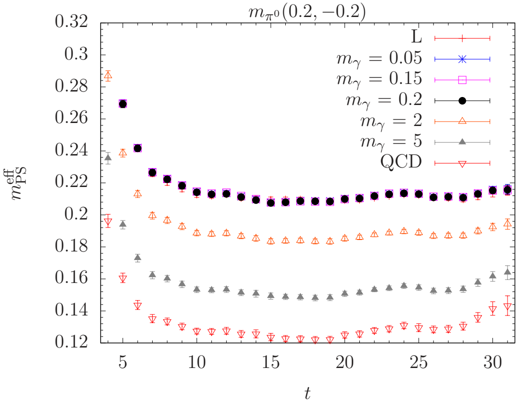
Finite volume and photon mass effects have been checked with PT formulae and negligible within errors

Dependence on m_γ

For $m_\gamma = 0.1$ the coeff. of linear t-term in eff. energies is suppressed

$$(m_\gamma^2 V)^{-1} \simeq 5 \times 10^{-5}$$

not visible in the effective masses for $m_\gamma \in [0.05, 0.1, 0.15, 0.2, 0.25, 2, 5]$



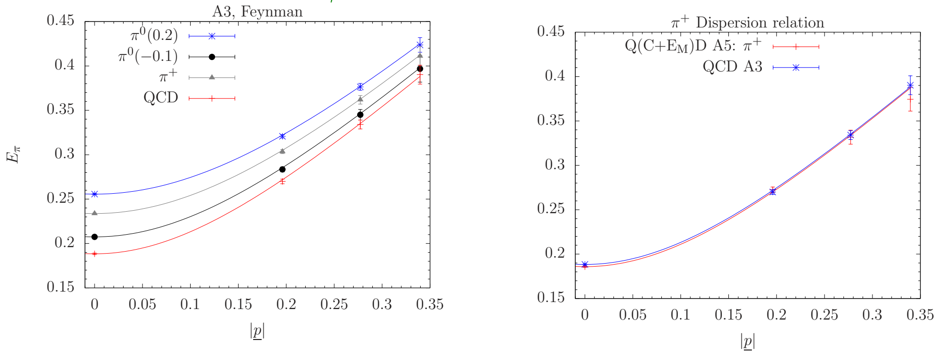
$\mathrm{QED_L}$ is consistent with $\mathrm{QED_M}$ for $m_\gamma \to 0$

Expectation:
photons decouple for $m_\gamma \to \infty$

[Appelquist and Carazzone Phys. Rev. D 11 (1975) 2856]

Our choices are $m_\gamma = 0.1, 0.25$

Dispersion relation $m_\gamma = 0.1$



No stiffness in $|\underline{p}|$

- $E_{\text{eff}}(t, \underline{p}) \overset{m_\gamma \to 0}{\simeq} \frac{(Q_u - Q_d)^2 e^2}{m_\gamma^2 V} t - \frac{d}{dt} \ln \langle \mathcal{O}(t, \underline{0}) \overline{\mathcal{O}}(0) \delta_{Q_{\text{T}}, \mathbf{0}} \rangle_{\text{TL}},$

All the effective energies agree with the continuum curve (solid lines)

Charged pion mass in A3 QCD matches the one in A5 Q(C+E$_\text{M}$)D

So far...

$m_\gamma \simeq 0.1$ seems to be a safe choice

- Negligible finite volume effects
- Negligible finite photon mass effects
- No subtle reduction to $\mathrm{QED_{TL}}$
- $\mathrm{QED_{L}}$ is consistent (for the spectrum and these parameters)

Pion masses in A5 Q(C+E)D "match" A3 QCD ones

- HVP depends strongly on pion masses
- Can give direct access to EM effects in the HVP

HVP

HVP tensor: $\Pi_{\mu\nu}(q) = \int \mathrm{d}^4x\, \mathrm{e}^{iq\cdot x} \langle V_\mu(x) V_\nu(0) \rangle$

Is the current still conserved in Q(C+E)D formal theory?

$$\mathbf{SU}(2)_\mathrm{L} \otimes \mathbf{SU}(2)_\mathrm{R} \otimes \mathbf{U}(1)_\mathrm{V}$$

$\downarrow$ explicit and spontaneous

$$\mathrm{QCD} : \mathbf{SU}(2)_\mathrm{V} \otimes \mathbf{U}(1)_\mathrm{V}$$

$\downarrow$ explicit

$$\mathrm{Q(C+E)D} : \mathbf{U}'(1)_\mathrm{V} \otimes \mathbf{U}(1)_\mathrm{V}$$

Combination of **1** and τ^3 in flavor is conserved

$$V_\mu(x) = \overline{\Psi}(x)\gamma_\mu \left[\frac{Q_u}{2} \left(\mathbf{1} + \tau^3\right) + \frac{Q_d}{2} \left(\mathbf{1} - \tau^3\right) \right] \Psi(x)$$

On the Lattice: 1-point-split current conservation implies $Z_V = 1$
no QED effects to take into account

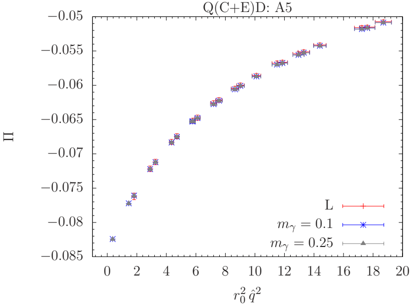
For completeness:

Neglecting quark-disconnected diagrams

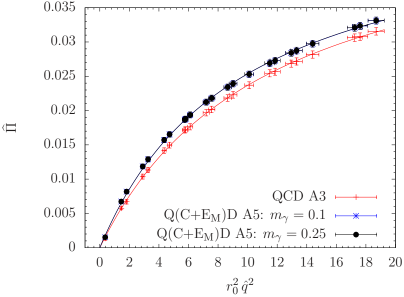
Electroquenched approximation

Scalar HVP

Agreement between $\mathrm{QED_L}$ and $\mathrm{QED_M}$



Matching gives direct access to EM eff.



r_0/a as any other gluonic scale does not receive QED contributions in the quenched approximation

For completeness:

- ZMS modification [Bernecker and Meyer Eur. Phys. J. A 47 (2011) 148]
- Padé fit R_{10} to extract $\Pi(0)$ [Blum et al. JHEP 1604 (2016) 063]
- Point sources are used

Strategy to extract EM effects for a_μ

First strategy

- Fit scalar HVP in Q(C+E)D and compute a_μ
- Fit scalar HVP in QCD and compute a_μ
- After extrapolation to infinite volume, physical point and continumm take the difference between QCD and Q(C+E)D results

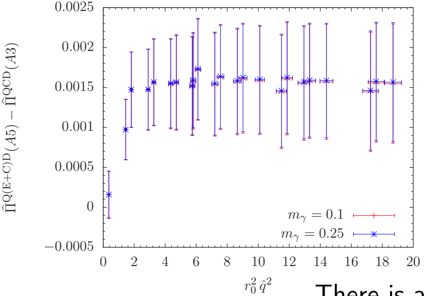
The effect can be washed out by the various systematics...

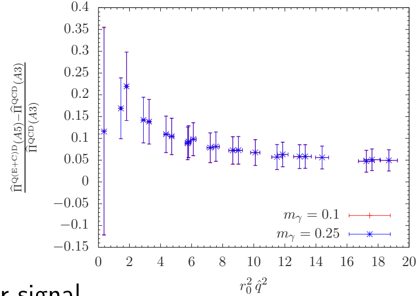
Second strategy

- Take $\widehat{\Pi}^{\mathrm{Q(C+E)D}} - \widehat{\Pi}^{\mathrm{QCD}} \equiv \delta\widehat{\Pi}$ at fixed pion masses
- Fit $\delta\widehat{\Pi}$ and plug it in $a_\mu^\delta = \int f(q)\delta\widehat{\Pi}$
- Extrapolate to infinite volume, physical point and continuum

Only one fit has to be performed to a slowly varying function

a_μ^δ PRELIMINARY estimates





There is a clear signal

$$a_\mu^\delta \times 10^{10} = \begin{cases} 68 \pm 28 & \text{for } R_{10} \\ 50 \pm 24 & \text{for } R_{11} \\ 35 \pm 14 & \text{for lin.-const.} \end{cases}$$

The effect is as large as the discrepancy between theory and experiment....

Still effects to quantify, e.g. in a and m_π (this could be large), so far $m_\pi \approx 460$ MeV, $a \approx 0.8$ fm

Conclusions

Extensive comparison between $\mathrm{QED_M}$ and $\mathrm{QED_L}$

A safe choice of m_γ is easy to find

- $\mathrm{QED_L}$ on the considered quantities (energies and HVP) is consistent

One lattice spacing

Unphysical pion masses

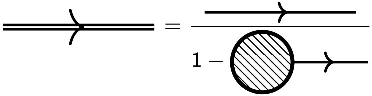
We could devise a setup to obtain a statistically significant signal for $\delta a_\mu^{\mathrm{HLO}}$ that relies on "matching" Q(C+E)D and QCD

Back-up slides

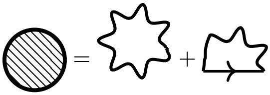
EM shift to the critical mass for Wilson fermions

Perturbation Theory: QED

Self-energy resummation :

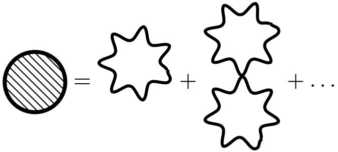


PT at 1-loop



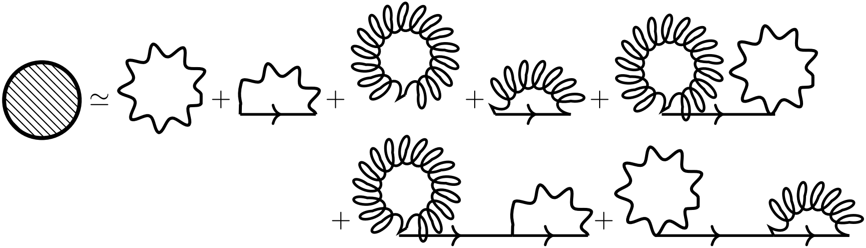
“Daisy” resummation

[Constantinou et al. Phys. Rev. D 74 (2006) 074503]



EM shift to the critical mass for Wilson fermions

Perturbation Theory: Q(C+E)D



With Daisy resummation we obtain

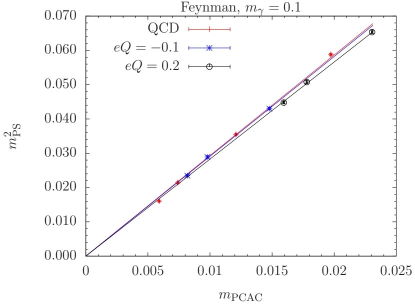
$$\Delta m_{\mathrm{c}}^{\mathrm{Q(C+E)D}} = \frac{1}{8k_{\mathrm{c}}}\left(1 - \mathrm{e}^{\Sigma_{\mathrm{T}}}\right)$$

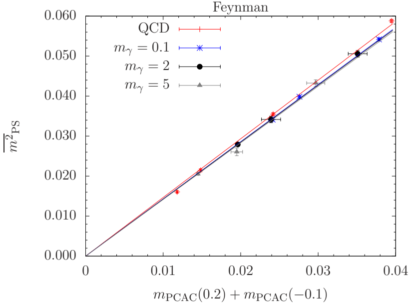
[Duncan et al. Phys. Rev. Lett. 76 (1996) 3894]

m_{PS} vs. m_{PCAC}

[Duncan et al. Phys. Rev. Lett. **76** (1996) 3894] in $\mathrm{QED_M}$

Square averaging respect the chiral symmetry expected from the full theory $\overline{m^2}_{\mathrm{PS}} = \frac{1}{2}\left(m^2_{\mathrm{PS}}(eQ = 0.2) + m^2_{\mathrm{PS}}(eQ = -0.1)\right)$





Same kind of linearity is found also for $\mathrm{QED_M}$

$$\overline{m^2}_{\mathrm{PS}} = m_{\mathrm{PCAC}}(eQ_1)B(eQ_1, eQ_2) + m_{\mathrm{PCAC}}(eQ_2)B(eQ_2, eQ_1)$$

Finite volume corrections in $\mathrm{QED_L}$

Scalar QED PT at one-loop

[Borsanyi et al. Science 347 (2015) 1452]

$$\delta_V m_{\pi^\pm}^{2,\mathrm{LO}} = m_{\pi^\pm}^2(L, T \to \infty) - m_{\pi^\pm}^2(L, T) \simeq \frac{Q^2 \alpha \kappa \, m_{\pi^\pm}(L, T)}{L}$$

Run	$m_{\pi^\pm}^2$	$\delta_V m_{\pi^\pm}^{2,\mathrm{LO}}$
A3	0.0537(4)	0.00015(1)
A4	.0398(4)	0.00013(1)
A5	.0342(3)	0.00012(1)

NEGLIGIBLE

Finite volume and photon mass corrections in $\mathrm{QED_M}$

[Endres et al. Phys. Rev. Lett. 117 (2016) no.7, 072002]

FINITE VOLUME

NRQED Analytic formulae for the extracted masses at each volume

$$\delta_V m_{\pi^\pm}^{\mathrm{LO}} = 2\pi Q^2 \alpha\, m_\gamma \left[\mathcal{I}_1(m_\gamma L) - \frac{1}{(m_\gamma L)^3}\right]$$

$$\delta_V m_{\pi^\pm}^{\mathrm{LO}}(m_\gamma = 0.1) = -.000097(1)$$

$$\delta_V m_{\pi^\pm}^{\mathrm{LO}}(m_\gamma = 0.25) = -.000023(1)$$

FINITE PHOTON MASS

Expansion in m_γ / M

$$\delta m_{\pi^\pm}^{\mathrm{LO}} = m_{\pi^\pm}(m_\gamma \to \infty) - m_{\pi^\pm}(m_\gamma) = -\frac{\alpha}{2} Q^2 m_\gamma$$

$$\delta m_{\pi^\pm}^{\mathrm{LO}}(m_\gamma = 0.1) = .00036(1)$$

$$\delta m_{\pi^\pm}^{\mathrm{LO}}(m_\gamma = 0.25) = .00091(1)$$

BOTH NEGLIGIBLE