June 23rd

The Search for Beauty-fully bound S-wave tetraquarks

Ciaran Hughes, Estia Eichten, Christine Davies

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The Search for $\bar{b}\bar{b}bb$ bound S-wave tetraquarks

Fermilab

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Why bother??

Why bother??

arXiv:1111.1867v2

Tetraquarks composed of 4 heavy quarks

A.V. Berezhnoy,[1,*] A.V. Luchinsky,[2,†] and A.A. Novoselov[2,‡]

[1] *SINP of Moscow State University, Russia*

[2] *Institute for High Energy Physics, Protvino, Russia*

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With hyperfine splitting one obtains the following masses of the T_{4b} states:

$$0^{++\prime}: \quad M = 18.754\,\text{GeV}, \quad M - M_{\text{th}} = -544.\,\text{MeV},$$
$$1^{+-\prime}: \quad M = 18.808\,\text{GeV}, \quad M - M_{\text{th}} = -490.\,\text{MeV},$$
$$2^{++}: \quad M = 18.916\,\text{GeV}, \quad M - M_{\text{th}} = -382.\,\text{MeV}.$$

It can be seen that in this case all the states are under the $\Upsilon(1S)$ pair production threshold $M_{\text{th}} = 2m_{\Upsilon(1S)}$.

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Tetraquark bound states in a Bethe-Salpeter approach

Walter Heupel,[1] Gernot Eichmann,[1] and Christian S. Fischer[1,2]

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[1] *Institut für Theoretische Physik, Justus-Liebig-Universität Giessen, D-35392 Giessen, Germany*
[2] *GSI Helmholtzzentrum für Schwerionenforschung GmbH, Planckstr. 1, D-64291 Darmstadt, Germany.*

(Dated: November 9, 2012)

We therefore read off the mass of an all-charm scalar tetraquark state to be at

$$m^{c}_{\text{Tetraquark}}(0^{++}) = 5.3 \pm (0.5)\,\text{GeV}\,, \qquad (20)$$

where the error is a guess based on our numerical and systematic uncertainties. This mass is considerably lower

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Tetraquarks composed of 4 heavy quarks

arXiv:1605.01134v1

Heavy-flavored tetraquark states with the $QQ\bar{Q}\bar{Q}$ configuration

Jing Wu[1], Yan-Rui Liu[1*]
[1] *School of Physics and Key Laboratory of Particle Physics and Particle Irradiation (MOE), Shandong University, Jinan 250100, China*

Kan Chen[2,3], Xiang Liu[2,3†]
[2] *School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China*
[3] *Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China*

Shi-Lin Zhu[4,5,6‡]
[4] *School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China*
[5] *Collaborative Innovation Center of Quantum Matter, Beijing 100871, China*
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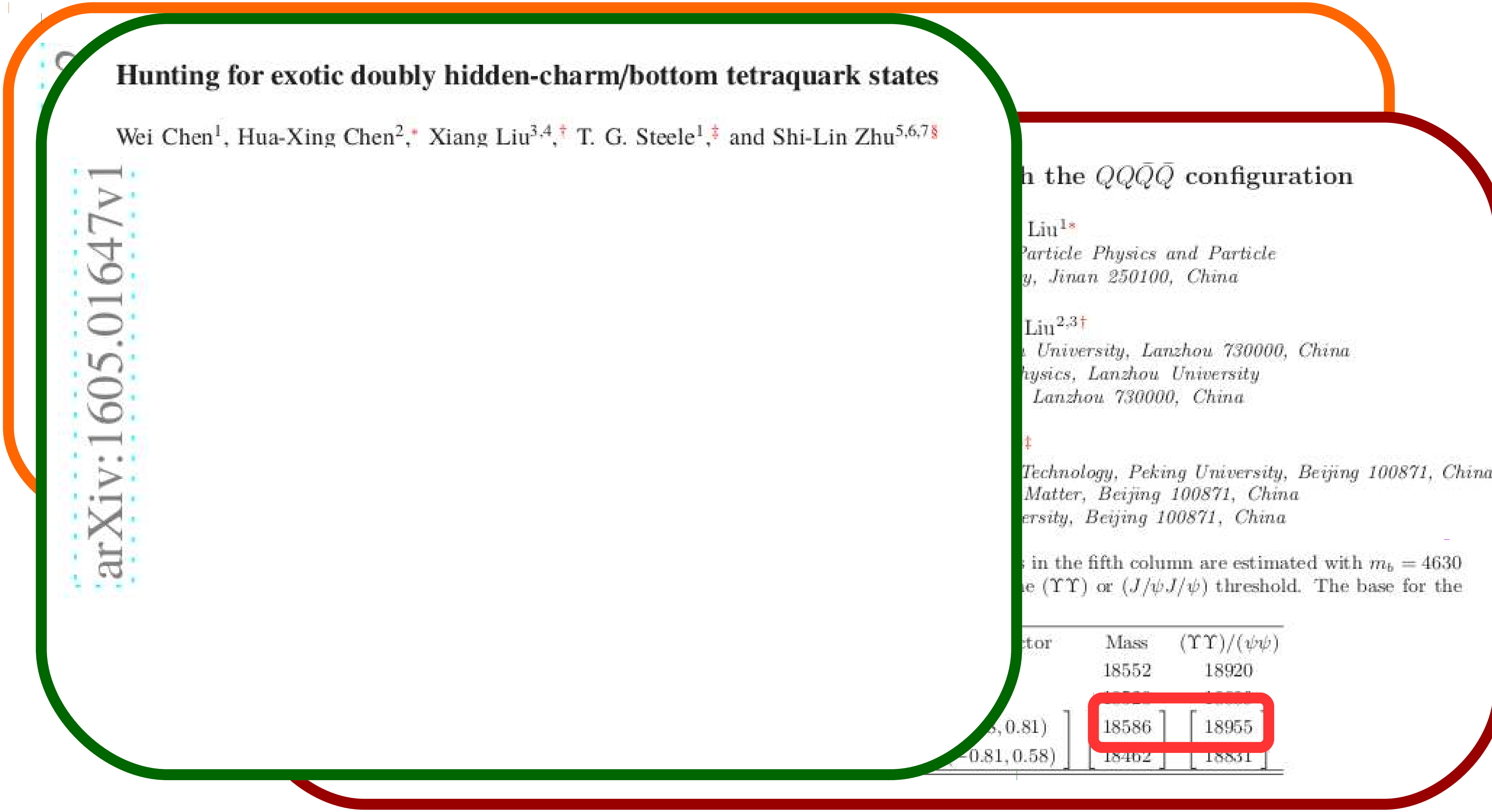
TABLE I: Results for the $bb\bar{b}\bar{b}$ and $cc\bar{c}\bar{c}$ systems in units of MeV. The masses in the fifth column are estimated with $m_b = 4630$ MeV and $m_c = 1430$ MeV. The last column lists masses estimated from the $(\Upsilon\Upsilon)$ or $(J/\psi J/\psi)$ threshold. The base for the $J = 0$ case is $(\phi_2\chi_3, \phi_1\chi_6)^T$.

System	J^{PC}	$\langle H_{CM}\rangle$	Eigenvalue	Eigenvector	Mass	$(\Upsilon\Upsilon)/(\psi\psi)$
	2^{++}	32.0	32.0	1	18552	18920
$(bb\bar{b}\bar{b})$	1^{+-}	0.0	0.0	1	[illegible]	[illegible]
	0^{++}	$\begin{pmatrix} -16.0 & 58.8 \\ 58.8 & 24.0 \end{pmatrix}$	$\begin{bmatrix} 66.1 \\ -58.1 \end{bmatrix}$	$\begin{bmatrix} (0.58, 0.81) \\ (-0.81, 0.58) \end{bmatrix}$	$\begin{bmatrix} 18586 \\ 18462 \end{bmatrix}$	$\begin{bmatrix} 18955 \\ 18831 \end{bmatrix}$

Why bother??



Hunting for exotic doubly hidden-charm/bottom tetraquark states

Wei Chen[1], Hua-Xing Chen[2,*] Xiang Liu[3,4,†] T. G. Steele[1,‡] and Shi-Lin Zhu[5,6,7§]

arXiv:1605.01647v1

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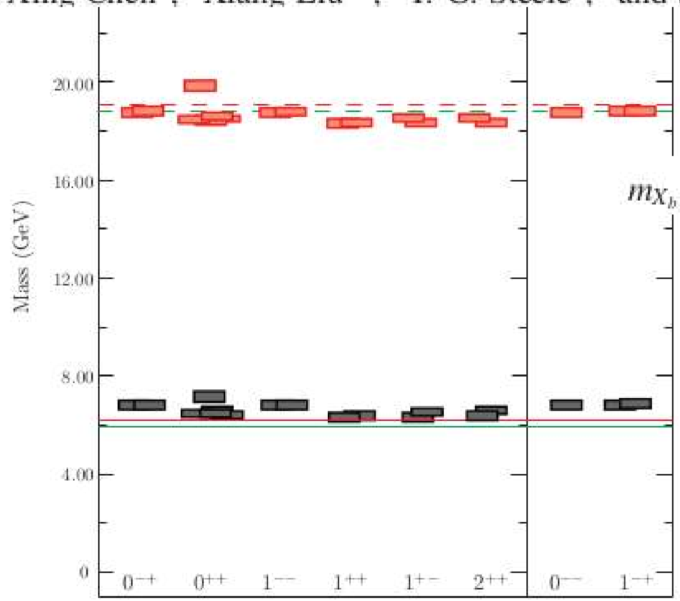

$m_{X_b} = (18.45 \pm 0.15)$ GeV

FIG. 3: Summary of the doubly hidden-charm/bottom tetraquark spectra labelled by J^{PC}. The red and black rectangles are the masses of the $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$ states, respectively. The vertical size of the rectangle represents the uncertainty of our calculation. The green and red solid (dashed) lines indicate the two-charmonium (bottomonium) thresholds $\eta_c(1S)\eta_c(1S)$ ($\eta_b(1S)\eta_b(1S)$) and $J/\psi J/\psi$ ($\Upsilon(1S)\Upsilon(1S)$), respectively.

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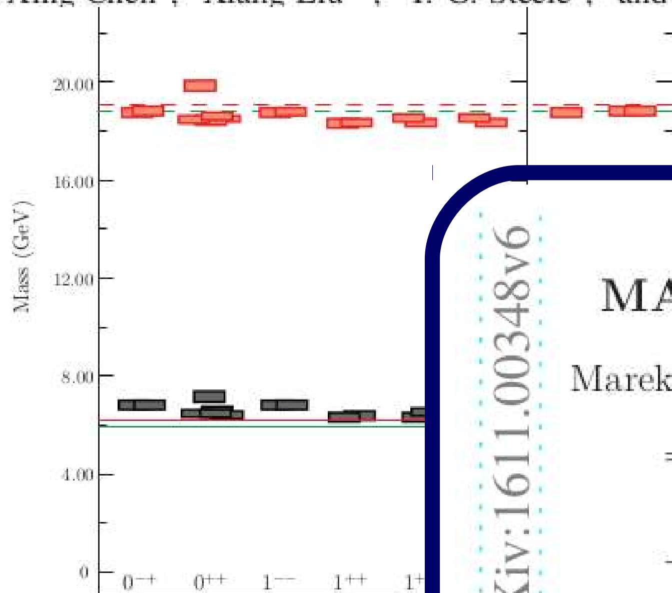

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...Liu[1*]
...Particle Physics and Particle

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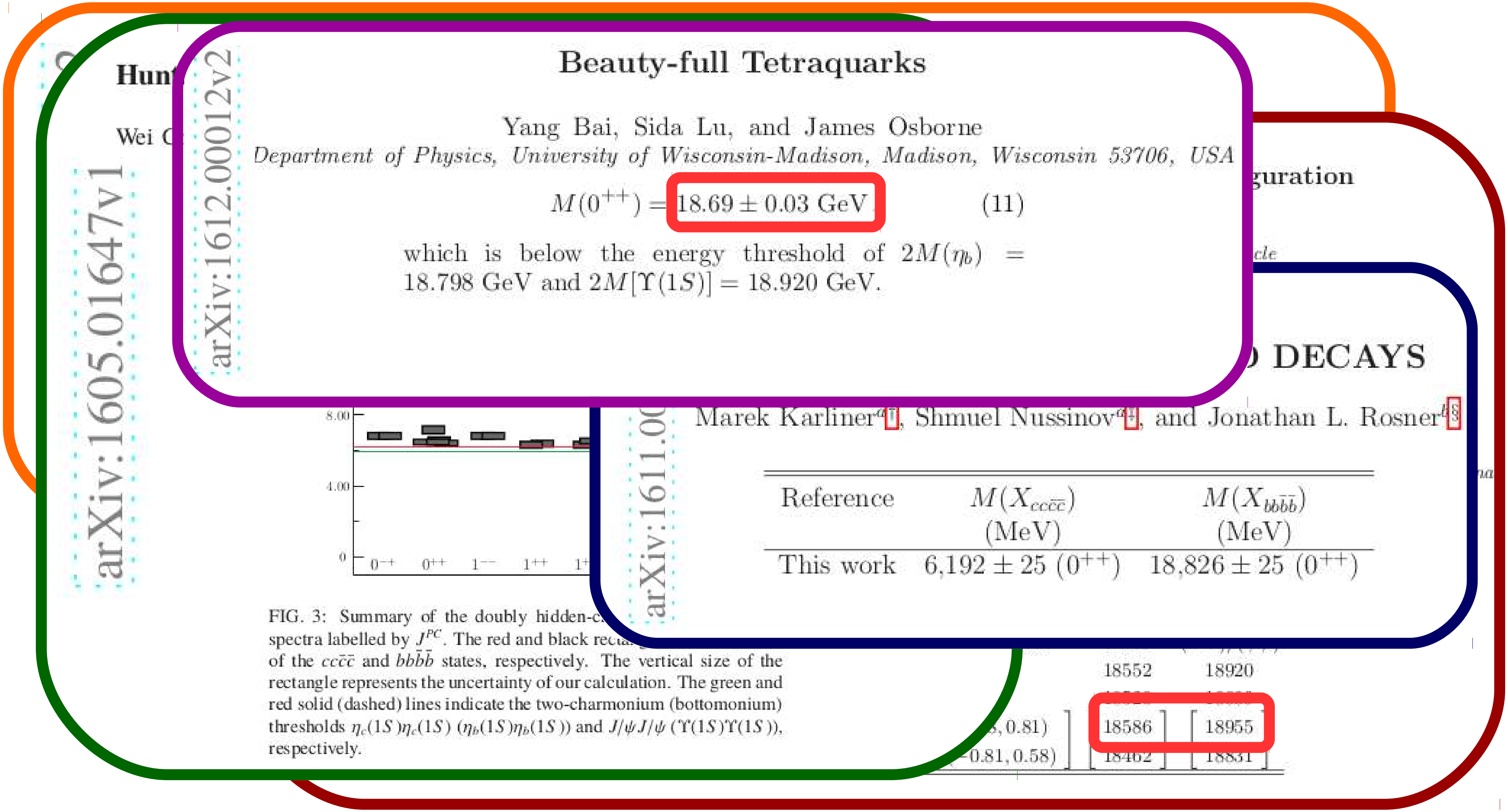
arXiv:1611.00348v6

$QQ\bar{Q}\bar{Q}$ STATES: MASSES, PRODUCTION, AND DECAYS

Marek Karliner[a†], Shmuel Nussinov[a‡], and Jonathan L. Rosner[b§]

Reference	$M(X_{cc\bar{c}\bar{c}})$ (MeV)	$M(X_{bb\bar{b}\bar{b}})$ (MeV)
This work	$6{,}192 \pm 25$ (0^{++})	$18{,}826 \pm 25$ (0^{++})

Why bother??


arXiv:1605.01647v1
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Wei C
8.00
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0^{-+} 0^{++} 1^{--} 1^{++} 1^{+}
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respectively.
arXiv:1612.00012v2
Beauty-full Tetraquarks
Yang Bai, Sida Lu, and James Osborne
Department of Physics, University of Wisconsin-Madison, Madison, Wisconsin 53706, USA
$M(0^{++}) = 18.69 \pm 0.03$ GeV (11)
which is below the energy threshold of $2M(\eta_b) =$ 18.798 GeV and $2M[\Upsilon(1S)] = 18.920$ GeV.
guration
cle
arXiv:1611.00
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But is it strong enough to bind $\bar{\mathrm{b}}\bar{\mathrm{b}}\mathrm{bb}$ into a stable-state below the $2\eta_{\mathrm{b}}$ threshold??

The One Question:

Does a $\bar{b}\bar{b}bb$ bound-state exist below $2\eta_b$ threshold?

- N.B.: Only interested in asking if there exists a stable (in QCD) state below-threshold: Not interested in quantifying finite-volume interactions at/above threshold.

Our Lattice Calculation

- We use the Highly-Improved Non-Relativistic QCD (NRQCD) methodology for b-quarks
- Four MILC $2+1+1$ ensembles (one with physical pions) and $a_{fm} \approx 0.12 \to 0.06$ fm
- Disconnected diagrams are OZI suppressed for heavy-quarks: Not Included

A Quick Glance: Interpolating Operators

- Two-meson (infinite-volume continuum) eigenstates look like:

$$|\hat{P}_{tot}, J^{PC}; \hat{k}_{rel}, {}^{2S_1+1}L_{1\,J_1}, {}^{2S_2+1}L_{2\,J_2}, L_{rel}\rangle$$

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- => $L_{rel} = 0$ as relative orbital angular momentum would increase internal energy. Similarly $L_1 = L_2 = 0$.
- =>$|\hat{P}_{tot} = 0, J^{PC}; \hat{k}_{rel}, {}^{2S_1+1}L_{1\,J_1}, {}^{2S_2+1}L_{2\,J_2}, L_{rel} = 0\rangle$

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- N.B., even though we create states with total momentum are rest, $\hat{P}_{tot} = 0$, states with back to back momenta, $\eta_b(\hat{k})\eta_b(-\hat{k})$,still contribute to correlator. We will come back to this!

Tetraquark S-wave Colour/Spin Operators

- S-wave tetraquark building blocks: $\bar{q}q \sim \eta_b = 0^{-+}, \Upsilon = 1^{--}$

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N.B., Due to fermions being identical, the Pauli exclusion principal =>
S-wave $\bar{3}(6)$ diquark can only be spin-triplet (spin-singlet). We study all systems of S-wave $\bar{b}\bar{b}bb$ at rest that can be made from diquarks

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- We can look for $\mathbf{0^{++}, 1^{+-}, 2^{++}}$ $\bar{b}\bar{b}bb$ tetraquarks

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- Interpolating Operators

$\mathbf{0}^{++}$	$\mathbf{1}^{+-}$	$\mathbf{2}^{++}$
$O^{0^{++}}_{\eta_b\eta_b}$	$O^{1^{+-}}_{\Upsilon\eta_b}$	$EO^{2^{++}}_{\Upsilon\Upsilon}$
$O^{0^{++}}_{\Upsilon\Upsilon}$	$O^{1^{+-}}_{D\bar{3}A3}$	$T_2O^{2^{++}}_{\Upsilon\Upsilon}$
$O^{0^{++}}_{D\bar{3}A3}$		$EO^{2^{++}}_{D\bar{3}A3}$
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Two-mesons correlators have four connected Wick-contractions (come back to this).

Diquark-AntiDiquark has one connected Wick-contraction (backup-slides).

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$O^{0^{++}}_{D6A\bar{6}}$		$T_2O^{2^{++}}_{D\bar{3}A3}$

- We include all possible local colour/spin operators and ensure Fierz relations are satisfied:

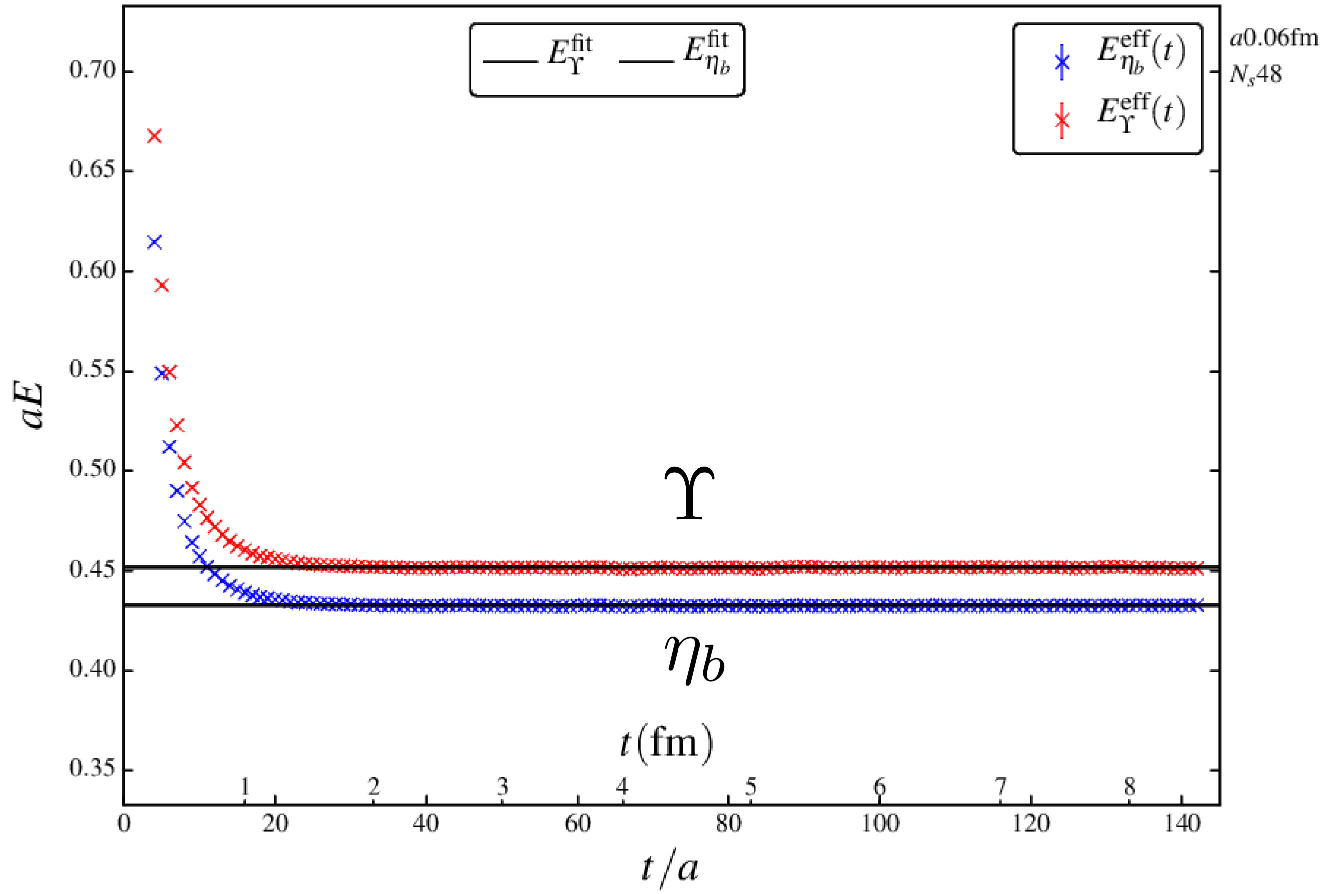
J^{PC}	Diquark-AntiDiquark	2Meson
0^{++}	$\bar{3}\times 3$	$-\frac{1}{2}\vert 0;\Upsilon\Upsilon\rangle + \frac{\sqrt{3}}{2}\vert 0;\eta_b\eta_b\rangle$
0^{++}	$6\times\bar{6}$	$\frac{\sqrt{3}}{2}\vert 0;\Upsilon\Upsilon\rangle + \frac{1}{2}\vert 0;\eta_b\eta_b\rangle$
1^{+-}	$\bar{3}\times 3$	$\frac{1}{\sqrt{2}}\left(\vert 1;\Upsilon\eta_b\rangle + \vert 1;\eta_b\Upsilon\rangle\right)$
2^{++}	$\bar{3}\times 3$	$\vert 2;\Upsilon\Upsilon\rangle$

Finally: The $\mathbf{a \approx 0.06}$ fm data

- Effective masses are a way to see what is in the data. Any drop of the effective mass below $2\eta_b$ would be a clear indication of a new bound-state. However, we fit the correlator data to extract fitted energies.

Finally: The $\mathbf{a \approx 0.06}$ fm data

- What does the single-meson data look like?



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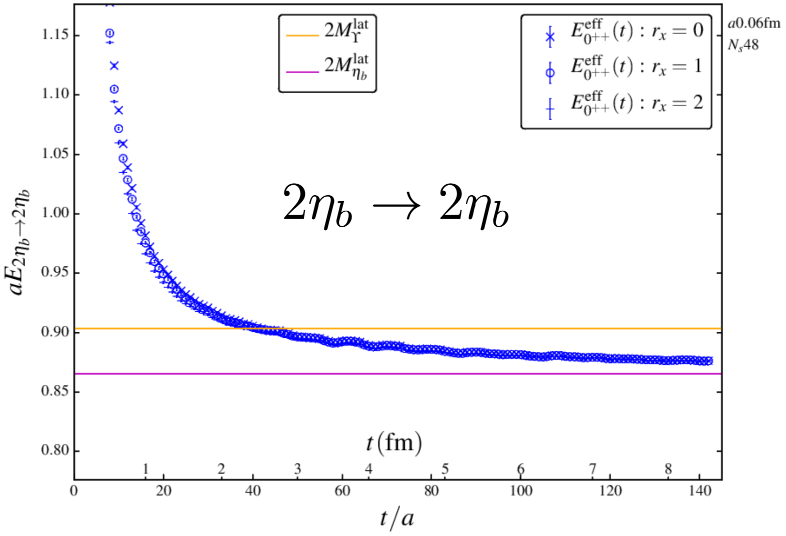

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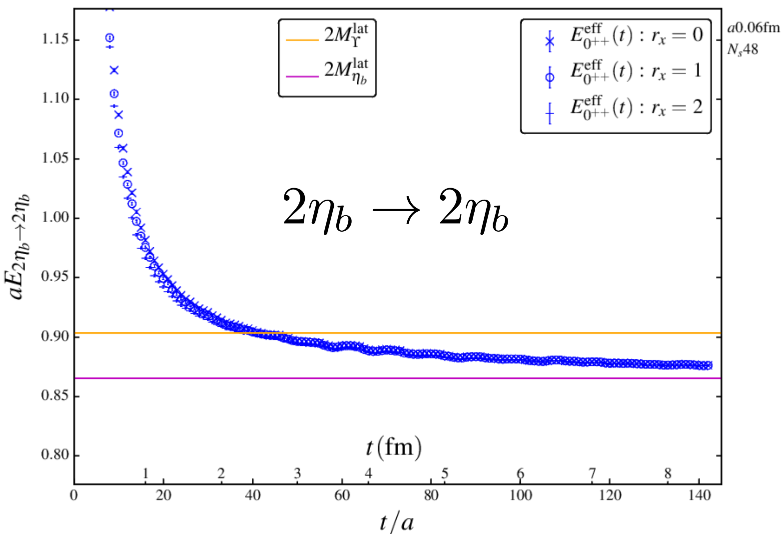

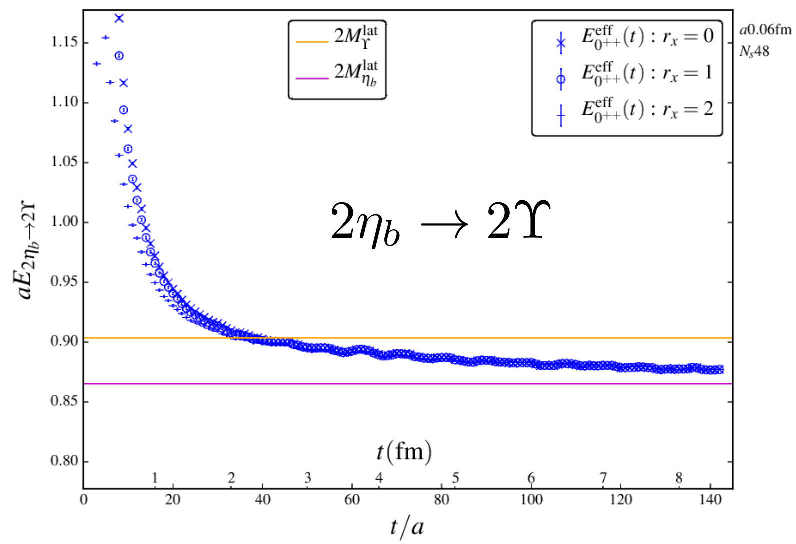

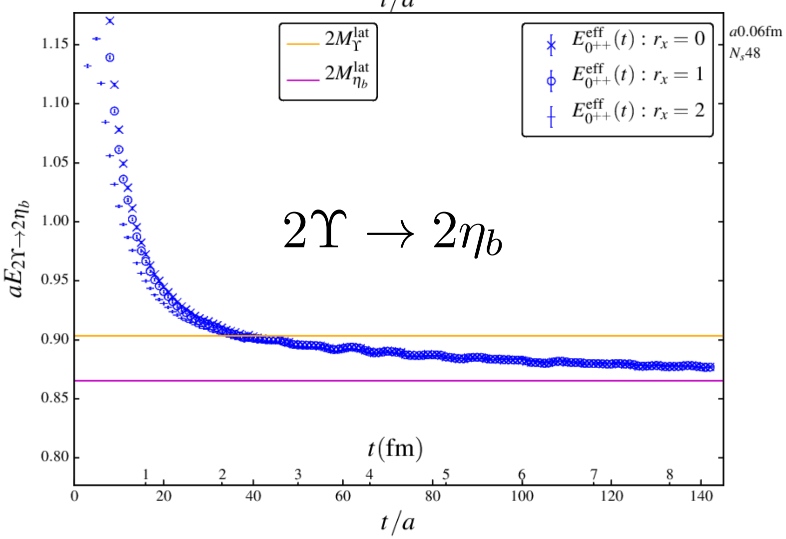

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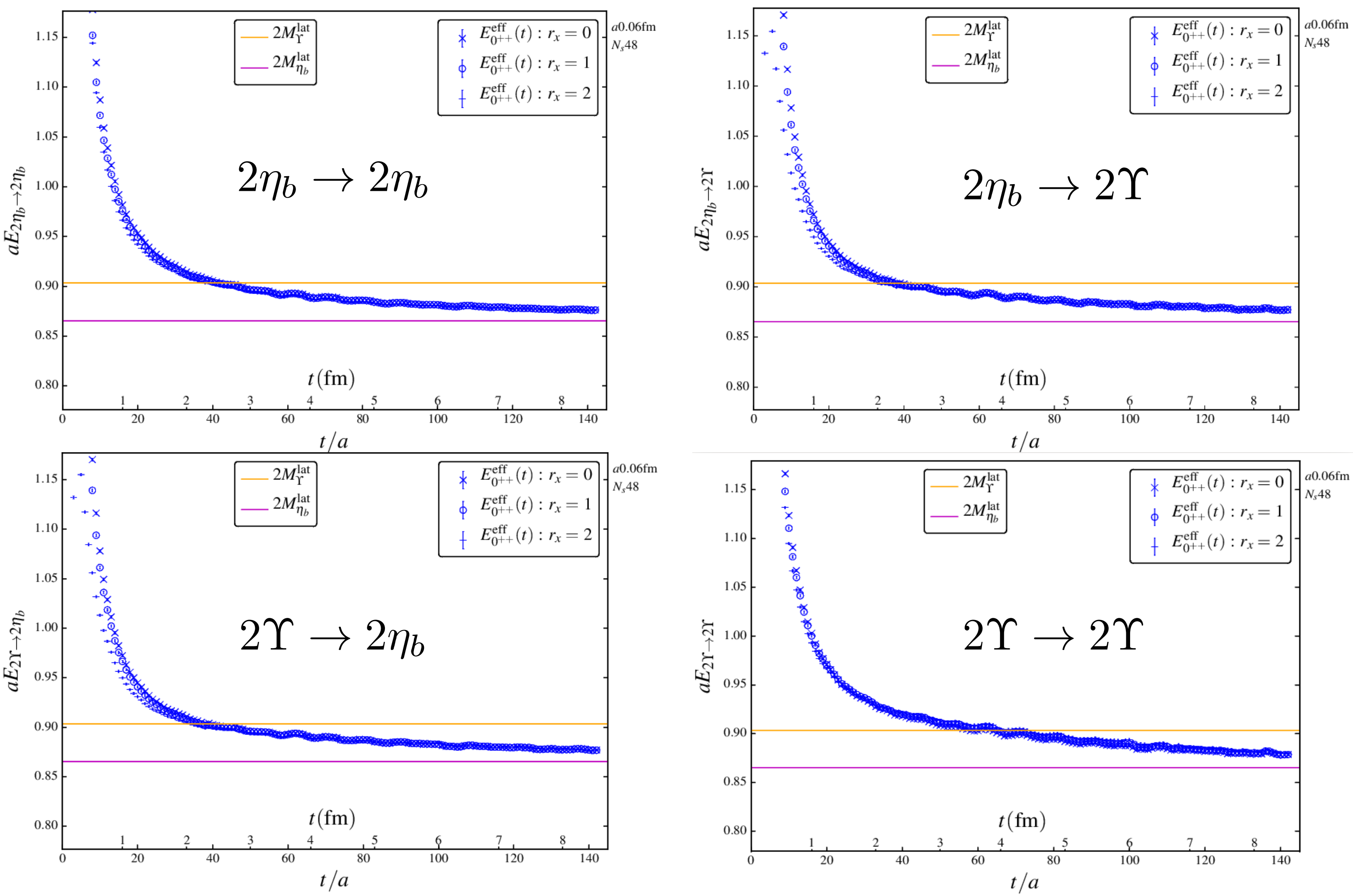

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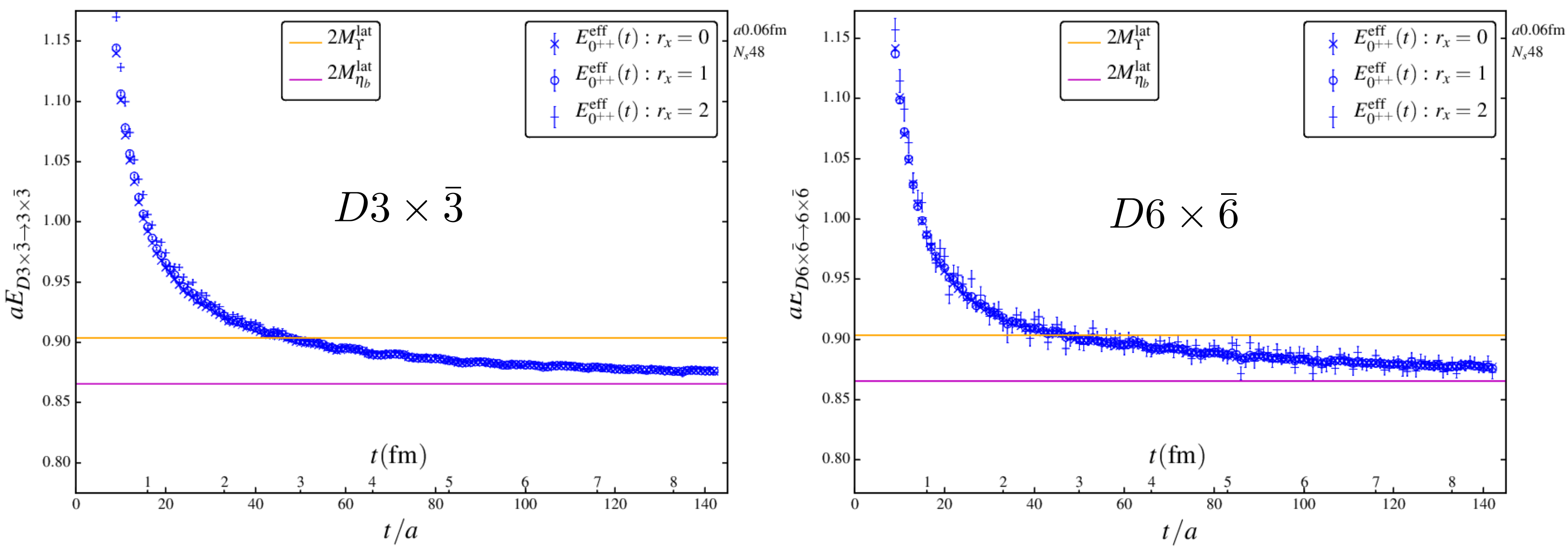

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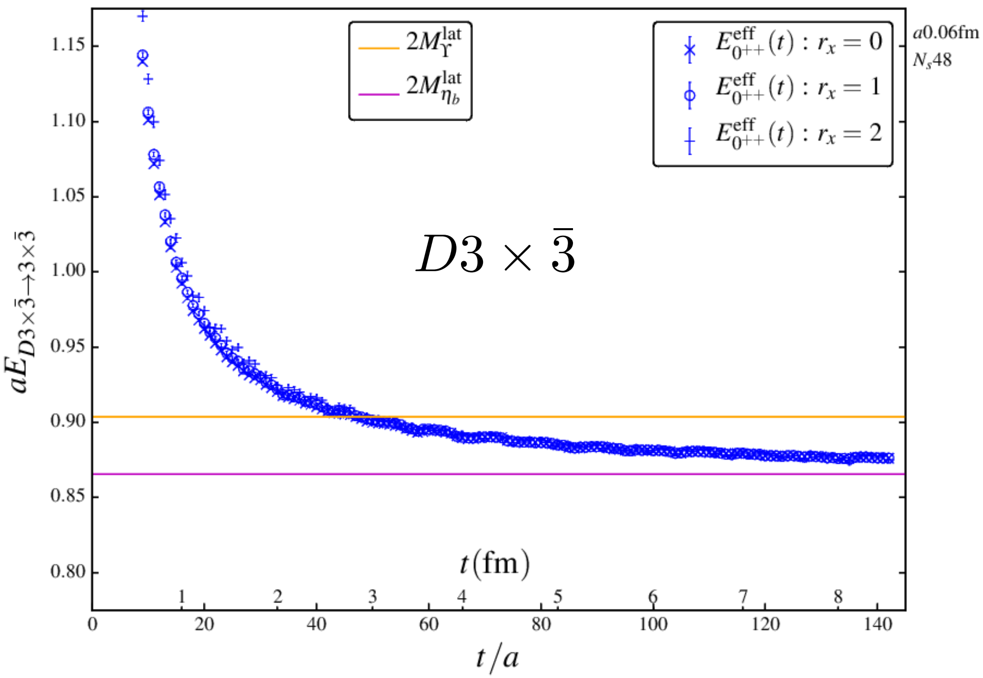

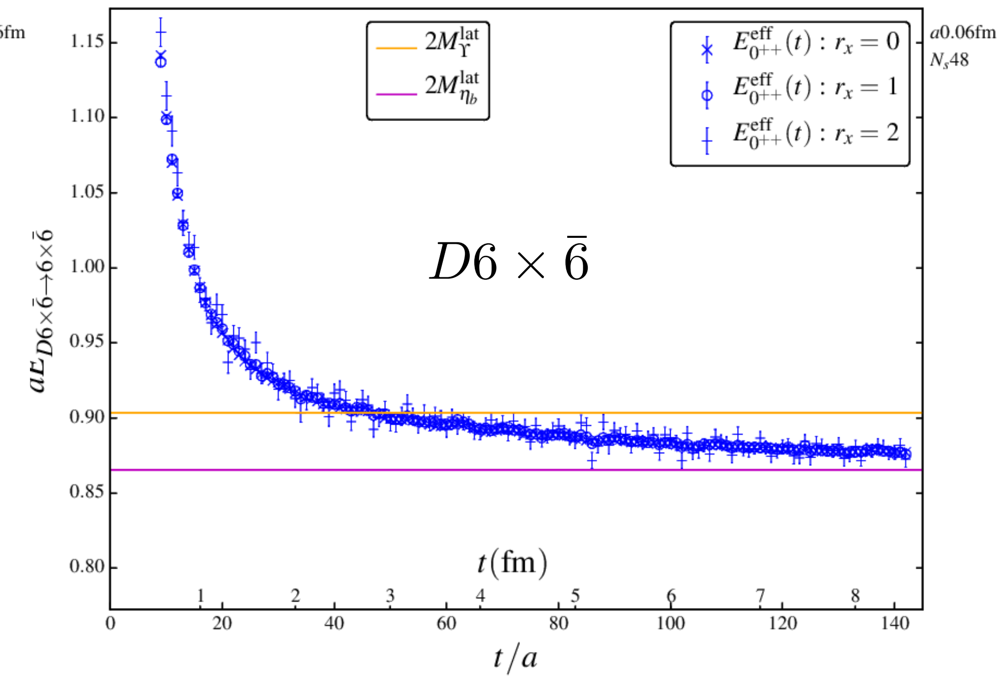

- Slow decay might make situation look unclear. Where does this slow decay come from?

Where does this slow decay come from?

- Operators overlap with the states with relative momenta back-to-back but overall momentum at rest. E.g.,: $|\hat{P}_{tot} = 0, 0^{++}; \eta_b(\hat{k})\eta_b(-\hat{k})\rangle$

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$$\sum_n \int d^3k Z_n(k) e^{-E_n(k)t}$$

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- The leading order non-rel. expansion of free two-meson energy:

$$E_n(k) = m'_n + \frac{|k|^2}{2m_n} \longleftrightarrow \frac{4\pi^2}{L^2 m_{\eta_b}} \sim \mathcal{O}(10MeV)!!!$$

- Within our statistical precision (and $\mathbf{t} > \mathbf{1}$ fm):

$$\sum_n \int d^3k Z_n(k) e^{-E_n(k)t} = \sum_n \left(\frac{\mathbf{m_n}}{\mathbf{2\pi t}}\right)^{\frac{3}{2}} Z_n^0 e^{-m'_n t} + \ldots$$

Will come back to FV interactions

Finally: The $\mathbf{a \approx 0.06}$ fm Data

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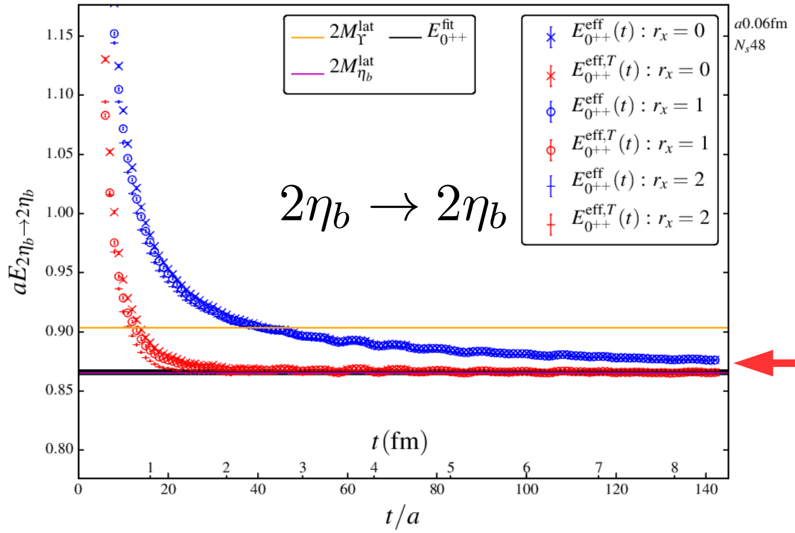

Shown by red curve

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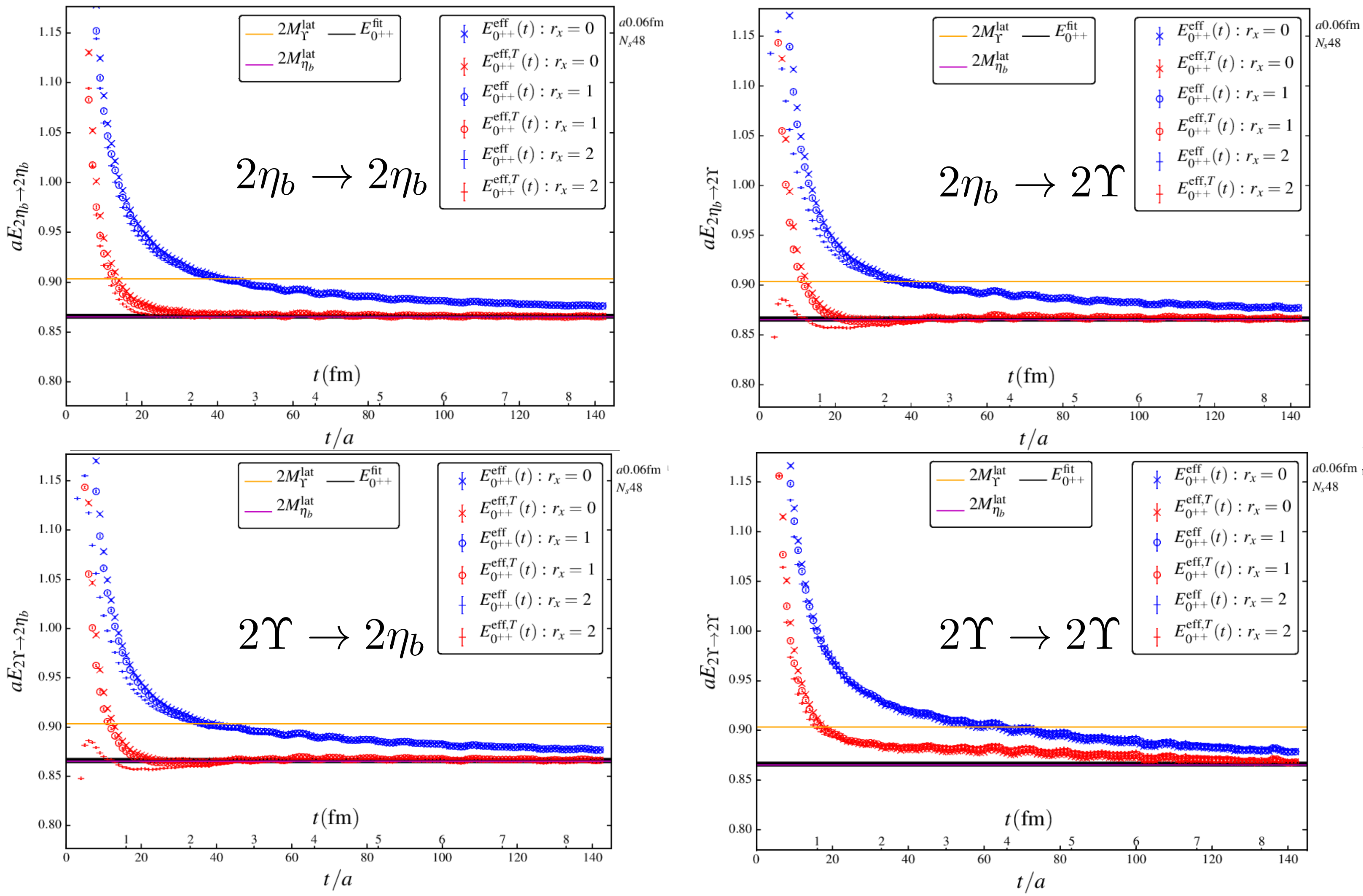

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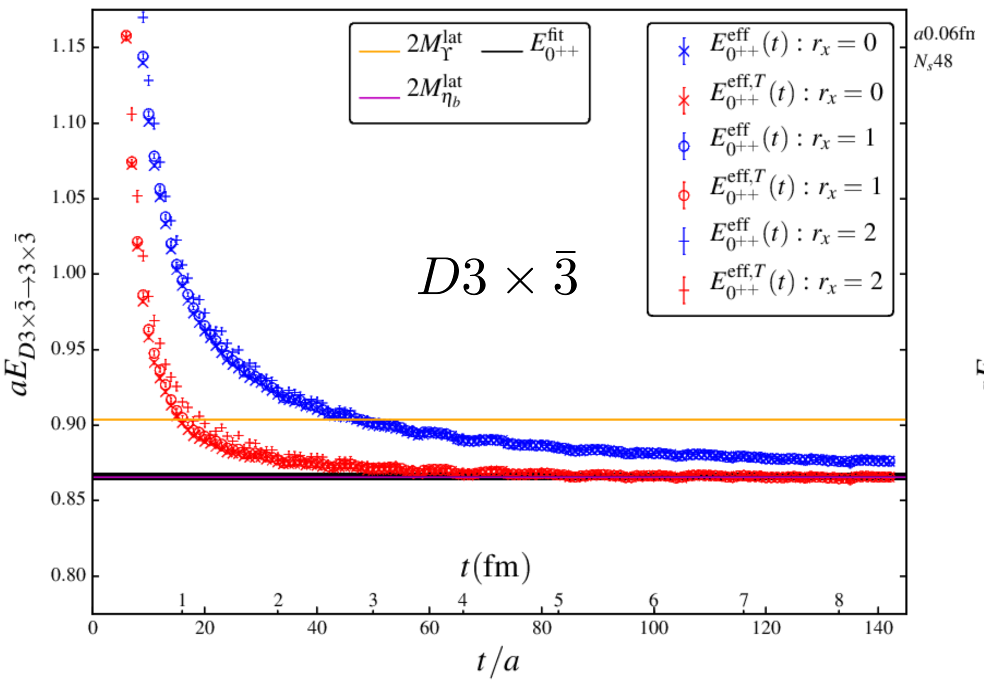
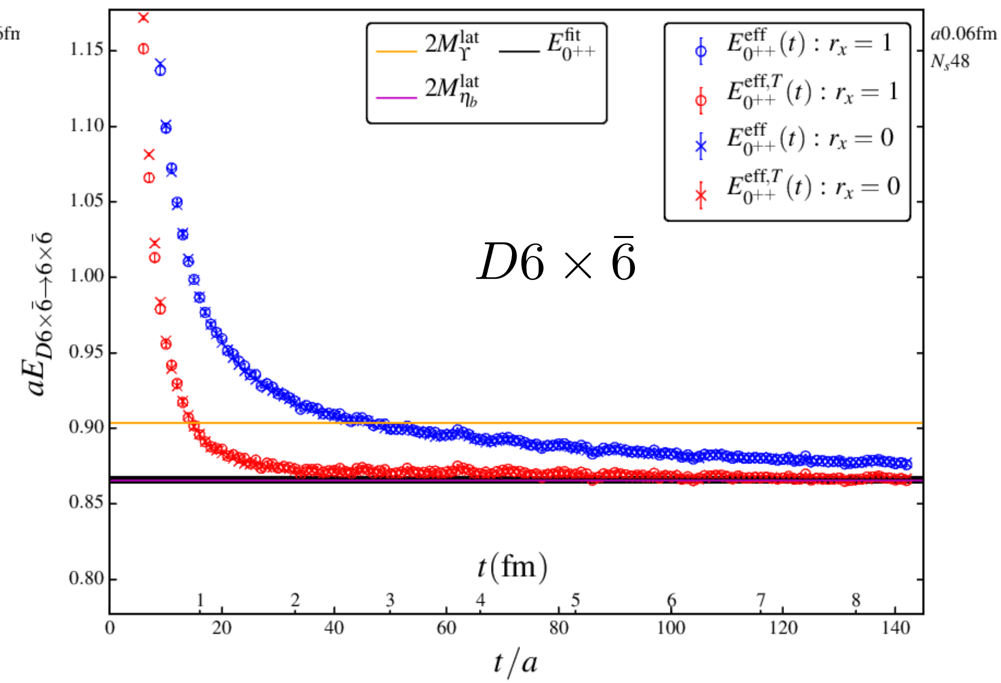
- And the 0^{++} Diquark-Antidiquark?





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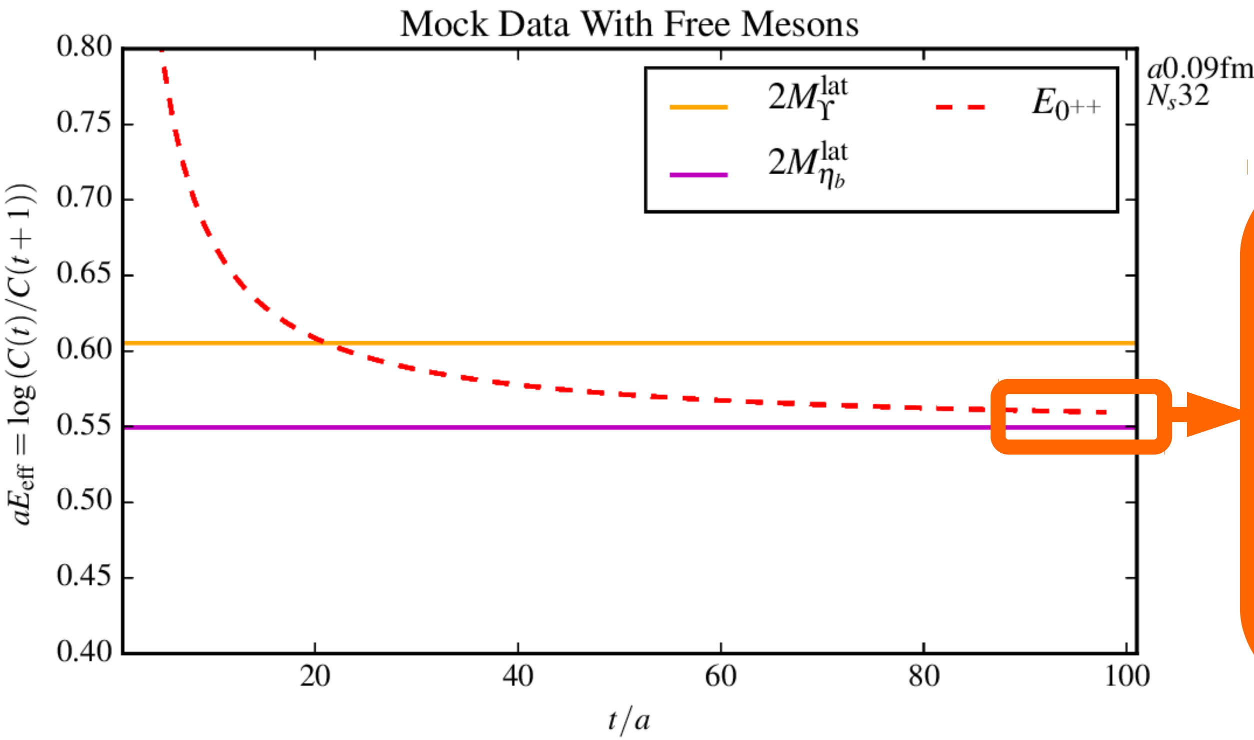

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- Is the long distance part of operators dominant?
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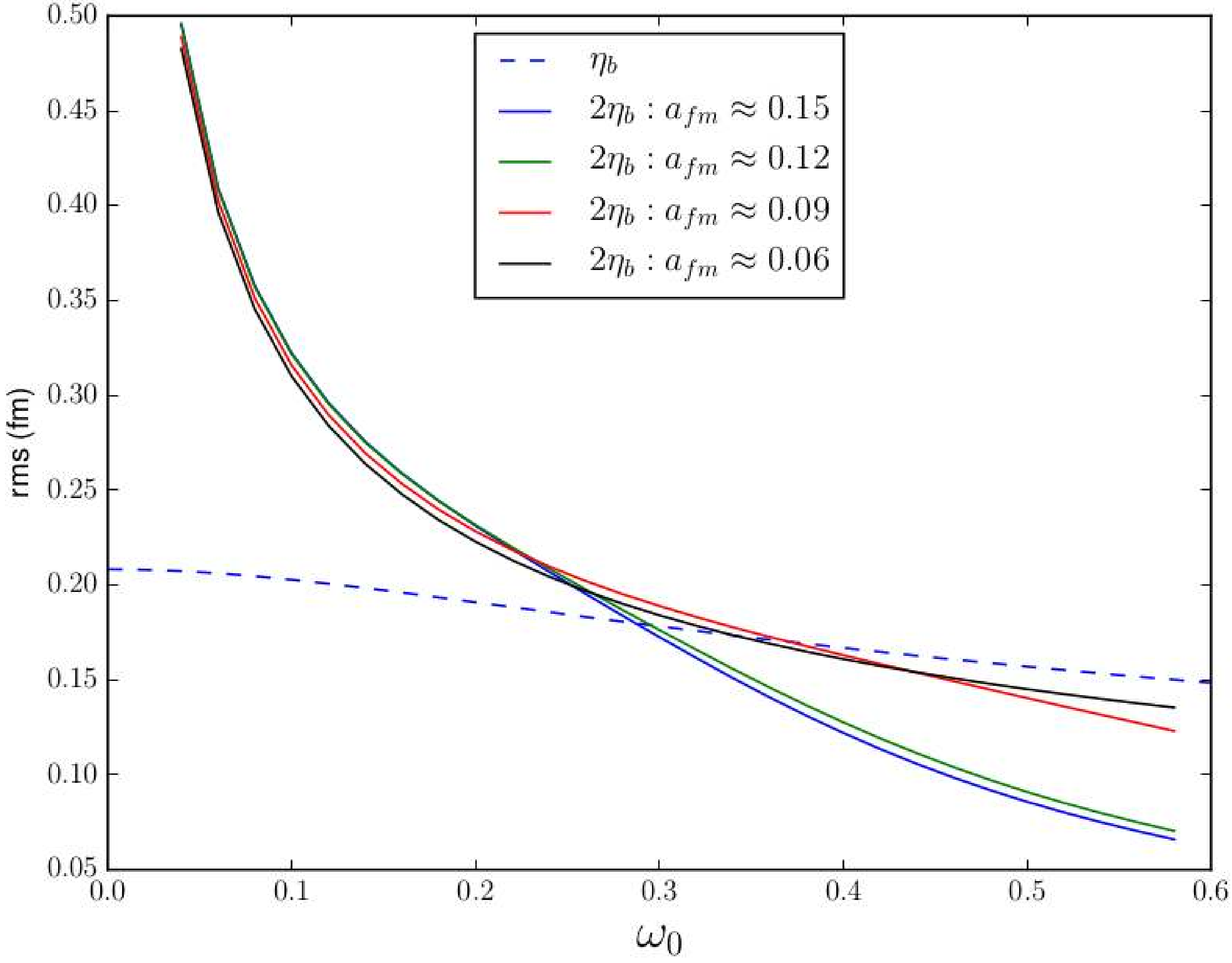
A Cute Trick

- Force quarks to be confined to a volume of our choice by introducing a harmonic oscillator potential into NRQCD Hamiltonian

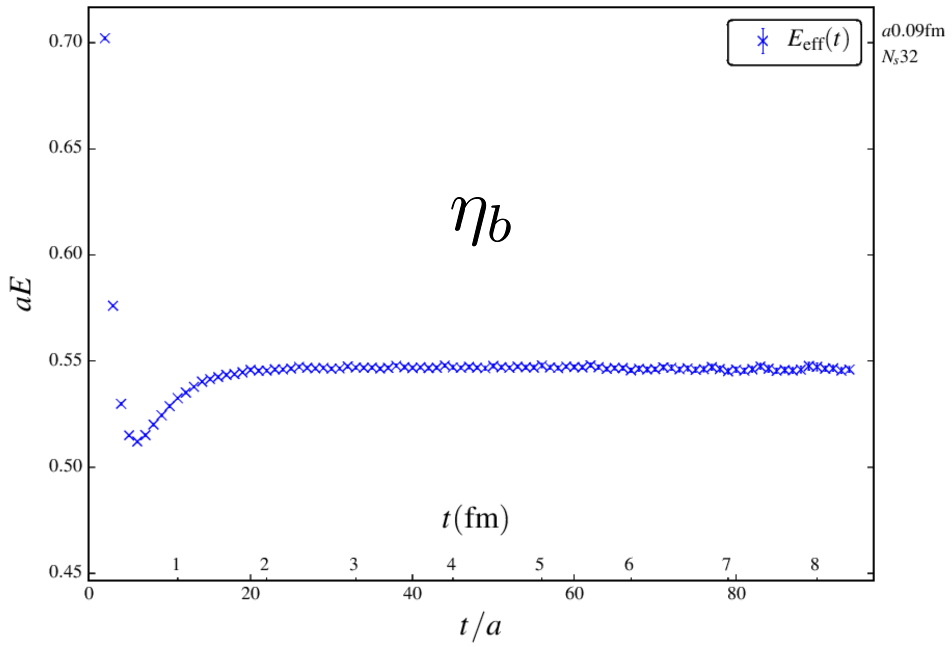
$$\frac{m\omega_0^2}{2}|\hat{x} - \hat{x_0}|^2$$

that pulls the quarks to x_0 with a coupling ω_0.

What Strengths (multiple) to use?



With Harmonic Oscillator: The $\mathbf{a_{fm} \approx 0.09}$ fm Data, $\omega_0 = \mathbf{0.35}$ GeV



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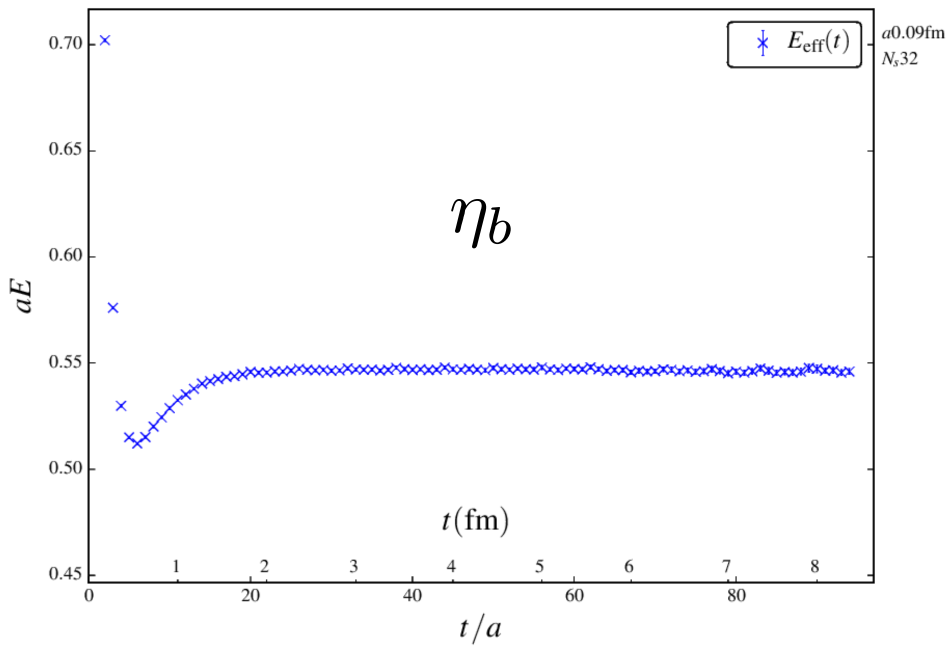
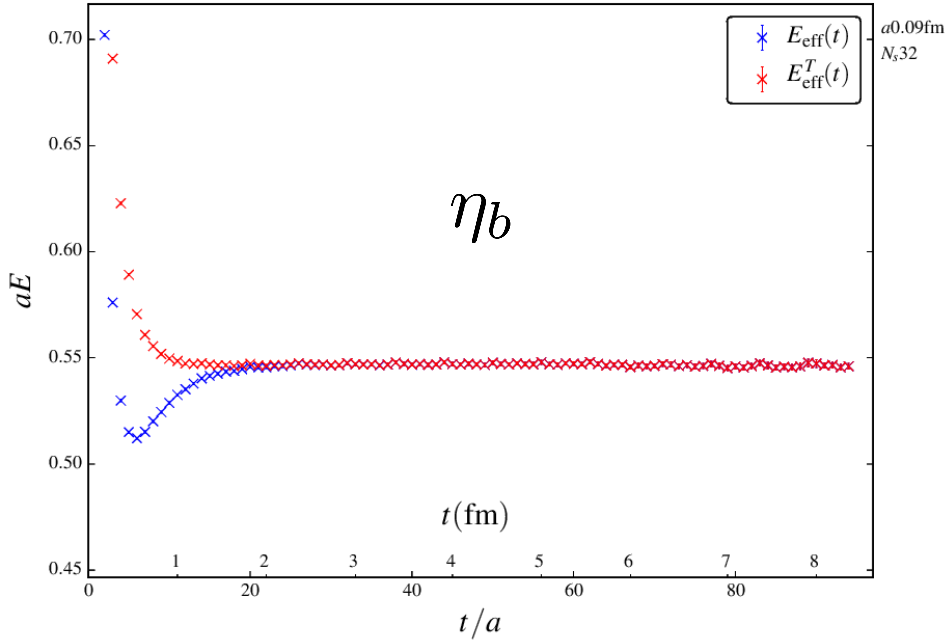


Dip due to dynamics. Fit function with H.O. looks like:

$$\sum_n Z_n^2 \left(\frac{1}{1 + e^{-2\omega_0 t}} \right) e^{-(m_n + \frac{3}{2}\omega_0)t}$$



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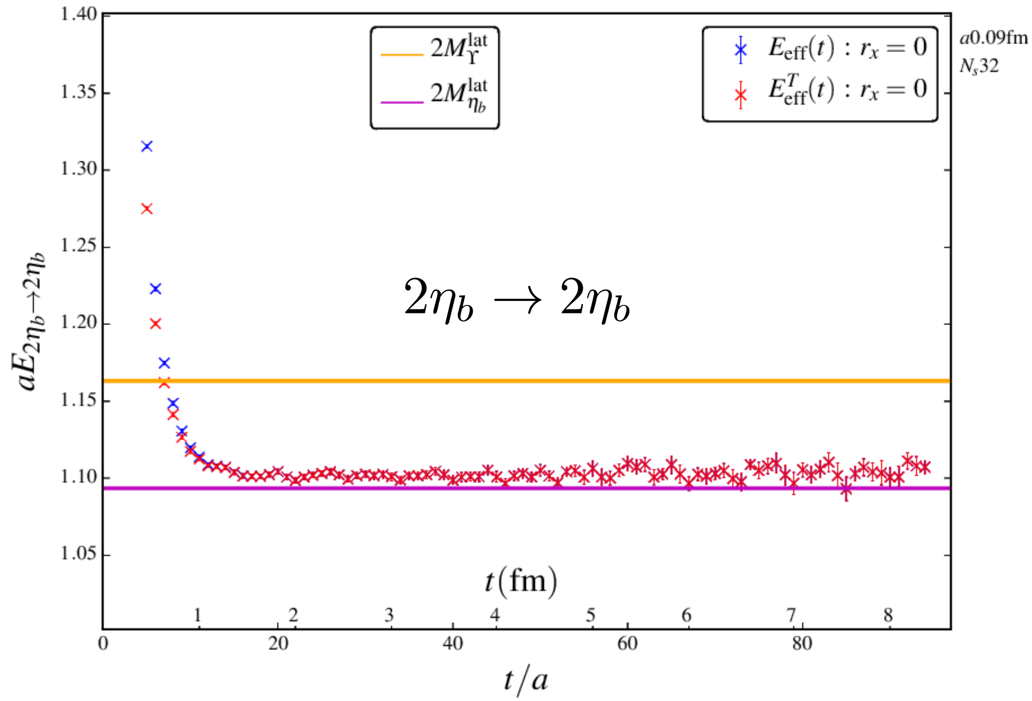

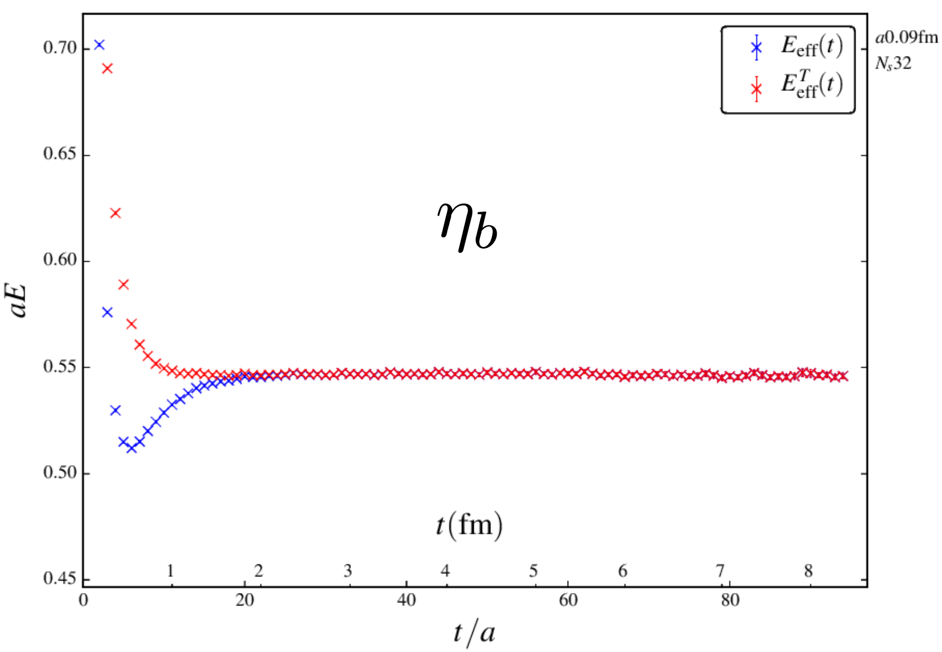

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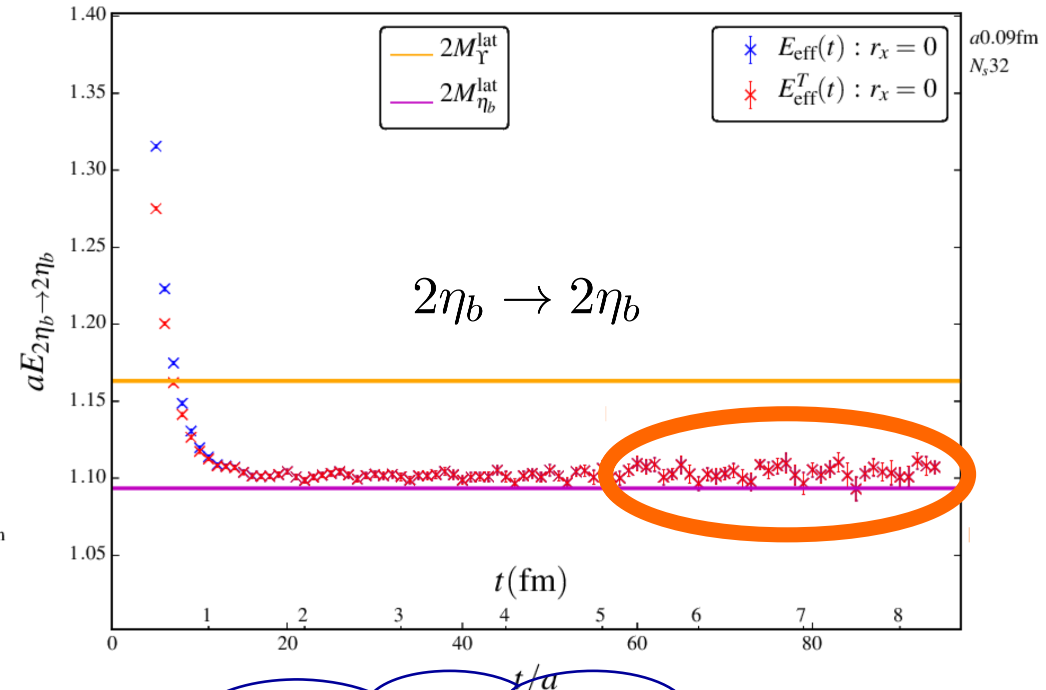
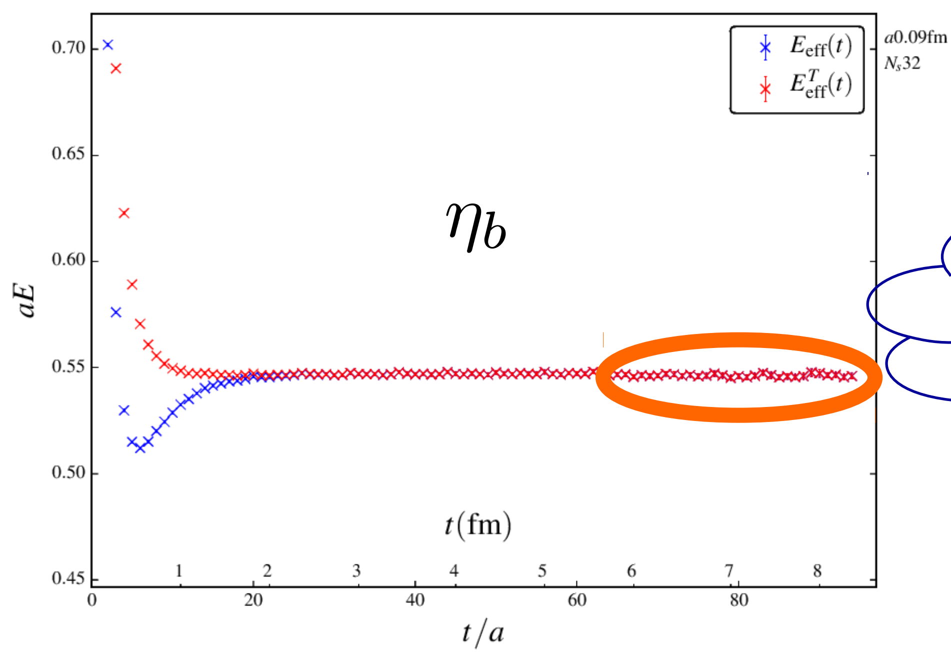




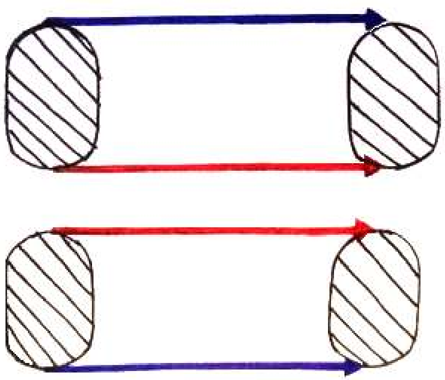
Where's this noise coming from?

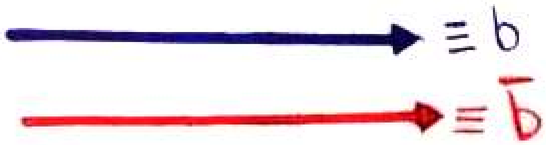
Two-Meson Connected Wick Contractions

- Two mesons with identical flavour have four connected Wick contractions:

M1

M2





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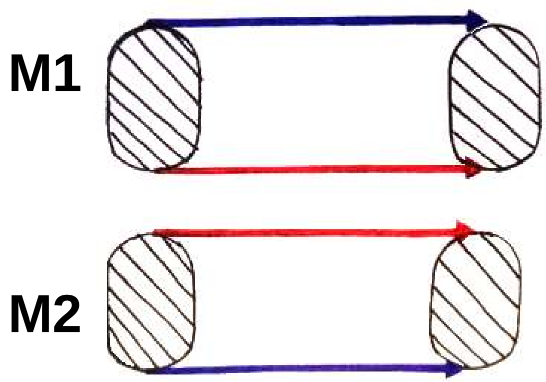

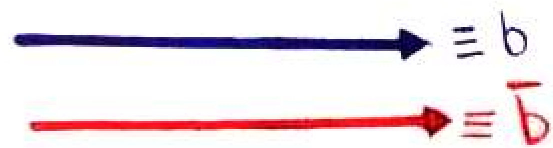

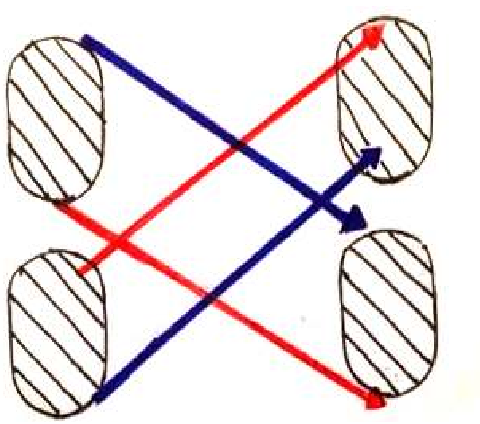

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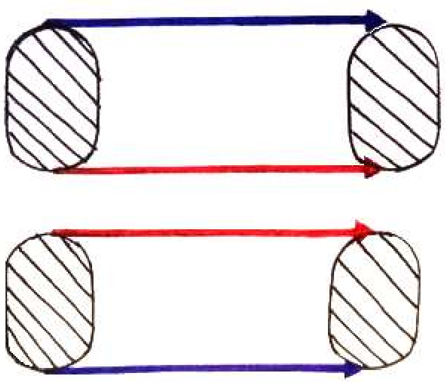

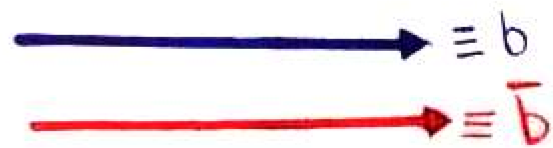

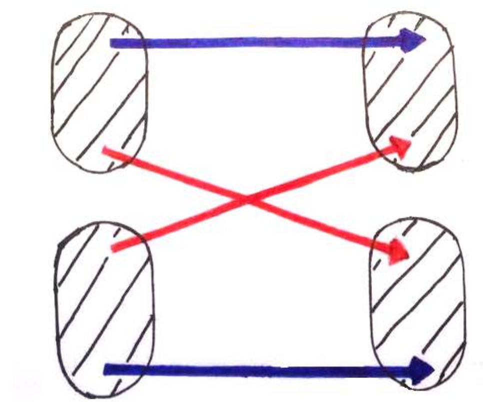

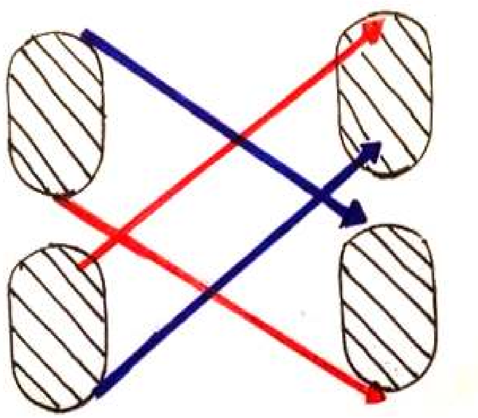

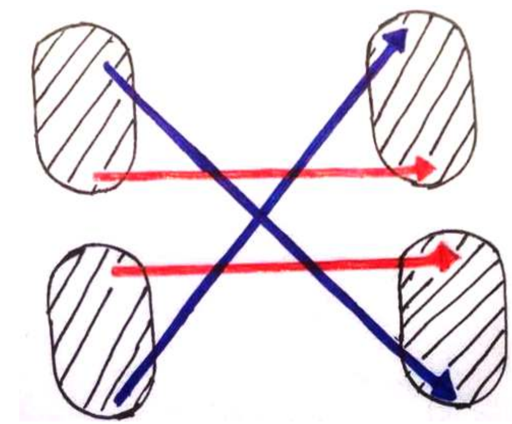

Why studying a single Wick contraction is a bad idea!

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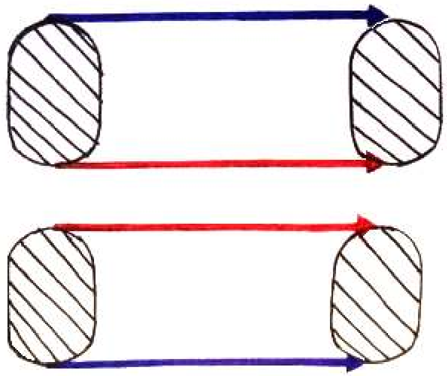

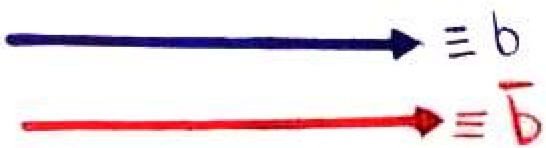

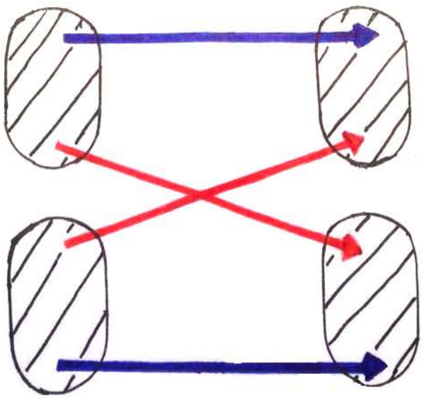

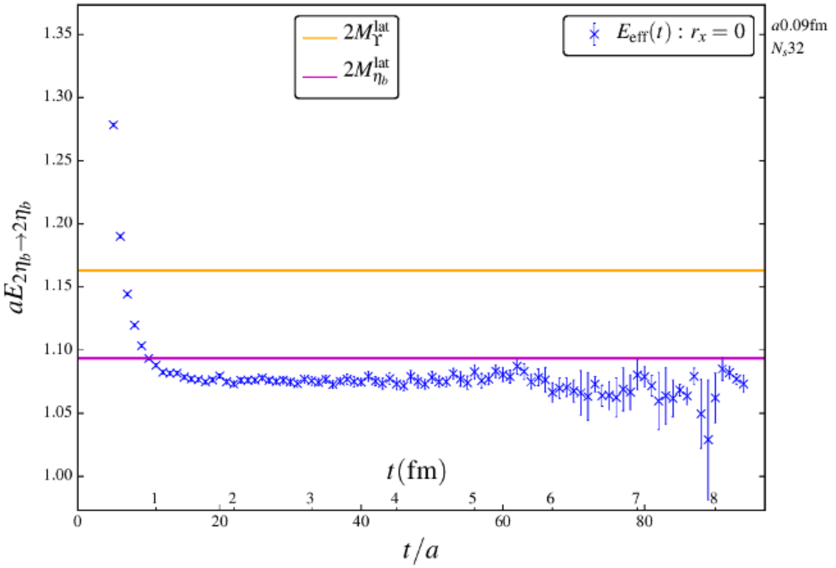

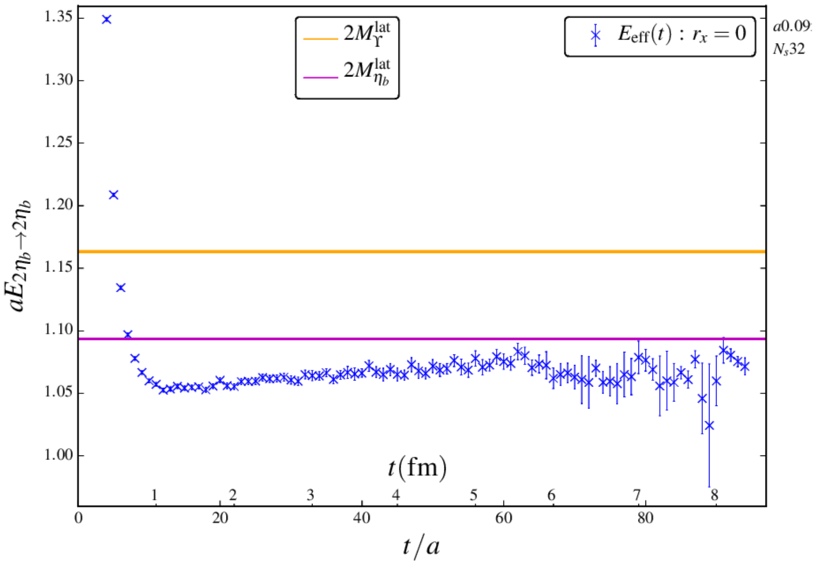

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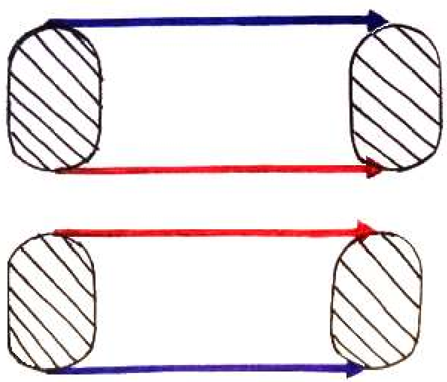

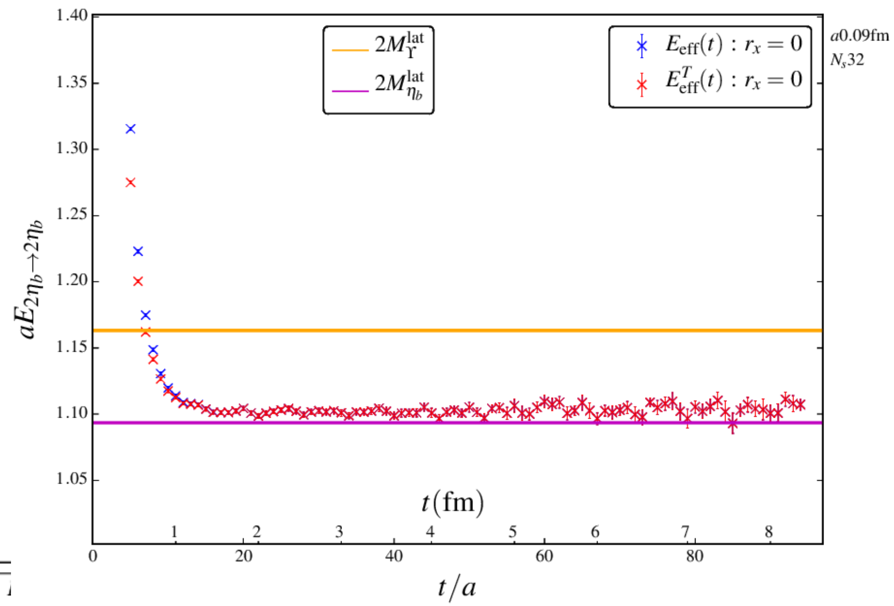

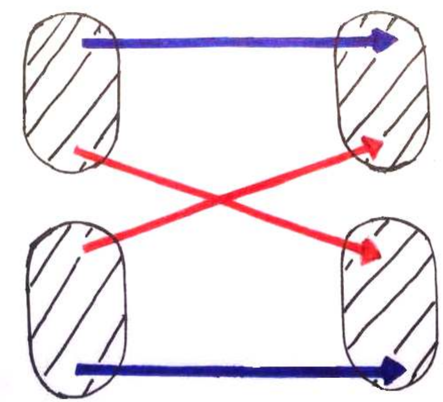

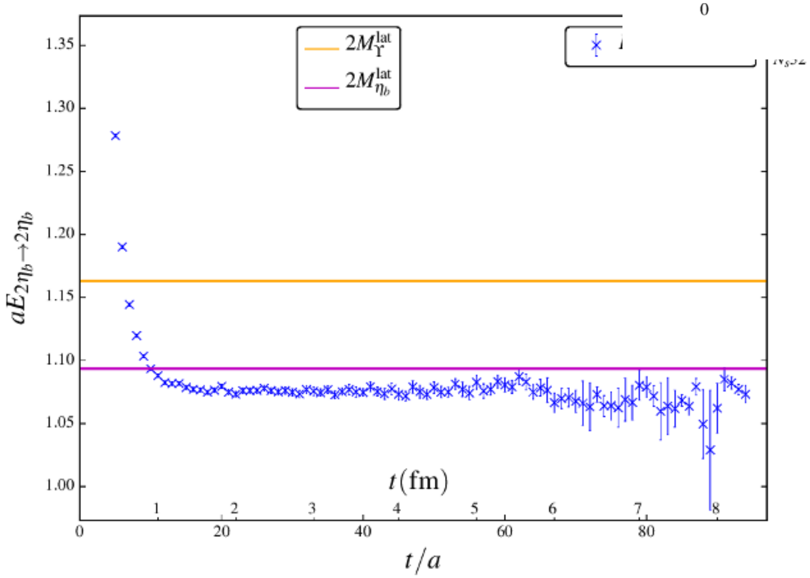

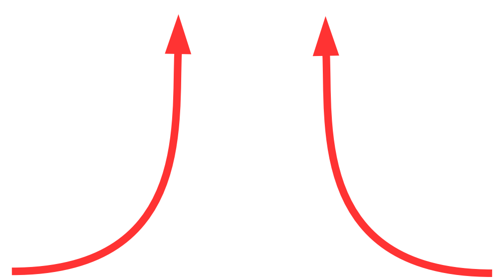

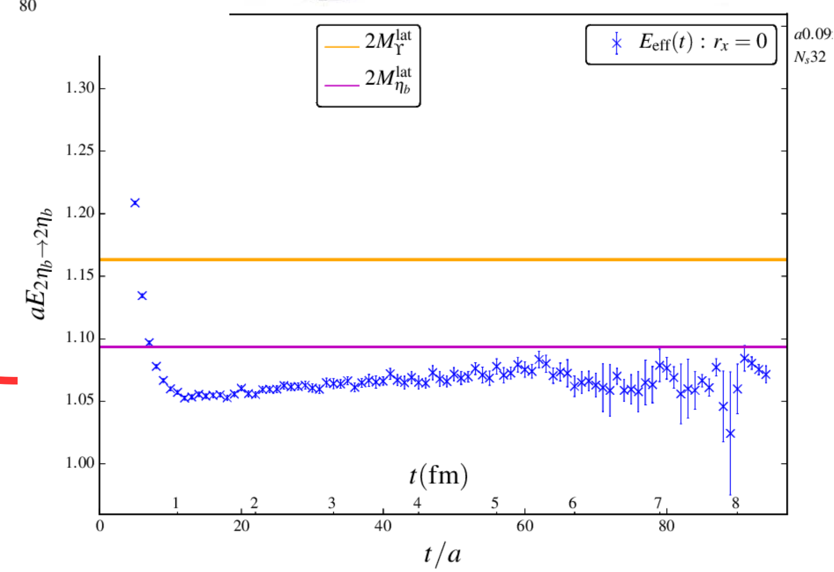

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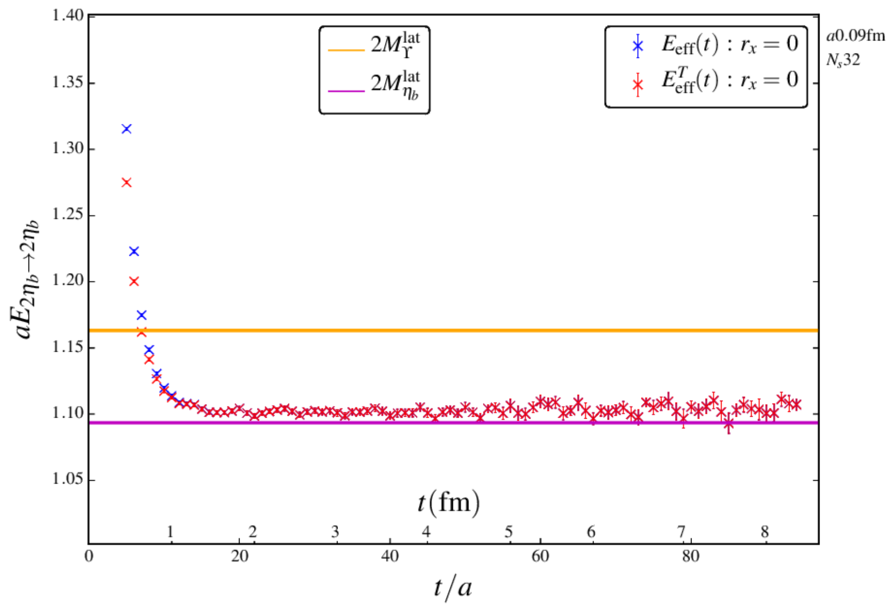

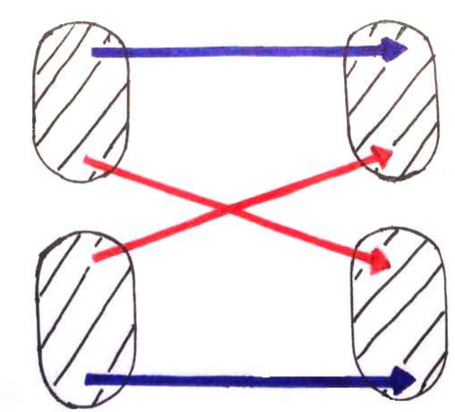

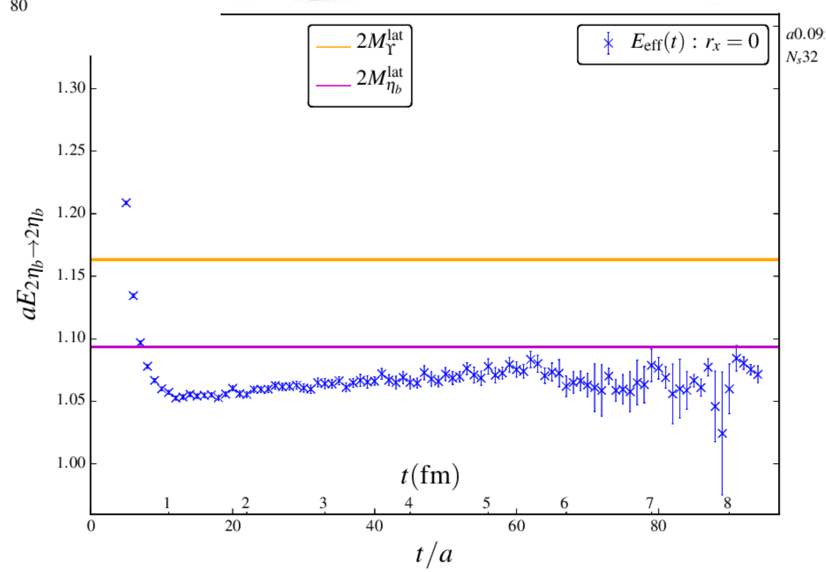

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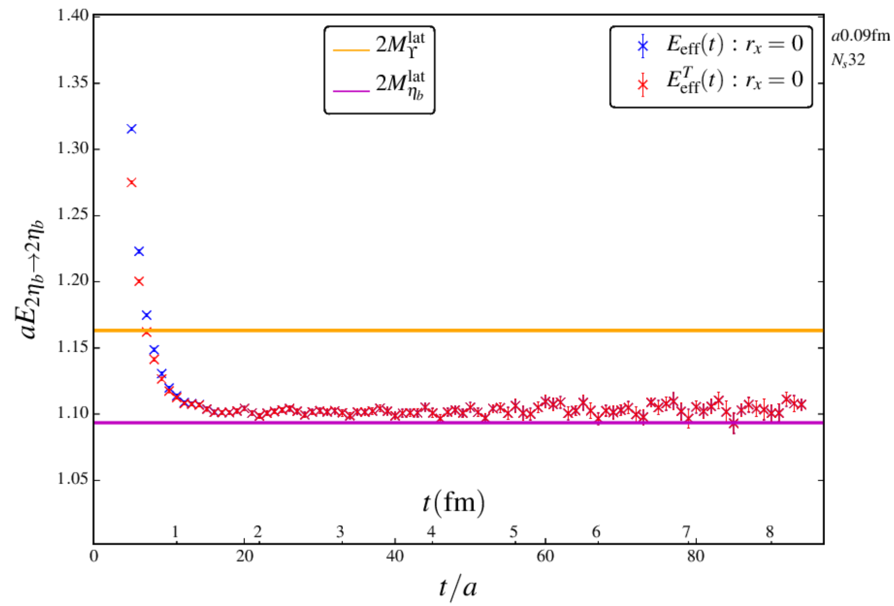

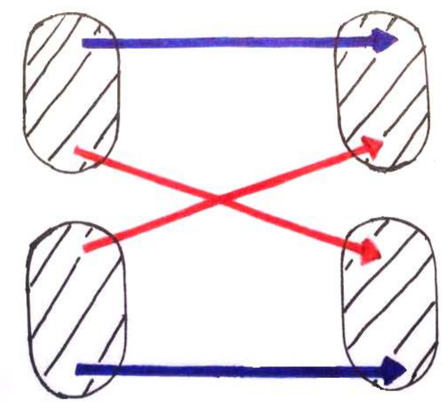

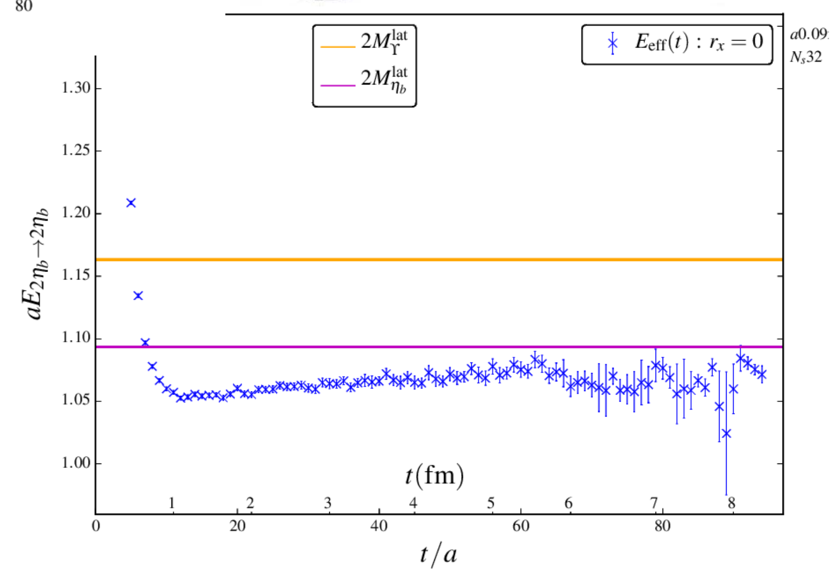

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“Does a $\bar{b}\bar{b}bb$ bound-state exist below $2\eta_b$ threshold?”

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- Questions!?! (chughes@fnal.gov)

Back Up Slides

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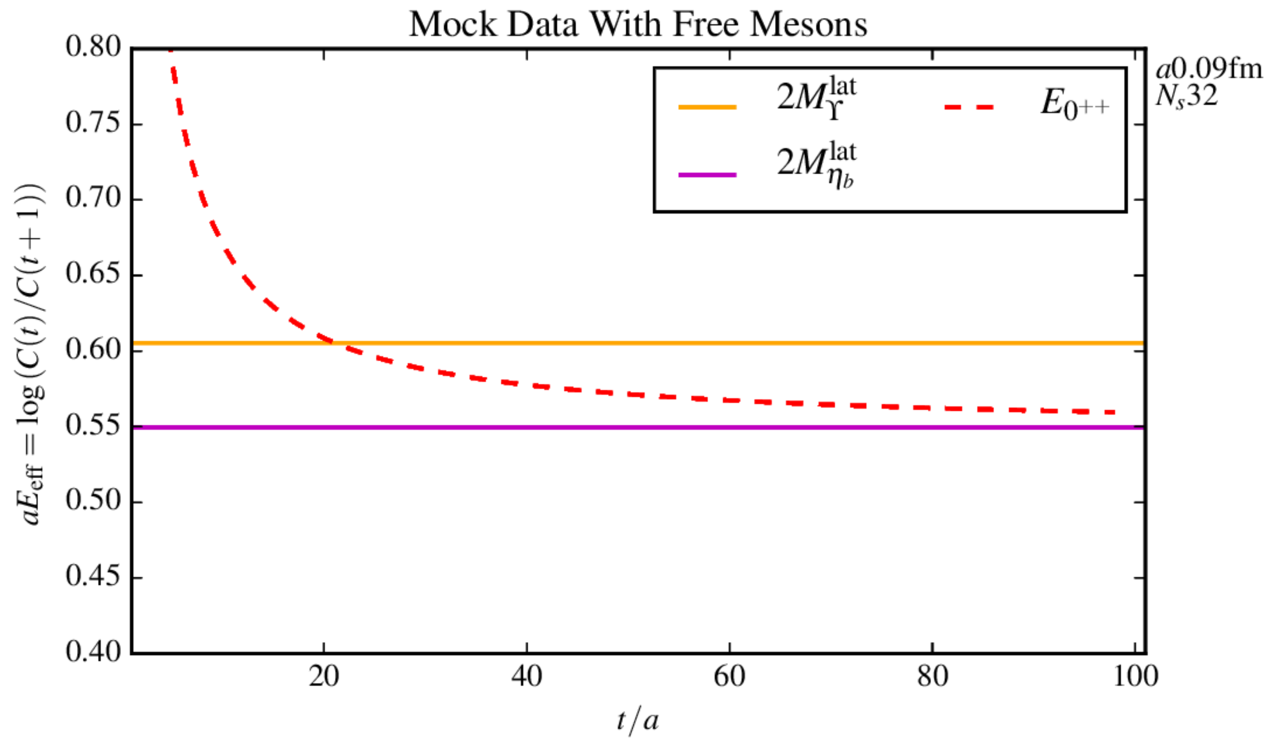

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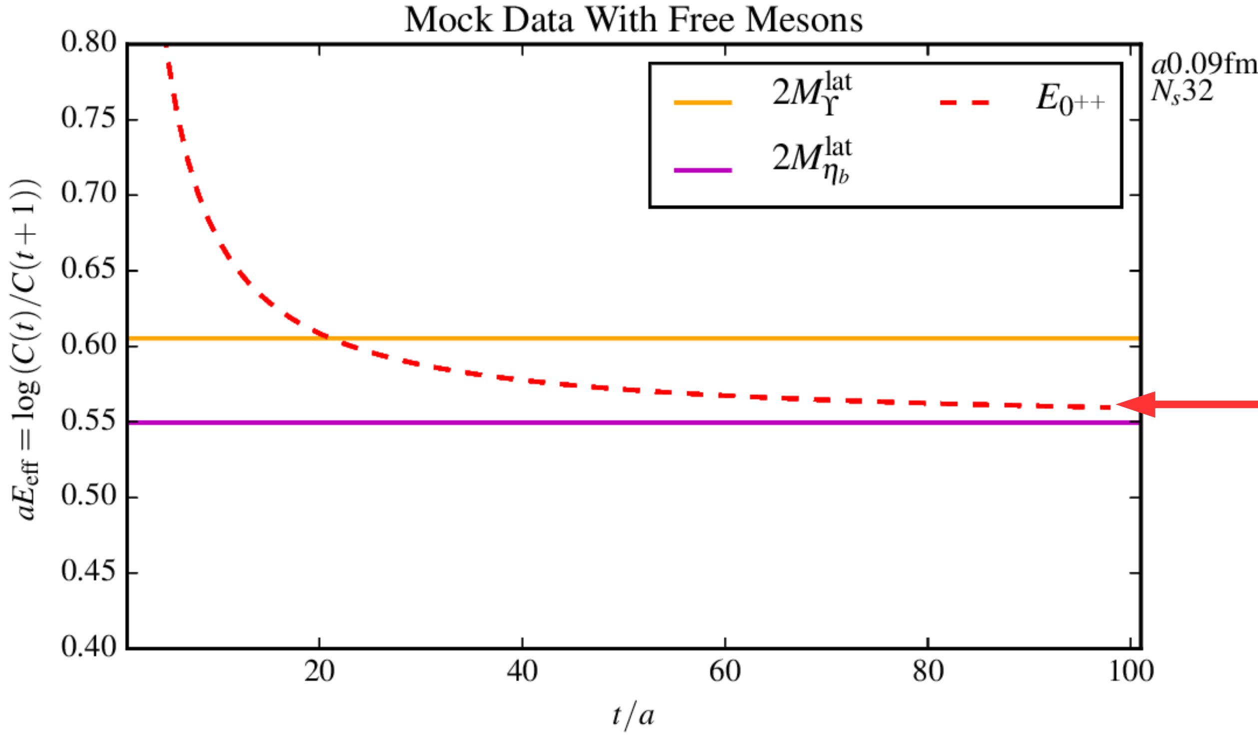

N.B.:Slow decay to ground state will be explained later!

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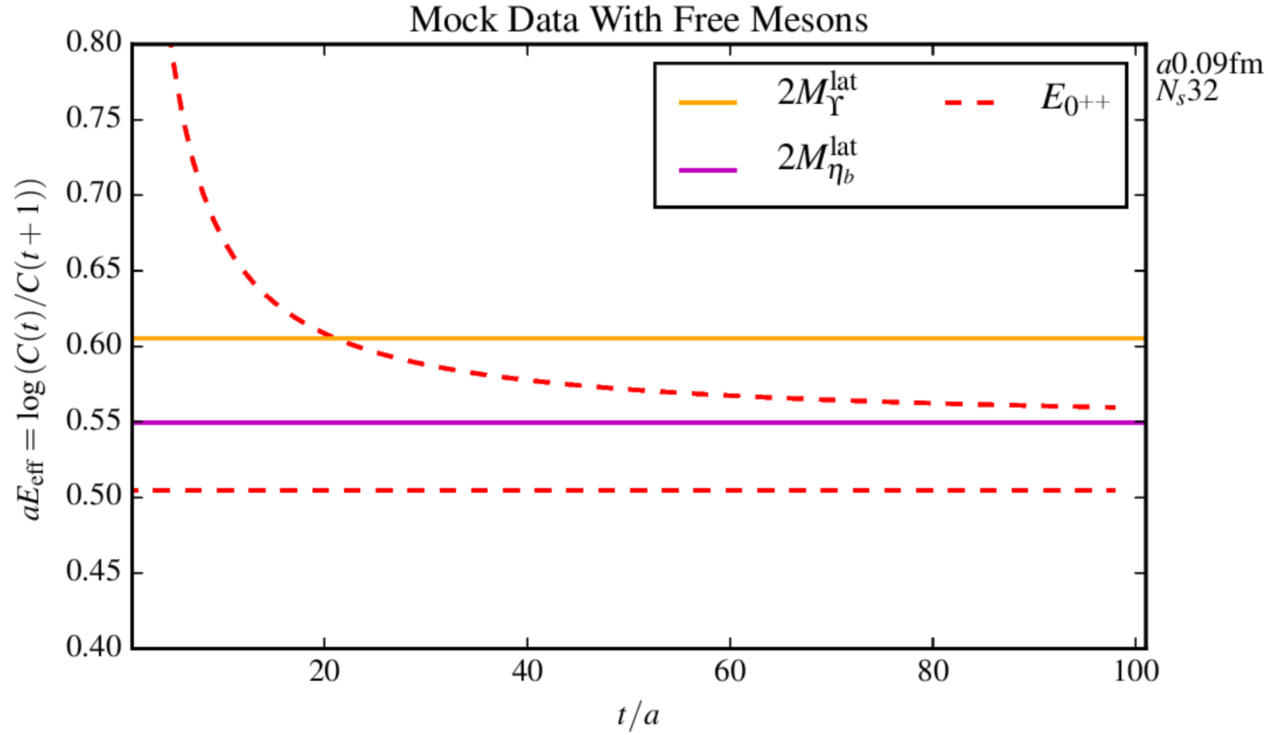

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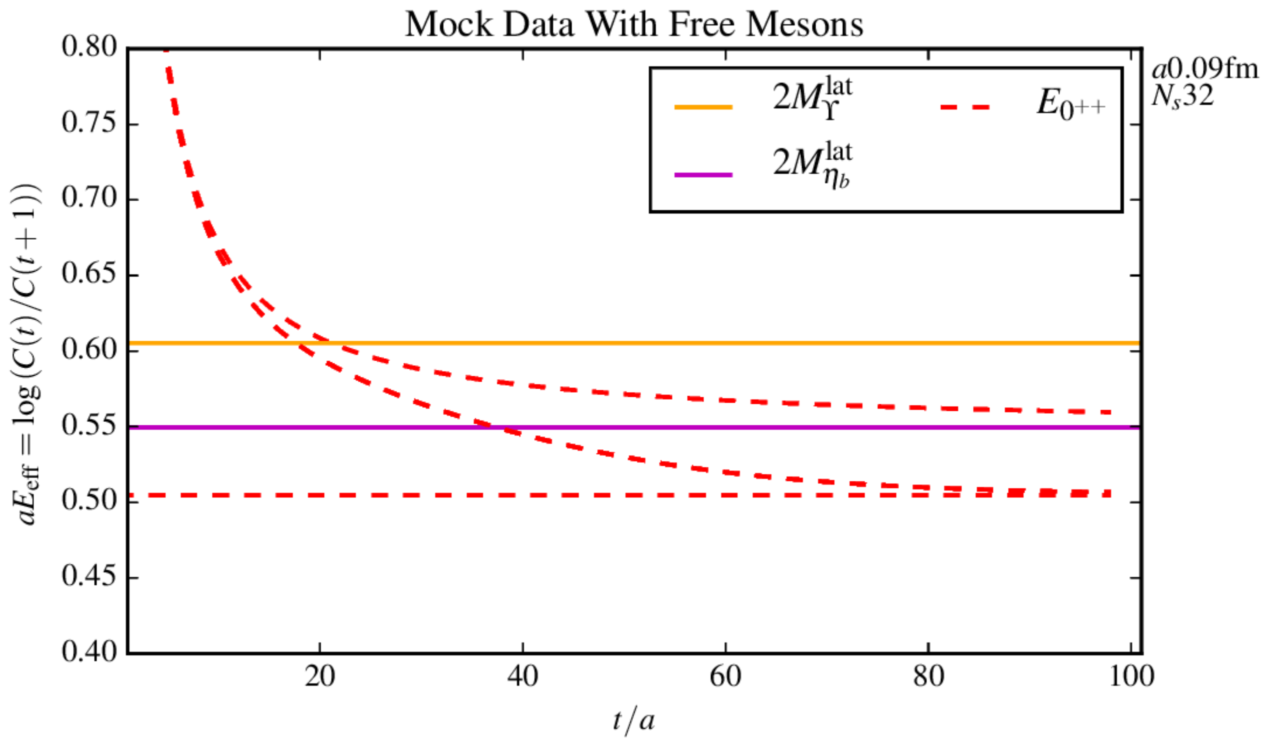

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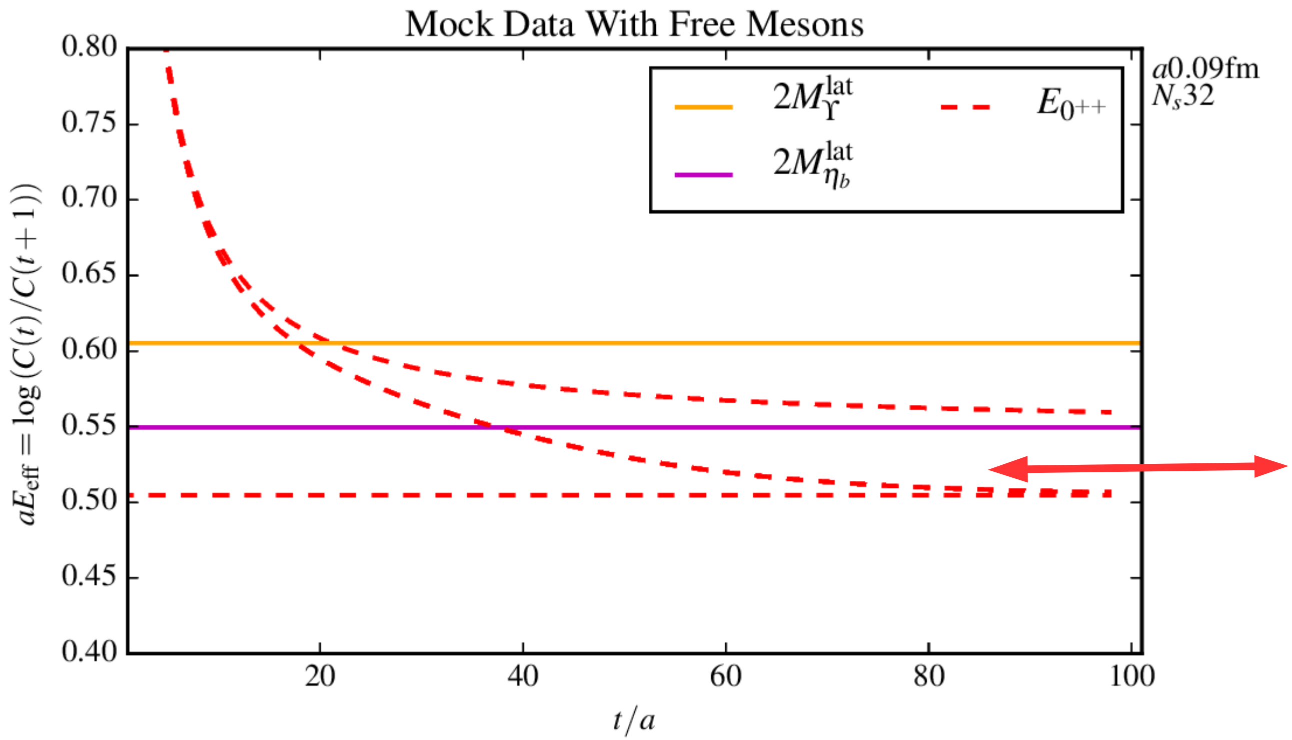

N.B.:This would be a clear signal for a new bound-state!!

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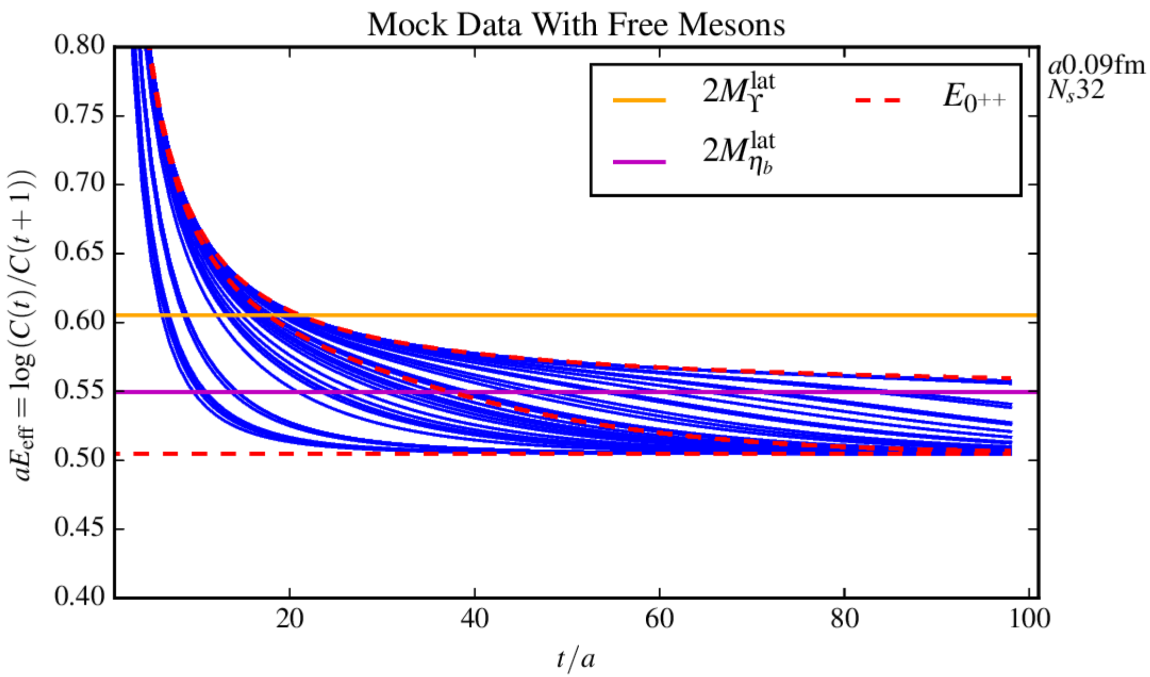

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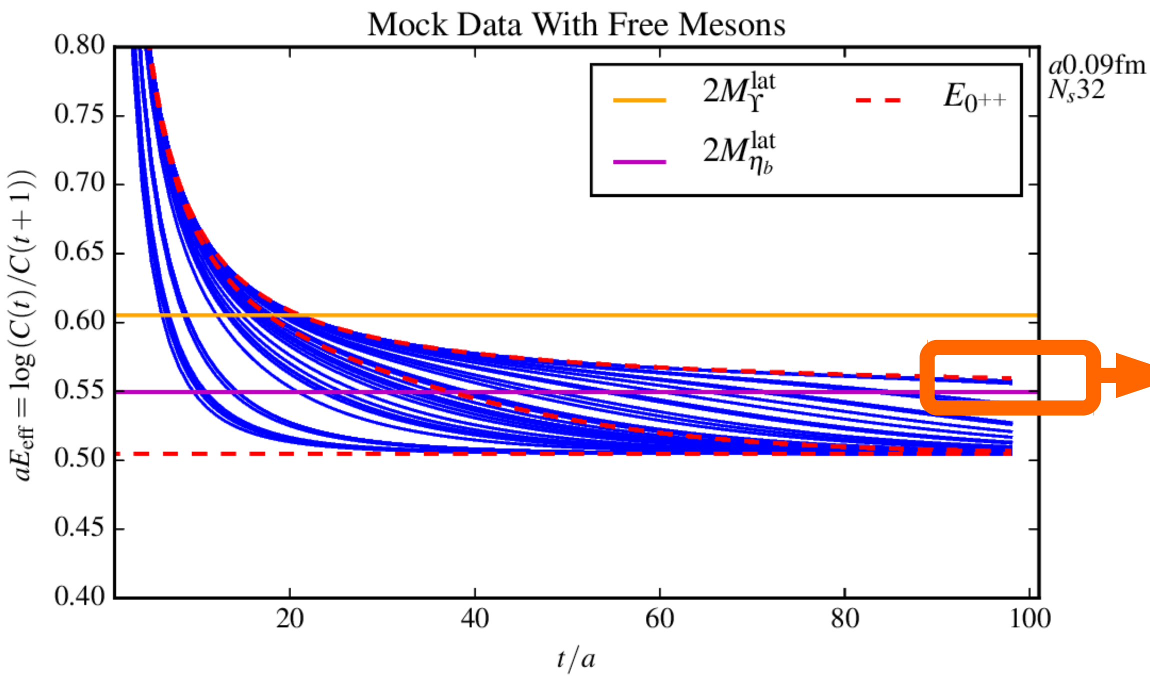

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i. Has amplitudes all suppressed (fine tuning)
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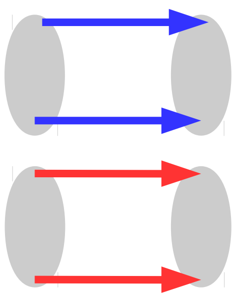
Diquark-Antidiquark Wick-Contractions

- The non-zero diquark-antidiquark correlators have four connected Wick contractions, which reduce to one:



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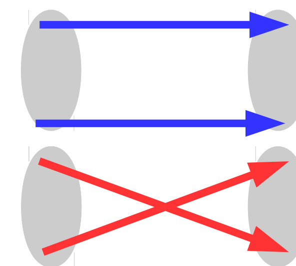

A Quick Glance: Interpolating Operators

- What are all the possible spin-colour combinations of two-identical quarks and two identical antiquarks in S-wave, e.g., $\bar{b}\bar{b}bb$.

Type (i) $\mathbf{q} \times \bar{\mathbf{q}}$

- => $q \times \bar{q} = 3 \times \bar{3} = 1 \oplus 8$ => Make singlet or octet mesons.
- => We can have $2\bar{q}q$ singlets which are 1×1 or $8 \times \bar{8}$
- Building block for these types of S-wave four-quark operators in irrep R:

$$O^{R}_{\eta_b} = G^{ABC}_{R} \bar{\Psi}^{B} \gamma^{5} \Psi^{C}$$

$$O^{i,R}_{\Upsilon} = G^{ABC}_{R} \bar{\Psi}^{B} \gamma^{i} \Psi^{C}$$

- To get all spin/colour $\bar{b}\bar{b}bb$ combos use the Clebsch-Gordan coefficients with circular basis of γ-matrices:

$$O^{J^{PC},M}_{4b} = \sum_{m_1,m_2} \langle J_1, m_1; J_2, m_2 | J, M \rangle O^{J_1,m_1,R} O^{J_2,m_2,\bar{R}}$$

A Quick Glance: Interpolating Operators

- What are all the possible spin-colour combinations of two-identical quarks and two identical antiquarks in S-wave, e.g., $\bar{b}\bar{b}bb$.

Type (ii) $\quad \mathbf{q}\times\mathbf{q}, \quad \bar{\mathbf{q}}\times\bar{\mathbf{q}}$

- $q \times q = 3 \times 3 = \bar{3} \oplus 6$
- $\bar{q} \times \bar{q} = \bar{3} \times \bar{3} = 3 \oplus \bar{6}$

- => Can have a Diquark-AntiDiquark $qq \times \bar{q}\bar{q}$ system in $\bar{3} \times 3$ or $6 \times \bar{6}$
- **N.B.**, Due to fermions being identical, the Pauli exclusion principal => S-wave $\bar{3}(6)$ Diquark can only be spin-triplet (spin-singlet).

A Quick Glance: Interpolating Operators

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- Building block for these types of S-wave four-quark operators in irrep R:

$$O_D^{i,\bar{3}} = G_{\bar{3}}^{ABC} \Psi_c^B \gamma^i \Psi^C$$

$$O_{AD}^{i,\bar{6}} = G_{\bar{6}}^{ABC} \bar{\Psi}^B \gamma^5 \bar{\Psi}_c^C$$

- To get all spin/colour $\bar{b}\bar{b}bb$ combos use the Clebsch-Gordan coefficients with circular basis of gamma matrices:

$$O_{4b}^{J^{PC},M} = \sum_{m_1,m_2} \langle J_1, m_1; J_2, m_2 | J, M \rangle O^{J_1,m_1,R} O^{J_2,m_2,\bar{R}}$$

Tetraquark S-wave Quantum Numbers

- With building blocks $\eta_b = 0^{-+}, \Upsilon = 1^{--}, qq = 1^{+}, 0^{+}$
  we can look for $\mathbf{0}^{++}, \mathbf{1}^{+-}, \mathbf{2}^{++}$ $\bar{b}\bar{b}bb$ tetraquarks

- $SO(3)$ breaks into lattice irreps: $$\begin{aligned} J &= 0 \rightarrow A_1(1) \\ J &= 1 \rightarrow T_1(3) \\ J &= 2 \rightarrow T_2(3) \oplus E(2) \end{aligned}$$

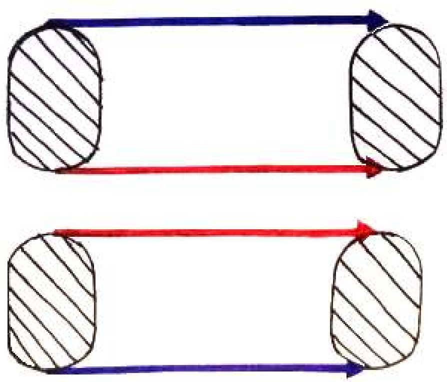
- Build operators which lie in lattice irreps: $O^{2^{++}}_{E/T_2} = O^{J=2,M=2} \pm O^{J=2,M=-2}$

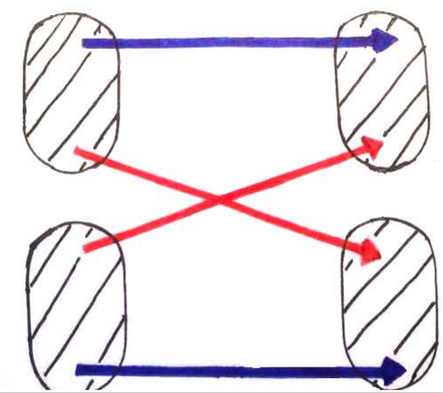
Why studying a single Wick contraction is a bad idea: Pure QCD

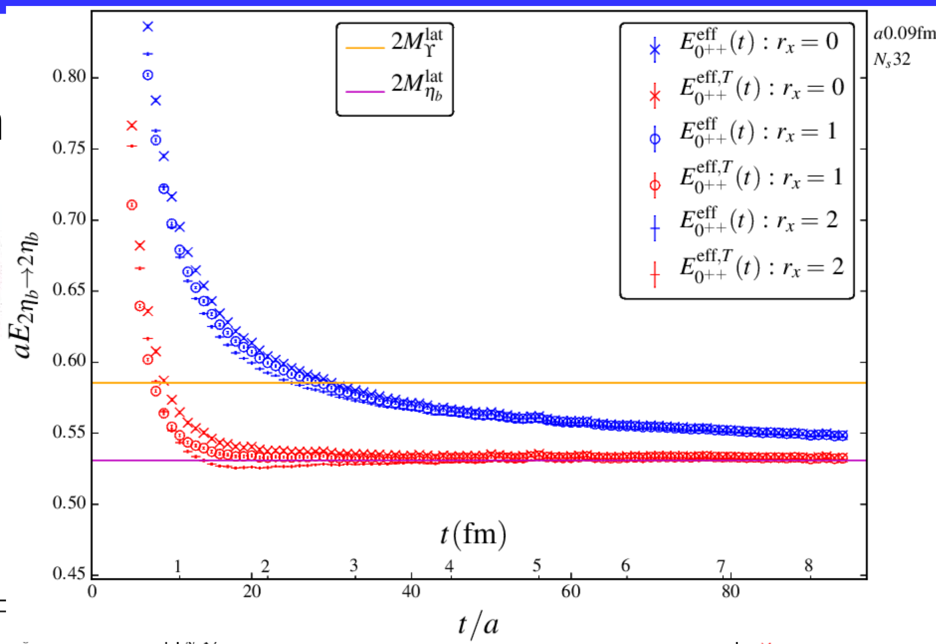
- Two mesons with … /ick contractions:

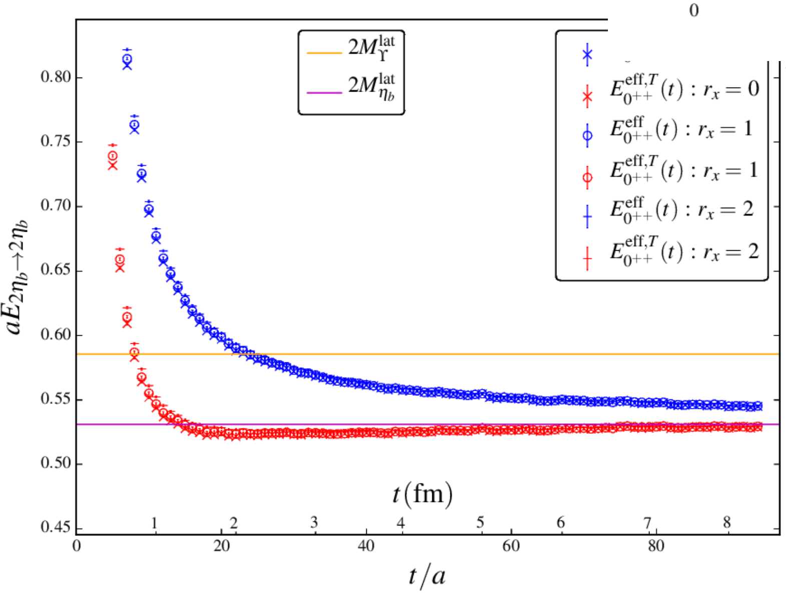
M1

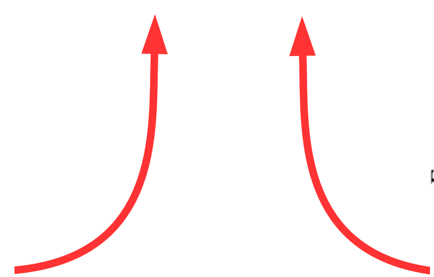
M2

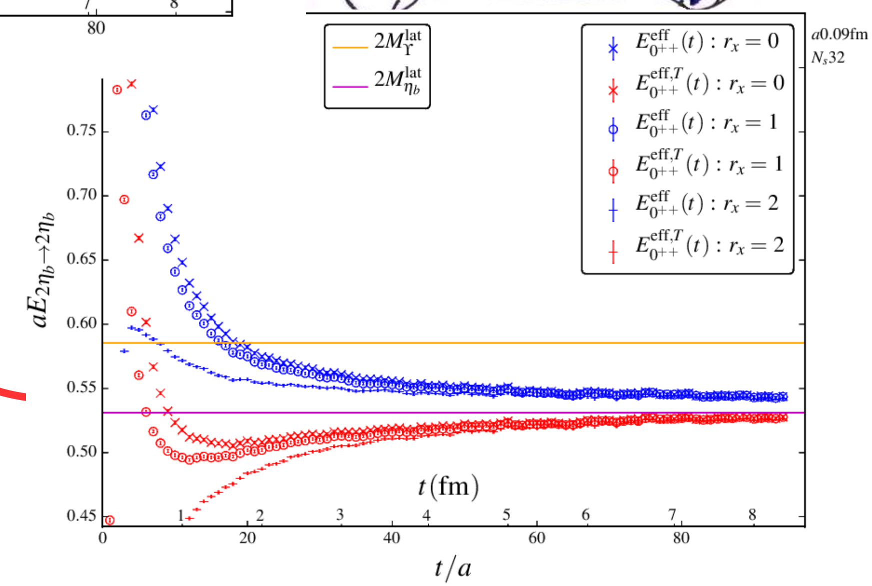




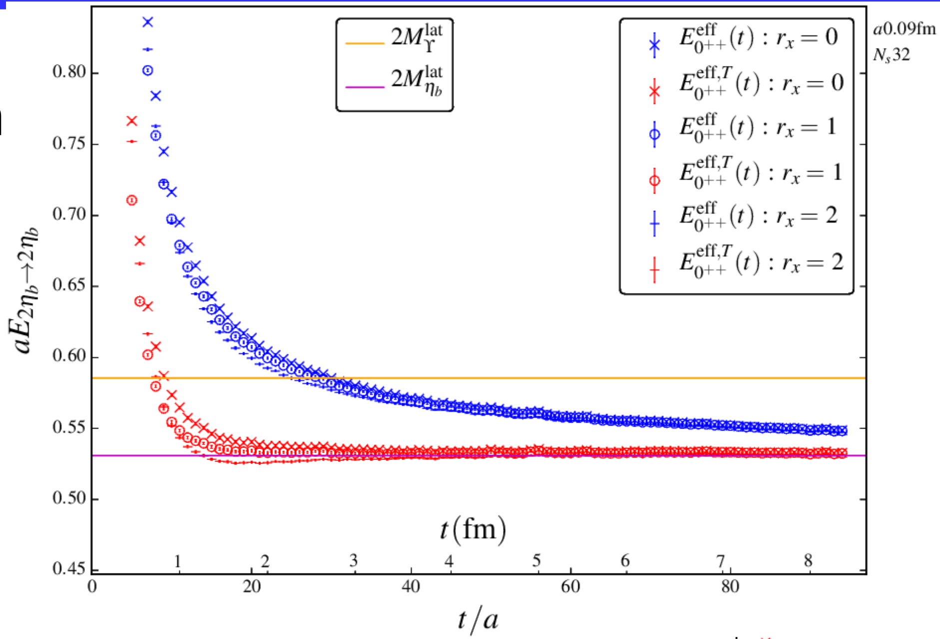




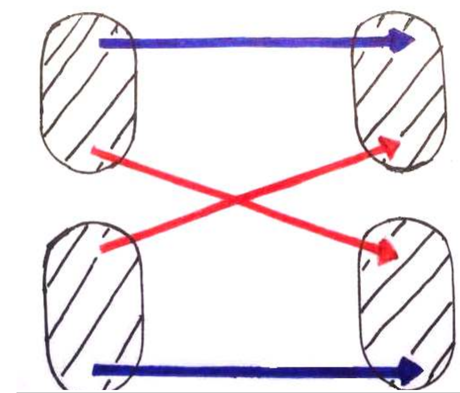


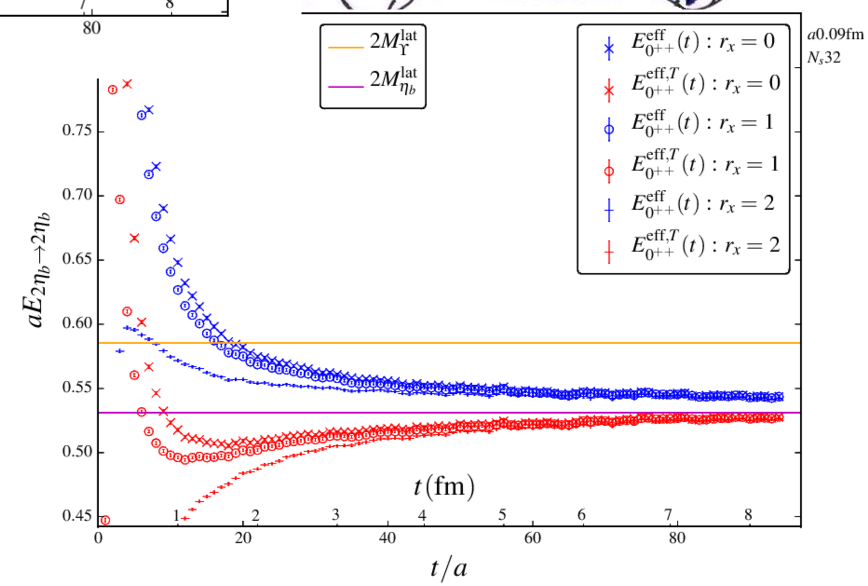


Why studying a single Wick contraction is a bad idea: Pure QCD

- Two mesons with 

/ick contractions:





Spin-flip interactions have negative sign and cause decay to threshold from below.
Models: Bound state can appear below threshold when including only a subset of interactions.