

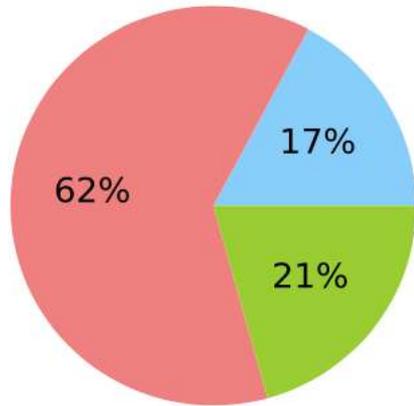
# **$D \rightarrow K l \nu$ semileptonic decay using lattice QCD with HISQ at physical pion masses**

Bipasha Chakraborty

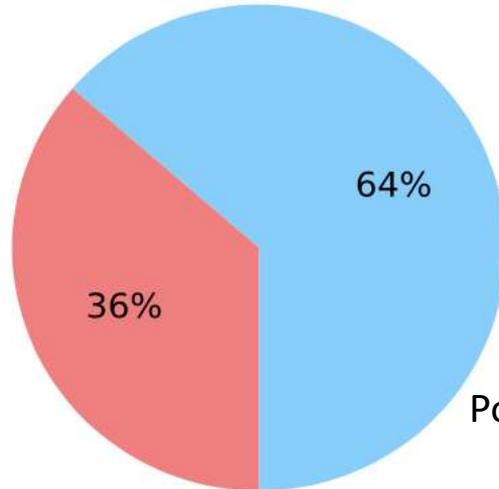
With HPQCD collaboration:

Christine Davies, Jonna Koponen, G. Peter Lepage

Leptonic



Semileptonic



$$V_{\text{CKM}} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

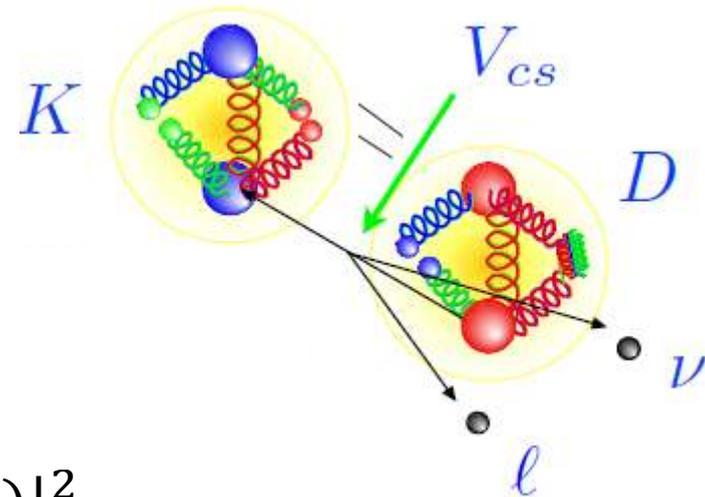
A red arrow points from the  $V_{cs}$  element in the matrix to the Semileptonic pie chart.

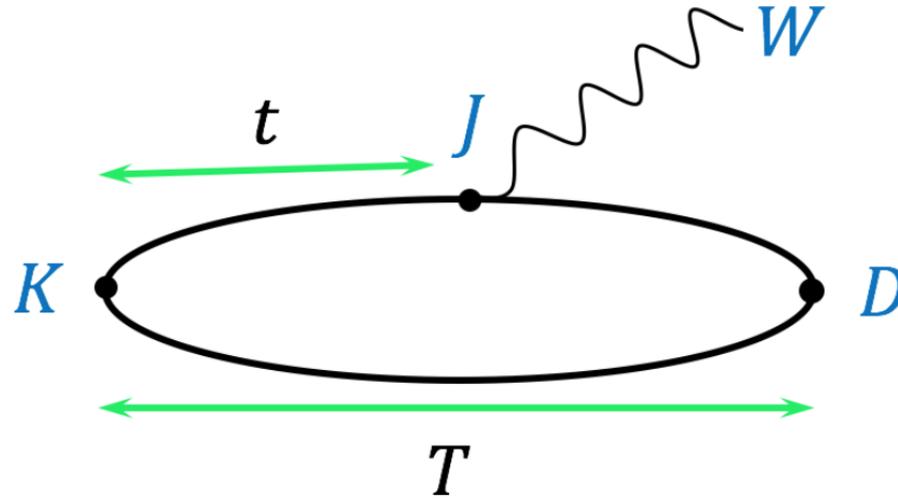
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Legend: Lattice (blue), Experiment (red), EM (green)

Comparison of  $|V_{CS}|$  Error-Radius proportional to total error

$$\frac{d\Gamma^{D \rightarrow K}}{dq^2} = \frac{G_F^2 p^3}{24\pi^3} |V_{CS}|^2 |f_+^{D \rightarrow K}(q^2)|^2$$





$$\langle K^- l^+ \nu | J_W | D^0 \rangle = \frac{G_F}{\sqrt{2}} V_{cs} \bar{v}(l) \gamma_\mu (1 - \gamma_5) u(\nu) \langle K^- | \bar{\psi}_s \gamma_\mu (1 - \gamma_5) \psi_c | D^0 \rangle.$$

$$\langle H_\mu \rangle = \langle K^- | \bar{s} \gamma_\mu (1 - \gamma_5) c | D^0 \rangle$$

# Lattice formalism

$$Z_{V,t} \times \langle K^- | V^\mu | D^0 \rangle = f_+^{D \rightarrow K}(q^2) \left[ p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2} q^\mu \right] \\ + f_0^{D \rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{q^2} q^\mu$$

$$\langle K | V^\mu | D \rangle = f_+^{D \rightarrow K}(q^2) \left[ p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2} q^\mu \right] \\ + f_0^{D \rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{q^2} q^\mu \rightarrow f_0(0) = f_+(0)$$

$$\langle K | S | D \rangle = f_0^{D \rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{m_{0c} - m_{0s}}$$

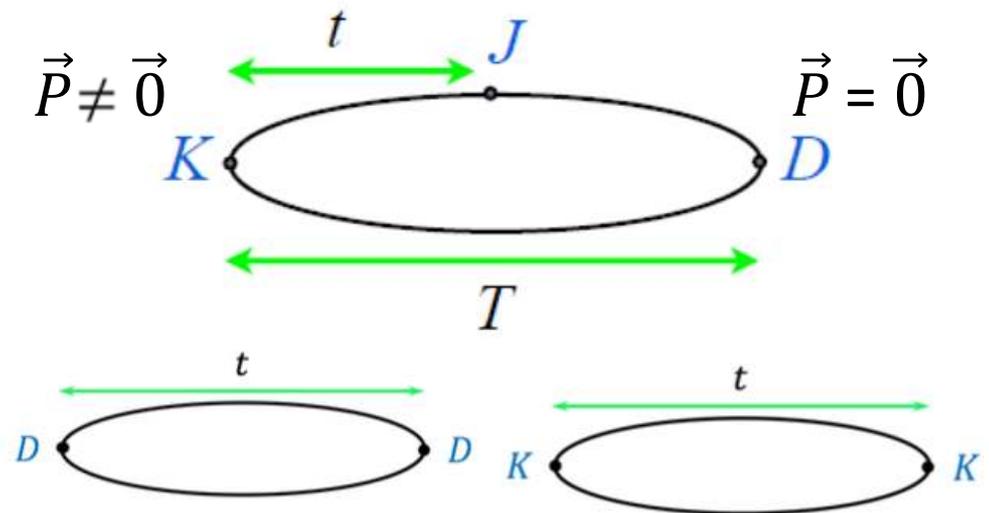
$$q^\mu = p_D^\mu - p_K^\mu$$

$$q^2 = q_{\max}^2 = (M_D - M_K)^2 :$$

$$\begin{aligned} Z_{V,t} \times \langle K^- | V^0 | D^0 \rangle &= f_+^{D \rightarrow K}(q^2) [M_D + M_K - (M_D + M_K)] \\ &\quad + f_0^{D \rightarrow K}(q^2) (M_D + M_K) \\ &= (M_D + M_K) \times f_0^{D \rightarrow K}(q_{\max}^2) \end{aligned}$$

Extract

- Local temporal vector current
- Non-goldstone D meson for that – local  $\mathbf{Y}_0 \mathbf{Y}_5$
- “Sequential technique” for three-points correlators



# Analysis Ingredients

MILC configurations: up/down, strange, charm quarks in the sea:  $m_u = m_d$

Multiple  $m_u/d$   
Including physical

Valence strange  
and charm quark  
masses tuned  
accurately

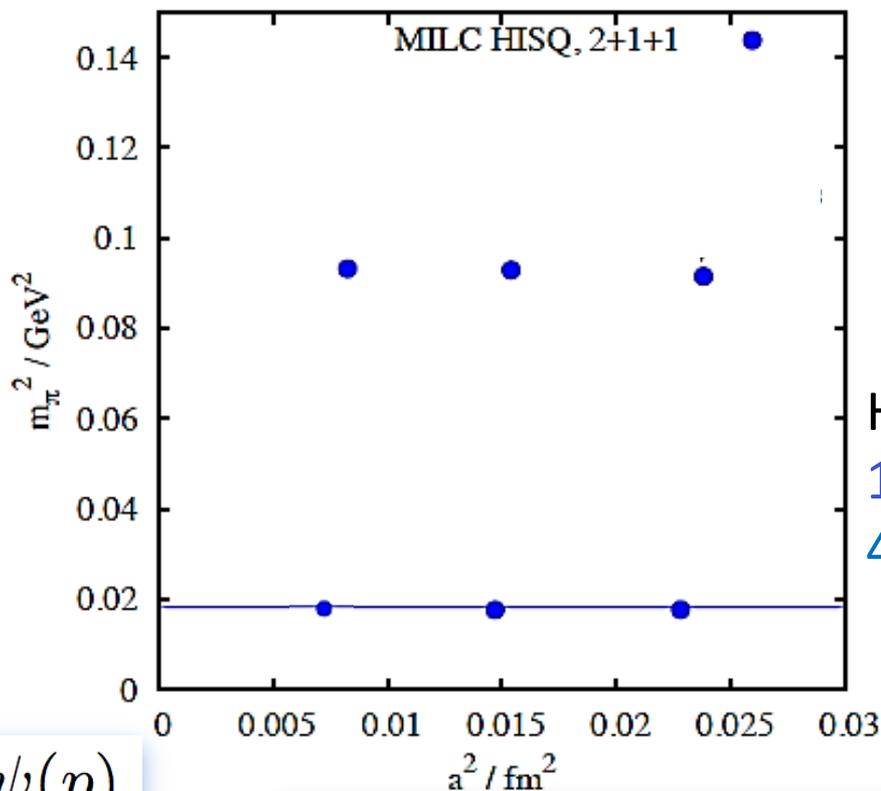
Twisted boundary  
Conditions:

$$\psi(p + L_\mu \hat{\mu}) = e^{i\theta_\mu} \psi(p).$$

Random wall sources:

$$\langle \eta^\dagger(i') \eta(i) \rangle = \delta_{ii'}$$

$$\eta(i_t) = \begin{cases} e^{i\theta} & \text{for } i_t = t_0, \\ 0 & \text{for } i_t \neq t_0. \end{cases}$$



Lattice spacing  
 $a \sim 0.09 - 0.15 \text{ fm}$

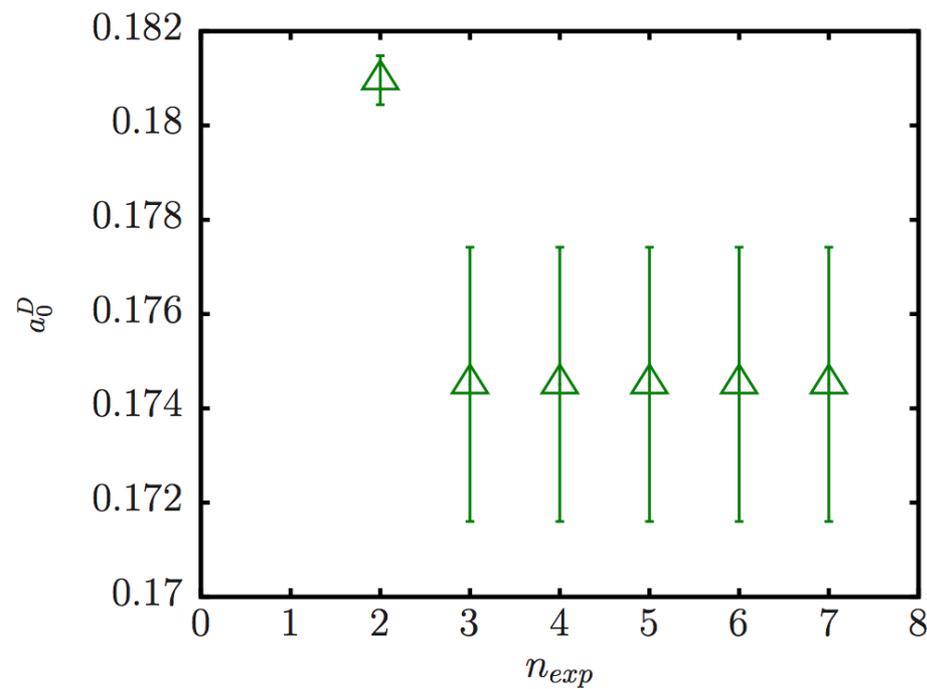
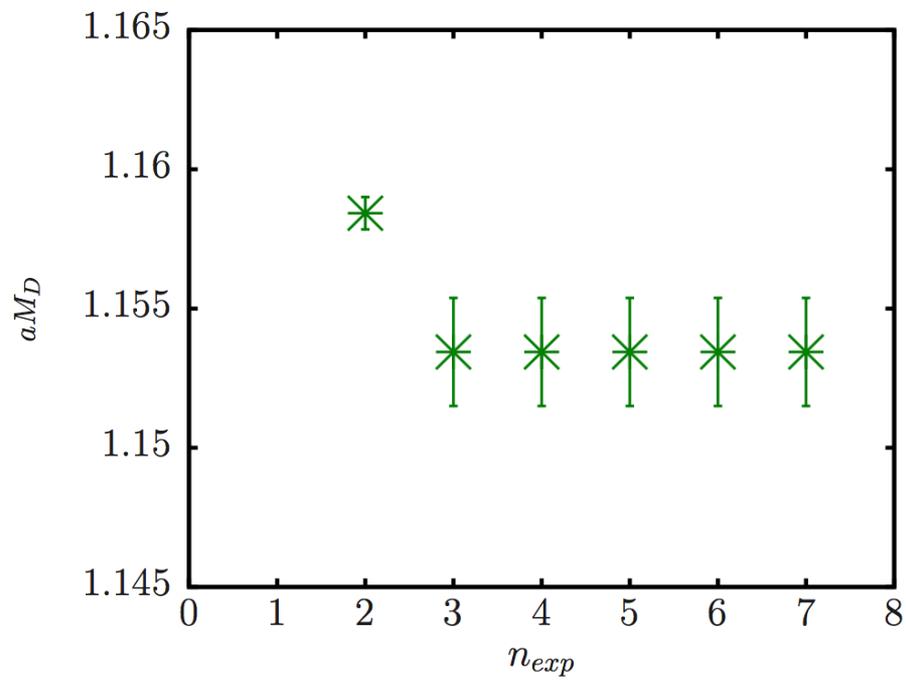
High statistics:  
1,000 configurations,  
4-16 time sources

# Multi-exponential Bayesian fitting

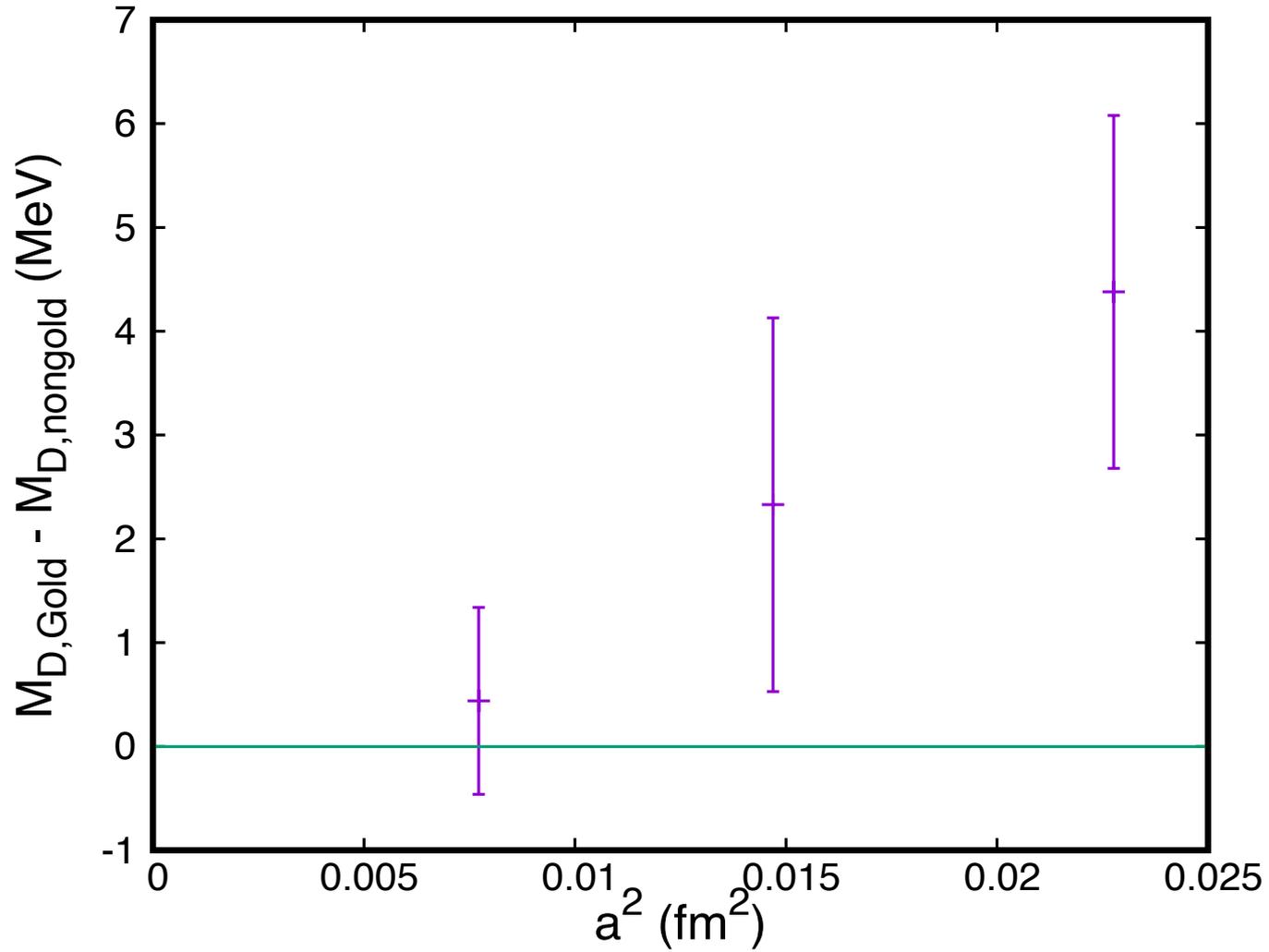
$$G^{2\text{pt}}(t; \vec{p}) = \sum_n a_n^2 (e^{-E_n t} + e^{-E_n(T-t)}) \\ + (-1)^t \sum_{n_o} a_{n_o}^2 (e^{-E_{n_o} t} + e^{-E_{n_o}(T-t)})$$

$$G^{3\text{pt}}(t; T) = \sum_{n_1, n_2} a_{n_1} a_{n_2} V_{n_1 n_2}^{nn} (e^{-E_{n_1} t} + e^{-E_{n_2}(T-t)}) \\ + (-1)^t \sum_{n_{1o}, n_2} a_{n_{1o}} a_{n_2} V_{n_{1o} n_2}^{on} (e^{-E_{n_{1o}} t} + e^{-E_{n_2}(T-t)}) \\ + (-1)^T \sum_{n_1, n_{2o}} a_{n_1} a_{n_{2o}} V_{n_1 n_{2o}}^{no} (e^{-E_{n_1} t} + e^{-E_{n_{2o}}(T-t)}) \\ + (-1)^{t+T} \sum_{n_{1o}, n_{2o}} a_{n_{1o}} a_{n_{2o}} V_{n_{1o} n_{2o}}^{oo} (e^{-E_{n_{1o}} t} + e^{-E_{n_{2o}}(T-t)})$$

# Check 1: Stability of the fits with multiple exponentials

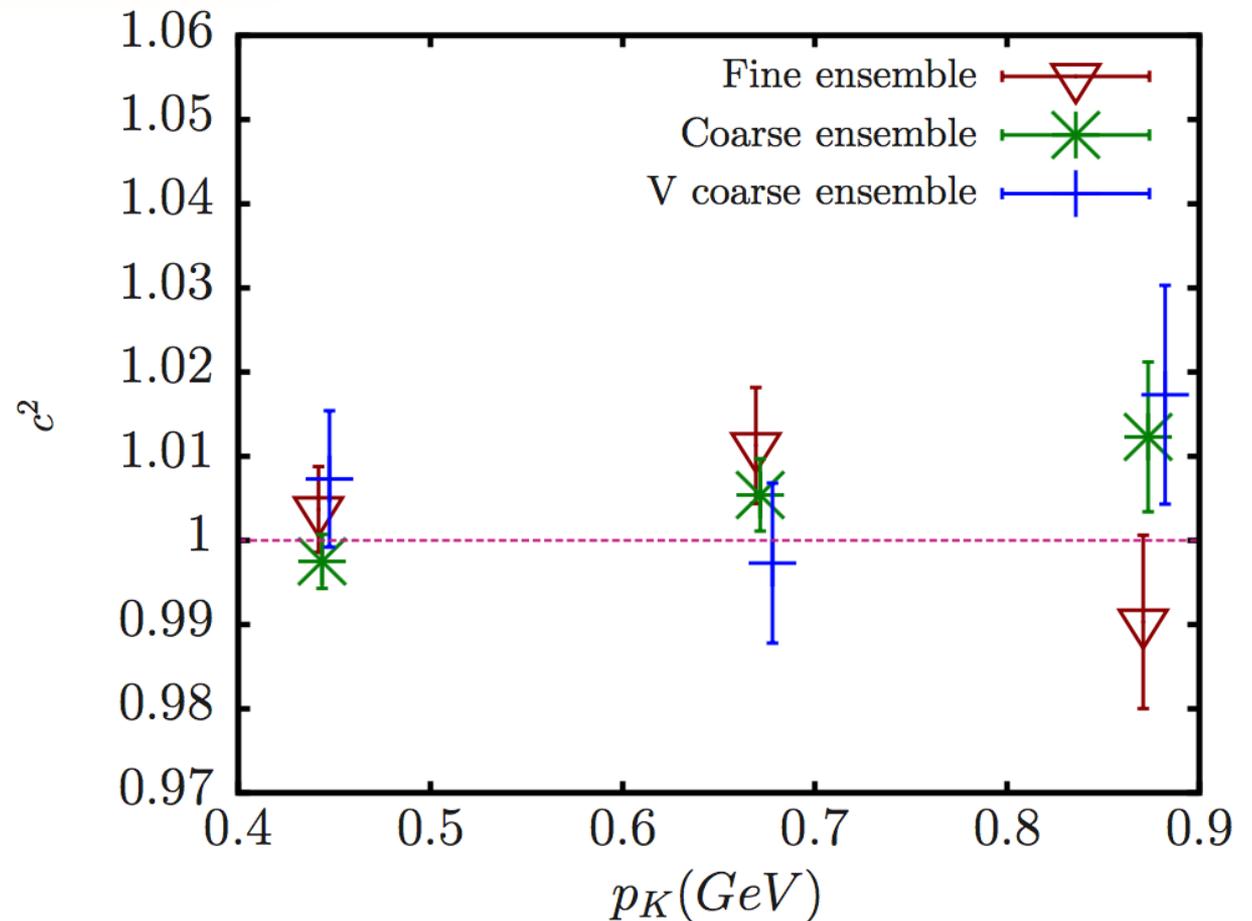


## Check 2: Mass difference: Goldstone and non-goldstone D mesons

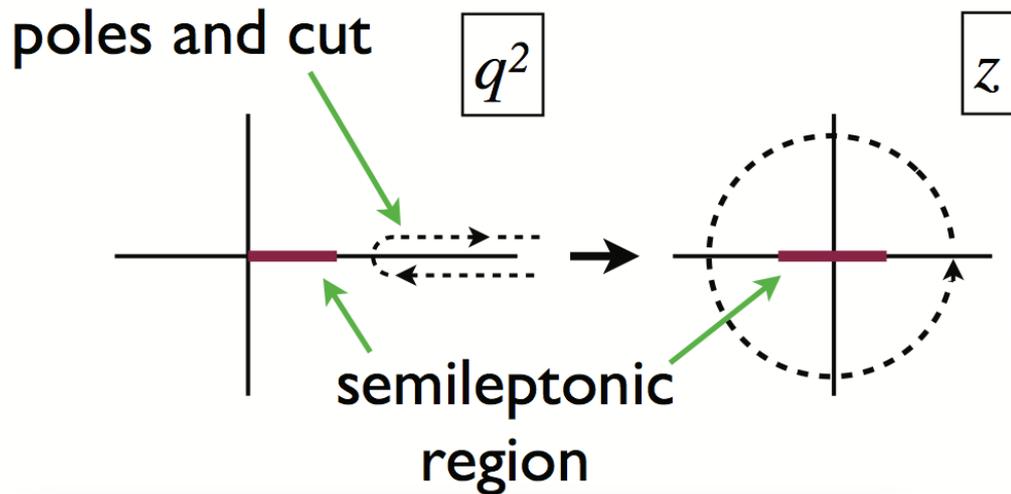


# Check 3 : Relativistic dispersion relation on lattice

$$c^2(\vec{p}) = \frac{E_K^2(\vec{p}) - M_K^2}{\vec{p}^2}$$



# Z-expansion



$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_{\pm} = (m_D \pm m_K)^2$$

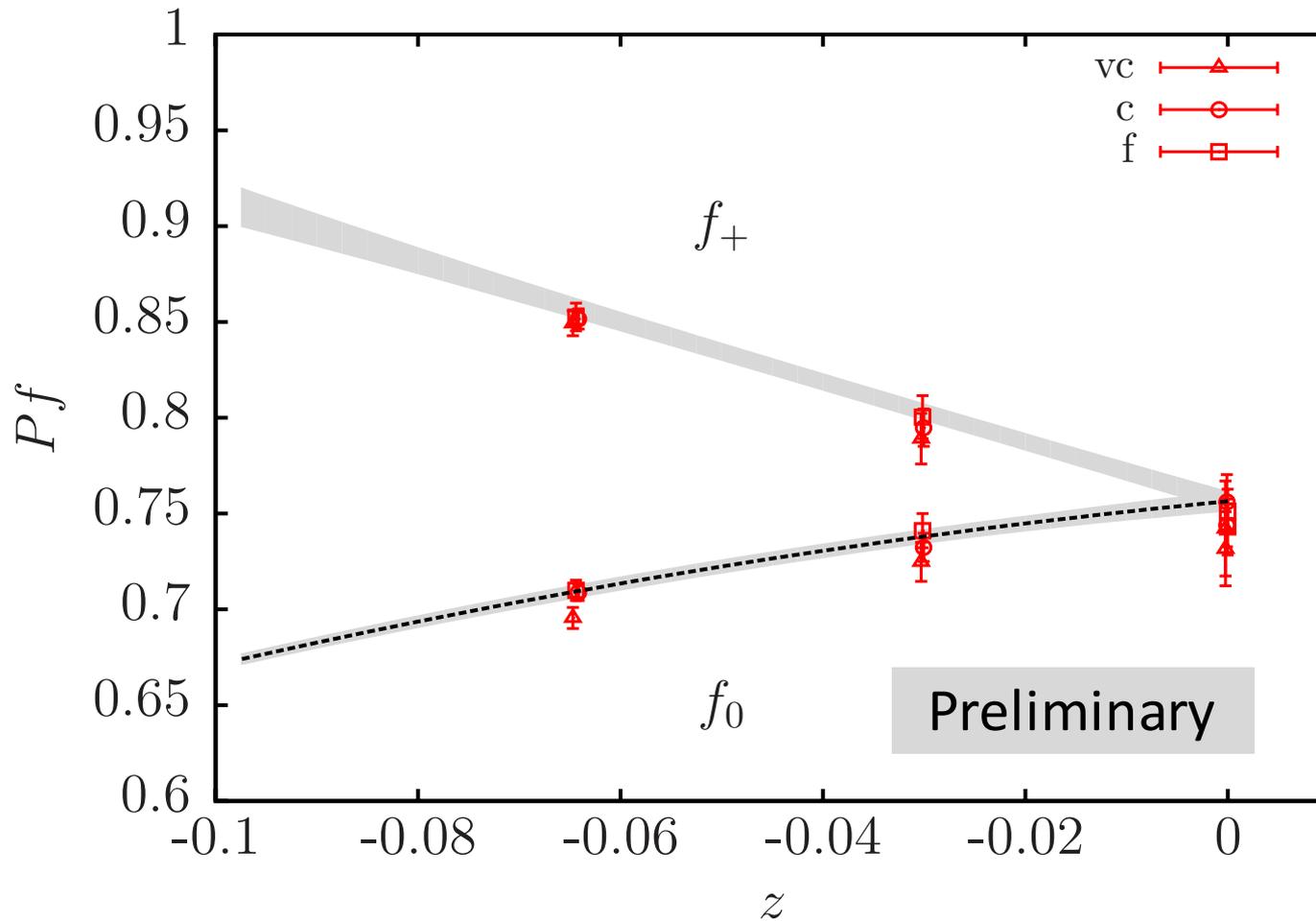
$$f(q^2) = \frac{1}{P(q^2)\Phi(q^2)} \sum_{n=0}^N b_n z^n$$

Pole masses

$$(1 - q^2/M_X^2)$$

$$b_n(a, m_l) = A_n \{ 1 + B_n a^2 + C_n a^4 + D_n \delta_l + E_n (\delta_l \ln[\delta_l] + F_n a^2 \delta_l) \}$$

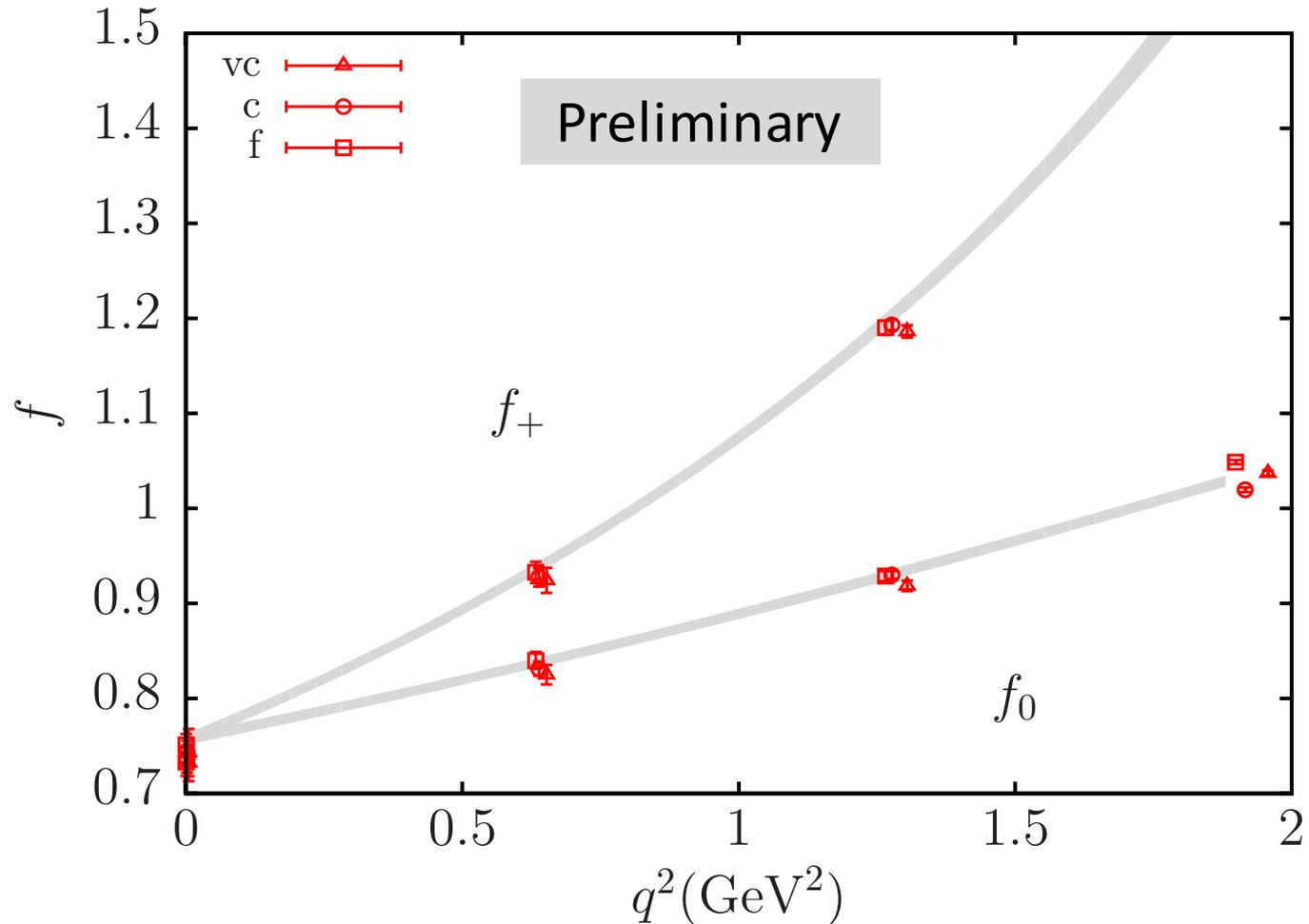
# Shape of the form factors: $D \rightarrow Kl\nu$

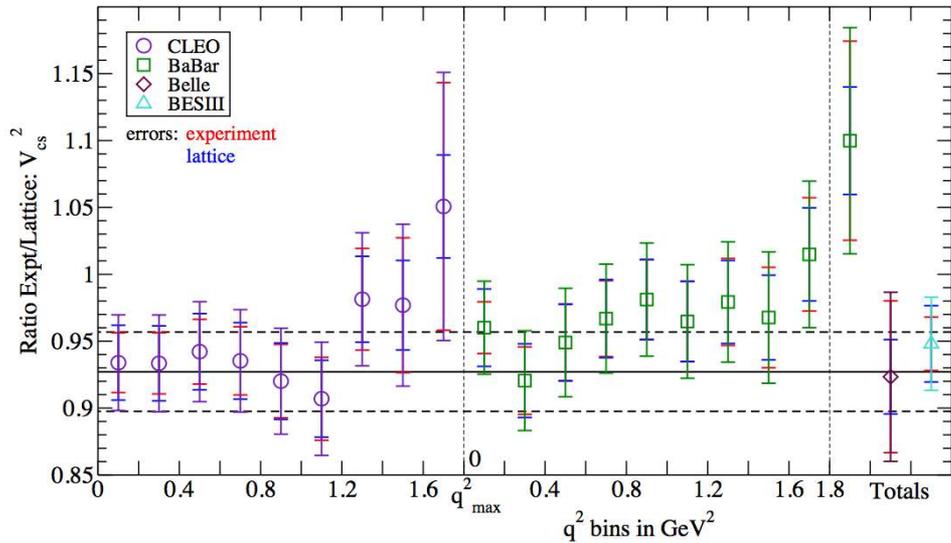


Only physical pion masses included; uncorrelated fit

# Shape of the form factors: $D \rightarrow Klv$

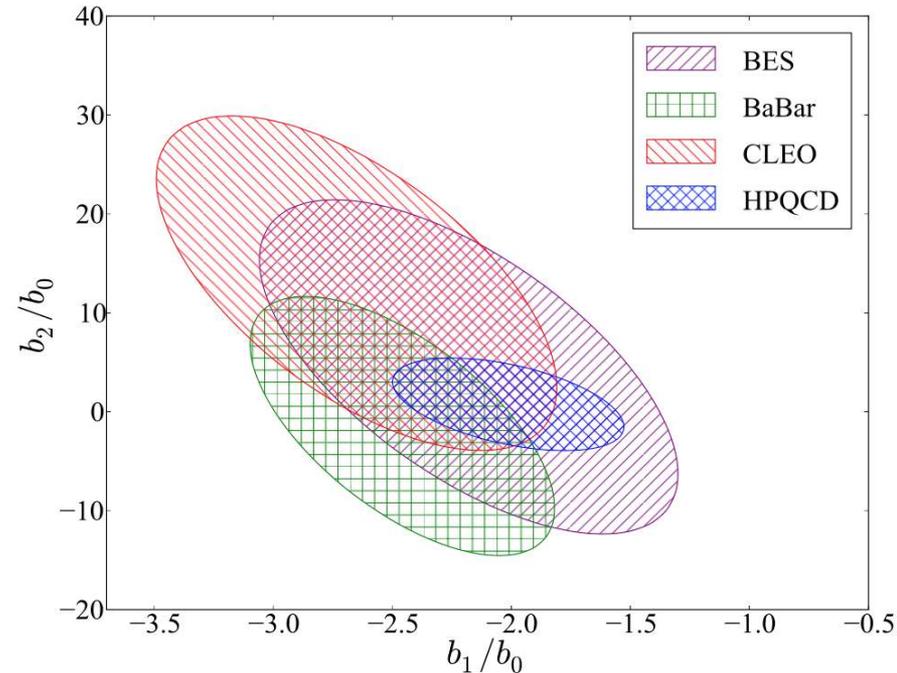
Converting back to 'q' space





Previous HPQCD results:  
 J. Koponen et. al. [arXiv:1305.1462](https://arxiv.org/abs/1305.1462)

Will be looking for  
 bin-to-bin correlation



➤ Add more ensembles –  
 pion mass dependence

➤ Refine analysis to  
 include all correlations

➤ Include a common Z-expansion with

$$t_0 = t_+ \left( 1 - \left( 1 - t_- / t_+ \right)^{1/2} \right)$$

MILC/Fermilab:

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ETMC:

**arXiv: 1306.03657**