Coupled-channel scattering from lattice QCD

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based on work with the Hadron Spectrum Collaboration
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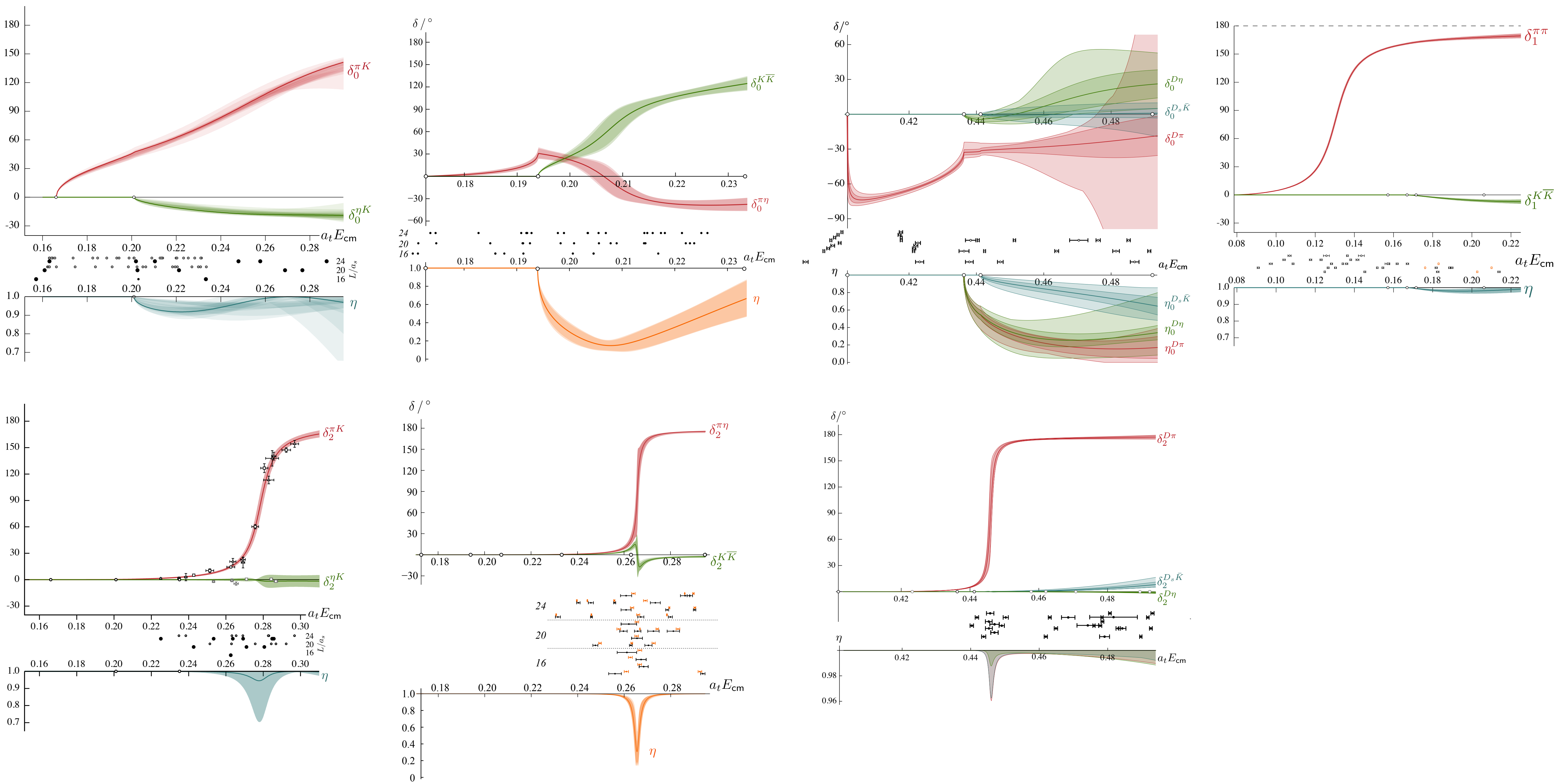
The University of Dublin

Coupled-channel scattering on the lattice

Several coupled-channel scattering systems have recently been computed:



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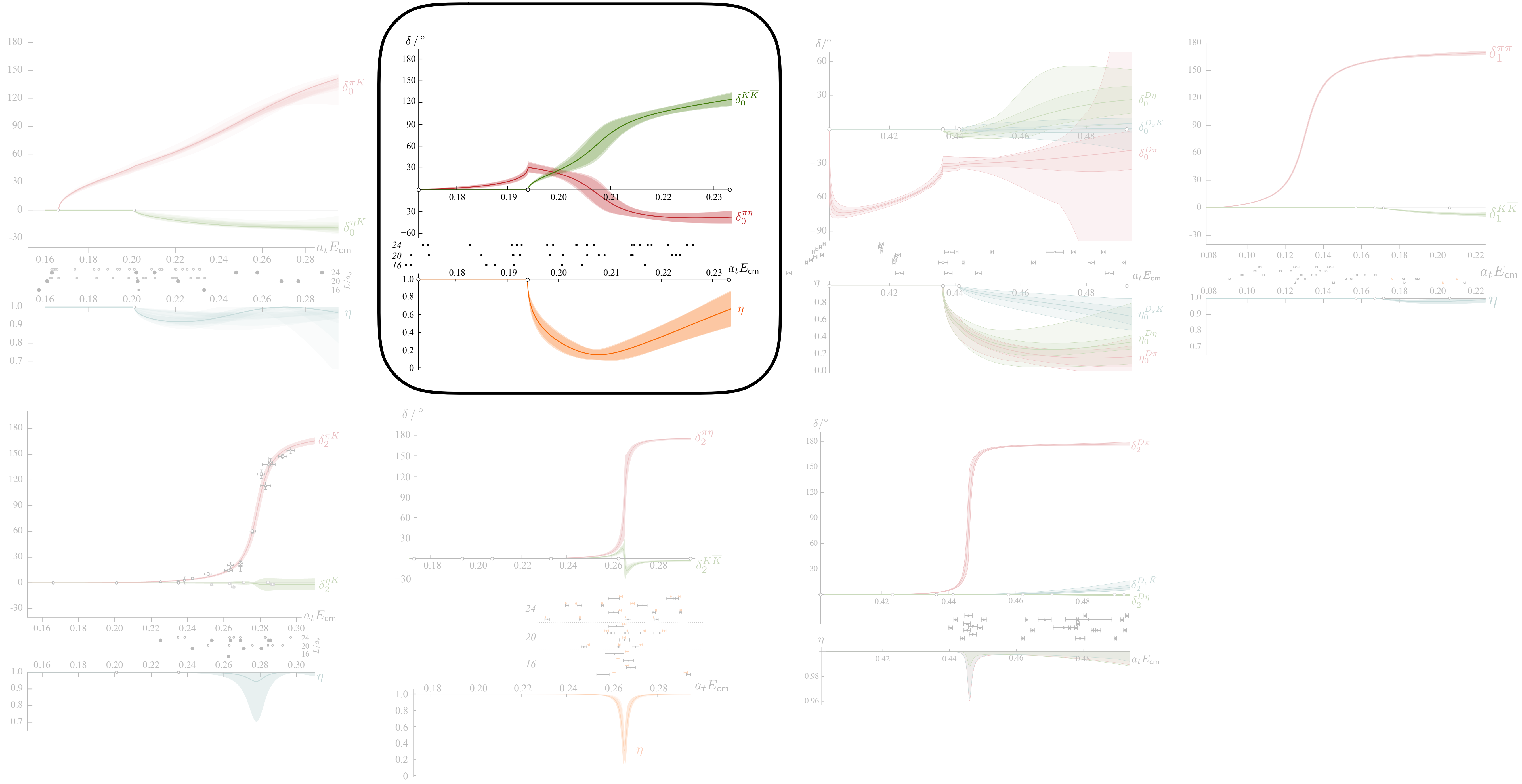
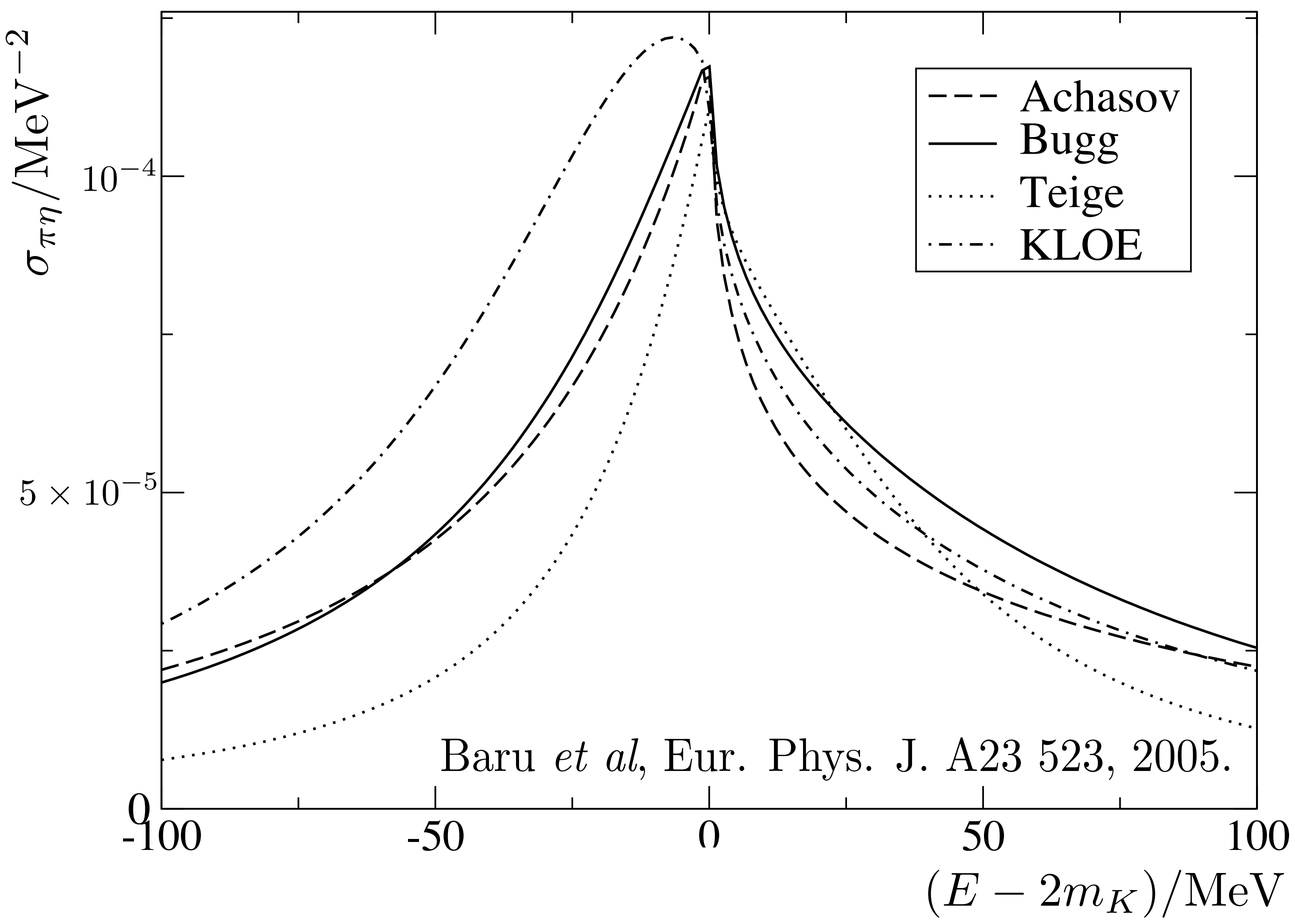


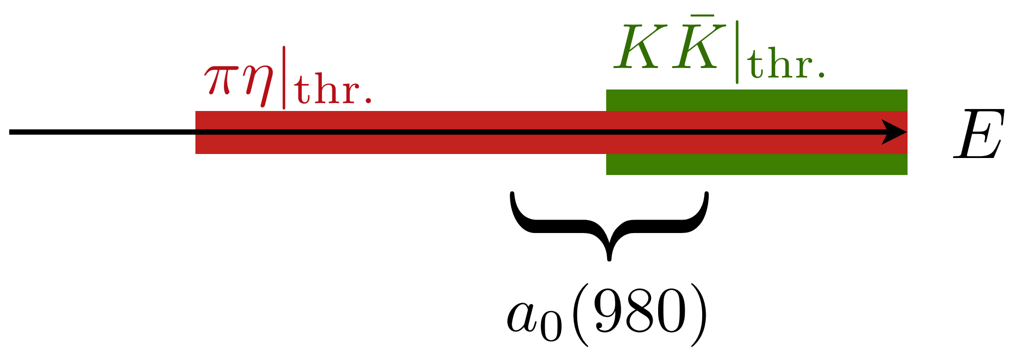
a_0 resonance

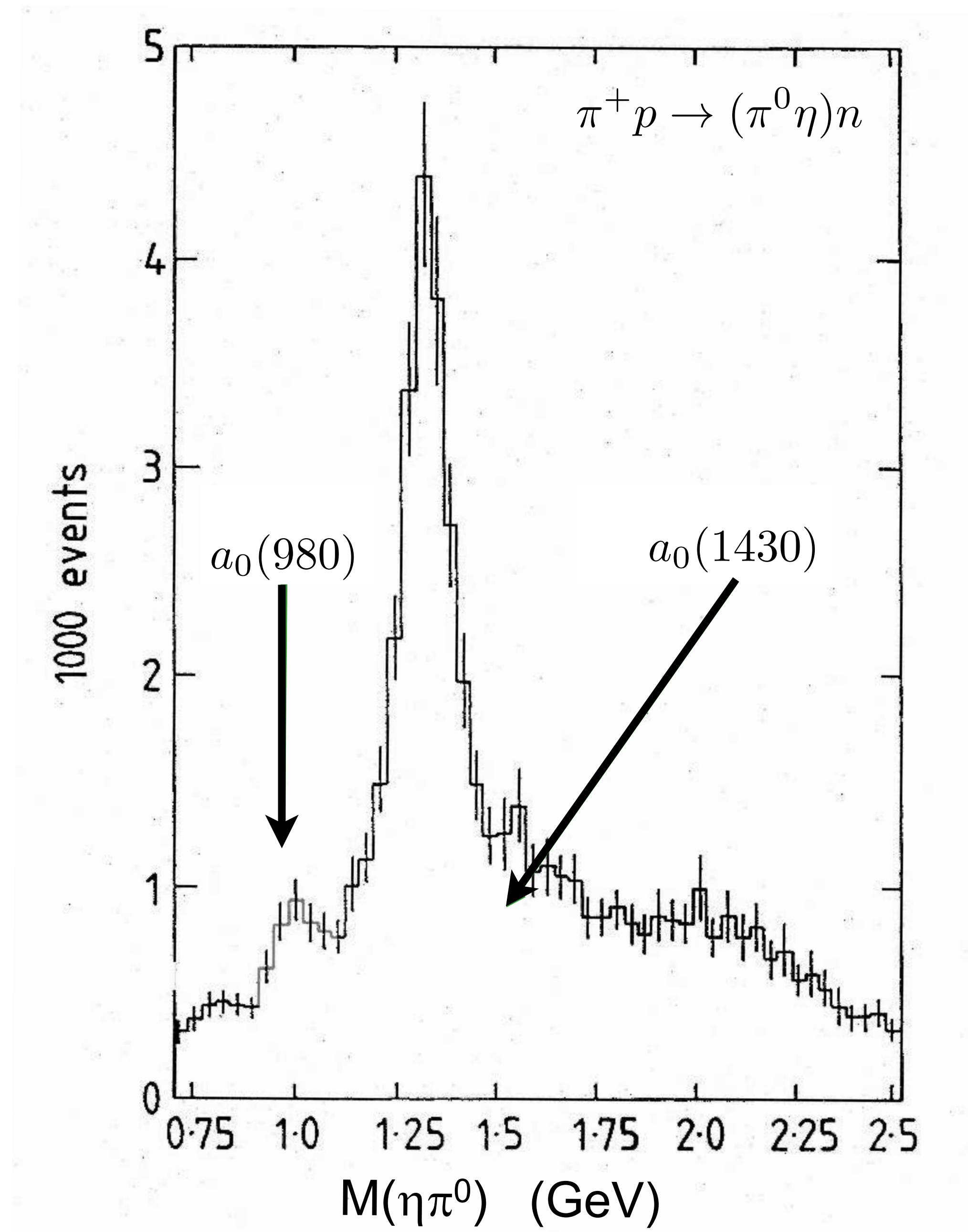
Dudek, Edwards and Wilson,
PRD 93 094506, arXiv:1602.05122

$$\pi\eta\text{-}K\bar{K}\text{-}\pi\eta'$$

$$I = 1 \;\; J = 0$$







GAMS, Alde *et al* PLB 203 397, 1988.

Correlation functions

Build a large basis of operators:

$$\bar{q}q \sim \bar{\psi}\Gamma D \ldots D\psi$$

$$\pi\eta \sim \sum_{\vec{p}_1+\vec{p}_2 \in \vec{P}} \mathrm{CG}\left(\vec{p}_1, \vec{p}_2, \Lambda^{\vec{P}}\right) \quad \pi(\vec{p}_1)\eta(\vec{p}_2)$$

$$K\bar{K} \sim \sum_{\vec{p}_1+\vec{p}_2 \in \vec{P}} \mathrm{CG}\left(\vec{p}_1, \vec{p}_2, \Lambda^{\vec{P}}\right) \quad K(\vec{p}_1)\bar{K}(\vec{p}_2)$$

& similarly for all other mesons

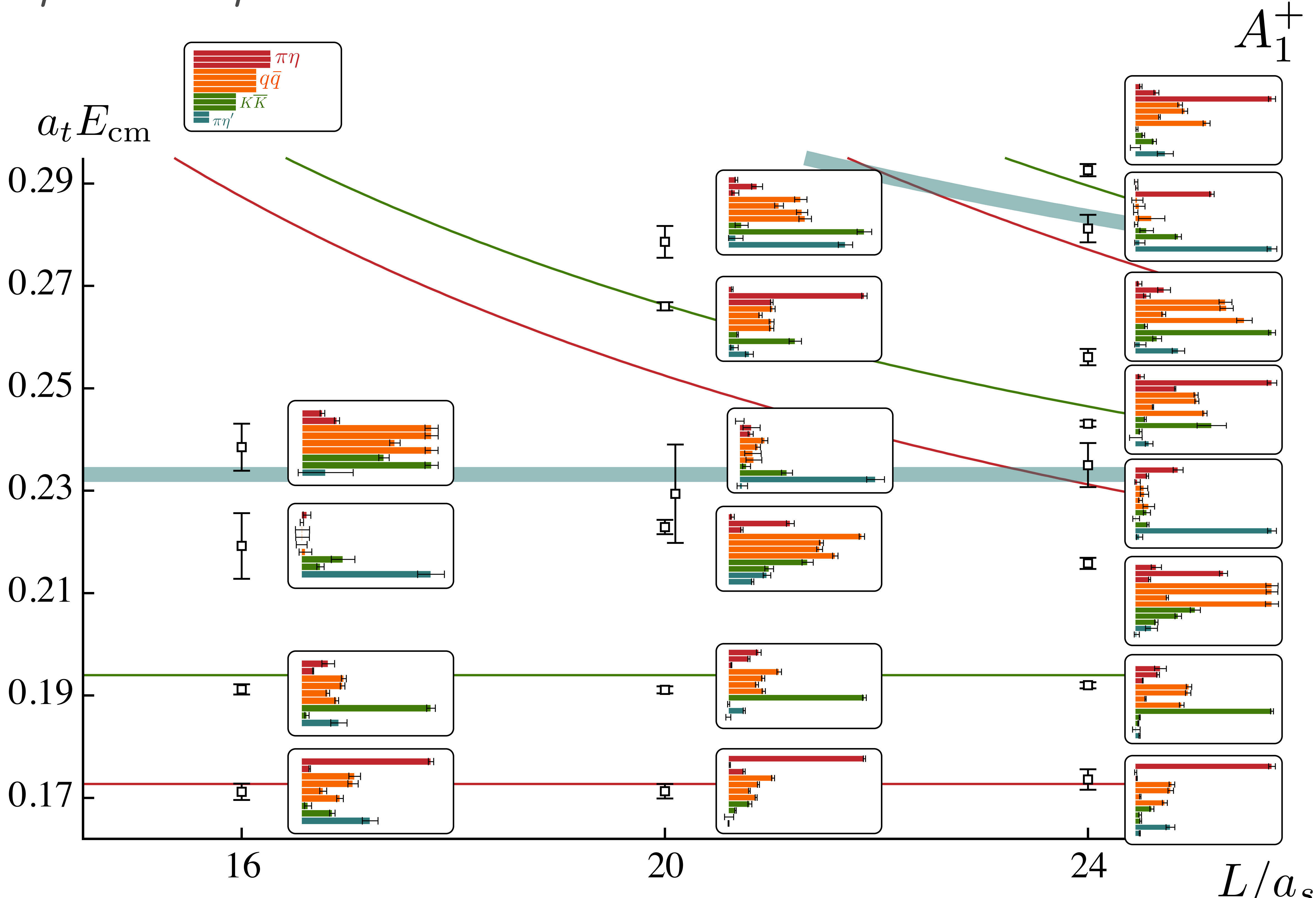
Compute the correlation matrix: $C_{ij}(t) = \langle 0| \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0) |0\rangle$

Solve a GEVP: $C_{ij}(t)v_j^{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t)C_{ij}(t_0)v_j^{\mathfrak{n}}$

Read off the energies $E_{\mathfrak{n}}$ $\quad \lambda_{\mathfrak{n}}(t) \sim e^{-E_{\mathfrak{n}}(t-t_0)}$

a_0 resonance

$\pi\eta$-$K\bar{K}$-$\pi\eta'$



$m_\pi = 391$ MeV

$\mathbf{a_0}$ resonance

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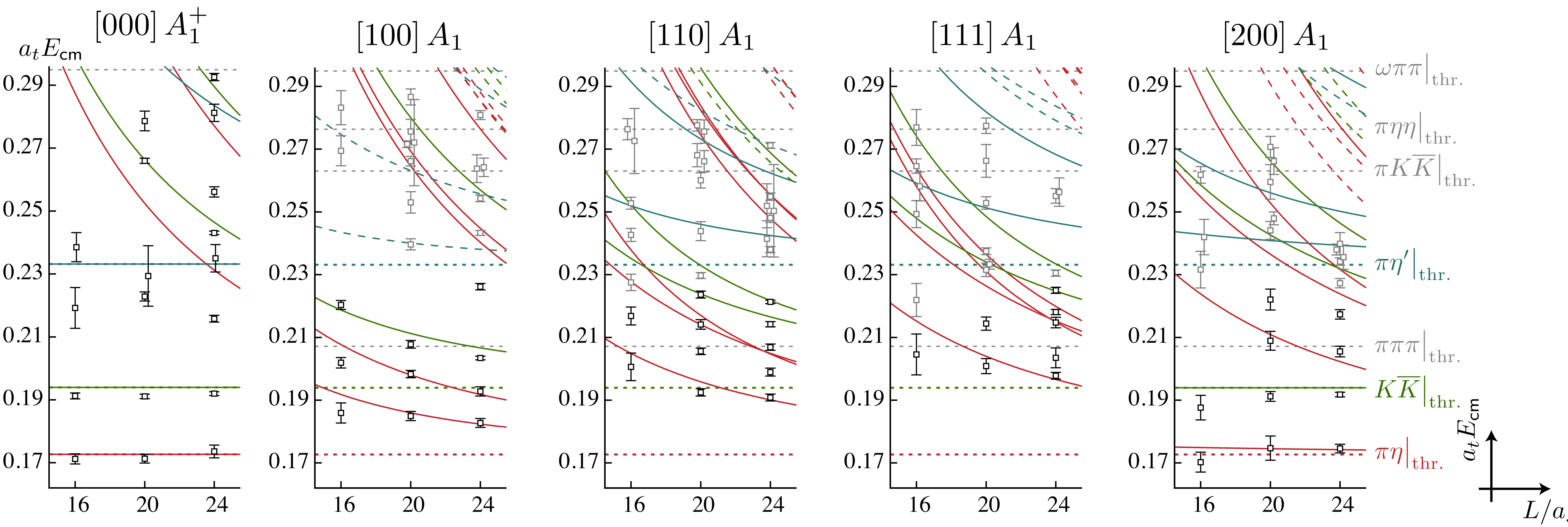

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Energy levels to scattering amplitudes

Direct extension of the elastic quantization condition derived by Lüscher

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

phase space

infinite volume scattering
t-matrix

known finite-volume
functions

Many derivations, **all in agreement**:

He, Feng, Liu 2005 - two channel QM, strong coupling
Bernard et al 2011 - two channel K-matrix for f_0 and a_0
Hansen & Sharpe 2012 - field theory, multiple two-body channels
Briceño & Davoudi 2012 - strongly-coupled Bethe-Salpeter amplitudes
Guo et al 2012 - Hamiltonian & Lippmann-Schwinger

Also derivations in specific channels, or for a specific parameterization of the interactions like NREFT, Chiral PT, Finite Volume Hamiltonian, etc.

Briceño 2014 - Most general condition for scattering of particles with arbitrary spins

Significant steps towards a general 3-body quantization condition have been made

Energy levels to scattering amplitudes

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}(E)\cdot\boldsymbol{t}(E)\cdot\left(\mathbf{1} + i\boldsymbol{\mathcal{M}}(E,L)\right)\right] = 0$$

- In the coupled-channel region, the t-matrix has multiple unknowns as a function of energy
- Use a parameterisation to interpolate the constraint provided by many energy levels
- One particularly useful form is the K-matrix - it also naturally respects S-matrix unitarity:

$$\boldsymbol{t}^{-1}(E) = \boldsymbol{K}^{-1}(E) + \boldsymbol{I}(E)$$

$$\mathrm{Im}I_{ij}(E) = -\rho_i\delta_{ij}$$

- Any function that is real for real E is permitted for K
- In practice low-order forms are sufficient:

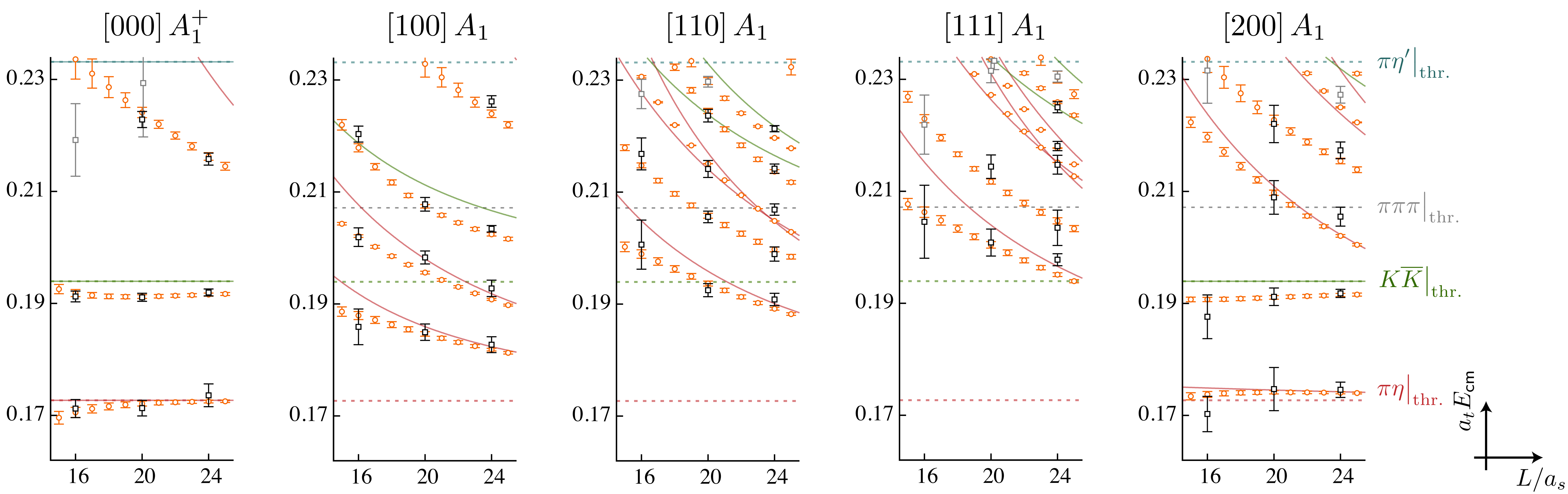
$$K_{ij} = \frac{g_i g_j}{m^2 - s} + \gamma_{ij}$$

- minimise m, g_i, γ_{ij}
- we also use many other forms

a₀ resonance - two channel region

$\pi\eta$-$K\bar{K}$

using 47 energy levels



$$K_{ij} = \frac{g_i g_j}{m^2 - s} + \gamma_{ij}$$

$$\begin{aligned}
m &= (0.2214 \pm 0.0029 \pm 0.0004) \cdot a_t^{-1} \\
g_{\pi\eta} &= (\ \ 0.091 \pm 0.016 \pm 0.009) \cdot a_t^{-1} \\
g_{K\bar{K}} &= (-0.129 \pm 0.015 \pm 0.002) \cdot a_t^{-1} \\
\gamma_{\pi\eta,\,\pi\eta} &= -0.16 \pm 0.24 \pm 0.03 \\
\gamma_{\pi\eta,\,K\bar{K}} &= -0.56 \pm 0.29 \pm 0.04 \\
\gamma_{K\bar{K},\,K\bar{K}} &= \ \ 0.12 \pm 0.38 \pm 0.08
\end{aligned}$$

$$\begin{bmatrix} 1 & 0.58 & -0.06 & -0.51 & 0.39 & 0.02 \\ & 1 & -0.63 & -0.87 & 0.84 & -0.49 \\ & & 1 & 0.52 & -0.68 & 0.83 \\ & & & 1 & -0.90 & 0.53 \\ & & & & 1 & -0.78 \\ & & & & & 1 \end{bmatrix}$$

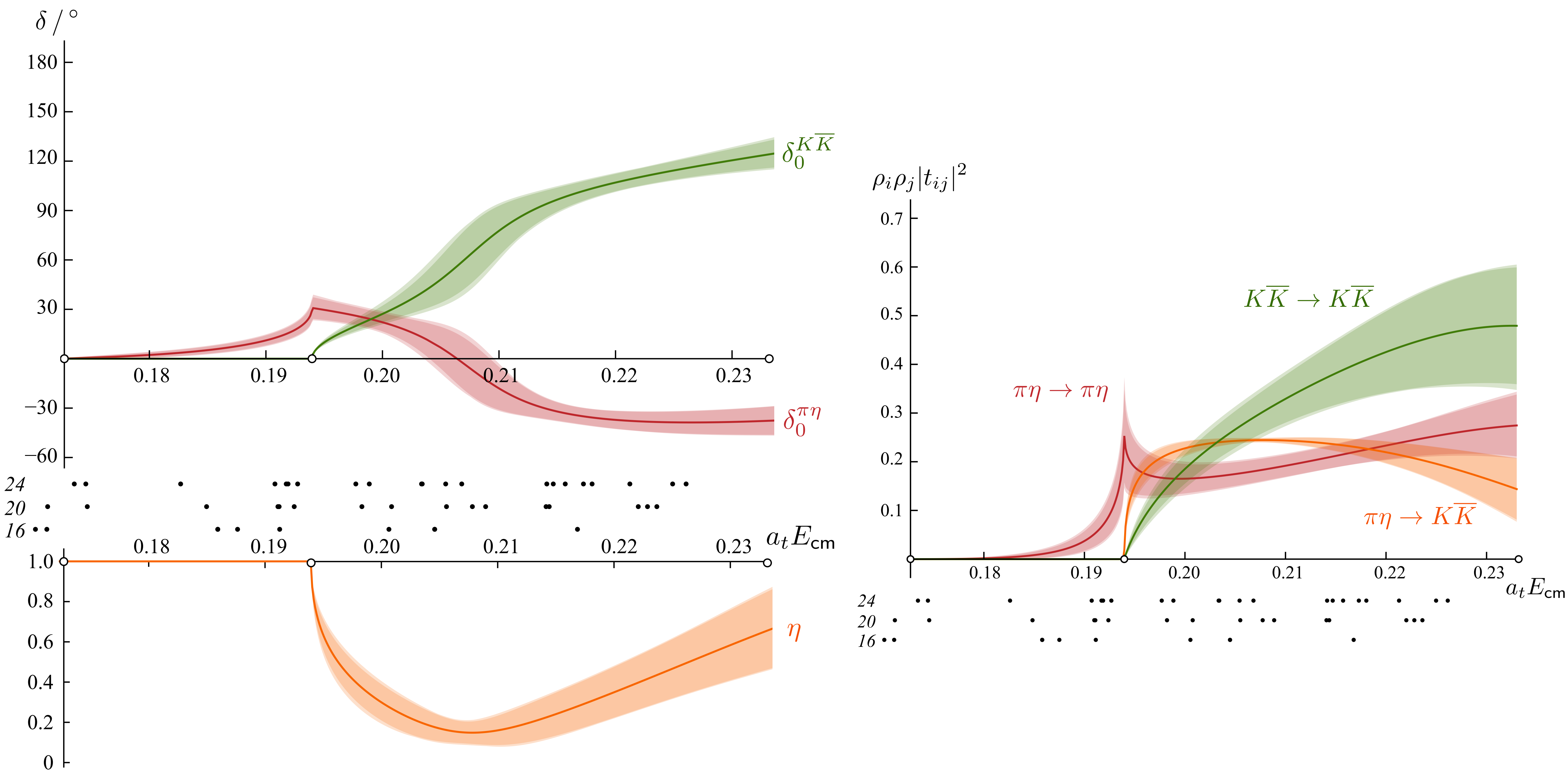
$$\chi^2/N_{\mathrm{dof}} = \frac{58.0}{47-6} = 1.41$$

$m_\pi = 391$ MeV

a_0 resonance - two channel region

S-wave $\pi\eta$-$K\bar{K}$

from 47 energy levels



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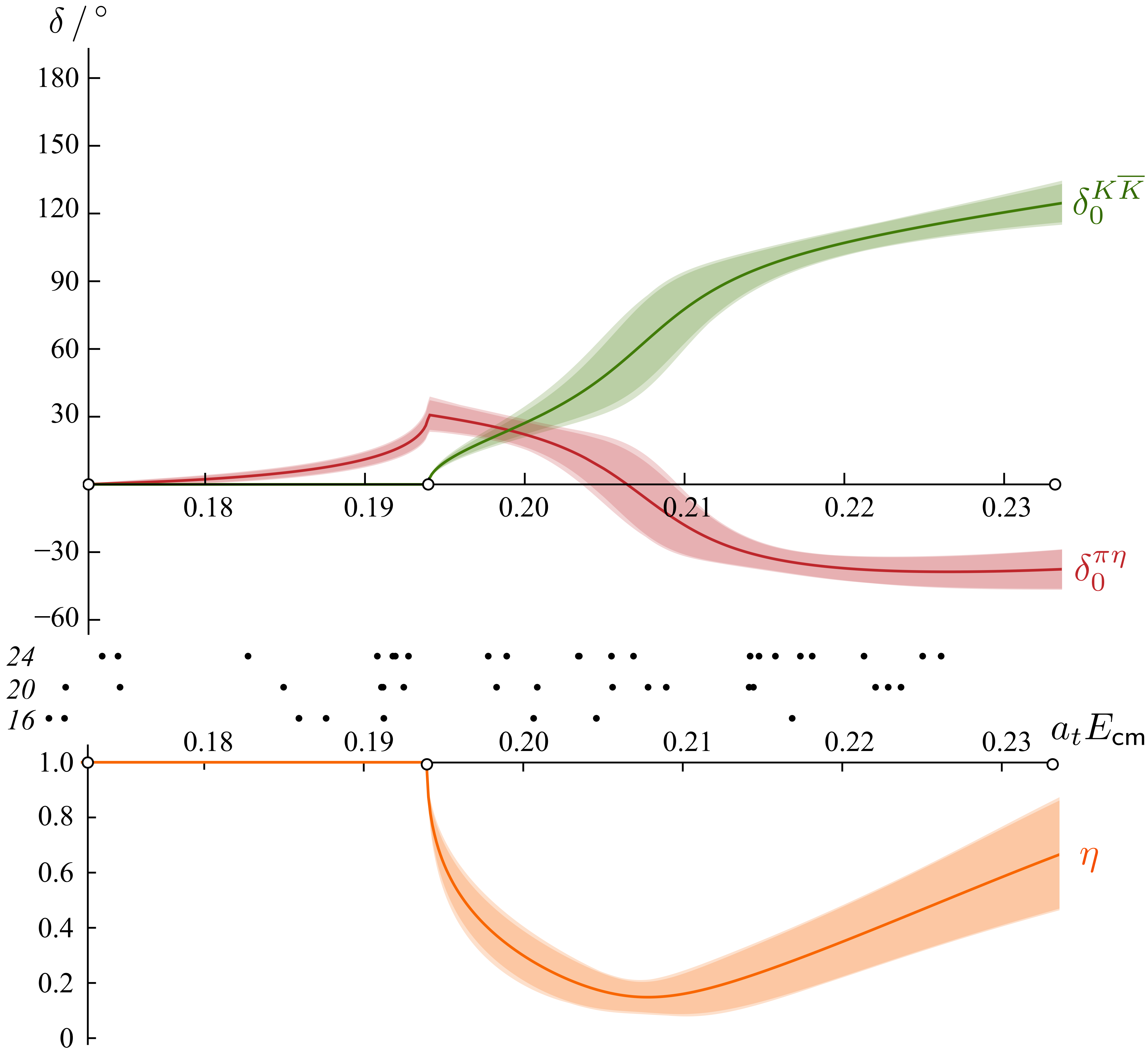
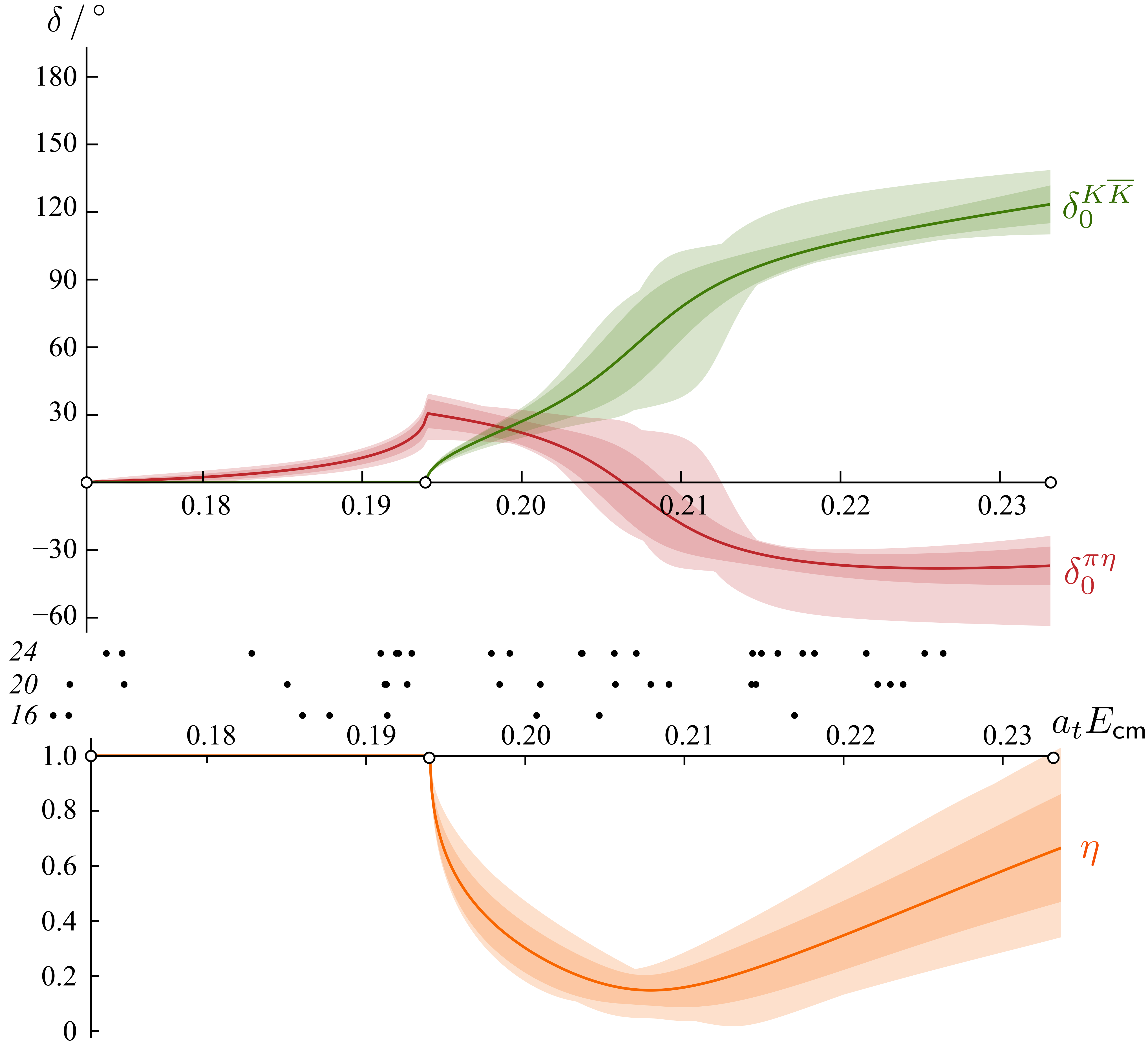


$\delta\,/\,^\circ$
180
150
120
90
60
30
−30
−60
0.18
0.19
0.20
0.21
0.22
0.23
$\delta_0^{K\overline{K}}$
$\delta_0^{\pi\eta}$
24
20
16
$a_t E_{\mathsf{cm}}$
1.0
0.8
0.6
0.4
0.2
0
η

$m_\pi = 391$ MeV



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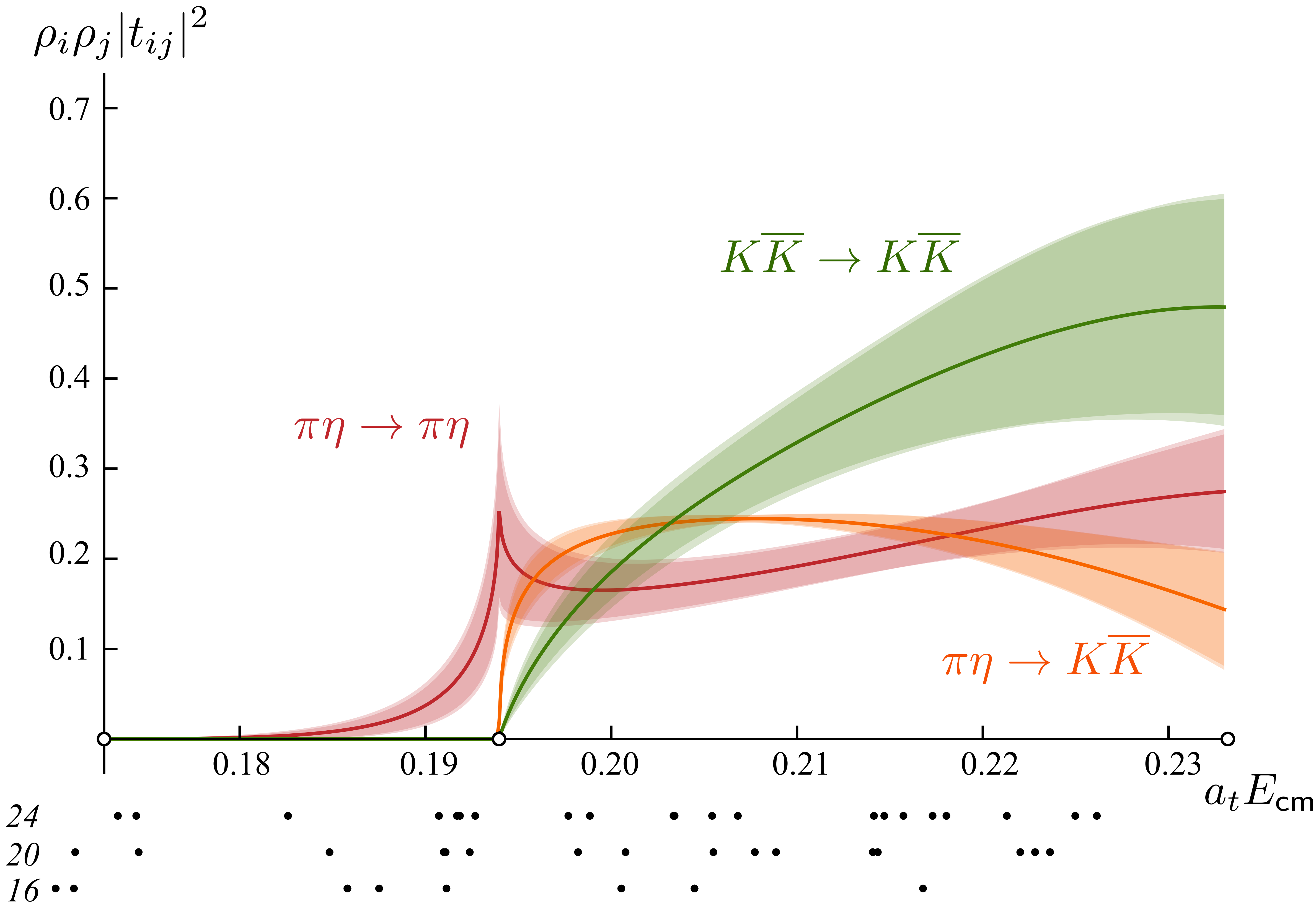
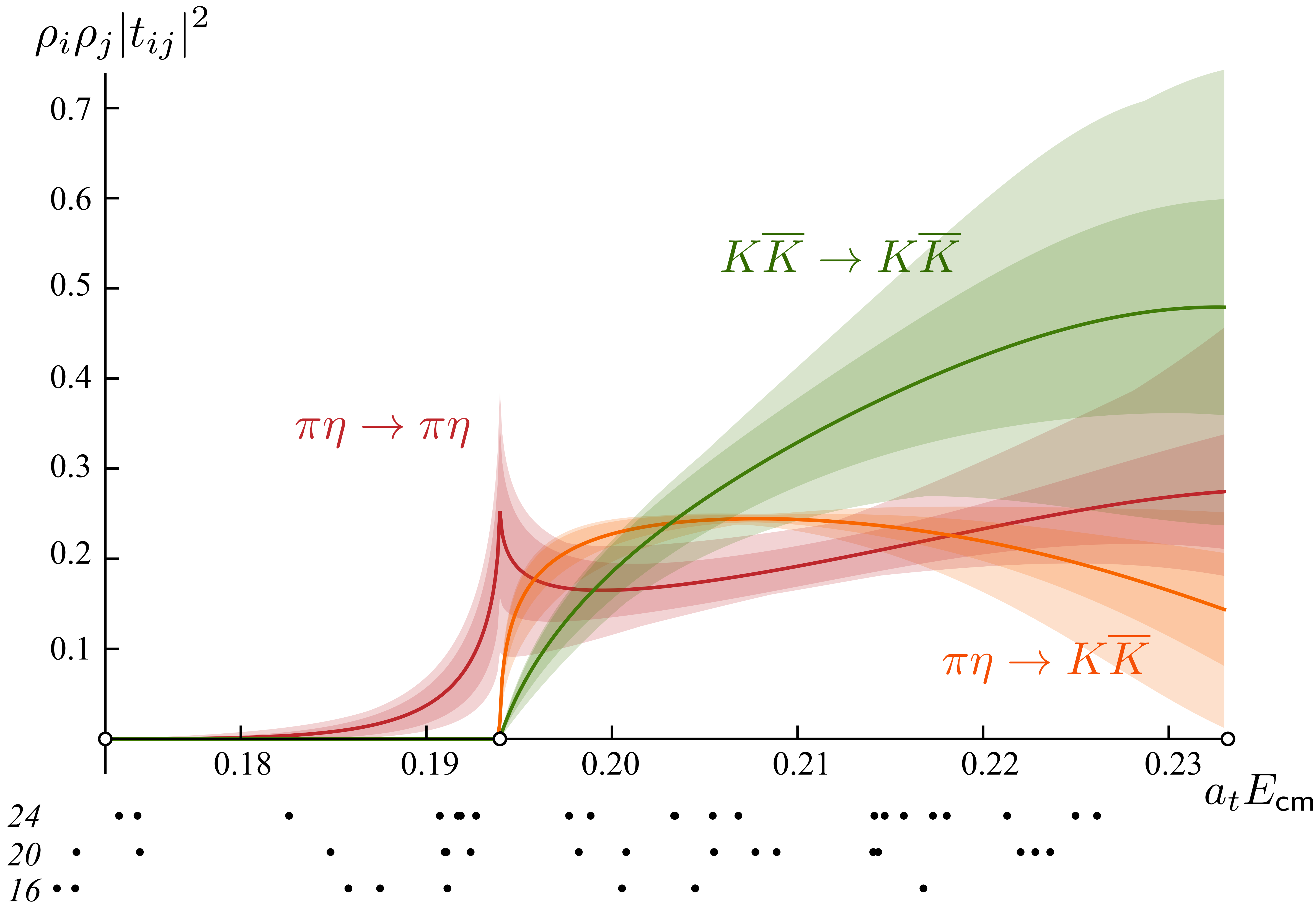


$\rho_i \rho_j |t_{ij}|^2$
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.18
0.19
0.20
0.21
0.22
0.23
$a_t E_{\mathsf{cm}}$
$K\overline{K} \to K\overline{K}$
$\pi\eta \to \pi\eta$
$\pi\eta \to K\overline{K}$
24
20
16

$m_\pi = 391$ MeV



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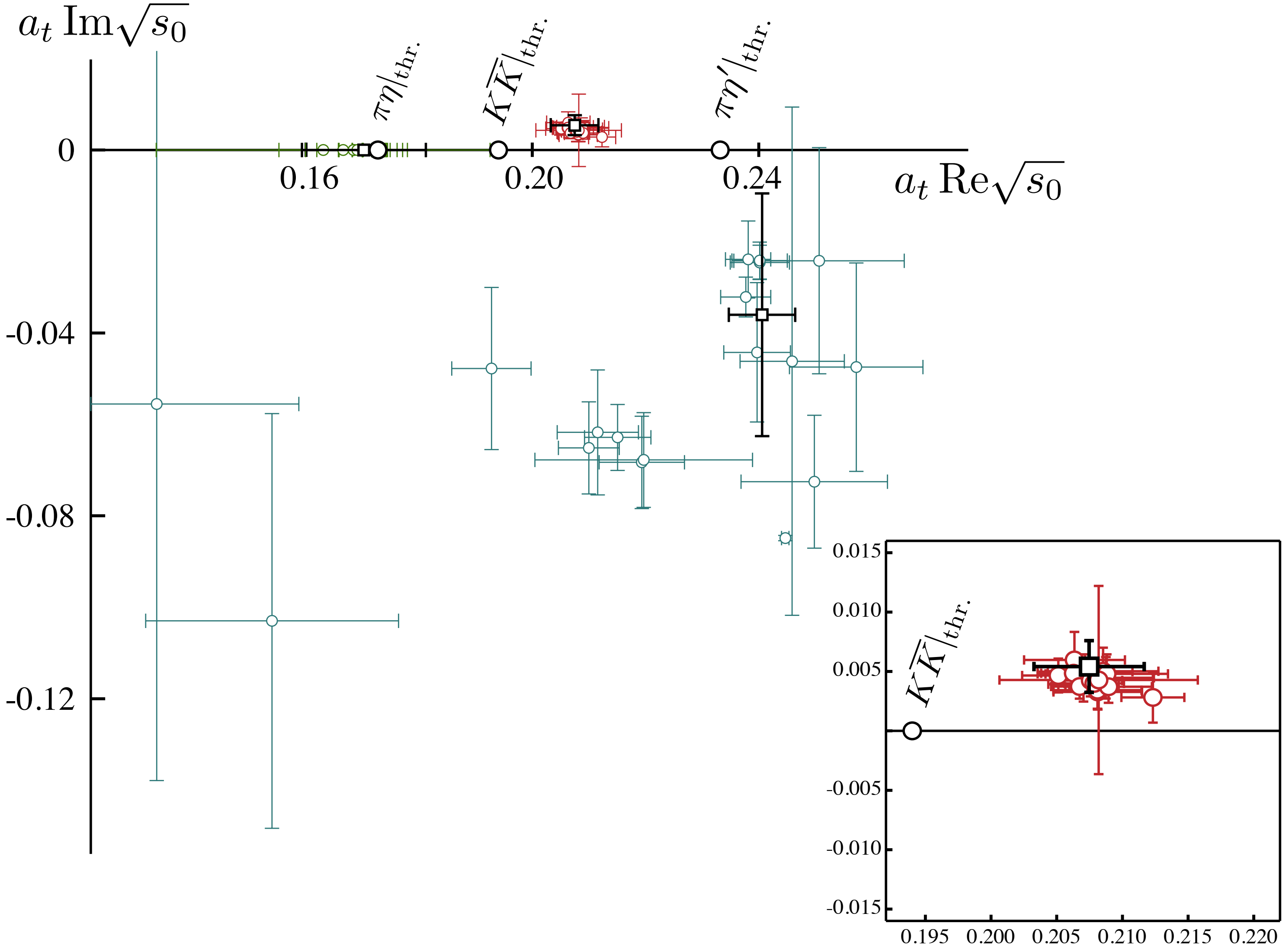
24
20
16

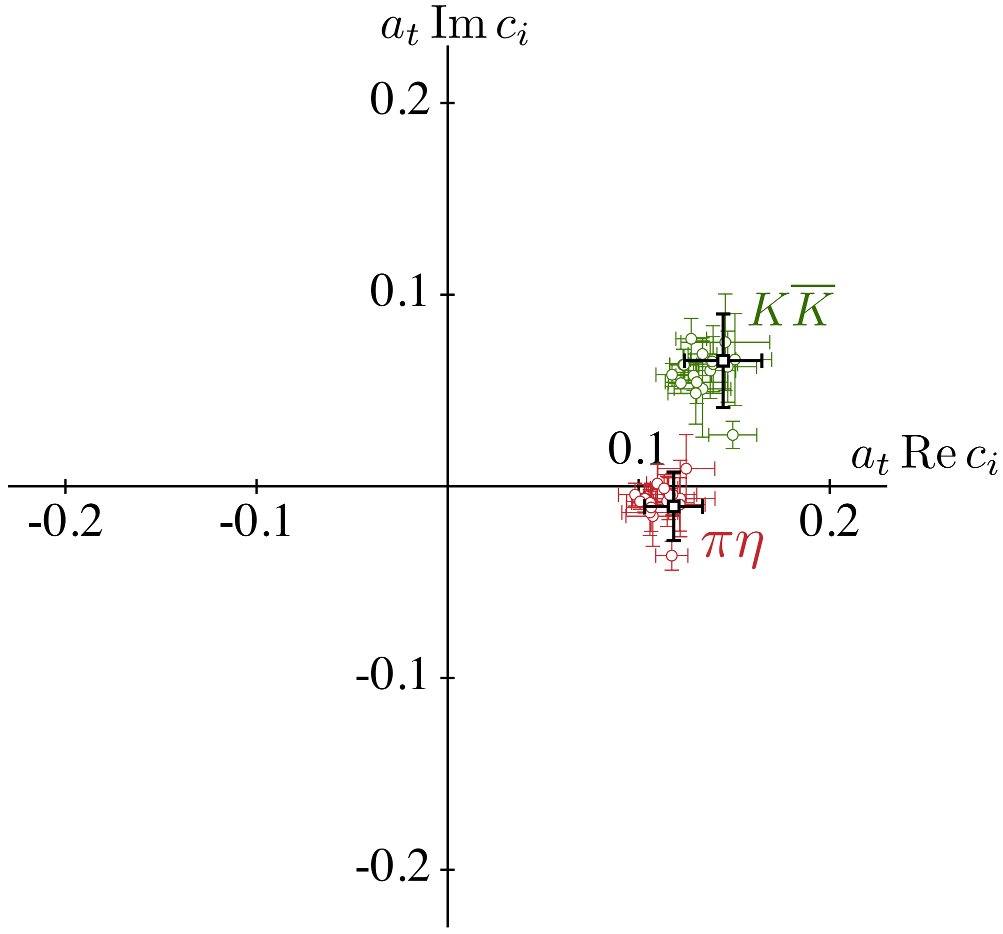
$m_\pi = 391$ MeV

a_0 resonance pole

$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$



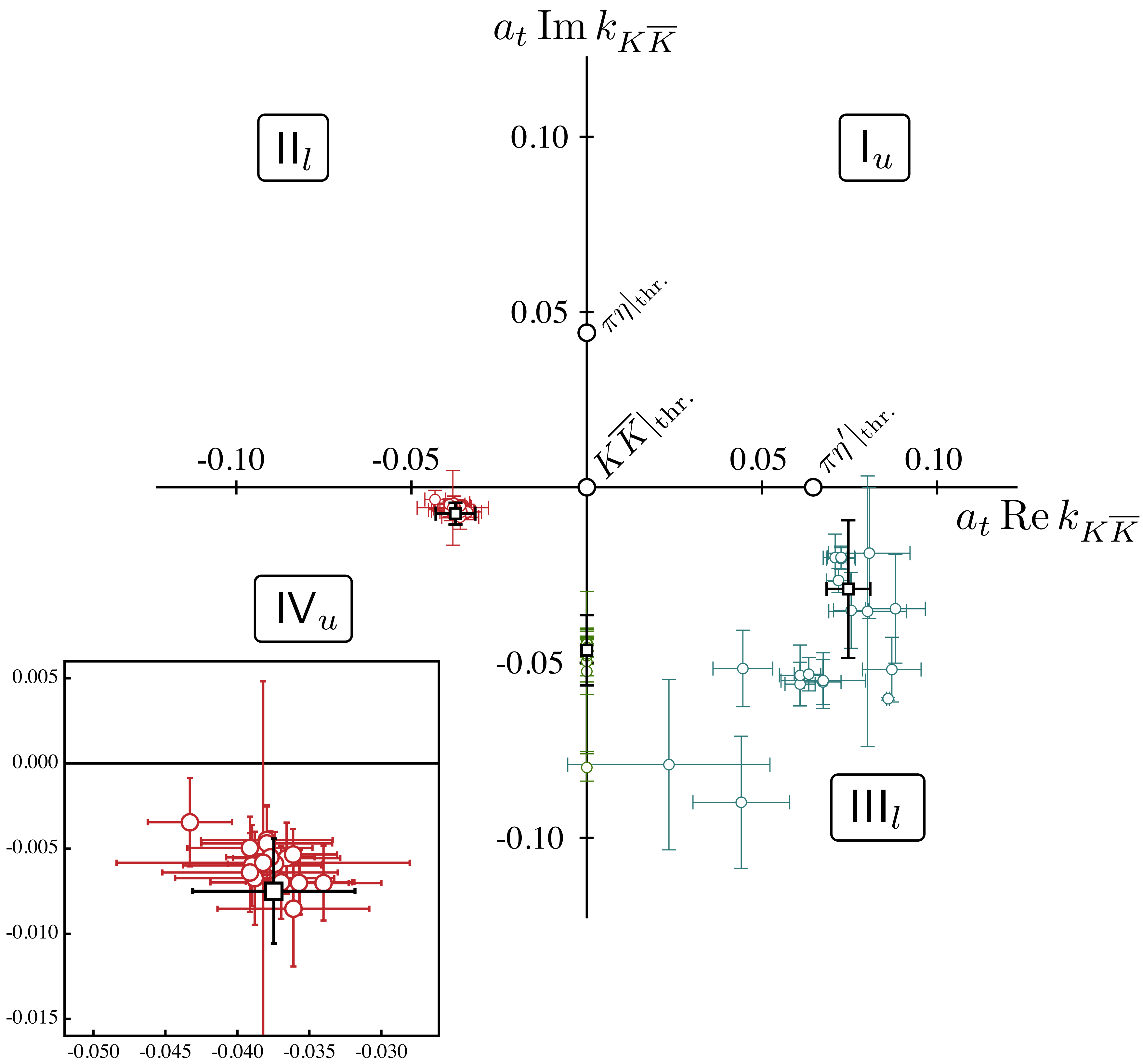
consistent *fourth* sheet pole



pole channel couplings

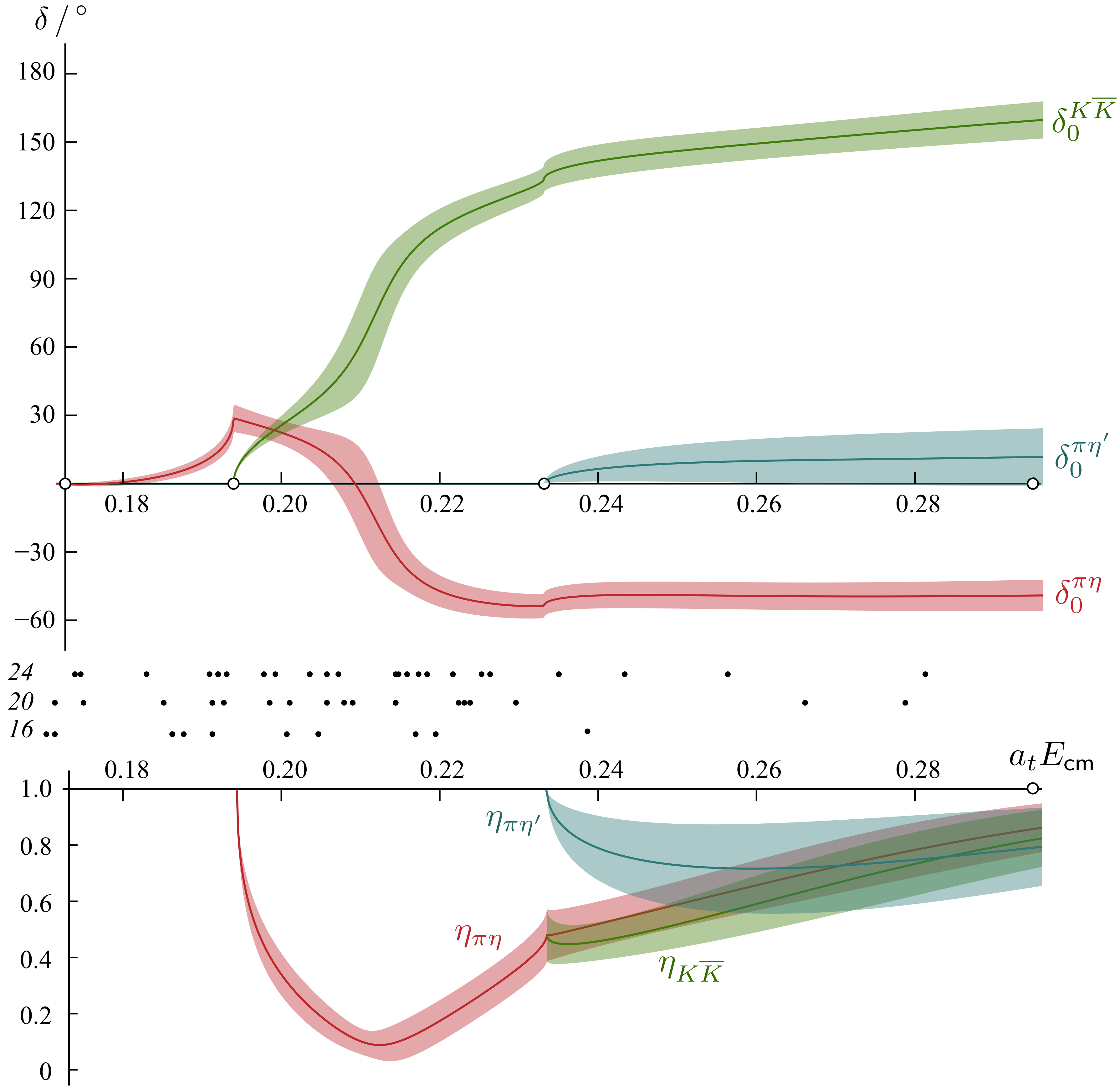
$m_\pi = 391$ MeV

Poles



$m_\pi = 391$ MeV

a_0 resonance - three channel region



$m_\pi = 391$ MeV

Future plans

Many interesting channels to investigate:

- $f_0(980)$
- Heavy quarks, charmonium
- These methods are also applicable to the scattering of spinning particles: $D\bar{D}^\star, \pi N$
- Investigating excited states as a function of pion mass can give insight.

Lots of work still needed:

- For lighter masses and higher energies, three-body effects must be understood.

Hadron Spectrum Collaboration

Special thanks to the others involved in these projects…

Jefferson Lab and around:

Raul Briceño
Jozef Dudek
Robert Edwards
David Richards

Trinity College Dublin:

Mike Peardon
Sinéad Ryan
Cian O'Hara
David Tims

Cambridge DAMTP:

Christopher Thomas
Graham Moir
Gavin Cheung
Antoni Woss

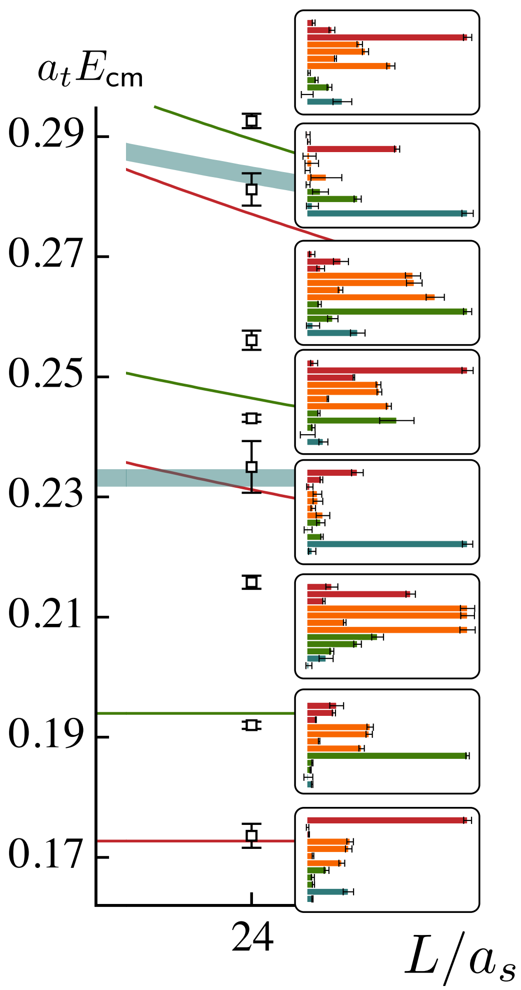
Tata Institute:

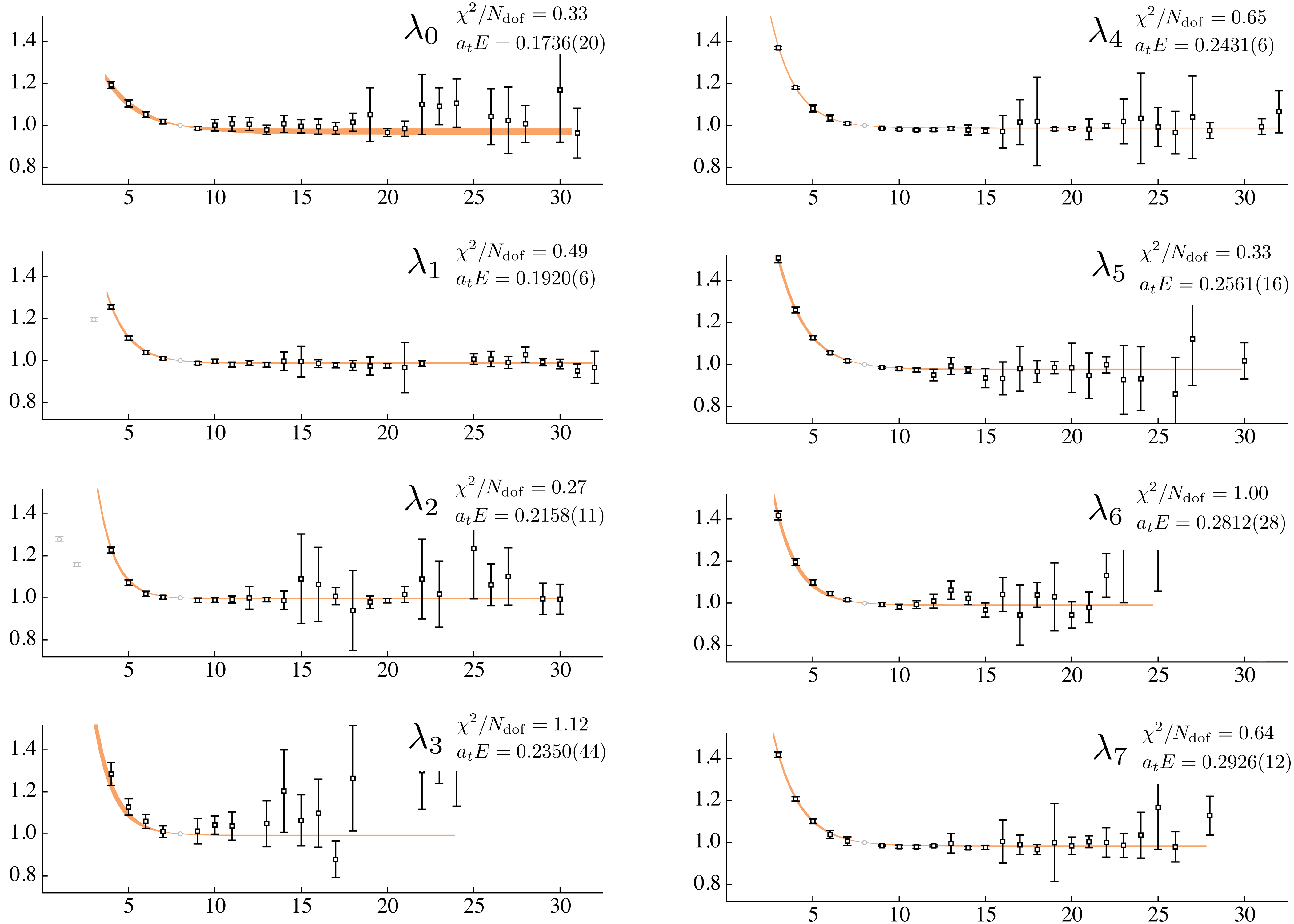
Nilmani Mathur

Backup

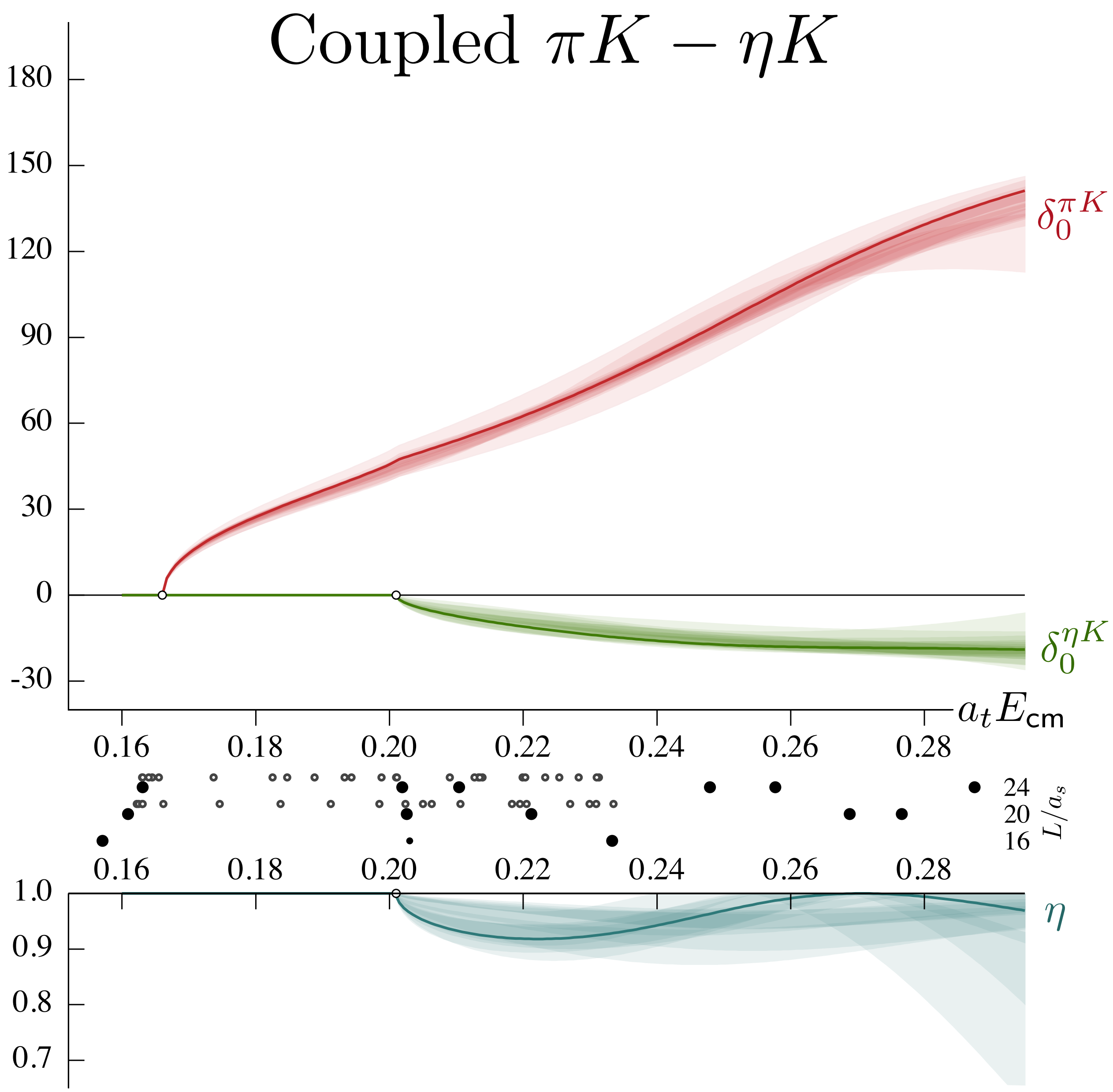
Principal Correlators

$A_1^+ \quad L/a_s = 24$

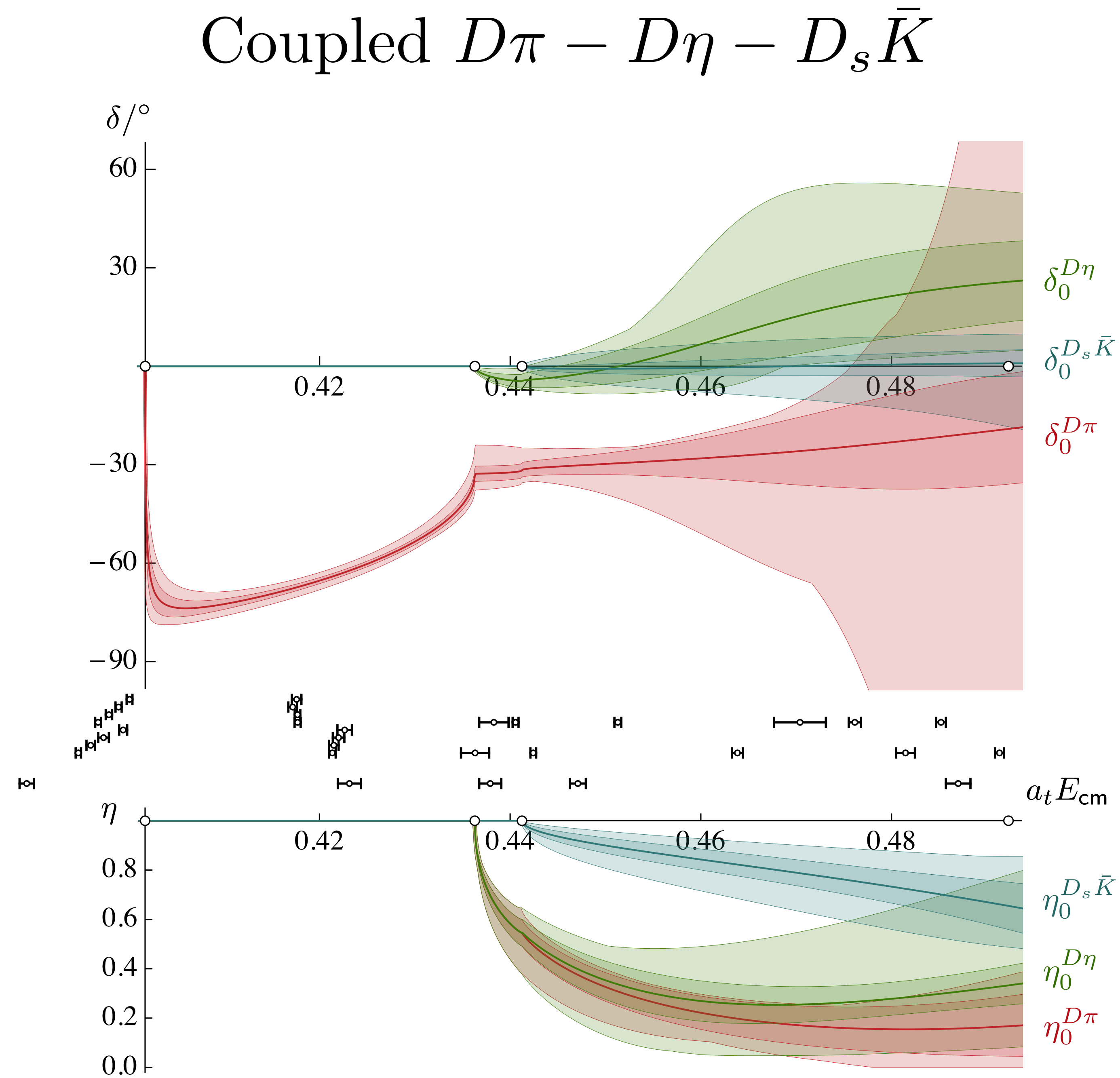




Other calculations



Combined S & P-wave analysis
80 energy levels from 3 volumes
arXiv:1406.4158, PRL 113 (2014) no.18, 182001

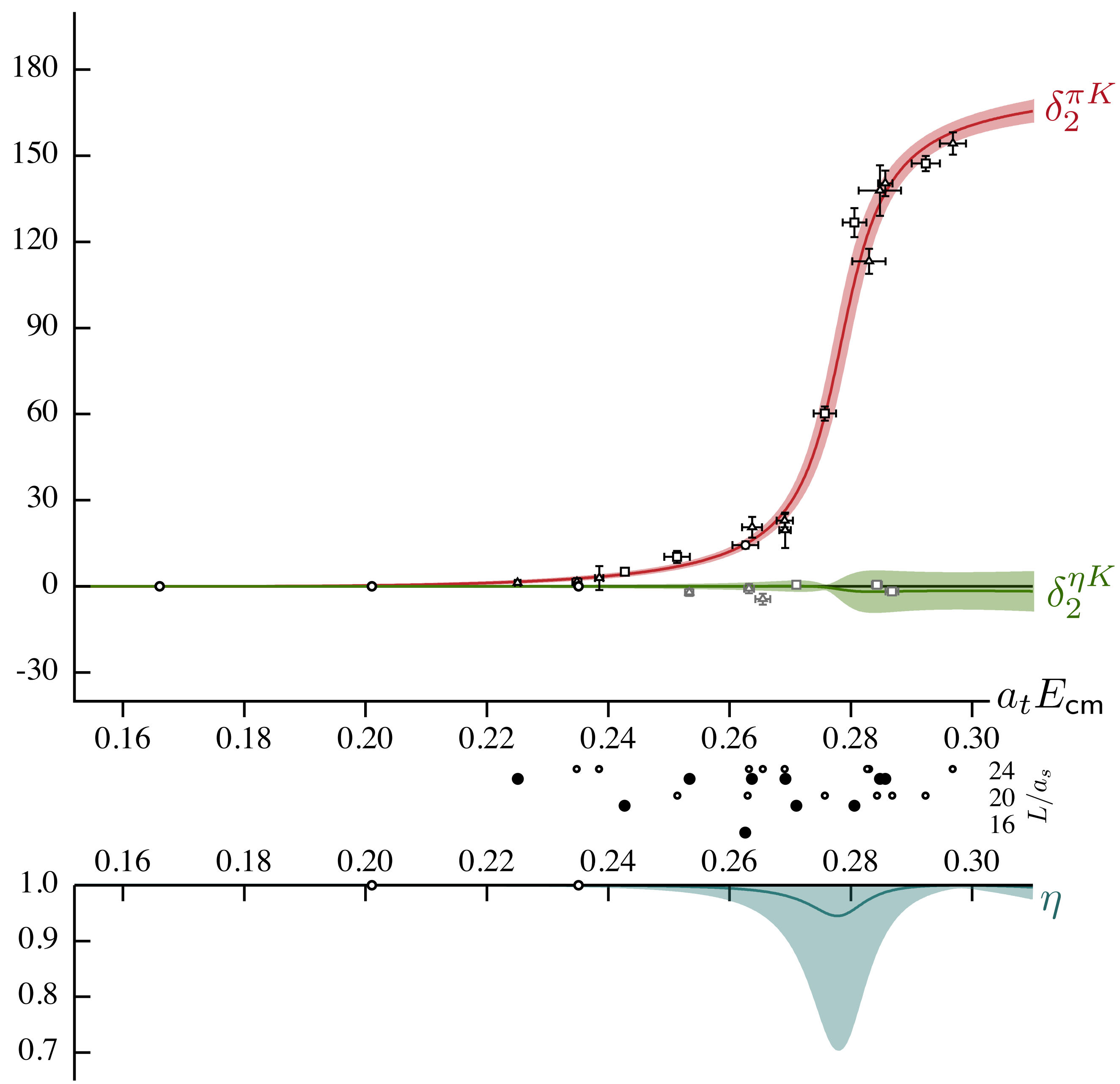


Combined S & P-wave analysis
3 coupled channels in S-wave
47 energy levels from 3 volumes
arXiv:1607.07093

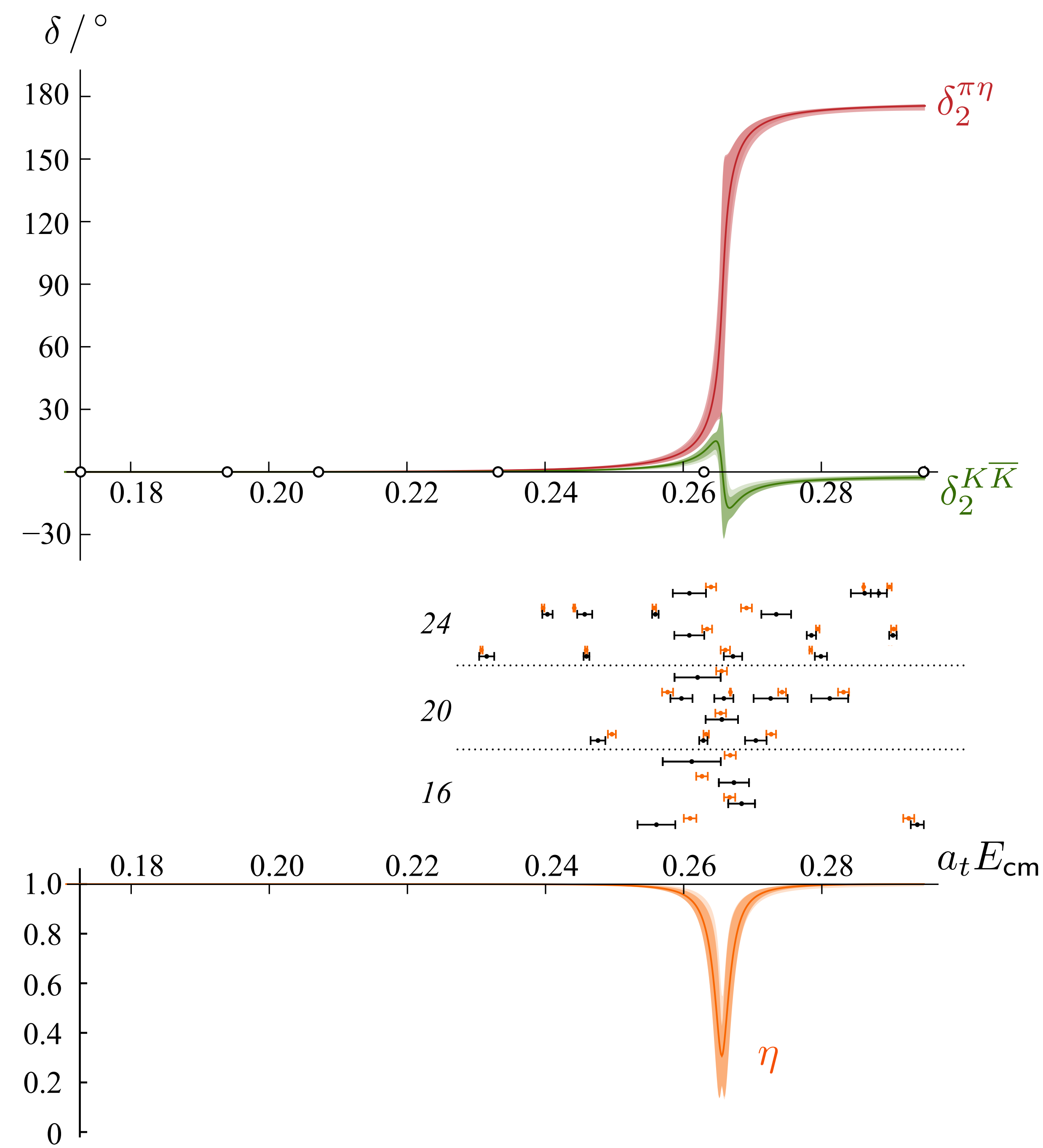
$m_\pi = 391$ MeV

Other calculations - D-waves

Coupled $\pi K - \eta K$



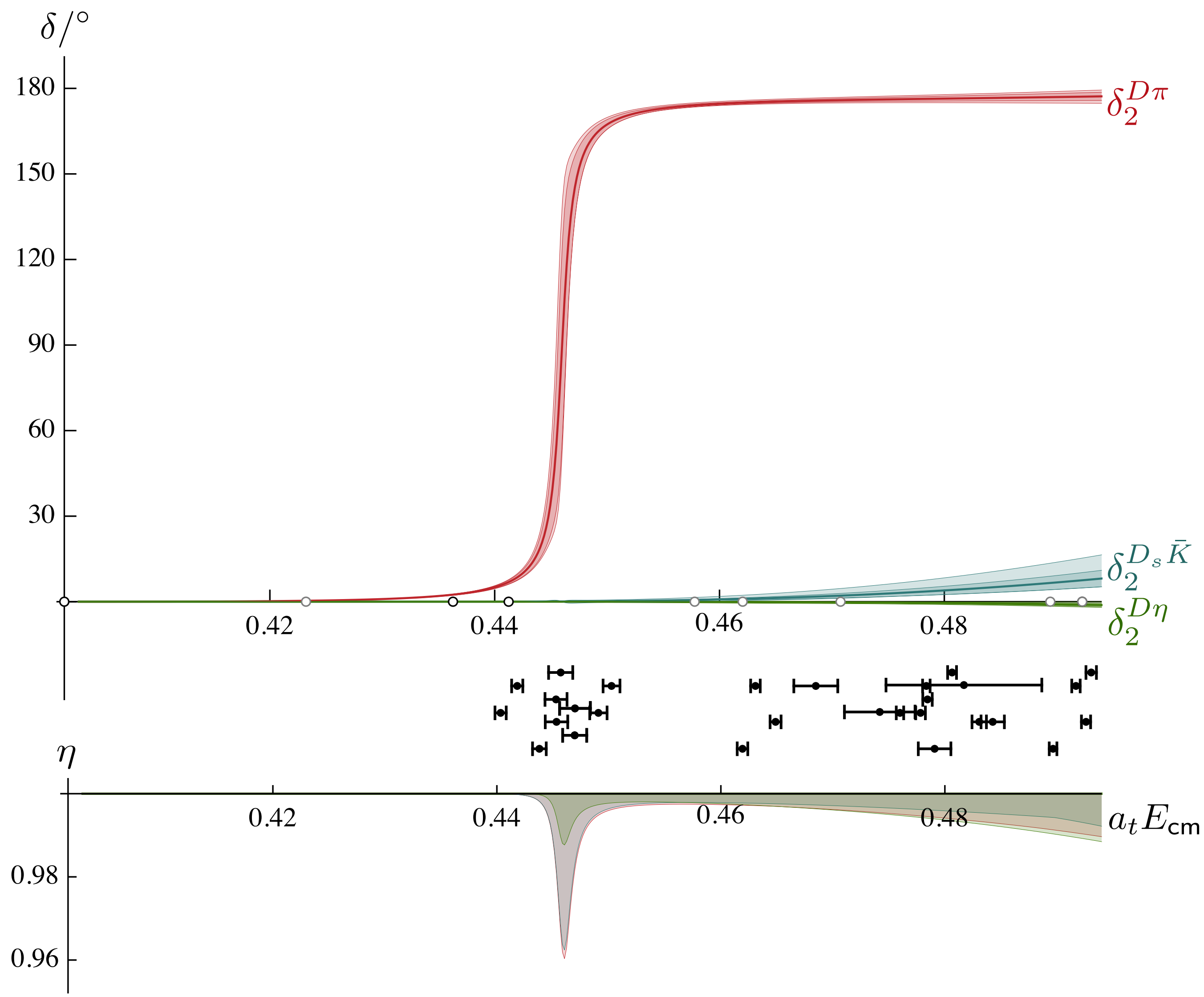
Coupled $\pi\eta - K\bar{K}$

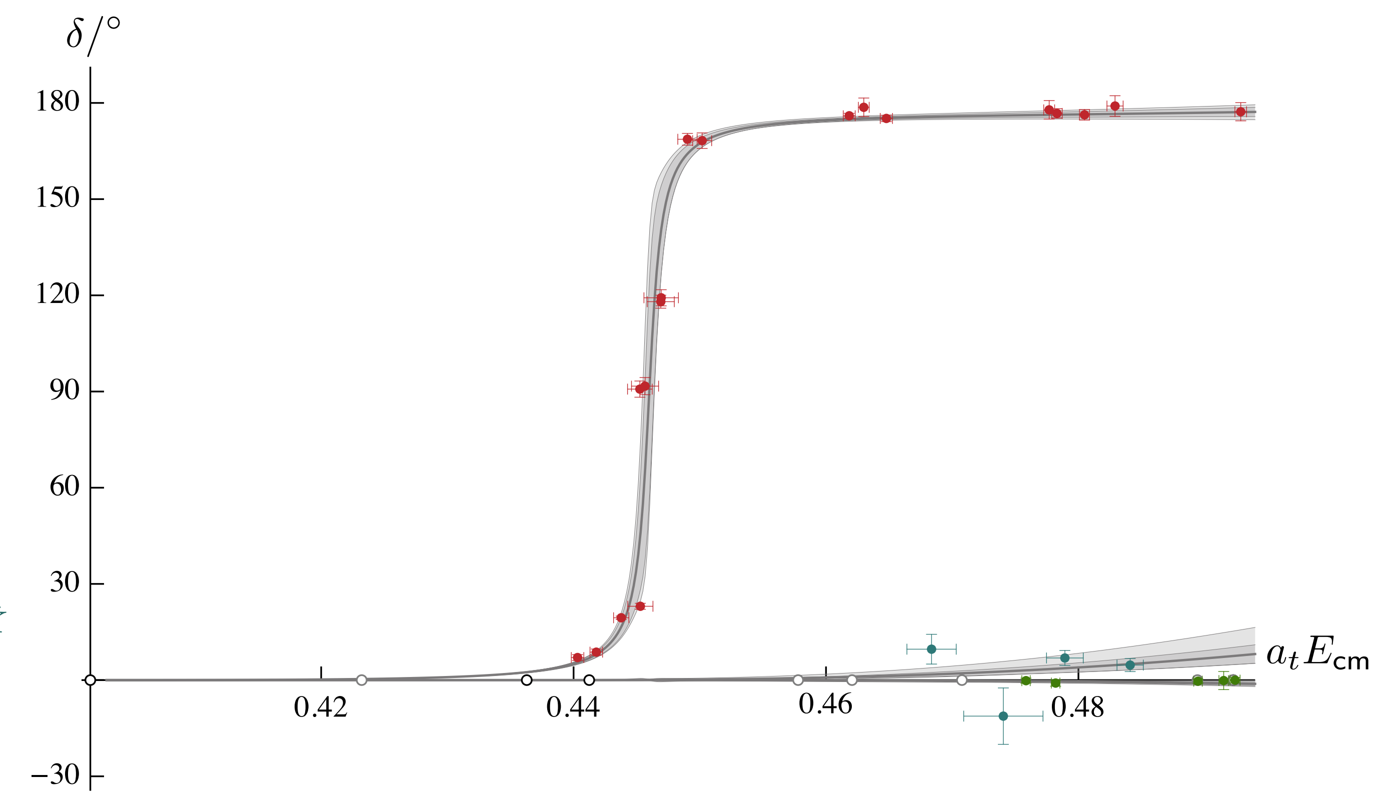


$m_\pi = 391$ MeV

Other calculations - D-waves

Coupled $D\pi - D\eta - D_s\bar{K}$





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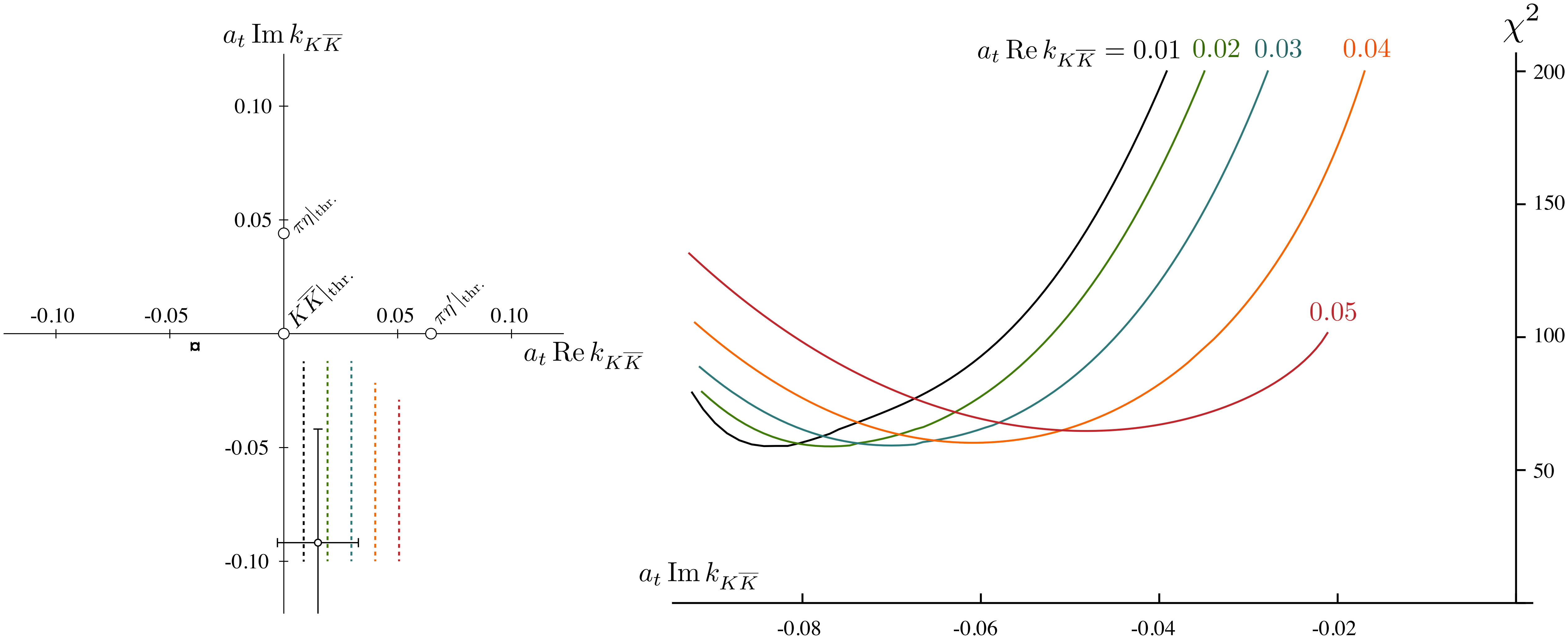
Poles

from a Jost function parameterisation



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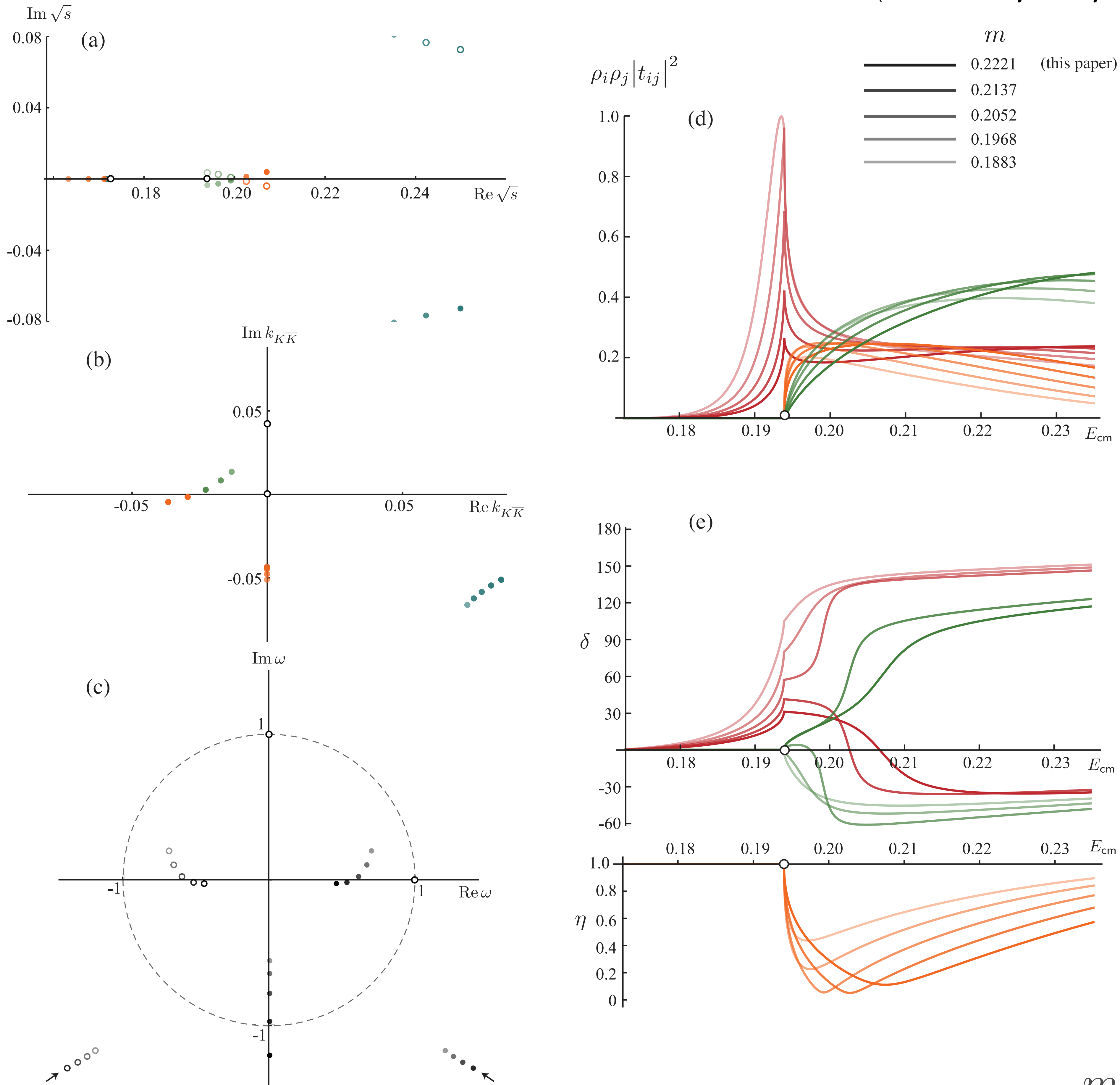
Poles

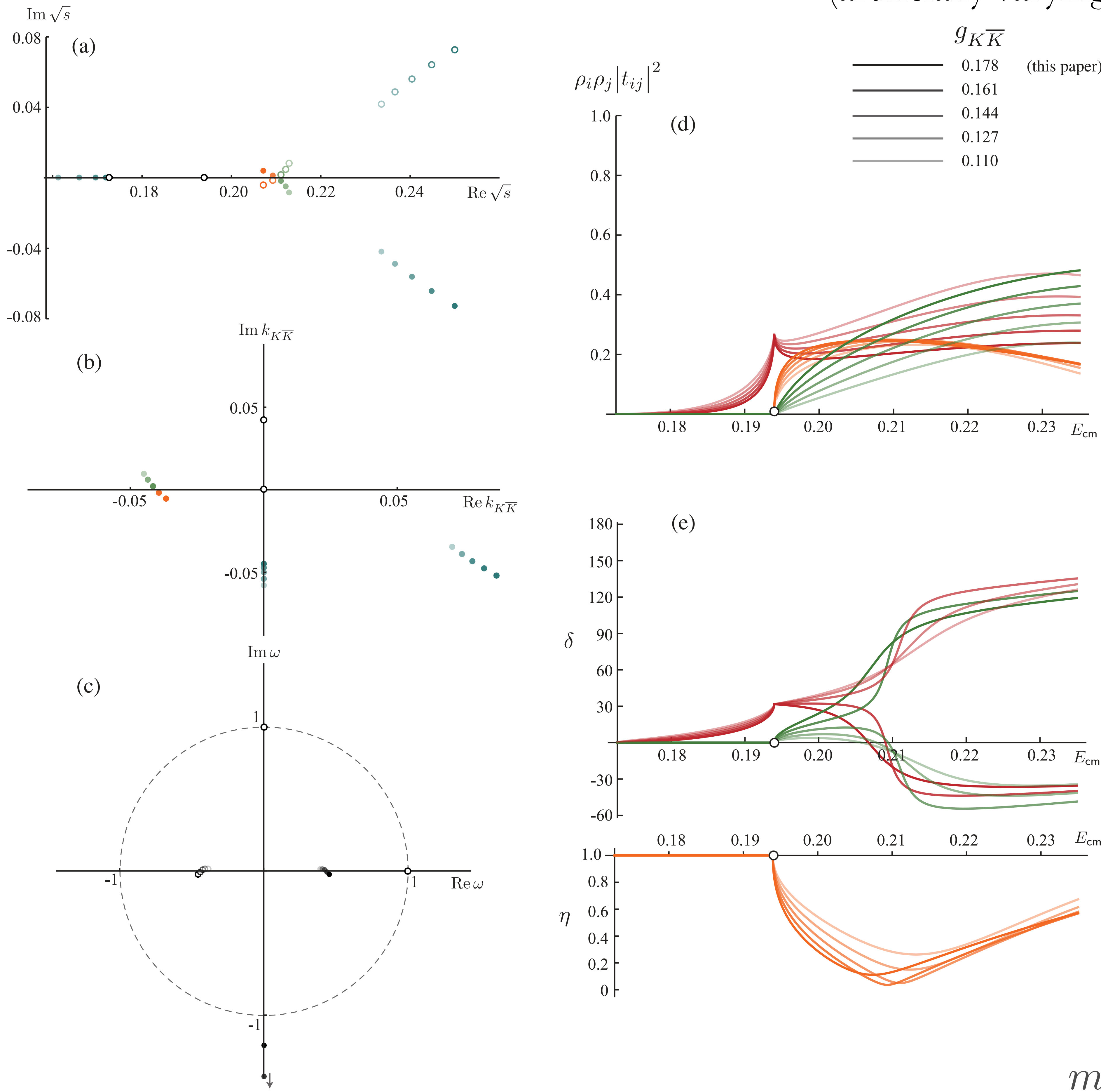
(artificially varying K-matrix m)



$m_\pi = 391$ MeV

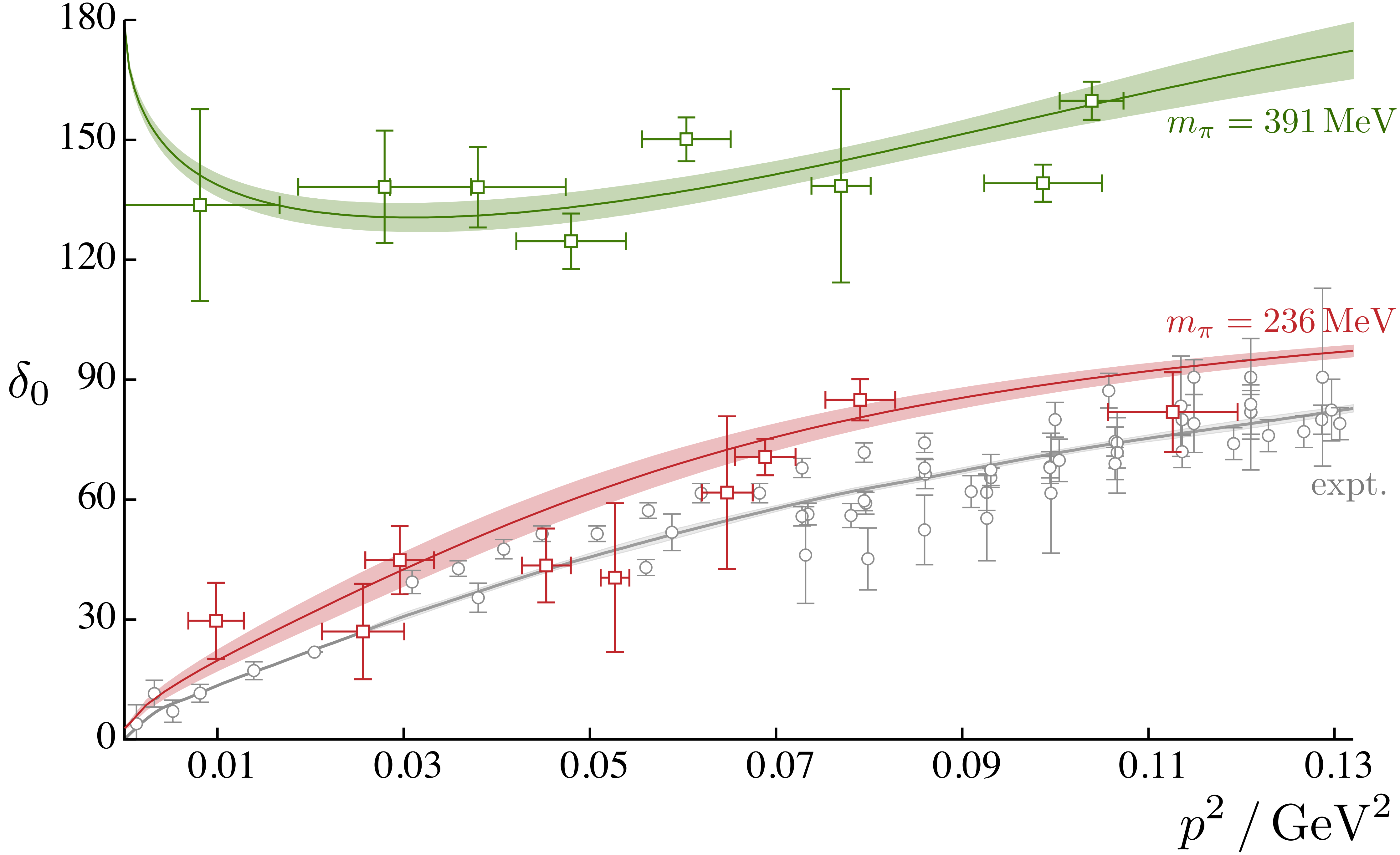
Poles

(artificially varying K-matrix gKK)



$m_\pi = 391$ MeV

The $f_0(500)/\sigma$ resonance



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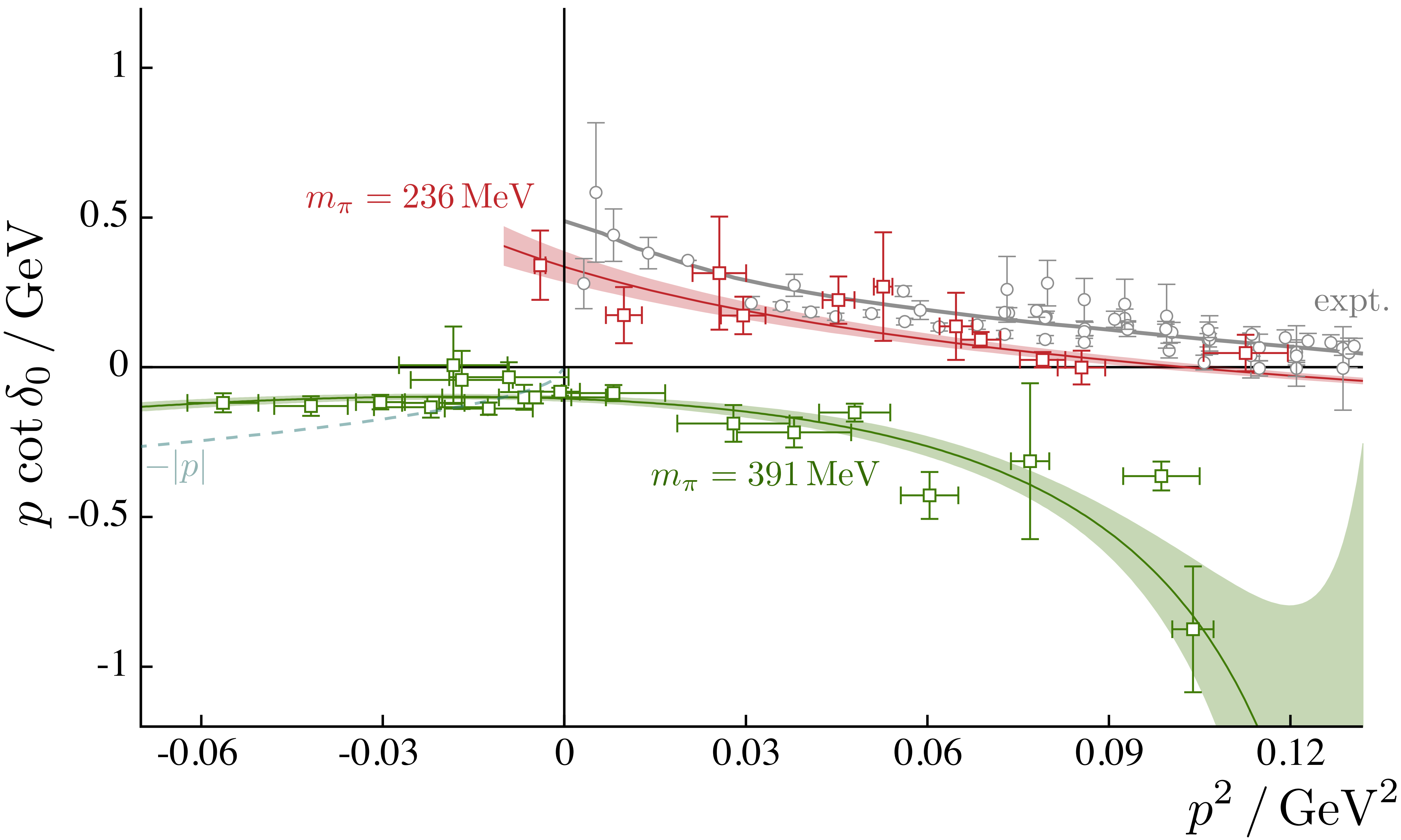
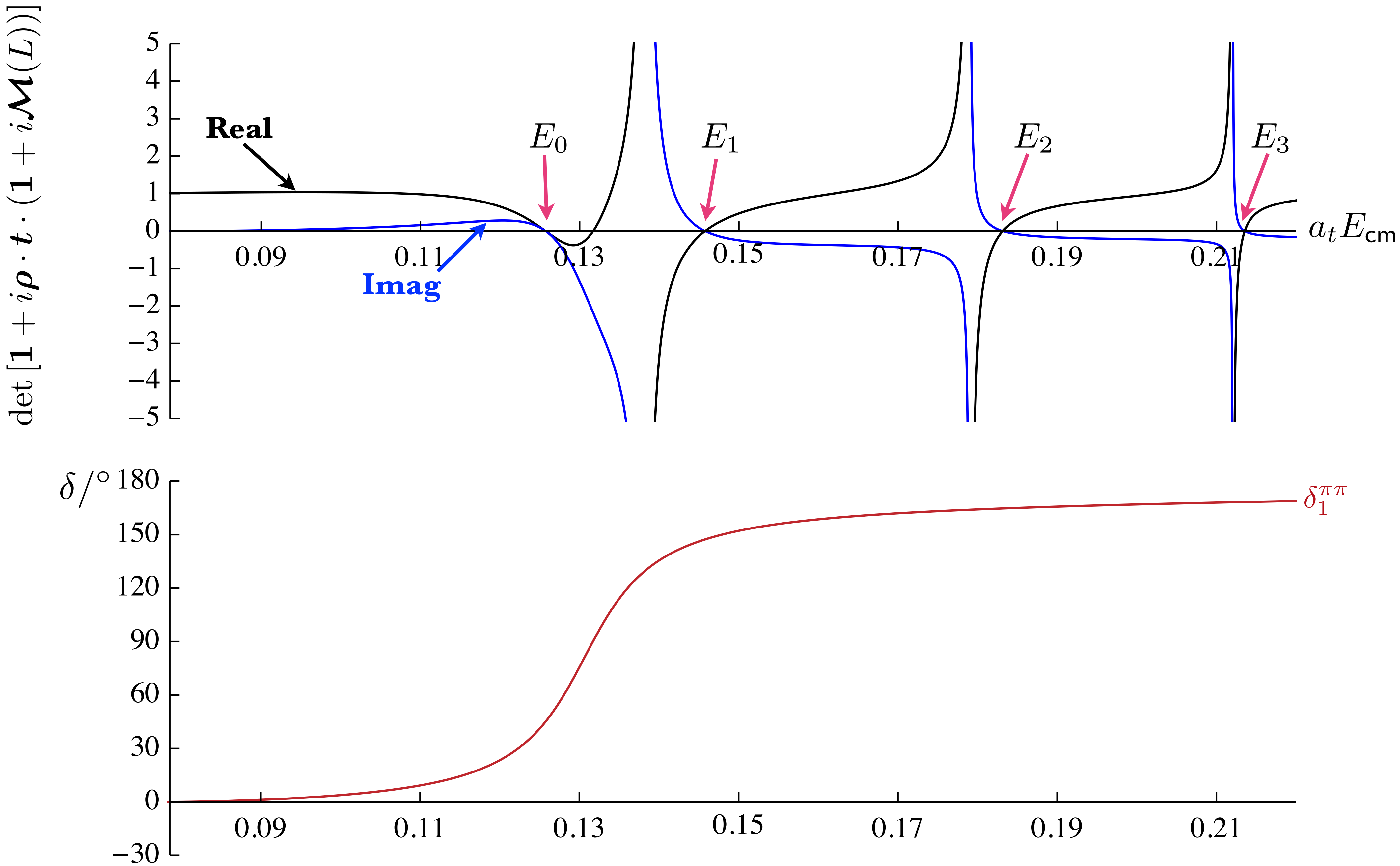


ρ resonance at $m_\pi = 236$ MeV

$$t = (\pi\pi \to \pi\pi)$$



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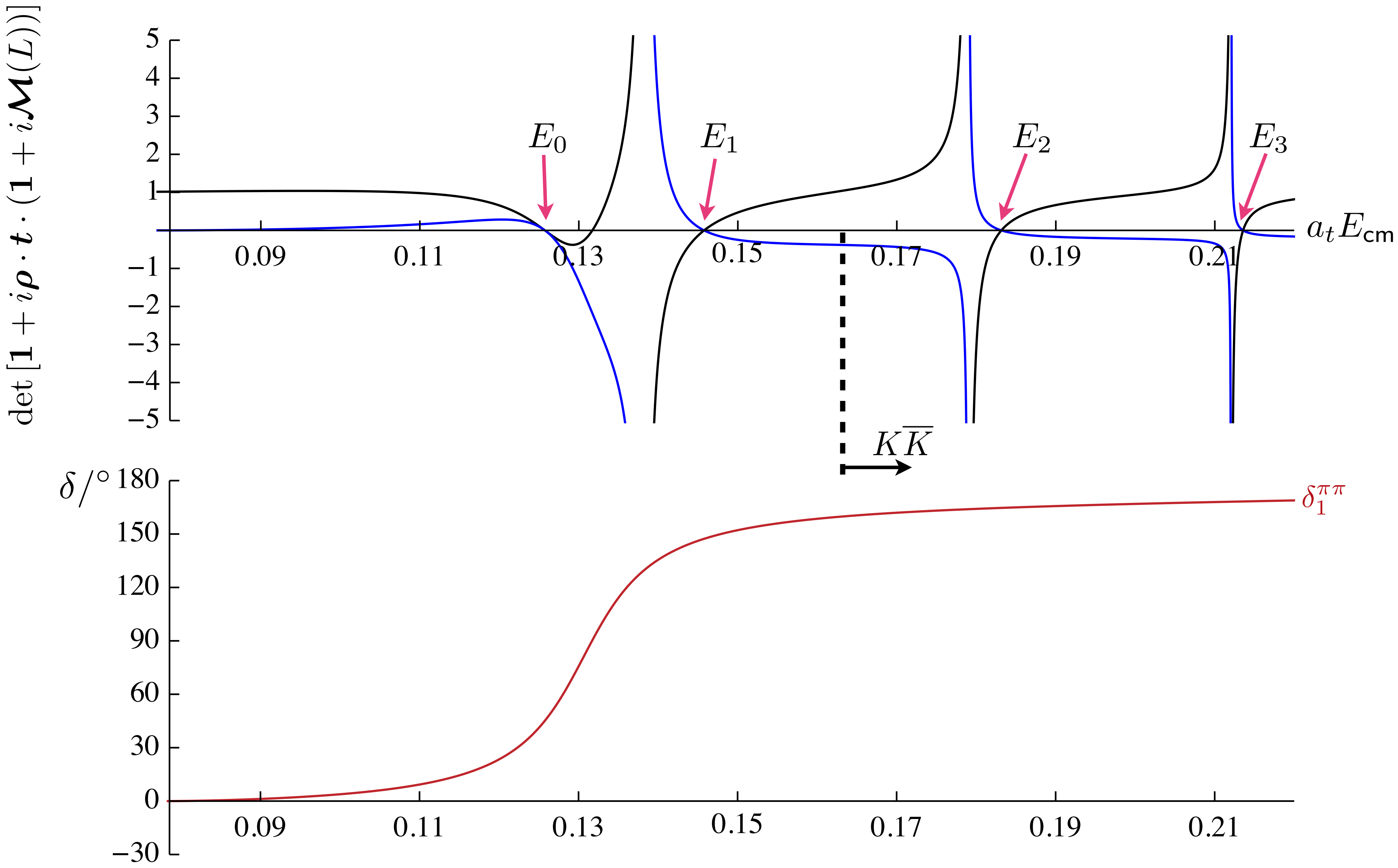
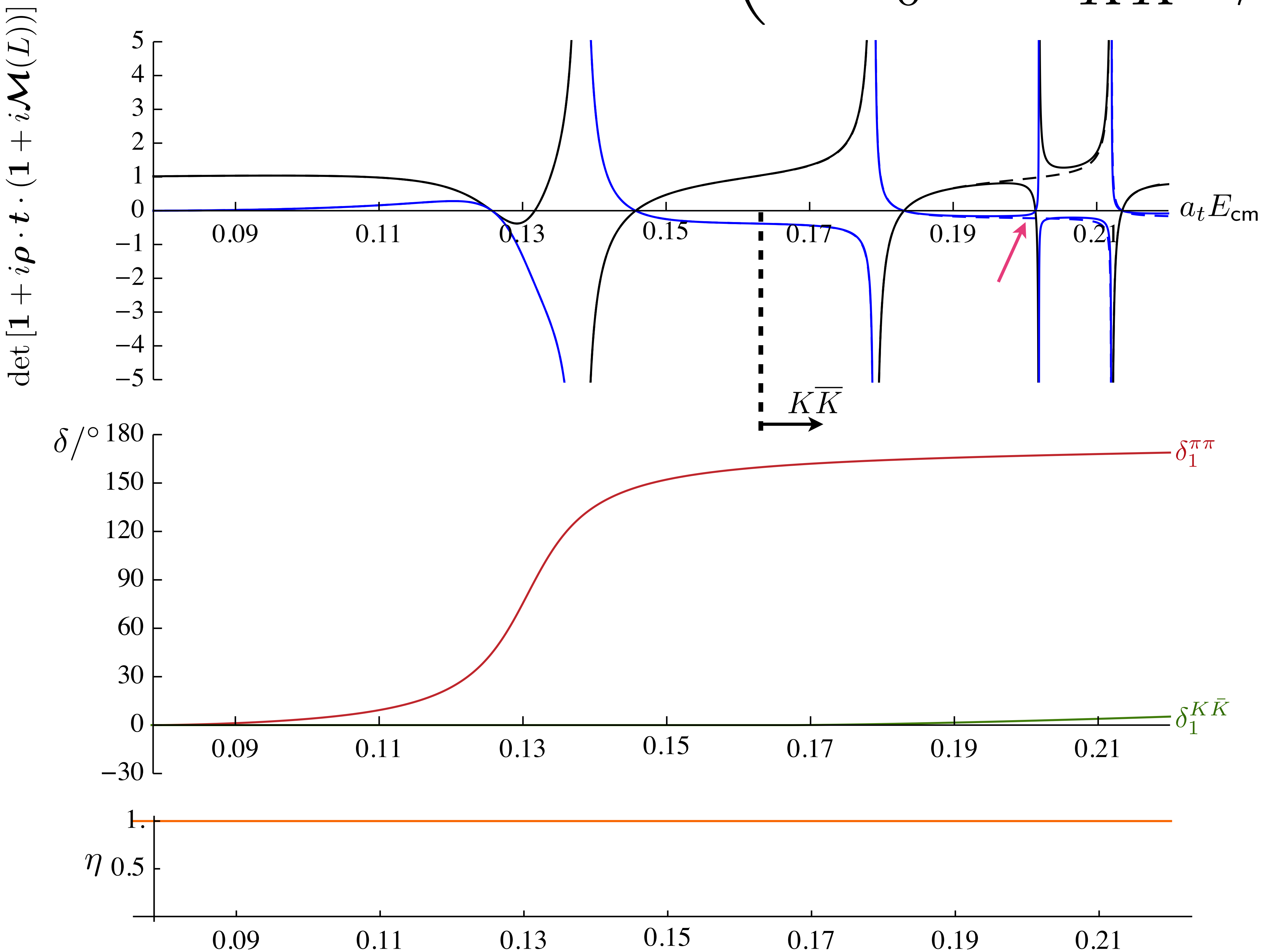


ρ resonance at $m_\pi = 236$ MeV

$$t = \begin{pmatrix} \pi\pi \to \pi\pi & 0 \\ 0 & K\bar{K} \to K\bar{K} \end{pmatrix}$$



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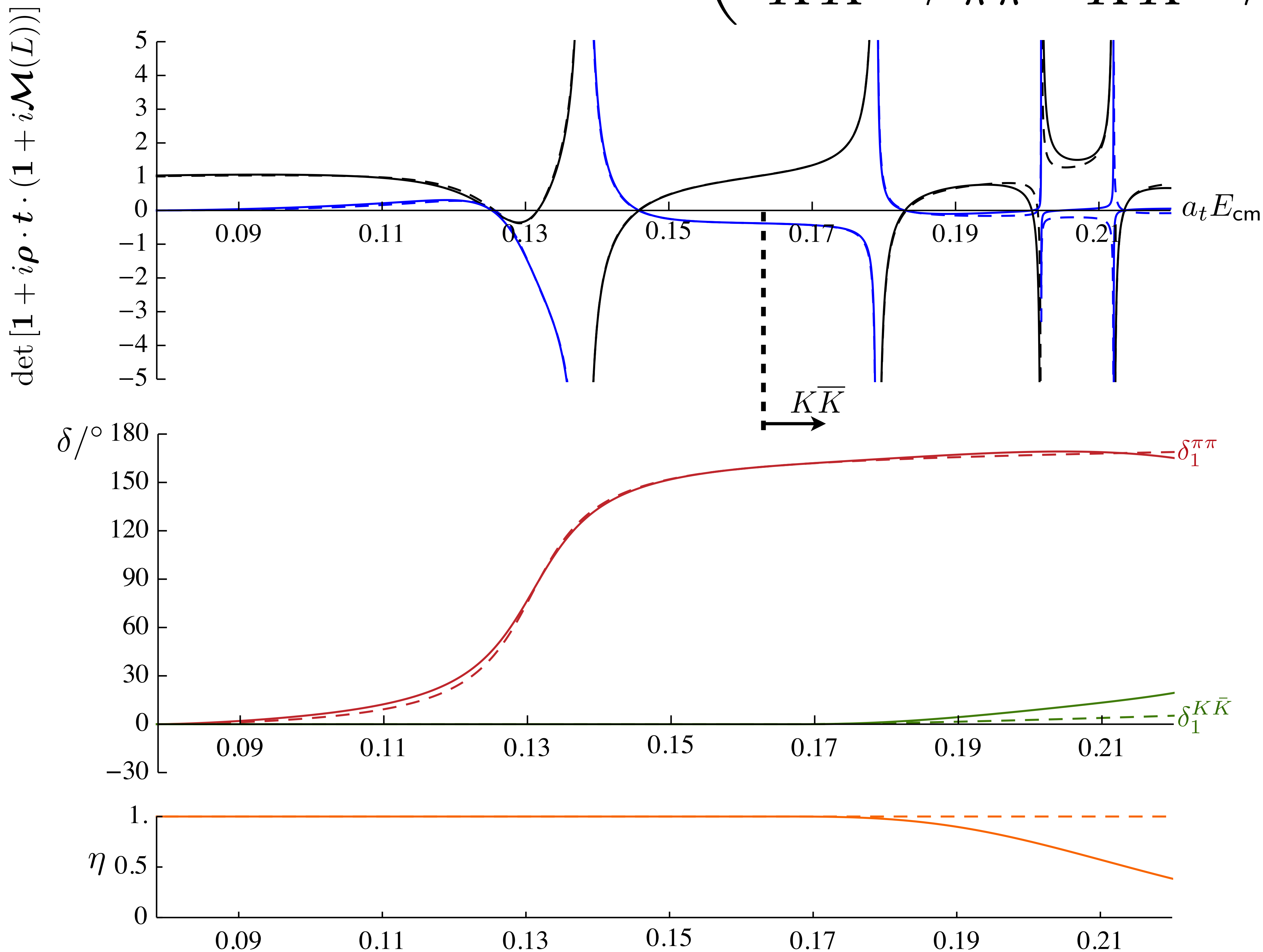

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