IS AXIAL U(1) ANOMALOUS AT HIGH TEMPERATURE?

HIDENORI FUKAYA (OSAKA UNIV.)

FOR JLQCD COLLABORATION

DO YOU THINK AXIAL U(1) ANOMALY CAN DISAPPEAR (AT FINITE T) ?

YES	NO
$U(1)_A$ sym. may be at some temperature.	$U(1)_A$ is always broken.

DO YOU THINK AXIAL U(1) ANOMALY CAN DISAPPEAR (AT FINITE T) ?

Many of you would say "NO!"

with reasonable reasons:

Anomaly = symmetry breaking at cut-off.

Anomalous Ward-Takahashi identity

$$\langle \partial_\mu J_5^\mu(x) O(x') \rangle_{fermion} = \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(x) \langle O(x') \rangle_{fermion} + \langle \delta_A O(x) \rangle_{fermion} \delta(x - x')$$

holds at any energy scale,

and for any gluon background.

DO YOU KNOW ANY OTHER ANOMALY WHICH CAN DISAPPEAR ?

YES	NO

DO YOU KNOW ANY OTHER ANOMALY WHICH CAN DISAPPEAR ?

The answer should be "YES".

Conformal anomaly in massless QCD

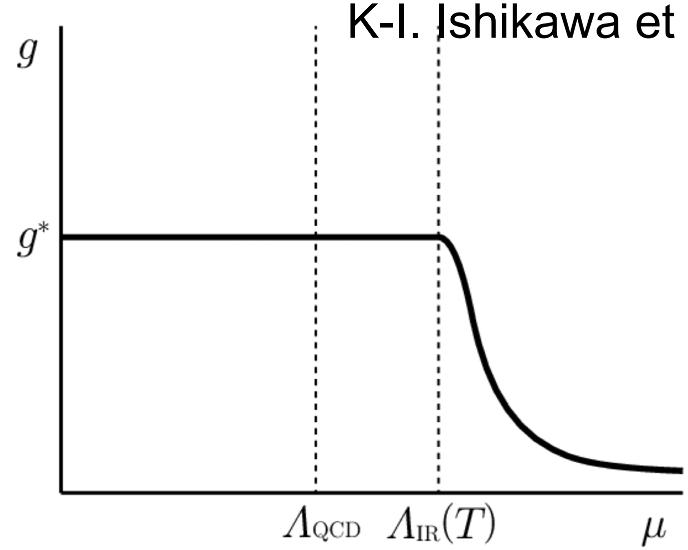
can disappear at IR

by tuning N_f

(conformal window).

Which Nf?→ nontrivial.

Figure from
K-I. Ishikawa et al. 2013



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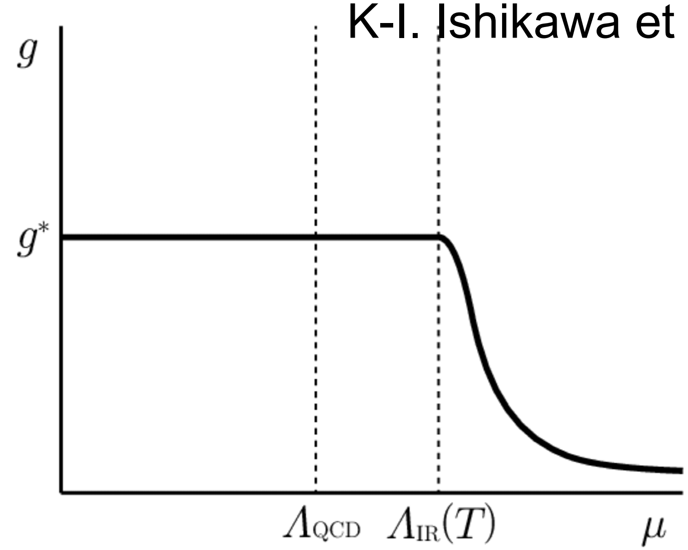

Why not axial U(1) (by tuning T ?)

WE ARE BIASED BY

$$\langle \partial_\mu J_5^\mu(x) O(x') \rangle_{fermion} - \langle \delta_A O(x) \rangle_{fermion} \delta(x - x') = \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(x) \langle O(x') \rangle_{fermion}$$

MAIN MESSAGE OF THIS TALK

In high T QCD, whether

$$\langle \langle \partial_\mu J_5^\mu(x) O(x') \rangle_{fermion} - \langle \delta_A O(x) \rangle_{fermion} \delta(x - x') \rangle_{gluons}$$

$$= \left\langle \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(x) \langle O(x') \rangle_{fermion} \right\rangle_{gluons} = 0???$$

or not is a non-trivial question, which can only be answered by carefully integrating over gluons (by lattice QCD).

In particular, good control of chiral symmetry (or continuum limit) is essential.

CAN AXIAL U(1) BE A SYMMETRY AT FINITE T? (MANY ANSWERS)

Before 2012

- Cohen 1996, 1998 (theory)
- Bernard et al. 1996 (staggered)
- Chandrasekharan et al. 1998 (staggered)
- HotQCD 2011 (staggered)
- Ohno et al. 2011 (staggered)
- and many others

Red: YES
Blue: NO
Green: Not (directly) answered but related

After 2012

- HotQCD 2012 (Domain-wall)
- Aoki-F-Taniguchi 2012 (theory)
- Ishikawa et al2013, 2014,2017. (Wilson)
- JLQCD 2013, 2016 (overlap)
- TWQCD 2013 (optimal DW)
- LLNL/RBC 2013 (Domain-wall) [may be at higher T]
- Pelisseto and Vicari 2013(theory)
- Bonati et al. 2014, 2016(staggered)
- Nakayama-Ohtsuki 2015, 2016(CFT)
- Sato-Yamada 2015(theory),
- Kanazawa & Yamamoto 2015, 2016 (theory)
- Dick et al. 2015 (OV in HISQ sea)
- Sharma et al. 2015, 2016 (OV in DW sea)
- Glozman 2015, 2016 (theory)
- Borasnyi et al. 2015 (staggered & OV)
- Brandt et al. 2016 (Wilson)
- Ejiri et al. 2016 (Wilson)
- Azcoiti 2016,2017(theory)
- Gomez-Nicola & Ruiz de Elvira 2017 (theory) ……

CONTENTS

1. Is $U(1)_A$ symmetry theoretically possible ?

2. Lattice QCD at high T with chiral fermions

3. Result 1: $U(1)_A$ anomaly

4. Result 2: topological susceptibility

5. Summary

$U(1)_A$ AND $SU(2)_L \times SU(2)_R$ SHARE DIM<=3 ORDER PARAMETER(S).

Among quark bi-linears $\langle \bar{q}\Gamma q(x) \rangle$

only $\langle \bar{q}q(x) \rangle$ can have a VEV :

No dim.<=3 operator breaks $U(1)_A$ without breaking $SU(2)_L x SU(2)_R$.

How about higher dim. operators ?

–> our work [Aoki, F, Taniguchi 2012]

DIRAC SPECTRUM AND SYMMETRIES [Aoki-F-Taniguchi 2012]

$$\langle \bar{q}q \rangle = \lim_{m\to 0} \int d\lambda\, \rho(\lambda) \frac{2m}{\lambda^2 + m^2} = \pi\rho(0)$$

[Banks-Casher 1980]

Our idea = generalization of BC relation

to higher dim operators (dim=6 operators were done by T.Cohen 1996) :

$$\delta_{SU(2)} \left\langle \frac{1}{V^{N'}} \prod_i^N \left(\int dV \bar{q}\Gamma_i q \right) \right\rangle = 0$$

→ Constraints on

$$\lim_{m\to 0} \rho(\lambda)$$

→

$$\delta_{U(1)_A} \left\langle \frac{1}{V^{N'}} \prod_i^N \left(\int dV \bar{q}\Gamma_i q \right) \right\rangle = 0\ ???$$

[Aoki-F-Taniguchi 2012]

OUR RESULT 1 : MANY ORDER PARAMETERS ARE SHARED.

(under some “reasonable” assumptions)

Constraint we find

$$\lim_{m\to 0}\langle\rho(\lambda)\rangle = c|\lambda|^{\gamma}(1+O(\lambda)),\ \gamma > 2$$

is strong enough to show

$$\delta_{U(1)_A}\left\langle \frac{1}{V^{N'}}\prod_i^N\left(\int dV\bar{q}\Gamma_i q\right)\right\rangle = 0 \text{ for } \Gamma_i{=}\tau^a \text{ and } \gamma_5\tau^a$$

for any N (up to 1/V corrections):

these order parameters are shared by $SU(2)_L$x$SU(2)_R$ and $U(1)_A$.

[Aoki-F-Taniguchi 2012]

OUR RESULT 2 : STRONG SUPPRESSION OF TOPOLOGICAL SUSCEPTIBILITY

We also find (in the thermodynamical limit)

$$\left(\frac{\partial}{\partial m}\right)^N \frac{\langle Q^2\rangle}{V} = 0 \quad \text{for any } N,$$

which implies $$\frac{\langle Q^2\rangle}{V} = 0 \quad \text{for } m <^{\exists} m_{cr}$$

Also suggests 1st order chiral transition ?

(There's no symmetry enhancement at finite quark mass.)

WHAT WE MEAN BY $U(1)_A$ "SYMMETRY"

We call it "symmetry" if

$$\langle \text{any } U(1)_A \text{ breaking} \rangle = \frac{1}{V^{\alpha}}, \quad \alpha > 0$$

*** The same as conformal symmetry at the IR fixed point.**

CONTENTS

✔ 1. **Is $U(1)_A$ symmetry theoretically possible ?**

   Our answer = YES. $SU(2)_L x SU(2)_R$ and $U(1)_A$ are connected through Dirac spectrum.

2. **Lattice QCD at high T with chiral fermions**

3. **Result 1: $U(1)_A$ anomaly**

4. **Result 2: topological susceptibility**

5. **Summary**

JLQCD FINITE T PROJECT (2009-)

Machines at KEK :

HITACHI SR16000

IBM BG/Q

Parallel talks

Y. Aoki [Mon, Nonzero-T]:
topology at finite T

K. Suzuki[Mon, Nonzero-T]:
U(1)anomaly at finite T

C. Rohrhofer [Wed, Nonzero-T]:
possible further symmetry enhancement at finite T

Simulation code = IroIro++

https://github.com/coppolachan/IroIro

JLQCD FINITE T PROJECT (2014-)

[JLQCD (Cossu et al.) 2015, JLQCD(Tomiya et al.) 2016]

We simulate 2-flavor QCD.

1. **good chirality :**

   Mobius domain-wall & overlap fermion w/ OV/DW reweighting (frequent topology tunnelings)

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3. **different lattice spacings : 0.07-0.1 fm.**

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3. **different lattice spacings : 0.07-0.1 fm.**

Other comments

T= 190-330MeV (Tc~180MeV) with Lt=8,10,12.

3-10 different quark masses (w/ reweighting).

long MD time 20000-30000 for reweighting.

VIOLATION OF CHIRAL SYMMETRY IN DOMAIN-WALL FERMIONS

[JLQCD (Cossu et al.) 2015, JLQCD(Tomiya et al.) 2016]

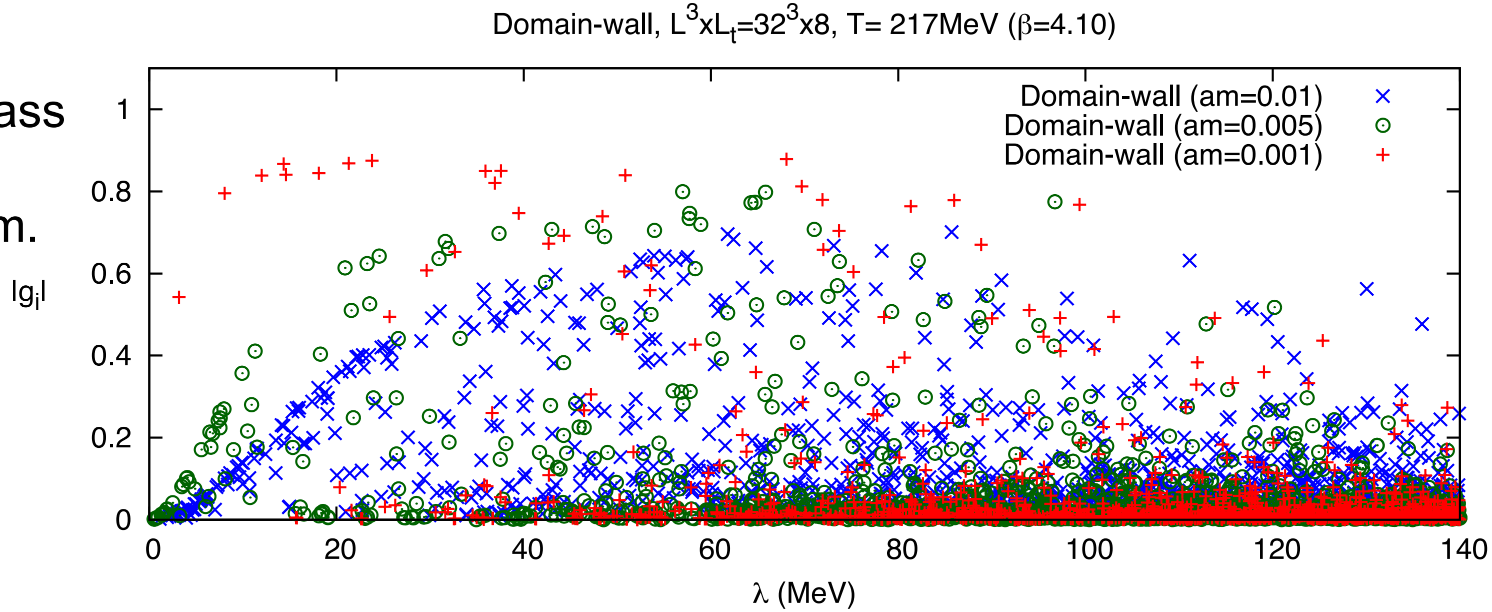
Examine GW relation for each eigen-mode of Mobius domain-wall Dirac operator:

$$g_i = \left(v_i^\dagger, \frac{D\gamma_5 + \gamma_5 D - aRD\gamma_5 D}{\lambda_i} v_i \right)$$

→ **very bad modes appear above Tc (∼180MeV).**

Cf.) residual mass is (weighted) average of them.

For T=0, gi are consistent with residual mass.



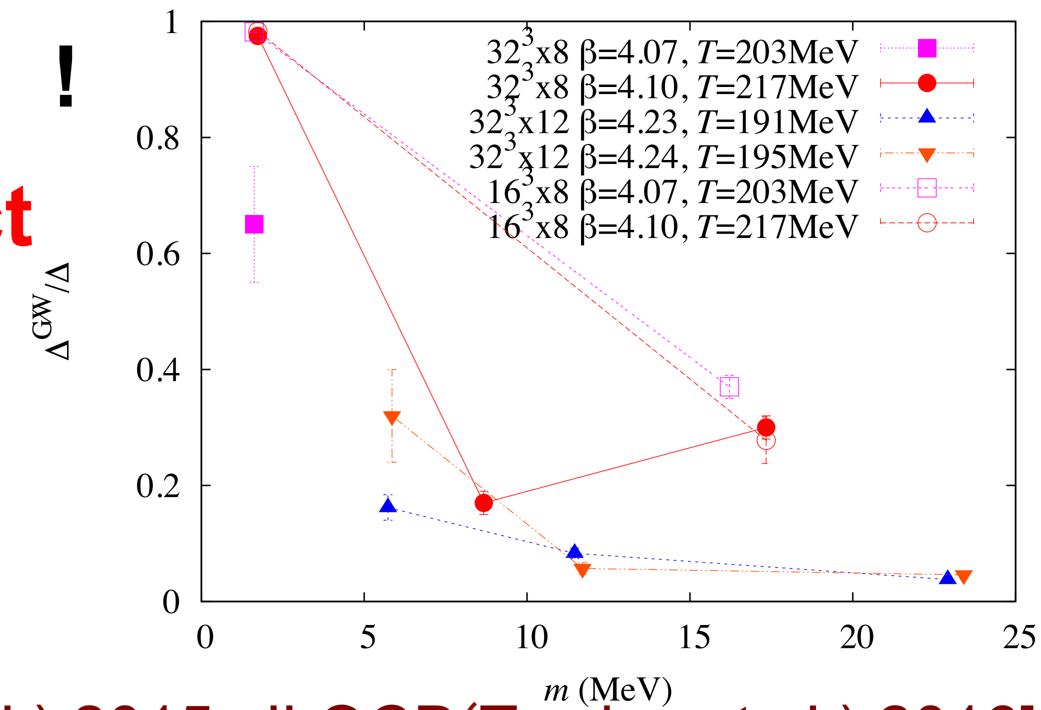
$U(1)_A$ ANOMALY IS SENSITIVE TO THE BAD MODES.

Mobius domain-wall fermion is not good enough (at high T) !

GW violation effect is 20%-100% .

(10 times of m_{res})

GW violation part in U(1)A susceptibility (definition will be given later.)

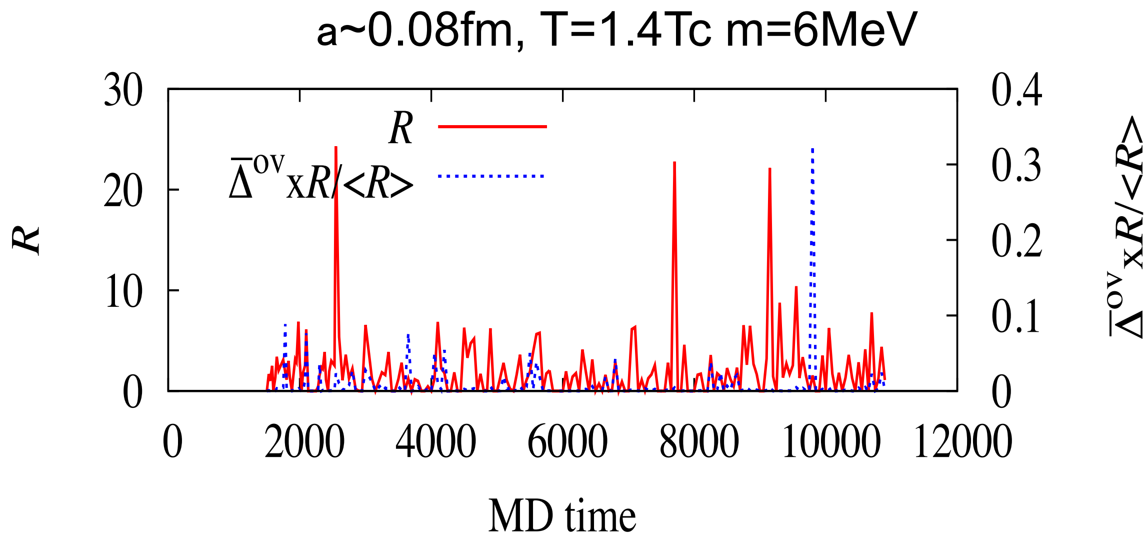


[JLQCD (Cossu et al.) 2015, JLQCD(Tomiya et al.) 2016]

OVERLAP/DOMAIN-WALL REWEIGHTING

(We employ overlap fermion in domain-wall sea)

$$R \equiv \frac{\det[D_{\rm ov}(m)]^2}{\det[D^{\rm 4D}_{\rm DW}(m)]^2}$$



$$D^{4D}_{\rm DW}(m) = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5 \frac{1-(T(H_M))^{L_s}}{1+(T(H_M))^{L_s}}$$ $m_{\rm res} \sim 1{\rm MeV}$

$$D_{\rm ov}(m) = \left[\frac{1+m}{2} + \frac{1-m}{2}\gamma_5\,{\rm sgn}(H_M)\right]$$ $m_{\rm res} = 0.$

*** The Mobius Kernel H_M (Brower et al. 2006) is different from original definitions by Kaplan 1992 & Neuberger 1998.**

“EFFICIENCY” OF OV/DW REWEIGHTING

$$\frac{N_{eff}}{N} = \frac{\langle R \rangle}{N max R}$$

On our 2-4 fm lattices at T=1.1-1.8Tc (Tc~180MeV)

a ~ 0.1 fm : O.K. for L=2 fm, $N_{eff}/N \sim 1/20$

but does not work for 4 fm. $N_{eff}/N < 1/1000.$

(→ we approximate it by O(10) low-modes.)

a ~ 0.08 fm : works well (3 fm). $N_{eff}/N \sim 1/10$

a ~ 0.07 fm : domain-wall & overlap are consistent (2.4, 3.6 fm). $N_{eff}/N > 1/10$

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Our focus in this talk

VALENCE OVERLAP IN DOMAIN-WALL SEA IS MORE DANGEROUS

Dirac spectrum

DW on DW confs

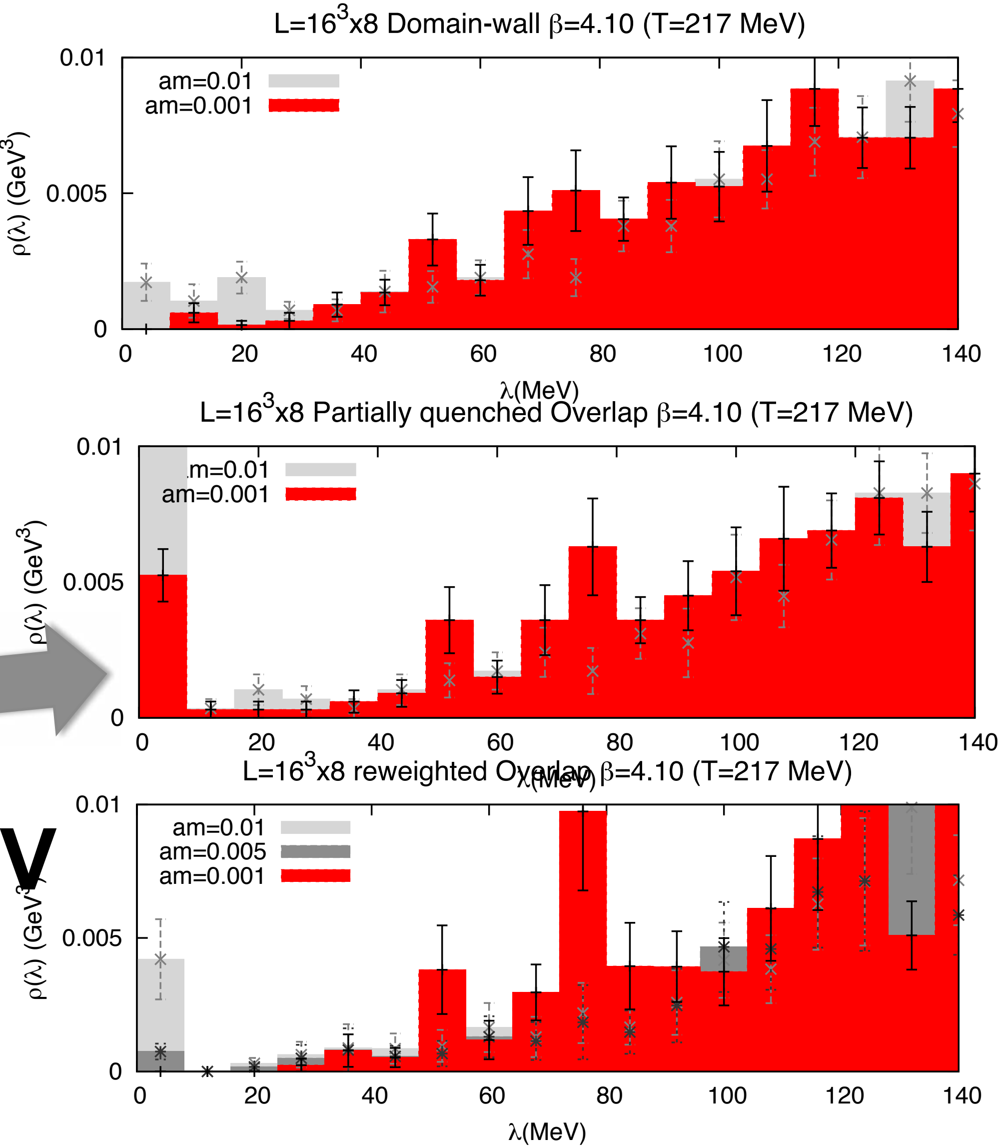
OV on DW confs

(partially quenched)

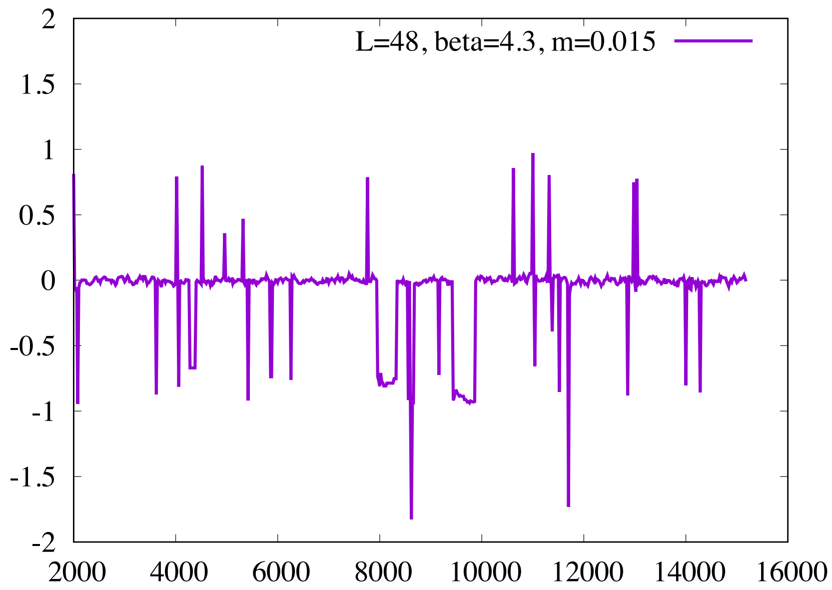
Fake zero modes

(partially quenched artifact)

OV on reweighted OV



OVERLAP/DOMAIN-WALL REWEIGHTING ALLOWS TOPOLOGY TUNNELINGS



Auto-correlation time of topology is O(100), small enough compared to our long trajectory length, 20000-30000 MD time.

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$U(1)_A$ at high T is sensitive to lattice artifact.

We need good chiral sym (or careful cont. limit.).

3. **Result 1: $U(1)_A$ anomaly**

4. **Result 2: topological susceptibility**

5. **Summary**

WHAT WE OBSERVE

Axial U(1) susceptibility

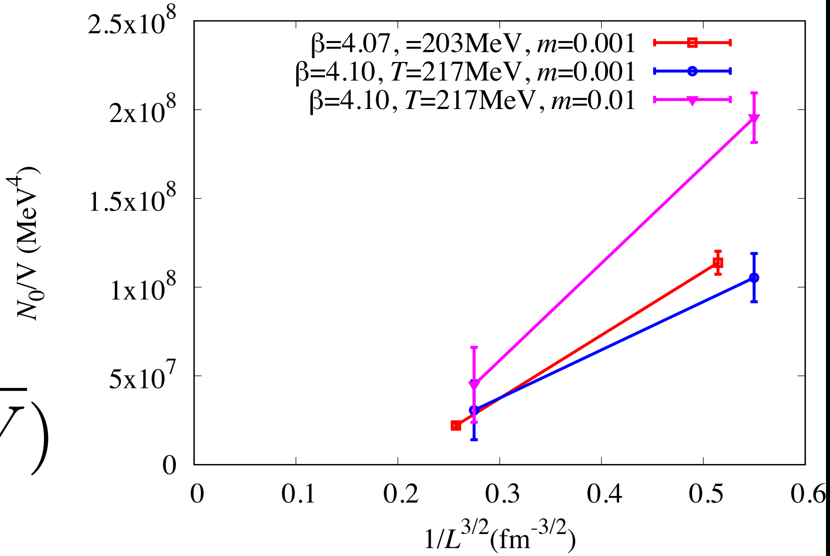
$$\Delta_{\pi-\delta} = \int d^4x \, [\langle \pi^a(x)\pi^a(0)\rangle - \langle \delta^a(x)\delta^a(0)\rangle] \, ,$$

$$\left(= \int_0^\infty d\lambda \, \rho(\lambda) \frac{2m^2}{(\lambda^2 + m^2)^2} \right)$$

We compute

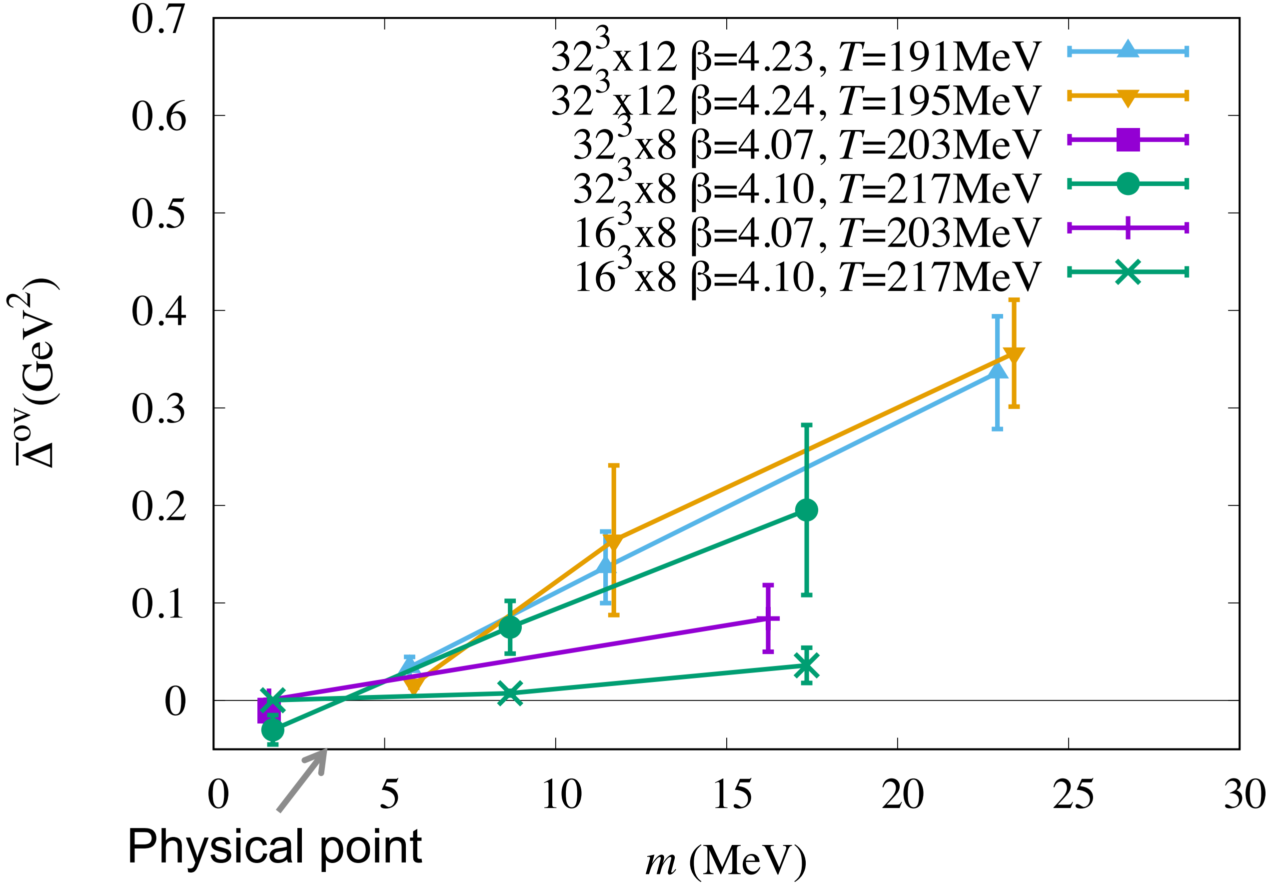
$$\bar{\Delta}^{\mathrm{ov}}_{\pi-\delta} \equiv \Delta^{\mathrm{ov}}_{\pi-\delta} - \frac{2N_0}{Vm^2} .$$

N_0 : # of zero modes $\quad (\sim 1/\sqrt{V})$



U(1)$_A$ ANOMALY VANISHES IN THE CHIRAL LIMIT

Coarse (a>0.08fm) lattice [JLQCD(Tomiya et al.) 2016]



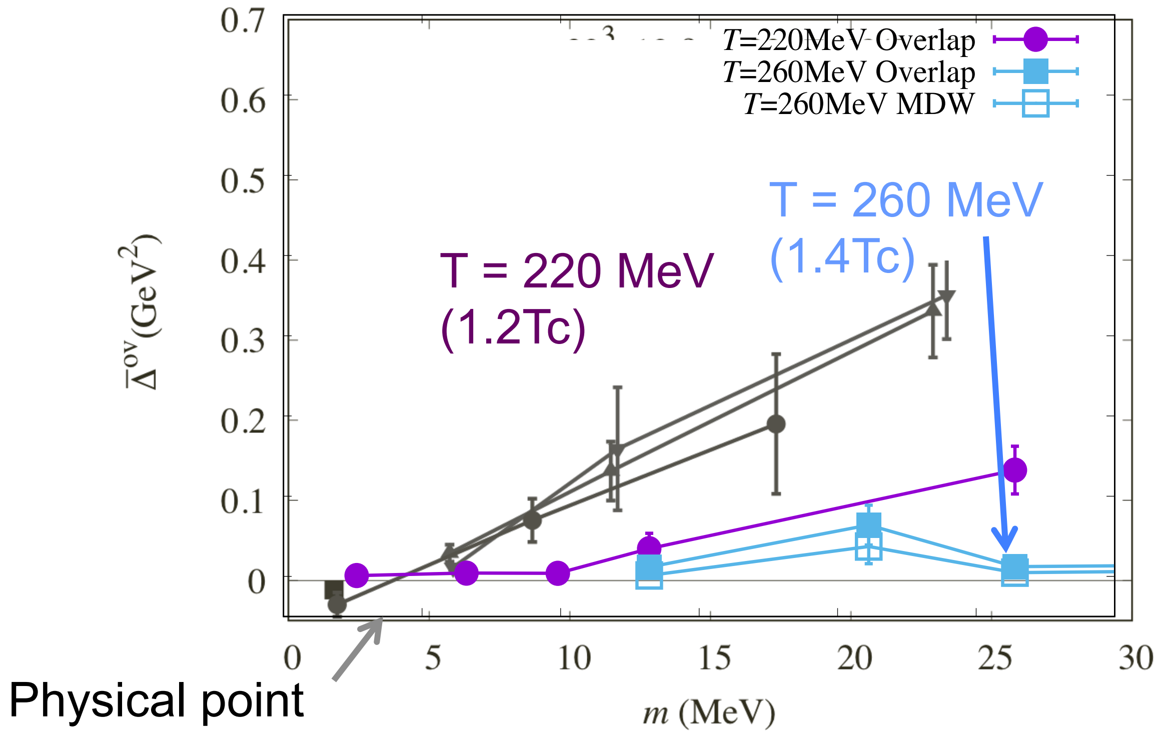
T = 1.1 – 1.2 Tc

L=16 (1.8fm) and 32 (3.6fm) results are consistent.

$(M_{screen}L > 5.)$

U(1) ANOMALY VANISHING ON FINE LATTICE (a~0.07fm)

[JLQCD, preliminary, Suzuki Mon non-zero T]



Consistent with coarse lattice,
Consistent with domain-wall fermion at T=260MeV (1.4Tc).

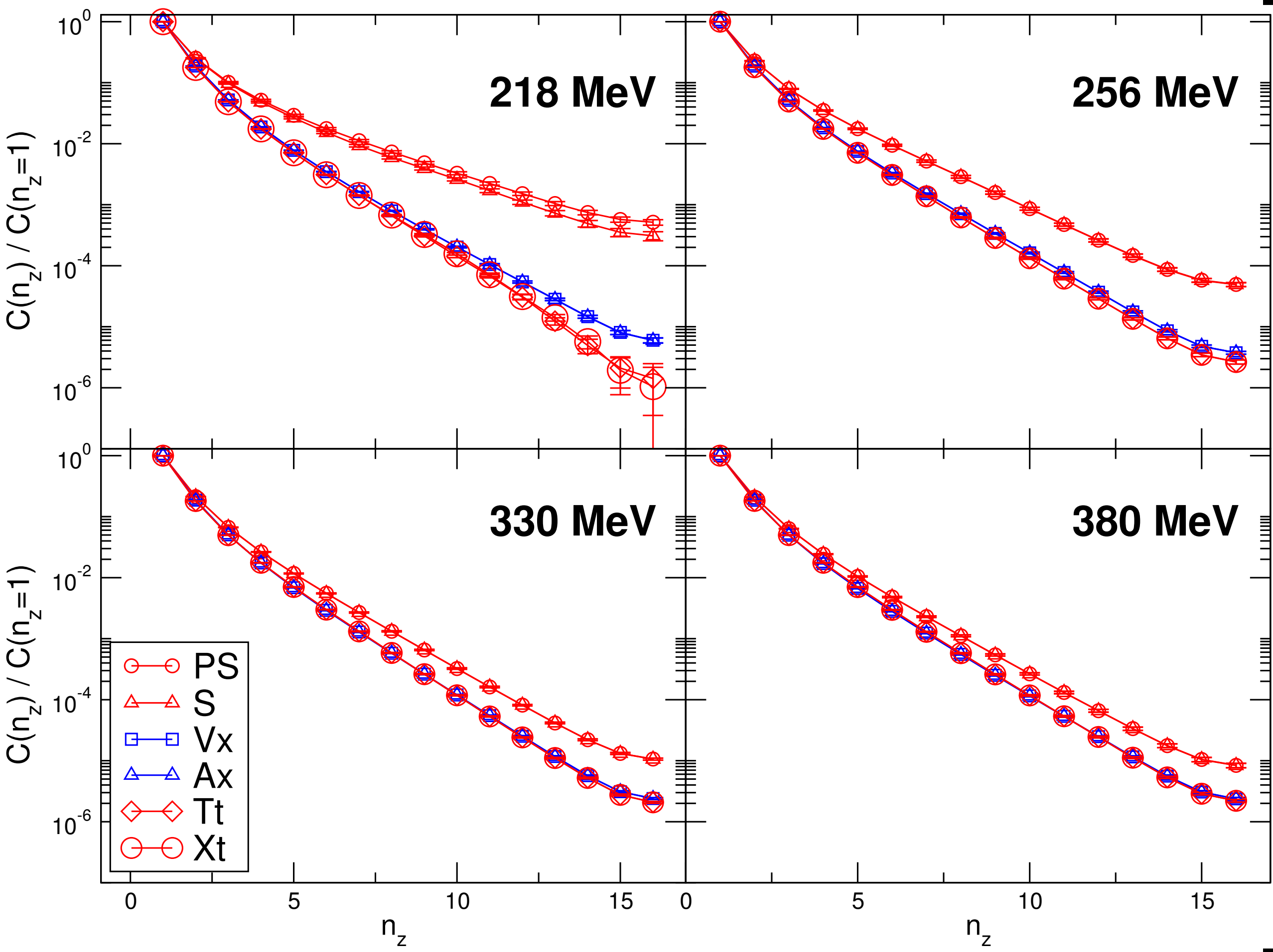
MESON CORRELATOR ITSELF SHOWS U(1) ANOMALY VANISHING

[C. Rohrhofer Wed, Nonzero-T preliminary]

SU(2)xSU(2) [blue] and $U(1)_A$ (red) partners are degenerate.

[similar results reported by Brandt et al. 2016]

Further enhancement to SU(4) ? [Glozman 2015]



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$U(1)_A$ at high T is sensitive to lattice artifact.
We need good chiral sym (or careful cont. limit.).

✔ 3. **Result 1: $U(1)_A$ anomaly**

$U(1)_A$ anomaly at T~1.1-1.4Tc (Tc~180MeV) in the chiral limit is consistent with zero.

4. **Result 2: topological susceptibility**

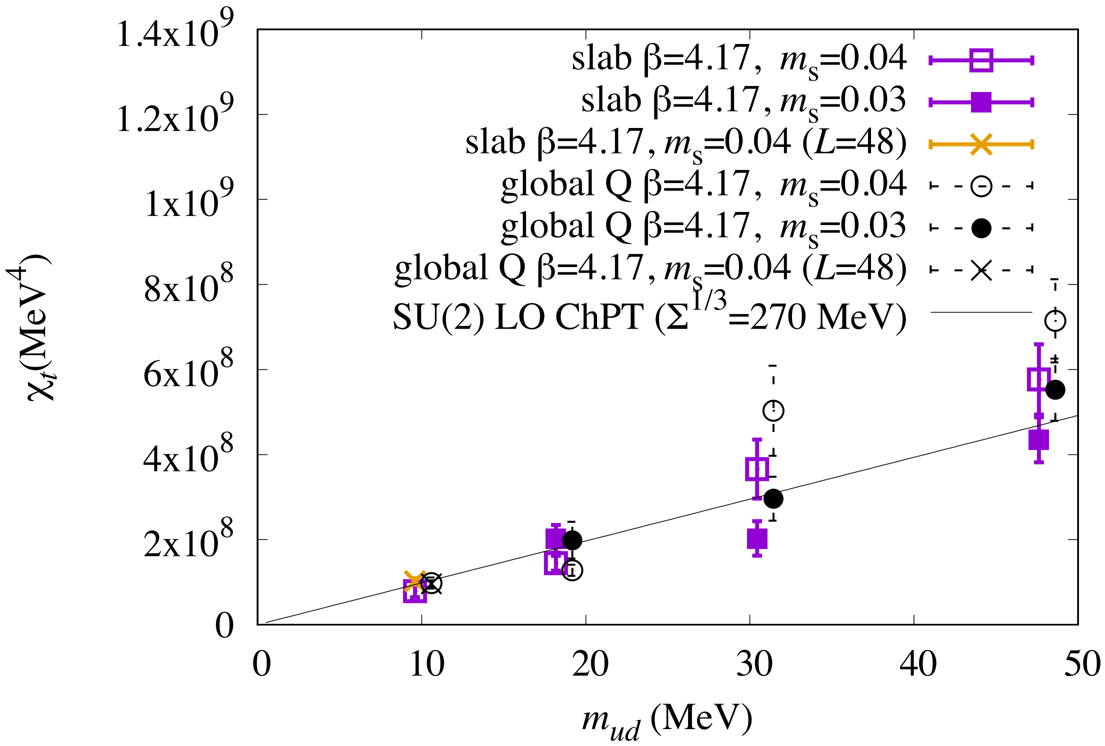
5. **Summary**

TOPOLOGICAL SUSCEPTIBILITY

$$\chi_t = \frac{\langle Q^2 \rangle}{V} \qquad Q = \frac{1}{32\pi^2} \int d^4x \mathrm{Tr} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

another direct probe for $U(1)_A$ anomaly.

T =0 results [JLQCD 2017 HF, Chiral Thu]

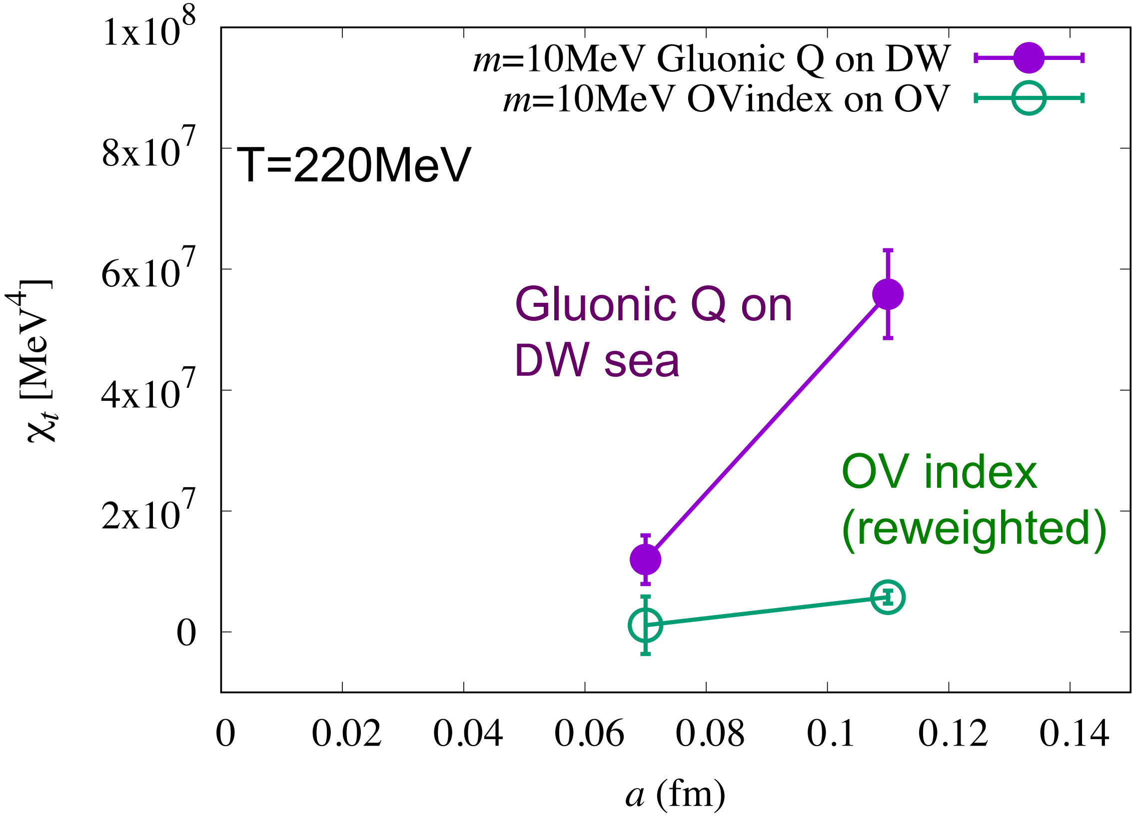


Mobius domain-wall fermion
Is good enough to
reproduce ChPT prediction.

TOPOLOGICAL SUSCEPTIBILITY

But above Tc, it is sensitive to lattice artifact.

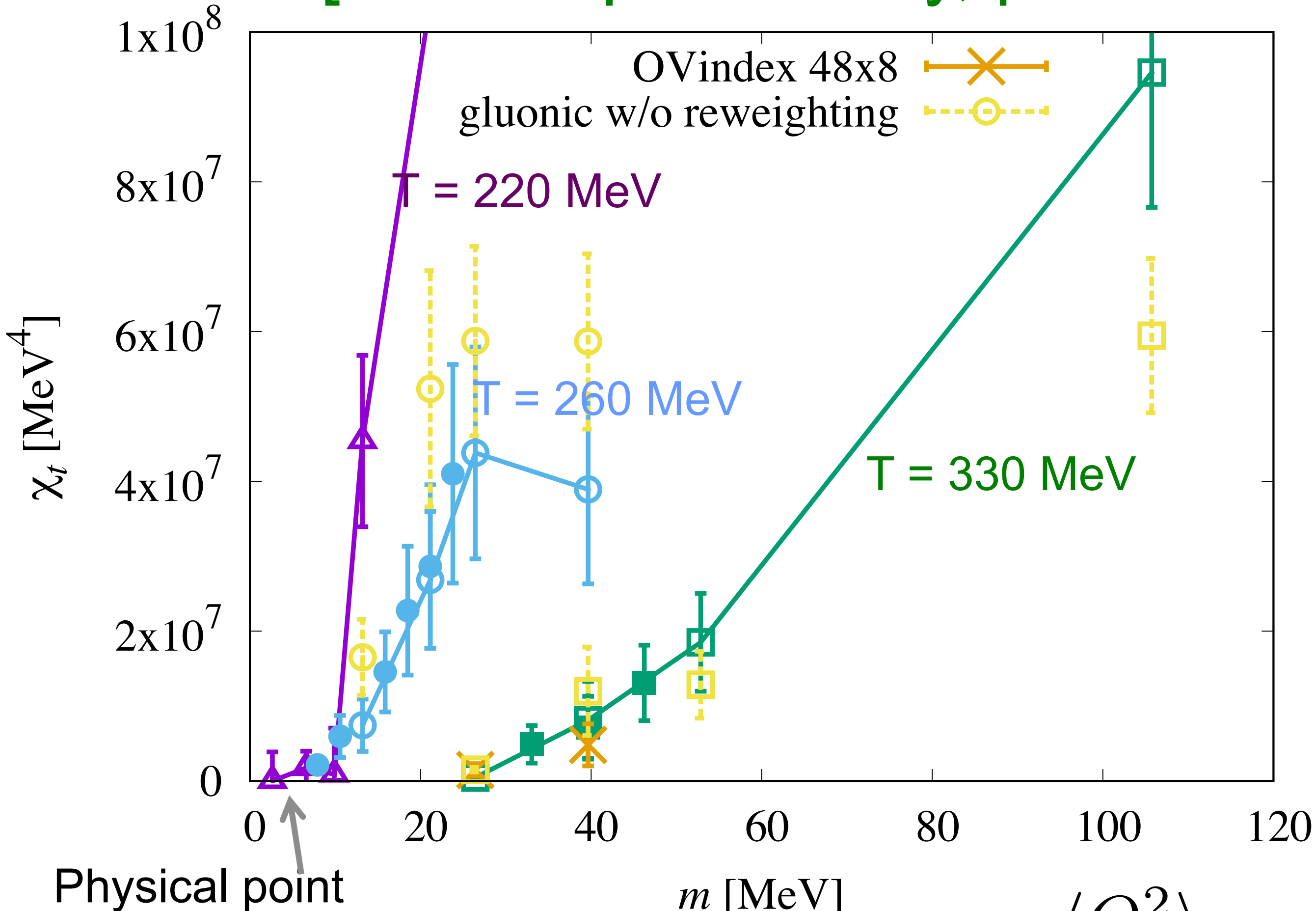
We need (reweighted) overlap fermion for a>0.08fm.



index of overlap Dirac operator is stable against lattice cut-off.

TOPOLOGICAL SUSCEPTIBILITY VANISHES *BEFORE* THE CHIRAL LIMIT

[JLQCD preliminary, parallel talk by Y. Aoki]



L=48 (3.6fm) & L=32 (2.4fm) results are consistent.

On our fine lattices (a~0.07fm) OV index and gluonic def. after Wilson flow ($\sqrt{8t} \sim 0.47$ fm) are also consistent.

agrees with our prediction

$$\frac{\langle Q^2 \rangle}{V} = 0 \quad \text{for } m <^{\exists} m_{cr}$$

[Aoki, F, Taniguchi 2012]

CAN AXION BE A DARK MATTER?

- If our result really indicates $\chi_t = 0$ at 1.8Tc, axion cannot be a dark matter.
- But Dilute Instanton Gas Approximation (DIGA) prediction is also small (< 1/100 of our first non-zero data). Our data cannot numerically exclude DIGA.
- Our simulation is 2-flavor. Any strange quark effect?

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$U(1)_A$ anomaly at T~1.1-1.4Tc (Tc~180MeV) in the chiral limit is consistent with zero.

✔ 4. **Result 2: topological susceptibility**

Topological susceptibility drops *before* the chiral limit.

5. **Summary**

SUMMARY

1. $U(1)_A$ anomaly at high T is a non-trivial problem.
2. $U(1)_A$ and $SU(2)_L xSU(2)_R$ order prms. connected.
3. $U(1)_A$ is sensitive to lattice artifact at high T
   -> We need overlap fermion for a>0.08 fm.

   For a=0.07 fm, Mobius domain-wall is O.K.
4. In our simulation with chiral fermions at 3 volumes and 3-10 quark masses at T=1.1-1.8Tc (Tc~180MeV), $U(1)_A$ anomaly disappears
   [before the chiral limit]
   (suggesting 1st order transition ?).

MAIN MESSAGE OF THIS TALK

In high T QCD, whether

$$\langle\langle\partial_\mu J_5^\mu(x)O(x')\rangle_{fermion} - \langle\delta_A O(x)\rangle_{fermion}\delta(x-x')\rangle_{gluons}$$

$$= \left\langle \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}(x)\langle O(x')\rangle_{fermion} \right\rangle_{gluons} = 0???$$

or not is a non-trivial question, which can only be answered by carefully integrating over gluons (by lattice QCD).

In particular, good control of chiral symmetry (or continuum limit) is essential.

BACK UP SLIDES

WHY BAD MODES ONLY ABOVE T_c?

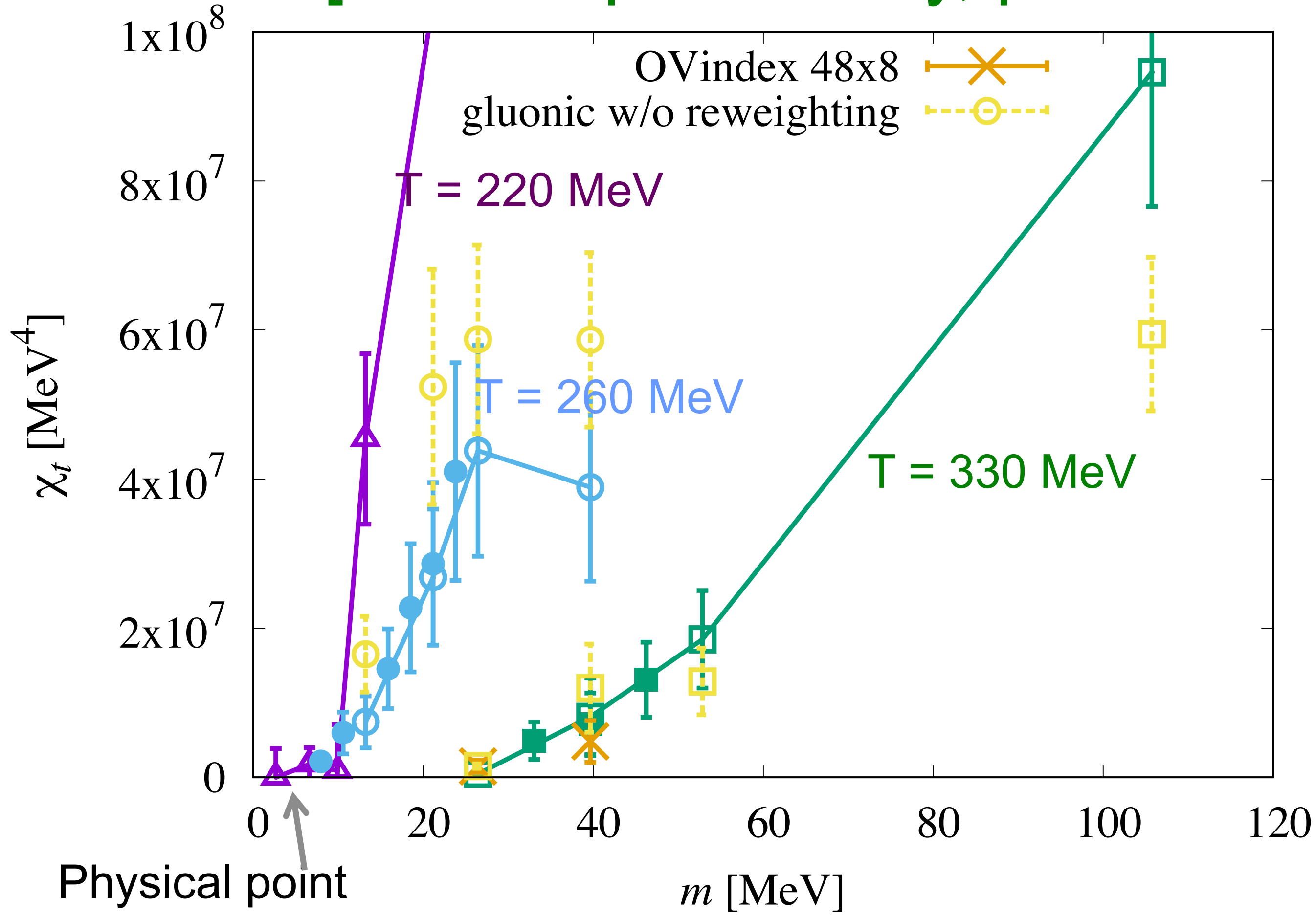
Suppose bad modes are always there but sparse.

At T=0, they mix with MANY good low-lying modes, then relative lattice artifact is comparable to residual mass.

At T>Tc, good modes are also SPARSE, the lattice artifacts remain large.

SHARP DROP IS NOT DUE TO TOPOLOGY FREEZING

[JLQCD preliminary, parallel talk by Y. Aoki]



Filled symbols = mass reweighting from heavier mass, where topology is fluctuating well.

COMPARISON WITH OTHER SPEAKERS

Bonati et al. 2015: bigger than DIGA

2+1-flavor staggered, gluonic Q, a=0.057-0.082fm 0.9-1.2Tc L=2.4-3.8fm.

Borsanyi et al. 2016: power consistent with DIGA.

Overlap (Q fixed) & staggered w/ OV/ staggered *zero-mode* reweighting, Ovindex, a= many points, T=100-3000MeV, L= many sizes

JLQCD 2017 [preliminary]: χt =0 but resolution is not enough to exclude DIGA.

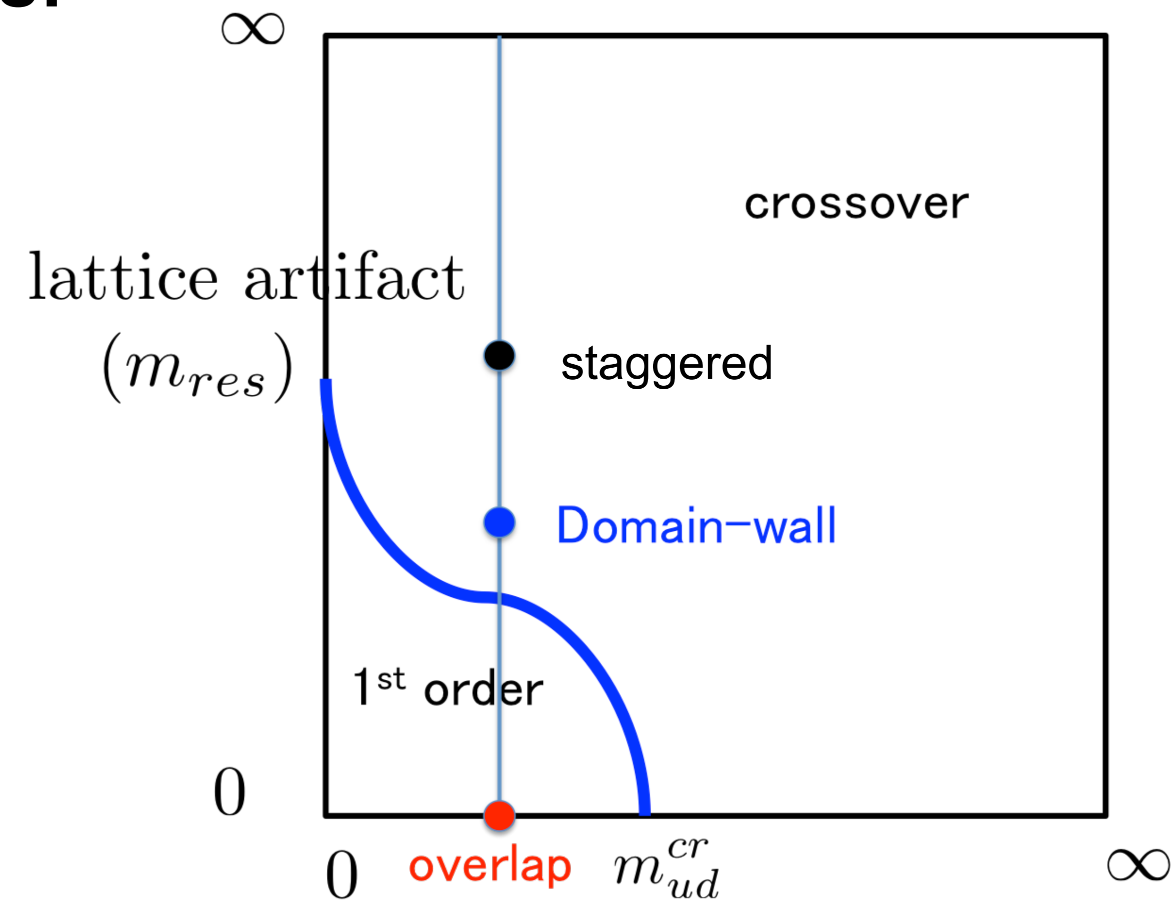
2-flavor Mobius domain-wall + reweighted overlap, Ovindex & gluonic Q, a=0.075-0.113 fm, T=200-330MeV, L=2.4-3.6fm.

WHY DIFFERENT ?

Overlap and non-chiral fermions may be in different phases:

Lattice artifact

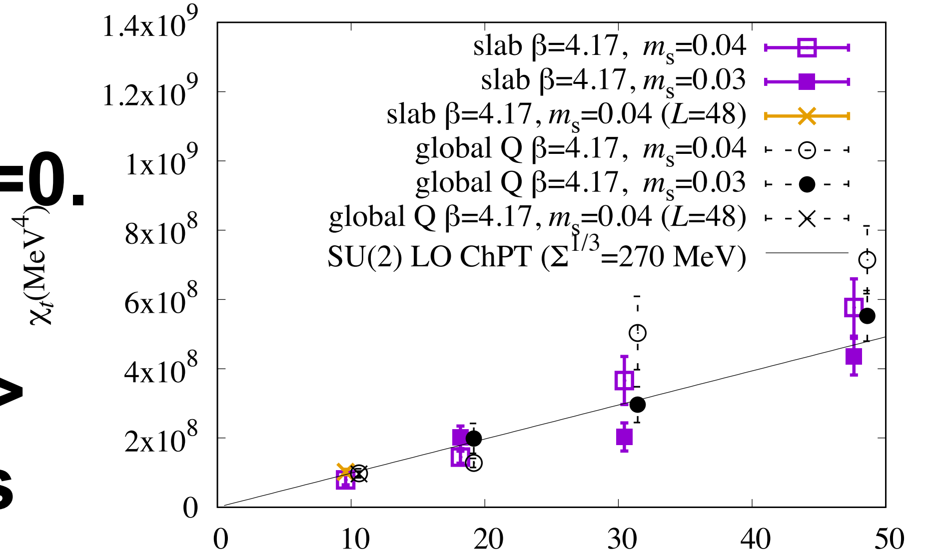
= another axis

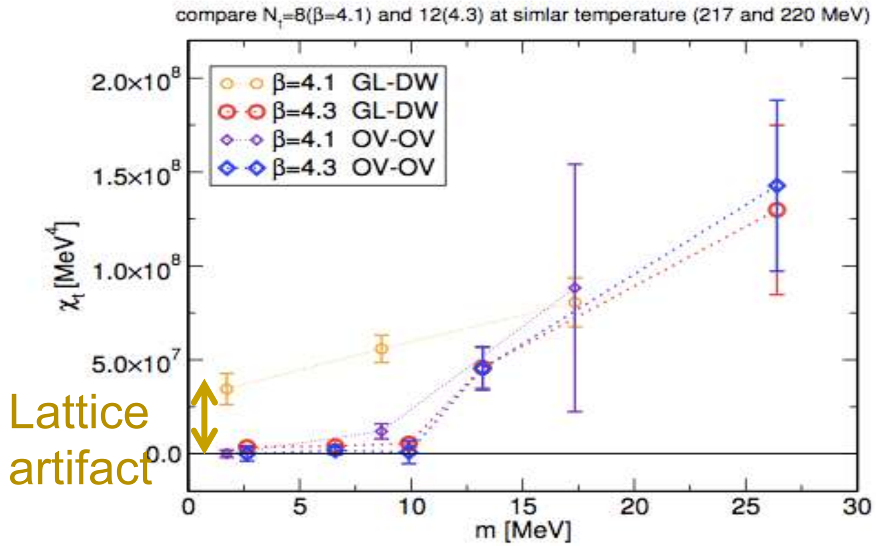


WHEN WE USE DOMAIN-WALL FERMIONS WE MUST

1. Check mass dependence at T=0.
2. Check mass dependence at T> Tc : if m=0 limit is consistent with zero.

Otherwise, your results could be contaminated by lattice artifacts.

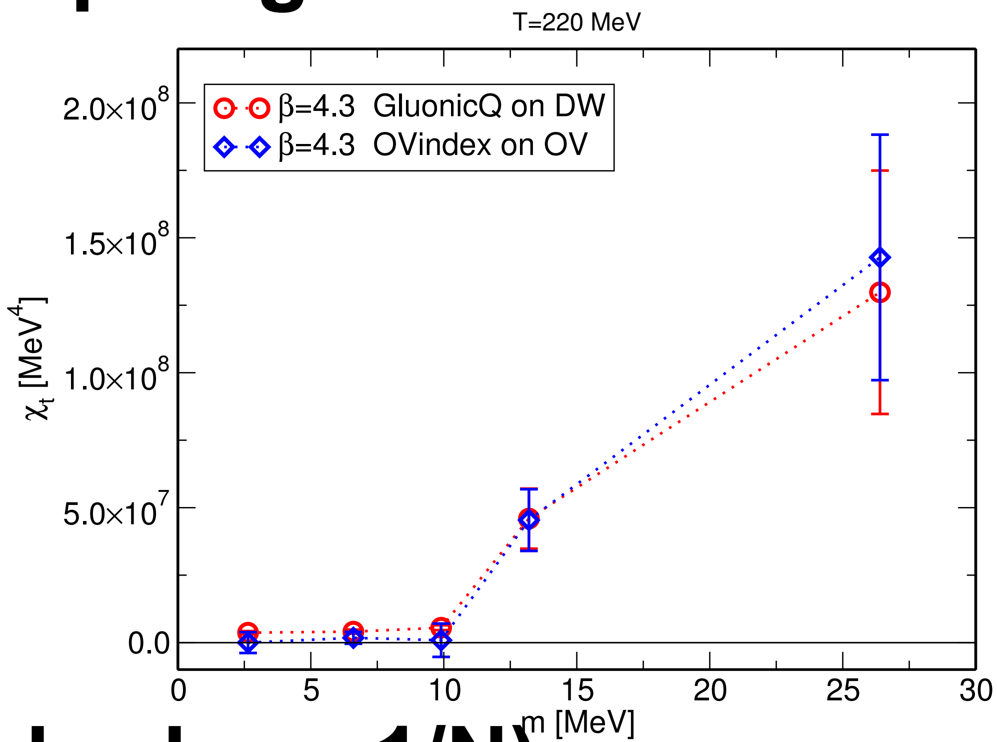




RESOLUTION OF <Q^2>

Topological charge is discrete.

→ resolution of topological susceptibility is limitted by # of confs.



We put max(standard err, 1/N) to estimate the statistical error.

ASSUMPTIONS IN AOKI-FUKAYA-TANIGUCHI 2012

1. **SU(2) x SU(2) fully recovered at Tc.**
2. if $\mathcal{O}(A)$ is m-independent

   $$\langle \mathcal{O}(A) \rangle_m = f(m^2) \qquad f(x) \text{ is analytic at } x = 0$$

3. if $\mathcal{O}(A)$ is m-independent and positive, and satisfies

$$\lim_{m \to 0} \frac{1}{m^{2k}} \langle \mathcal{O}(A) \rangle_m = 0$$

$$\Rightarrow \langle \mathcal{O}(A) \rangle_m = m^{2(k+1)} \underbrace{\int \mathcal{D}A \, \hat{P}(m^2, A) \mathcal{O}(A)}_{\text{finite}}$$

$$\Rightarrow \langle \mathcal{O}(A)^l \rangle_m = m^{2(k+1)} \int \mathcal{D}A \, \hat{P}(m^2, A) \mathcal{O}(A)^l = O(m^{2(k+1)})$$

4. $$\rho^A(\lambda) \equiv \lim_{V \to \infty} \frac{1}{V} \sum_n \delta\left(\lambda - \sqrt{\bar{\lambda}_n^A \lambda_n^A}\right) = \sum_{n=0}^{\infty} \rho_n^A \frac{\lambda^n}{n!} \quad \text{at } \lambda = 0 \ (\lambda < \epsilon\ )$$

(4 can be removed.)

SUM OF NON-ZERO QUANTITY NONZERO ?

NOT always.

Example: chiral condensate

$$\langle \bar{q}q \rangle = \frac{\int dA \mathrm{Tr} D^{-1} \det D e^{-S_G}}{Z}$$

can be zero and non-zero. How about

$$\langle \langle \partial_\mu J_5^\mu(x) O(x') \rangle_{fermion} - \langle \delta_A O(x) \rangle_{fermion} \delta(x - x') \rangle_{gluons}$$

$$= \left\langle \frac{1}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}(x) \langle O(x') \rangle_{fermion} \right\rangle_{gluons} = 0???$$

OUR OVERLAP DIRAC OPERATOR

$$D_{\rm ov}(m) = \sum_{|\lambda_i^M|<\lambda_{\rm th}^M} \left[\frac{1+m}{2} + \frac{1-m}{2}\gamma_5\,{\rm sgn}(\lambda_i^M)\right] |\lambda_i^M\rangle\langle\lambda_i^M|$$

$$+ D_{\rm DW}^{\rm 4D}(m) \left[1 - \sum_{\lambda_i^M<|\lambda_{\rm th}^M|} |\lambda_i^M\rangle\langle\lambda_i^M|\right],$$

λ_i^M : eigenvalue of H_M.

PHASE DIAGRAM?