Topological susceptibility in 2+1-flavor QCD with chiral fermions

OSAKA UNIVERSITY
Live Locally, Grow Globally

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S. Hashimoto and T. Kaneko
[JLQCD collaboration]

[arXiv:1705.10906]

JLQCD collaboration

We simulate 2+1-flavor QCD with Mobius domain-wall fermions.

Computers @KEK: SR16000 (55 TFLOPS) + BG/Q (1.2 PFLOPS)
Lattice cut-off : 2.4, 3.6, 4.2 GeV
Lattice size : 32^3x64, 48^3x96, 64^3x128 (2.6 fm ~ 4 fm)
pion mass : 230-500 MeV.


Hitachi SR16000


IBM Blue Gene/Q

Talks by my colleagues

Y. Aoki [Mon, Nonzero-temp.]: topology at finite T

B. Colquhoun[Mon, Weak decays]: B semileptonic decay

HF[Thu, plenary]: U(1) anomaly at finite T

S. Hashimoto[Wed, Weak decays]: Inclusive B decay

T. Kaneko[Tue, Weak decays]: D semileptonic decays

K. Nakayama[Wed, SMprm]: Anomalous dim. from eigenvalue

C. Rohrhofer[Wed, Nonzero-temp.]: possible symmetry enhancement at finite T

K. Suzuki[Mon, Nonzero-temp]: U(1)anomaly at finite T

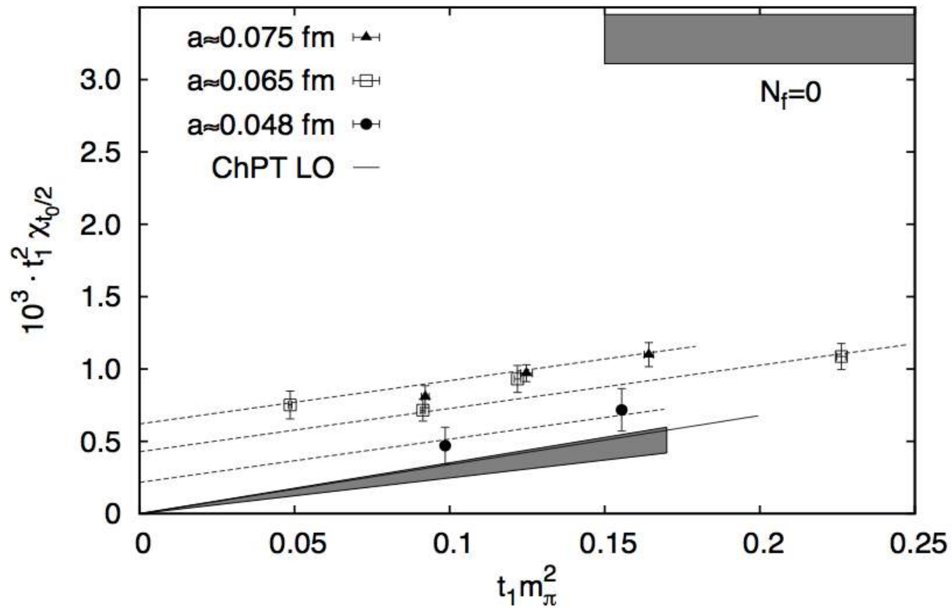
Motivation: Topological susceptibility is difficult.

$$\chi_t = \frac{\langle Q^2 \rangle}{V} \qquad Q = \frac{1}{32\pi^2}\int d^4x \mathrm{Tr}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

Operator has no explicit quark mass dependence.

$\rightarrow$ m-dependence = 1-loop & higher $\sim O(\hbar)$

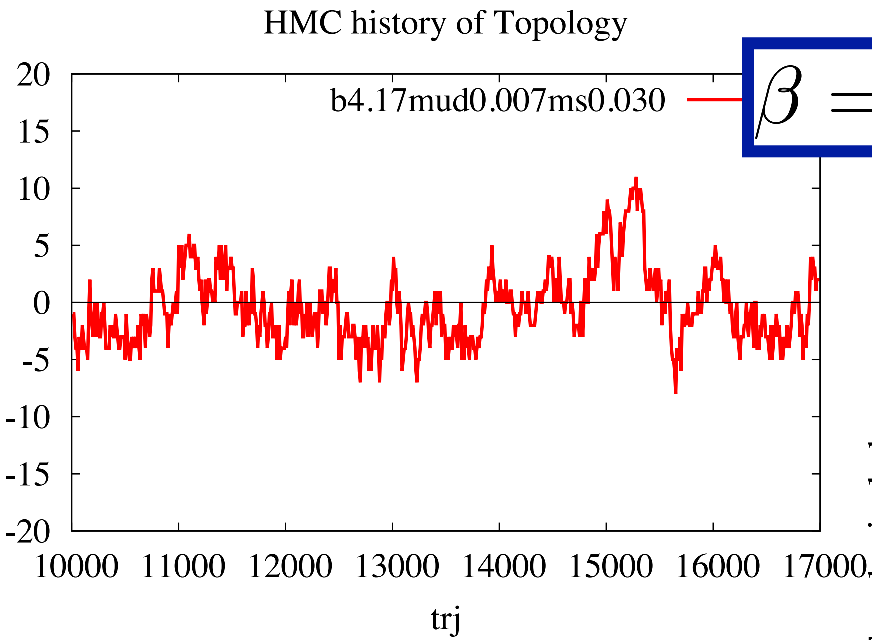
$\rightarrow$ relative cut-off effect is large.



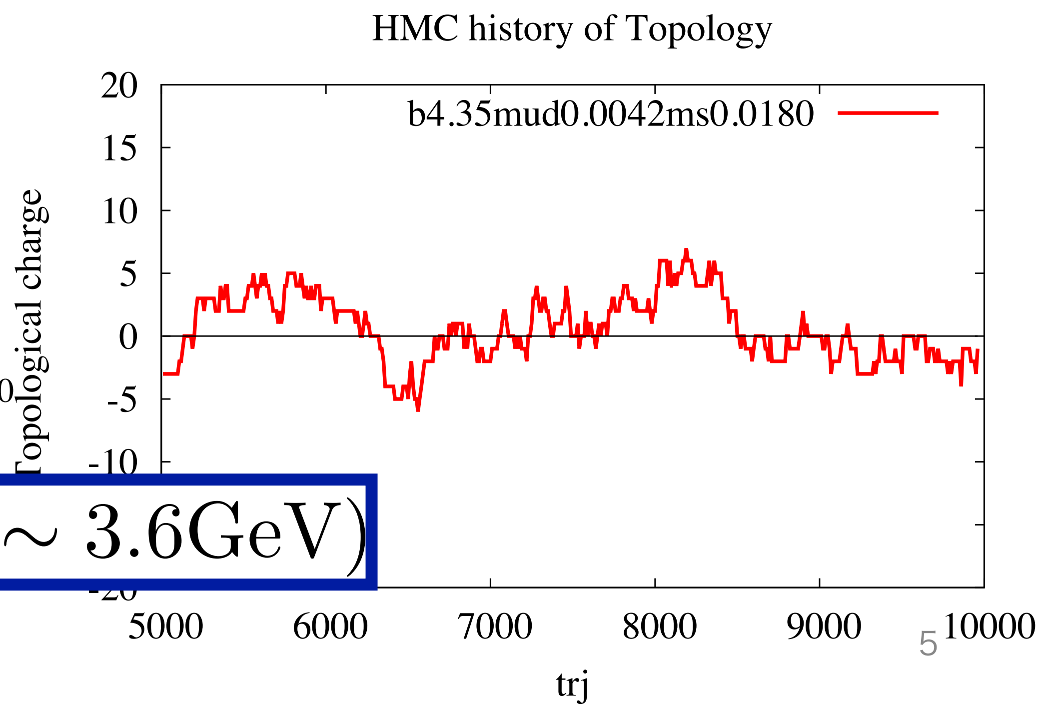
Cf. Results w/ Wilson fermions
Bruno, Schaefer, Sommer 2014

Motivation: Topological susceptibility is difficult.

It suffers from long auto-correlation on fine lattice



$$\beta = 4.17\ (a^{-1} \sim 2.4\mathrm{GeV})$$



$$\beta = 4.35\ (a^{-1} \sim 3.6\mathrm{GeV})$$

[JLQCD 2017]

What's new in our work arXiv:1705.10906

1. Mobius domain-wall fermion

   good chirality, topology tunnelings, large V, fine lattice spacings.

2. "Slab" sub-volume method [Bietenholz et al. 2015]

   statistically less noisy, shorter auto-correlation than global topology.

3. Chiral perturbation theory

   to find good combination of observables:

   $$\frac{\chi_t}{M_\pi^2 F_\pi^2}$$ is free from chiral logs.

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- ✔ 1. Introduction
- 2. Chiral perturbation for χ_t
- 3. Slab sub-volume method
- 4. Numerical simulations
- 5. Summary

1-loop SU(2) ChPT

[Gasser and Leutwyler 1984]

$$M_\pi^2 = M^2\left[1+\frac{M^2}{F^2}\left\{\frac{1}{32\pi^2}\ln\frac{M^2}{M^2_{\rm phys}}+2l_3^r\right\}\right], F_\pi^2 = F^2\left[1+\frac{M^2}{F^2}\left\{-\frac{1}{8\pi^2}\ln\frac{M^2}{M^2_{\rm phys}}+2l_4^r\right\}\right],$$

$$\chi_t = \frac{M^2F^2}{4}\left[1-\frac{3M^2}{32\pi^2F^2}\ln\frac{M^2}{M^2_{\rm phys}}+\frac{2M^2}{F^2}(l_3^r-l_7^r+h_1^r-h_3^r)\right],$$

The ratio,

[Mao-Chiu 2009, Aoki-F 2009 Guo-Meiner 2015]

$$\frac{\chi_t}{M_\pi^2F_\pi^2} = \frac{1}{4}\left[1+\frac{2M^2}{F^2}(-l_4^r-l_7^r+h_1^r-h_3^r)\right],$$

is free from chiral logs (and UV divs.), and finite V corrections (at 1-loop).

1-loop SU(3) ChPT

Moreover, the chiral limit is stable against strange quark effect:

SU(3) ChPT formula

$$\frac{\chi_t}{M_\pi^2 F_\pi^2} = \frac{1}{4}\left[1 + \frac{2M_\pi^2 l'_{(\mathrm{eff})}}{F_\pi^2} + \mathcal{O}(M_\pi^4)\right]$$

where $l'_{(\mathrm{eff})} = -\frac{1}{4M_{ss}^2}\left(F_\pi^2 + \Delta(M_K^2) + \frac{1}{2}\Delta(M_\eta^2)\right) + 36L_7 + 4L_8^r.$

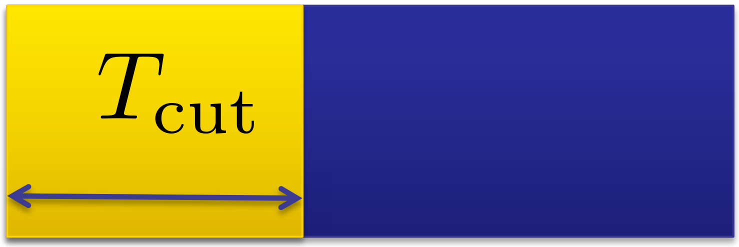
$\rightarrow$ From χ_t, M_π, F_π at each simulation point, we can precisely estimate the physical point of χ_t.

Contents

Topological susceptibility from slab

$$\langle Q^2_{\rm slab}(T_{\rm cut})\rangle \equiv \left\langle \left(\int_0^{T_{\rm cut}} dx_0 \int d^3x\, q^{\rm lat}(x) \right)^2 \right\rangle .$$

[Bietenholz et al. 2015]



$$q(x) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}_{clover}F^{\rho\sigma}_{clover}(x)$$

after Wilson flow time $\sqrt{8t} \sim\ 0.5\ \mathrm{fm}$

Advantage : it is always positive.

In infinite volume and with infinite statistics,

$$\langle Q^2_{\rm slab}(T_{\rm cut})\rangle = \frac{T_{\rm cut}}{T}\chi_t V$$

Finite V correction from 1st excited state

$$\int d^3x \langle q(x)q(0)\rangle = A\cosh(m_{\eta'}(x_0 - T/2))$$

$$\langle Q^2_{\rm slab}(T_{\rm cut})\rangle = (\chi_t V)\times\frac{T_{\rm cut}}{T} + \frac{A}{2}(1-e^{-m_{\eta'}T_{\rm cut}})(1-e^{-m_{\eta'}(T-T_{\rm cut})})$$

$$\simeq const. \text{ for } 0 \ll T_{\rm cut} \ll T$$

From linear + const. behavior, we can extract topological susceptibility from the slope:

$$\chi_t^{\rm slab} = \frac{T}{V}\left[\frac{\langle Q^2_{\rm slab}(t_1)\rangle - \langle Q^2_{\rm slab}(t_2)\rangle}{t_1 - t_2}\right].$$

What we expect in slab sub-volume

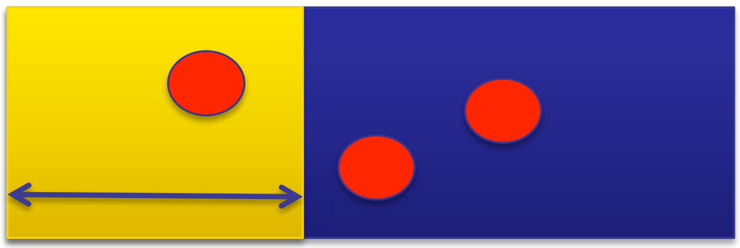
1. Increase statistics by $\dfrac{V}{V_{\mathrm{instanton}}}$

   If V = ∞, one configuration is enough.

   [Luescher plenary]

2. Shorter auto-correlation than global Q:

   no topology barrier in a sub-volume



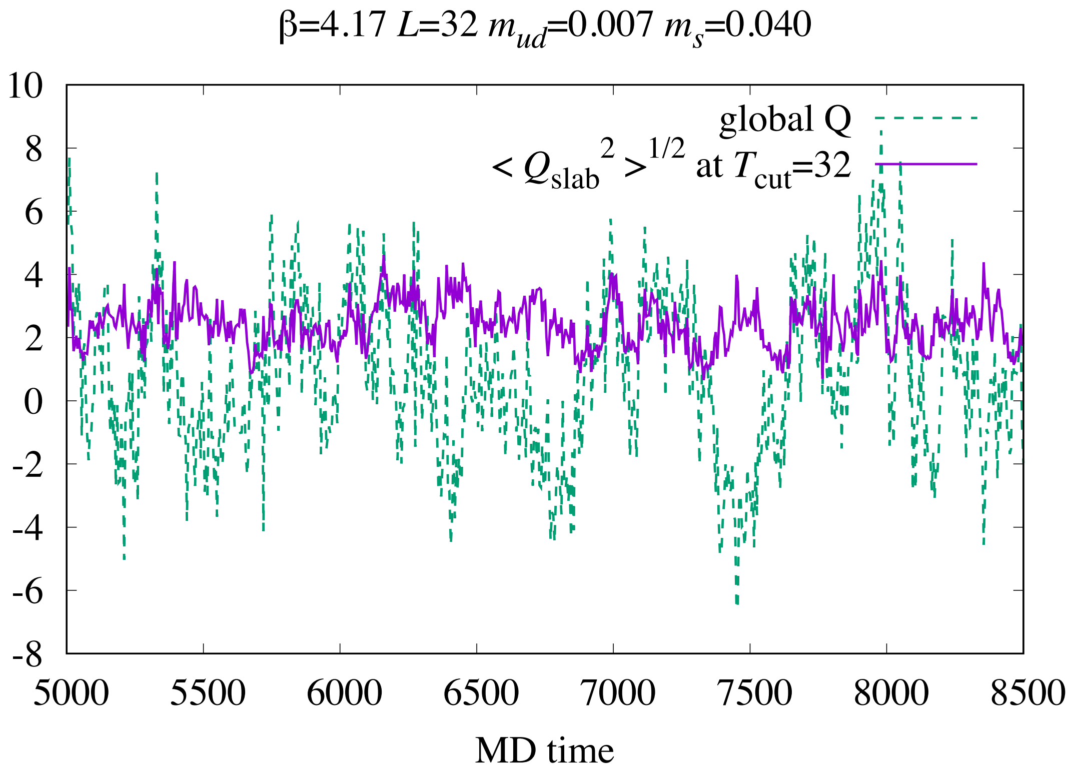
T_{cut}

Contents

- ✔ 1. Introduction
- ✔ 2. Chiral perturbation for χ_t
  - $\chi_t / M_\pi^2 F_\pi^2$ is free from chiral logs.
- ✔ 3. Slab sub-volume method :we expect
  - better statistics & shorter auto-correlations
- 4. Numerical simulations
- 5. Summary

Result at β=4.17 (a~0.08 fm)

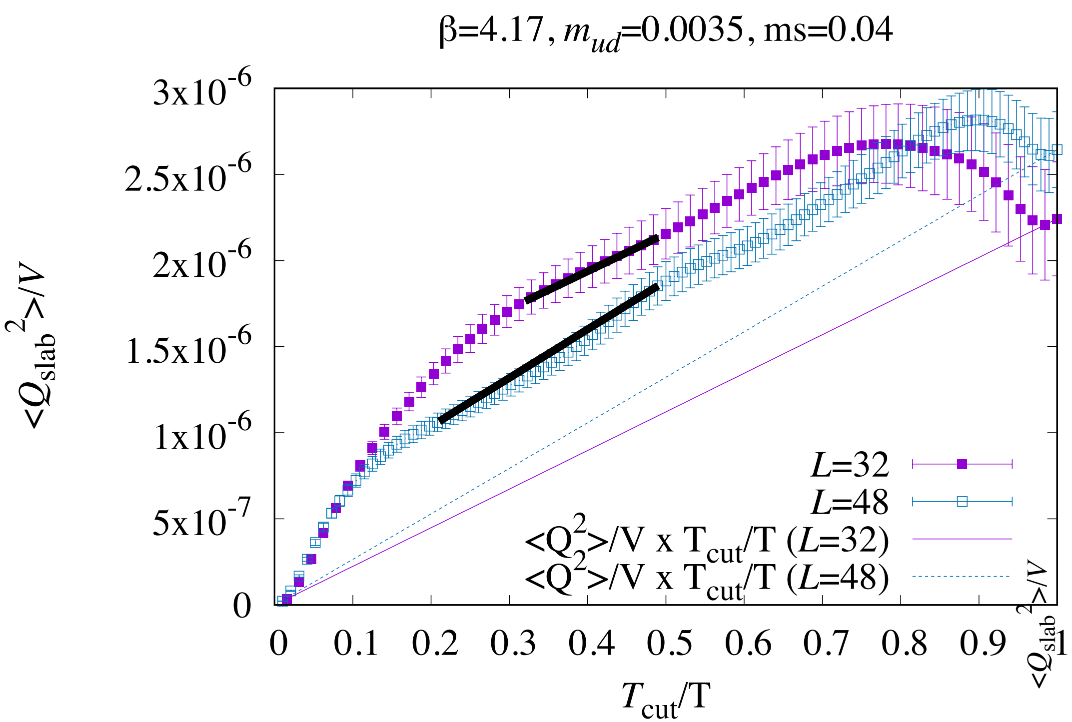
Both of global and sub-volume topological fluctuations are active.



Let us check the consistency with each other, and with ChPT.

Result at β=4.17 (a~0.08 fm)

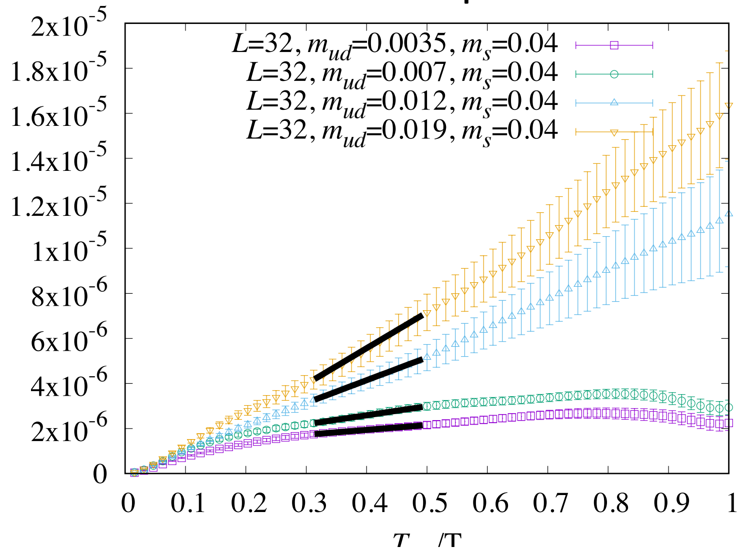
Linear+const. behavior is seen.



Consistent with global Q

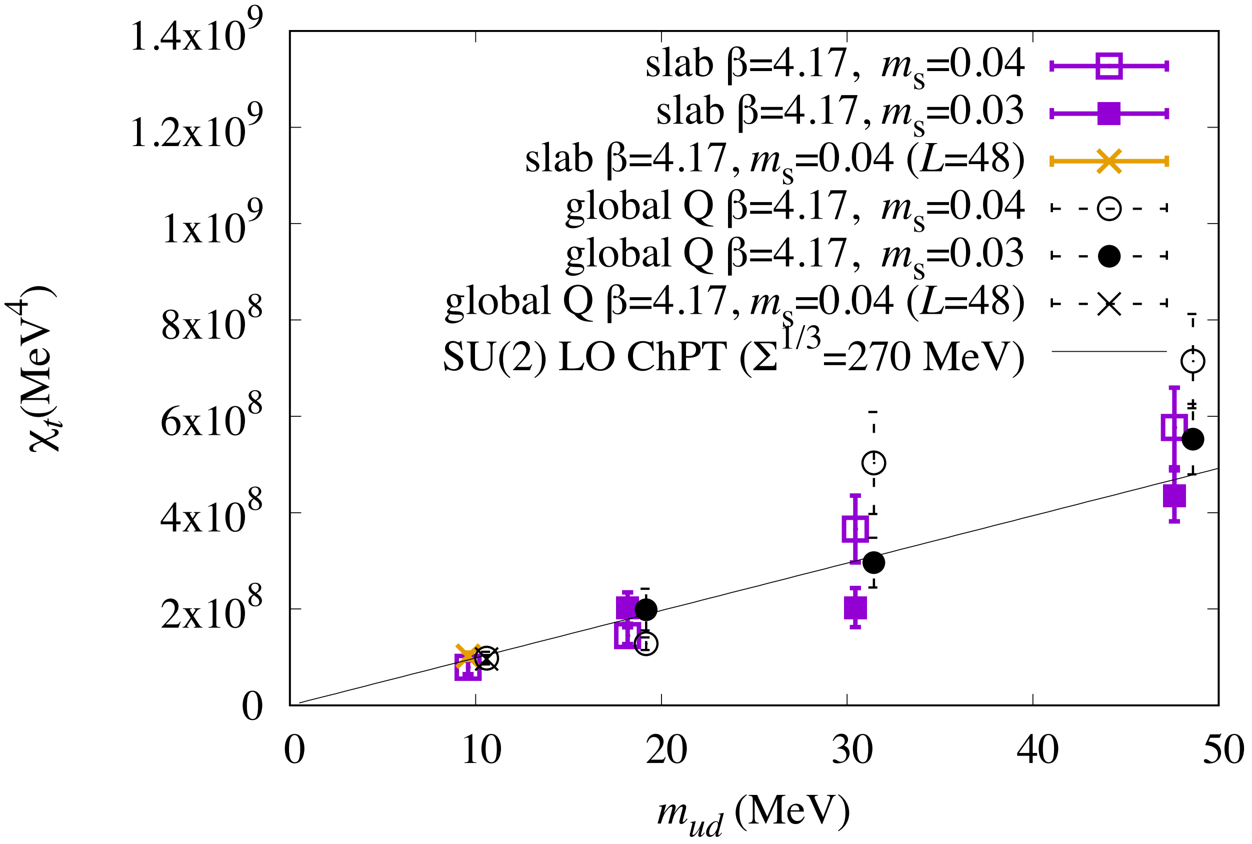
L=32 (2.6fm) and L=48 (3.9 fm) results agree.

Data at heavier quark mass



Result at β=4.17 (a~0.08 fm)

$$\chi_t^{\rm slab} = \frac{T}{V}\left[\frac{\langle Q^2_{\rm slab}(t_1)\rangle - \langle Q^2_{\rm slab}(t_2)\rangle}{t_1 - t_2}\right].$$ is consistent with ChPT.

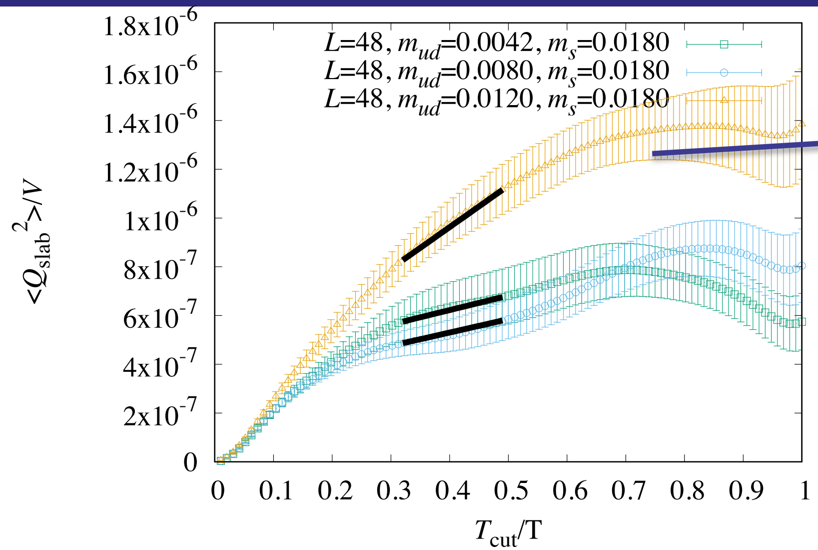


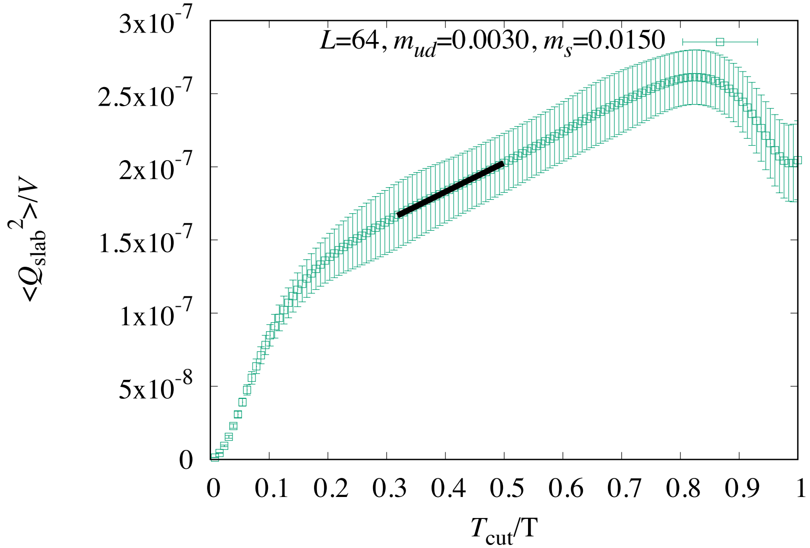
Also consistent with

$$\frac{\langle Q^2\rangle}{V}$$

from global topology.

Result at β >=4.35 (a<=0.05fm)





Some curvature is seen, due to effect from global topology, whose autocorrelation time is long.

Careful error estimate is needed.

Autocorrelation and error estimate

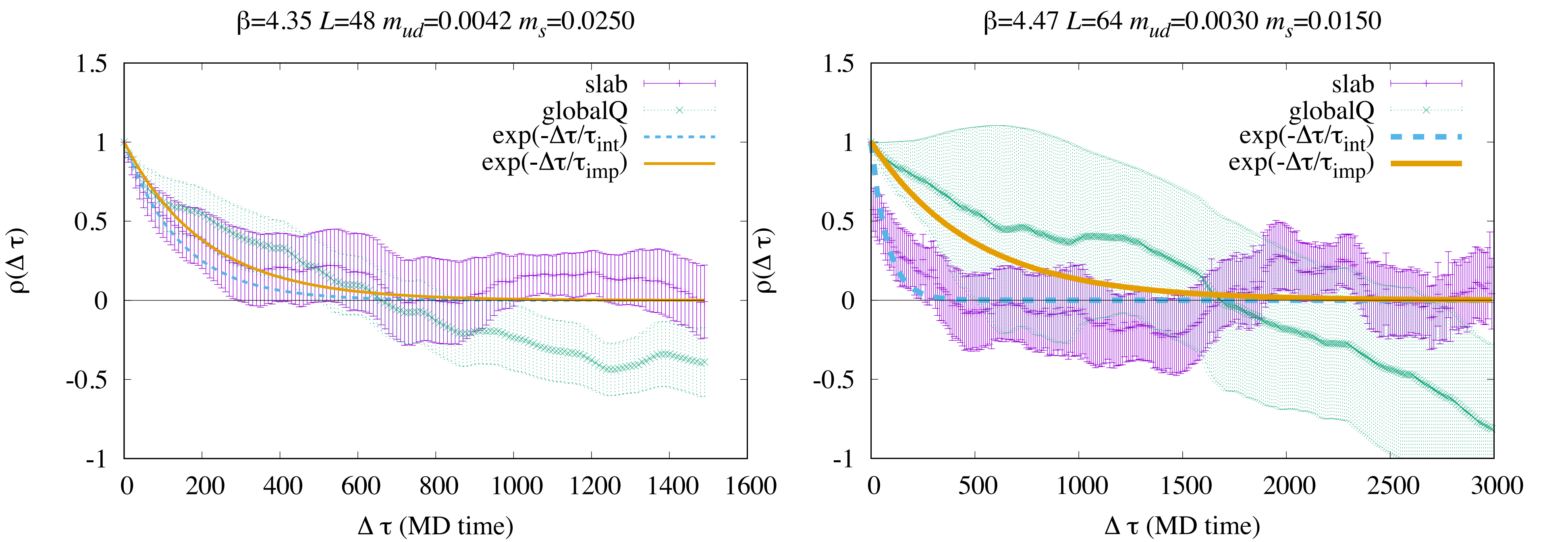
ALPHA collaboration method[Schaefer et al. 2010]:

Assuming a double exponential structure,

$$\rho(\tau) = A \exp(-\tau/\tau_{naive}) + B \exp(-\tau/\tau_Q)$$

slow mode due to global topology

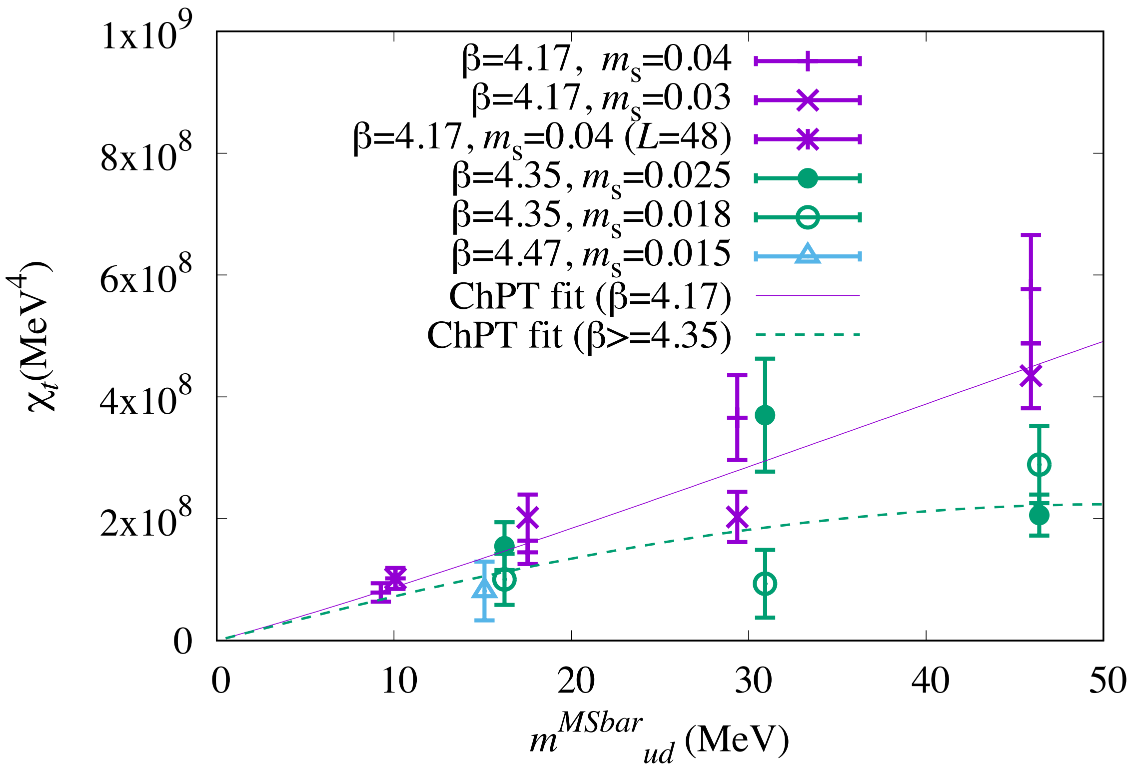
autocorrelation time is estimated -> statistical errors



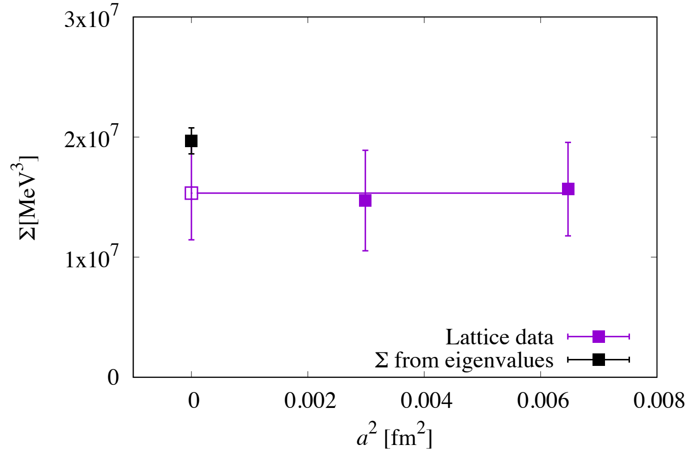
Chiral and continuum extrapolation

$$\chi_t = \frac{m_{ud}\Sigma}{2}\left\{1 - \frac{3m_{ud}\Sigma}{16\pi^2 F^4_{\rm phys}}\ln\left(\frac{2m_{ud}\Sigma}{F^2_{\rm phys}M^2_{\rm phys}}\right) + \frac{4m_{ud}\Sigma}{F^4_{\rm phys}}l\right\},$$

$$l = l_3^r - l_7^r + h_1^r - h_3^r$$

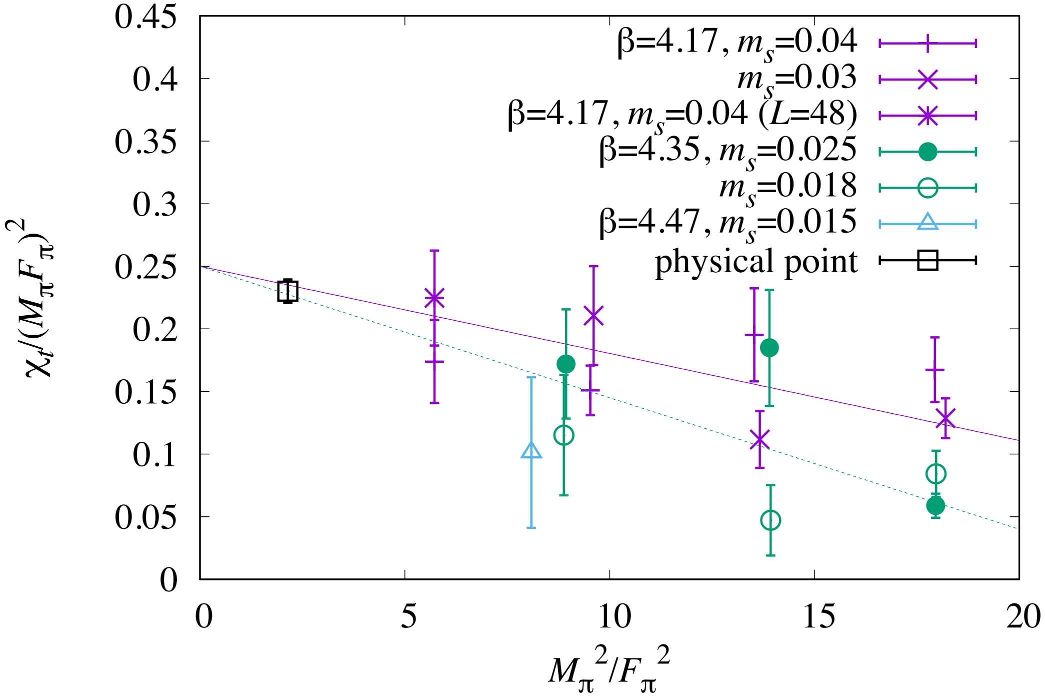


Continuum limit of chiral condensate

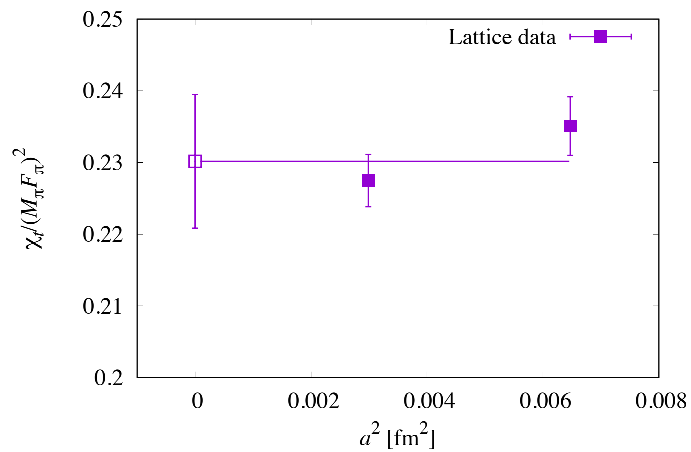


Ratio with pion mass and decay constant

$$\frac{\chi_t}{M_\pi^2 F_\pi^2} = \frac{1}{4}\left[1 + \frac{2M^2}{F^2}(-l_4^r - l_7^r + h_1^r - h_3^r)\right],$$



Continuum limit of the ratio at physical point



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  $\chi_t/M_\pi^2 F_\pi^2$ is free from chiral logs.
- ✔ 3. Slab sub-volume method :

  better statistics & shorter auto-correlations.
- 4. Numerical simulations

  Consistent with ChPT, good precision for $\chi_t/M_\pi^2 F_\pi^2$
- 5. Summary

Summary

Using

1. Good chiral sym. with domain-wall fermion,
2. Slab sub-volume method (to reduce statistical noise and auto-correlation)
3. Ratio with $M_\pi^2 F_\pi^2$ (chiral limit is stable),

we obtain [JLQCD, 1705.10906]

$$\chi_t = 0.230(01)(01)(09) M_\pi^2 F_\pi^2 \text{ (at physical point)},$$

$$\Sigma^{\overline{\rm MS}}(2\text{GeV}) = [248(11)(15)(08)\text{MeV}]^3,$$

$$(l_3^r - l_7^r + h_1^r - h_3^r) = 0.0025(24)(150)(22),$$

$$(-l_4^r - l_7^r + h_1^r - h_3^r) = -0.019(01)(09)(01).$$