On the nature of phases at finite isospin chemical potential

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** Work in progress with G. S. Bali (Regensburg) & G. Endrődi (Frankfurt) and N. Mathur (TIFR)*

Introduction : Why μ_I ?

- Protons convert to neutrons and neutrinos via electron capture in the core of neutron stars, leading to high baryon density with considerable I_3-finite isospin, $i.e.$, both $\mu_B \neq 0 \neq \mu_I$.

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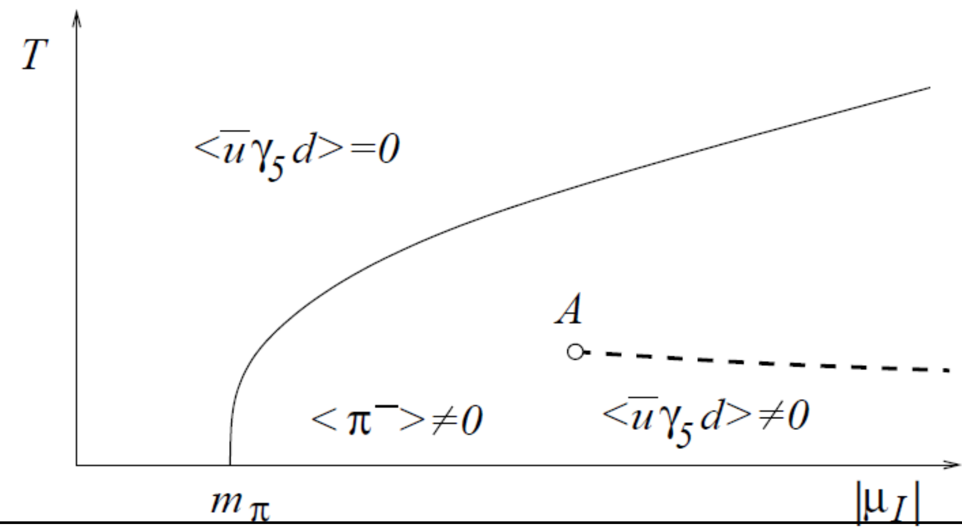
- Son & Stephanov (Son-Stephanov PRL '01, Phys.Atom.Nucl. '01) studied analytically limiting cases of QCD for $\mu_B = 0$ but $\mu_I \neq 0$, for which there is no sign/phase problem, to conjecture the following interesting phase diagram.



- Using μ_u and μ_d as light quark chemical potentials, one has $\mu_B = 3(\mu_u + \mu_d)/2$ & $\mu_I = (\mu_u - \mu_d)/2$ or alternatively, $\mu_u = \mu_B/3 + \mu_I$ & $\mu_d = \mu_B/3 - \mu_I$.

- For QCD with two flavours, quarks are two component spinors in flavour space, leading to a quark matrix :

$$M = \begin{pmatrix} D\!\!\!/(\mu_I) + m & \lambda\gamma_5 \\ -\lambda\gamma_5 & D\!\!\!/(-\mu_I) + m \end{pmatrix}$$

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- Kogut-Sinclair(PRD '02) employed the standard techniques for staggered fermions,

$$S_F = \sum_{sites} \bar{\chi} \left[D\!\!\!/(\tau_3\mu_I) + m + i\lambda_I \epsilon \tau_2 \right] \chi \ ,$$

and showed that the fermion determinant is positive definite, and the usual techniques do work to simulate the theory.

- They also worked out the symmetry-breaking patterns and the corresponding observables which are signals for them, and obtained the first numerical results for the phase transition at zero and finite temperature.

- These are the chiral condensate, the pion condensate, and the isospin density,

$$\langle \bar{\psi}\psi \rangle = \frac{T}{V}\frac{\partial \ logZ}{\partial m}, \langle \pi \rangle = \langle \bar{\psi}_u \gamma_5 \psi_d - \bar{\psi}_d \gamma_5 \psi_u \rangle = \frac{T}{V}\frac{\partial \ logZ}{\partial \lambda}, \langle n_I \rangle = \frac{T}{V}\frac{\partial \ logZ}{\partial \mu_I}$$

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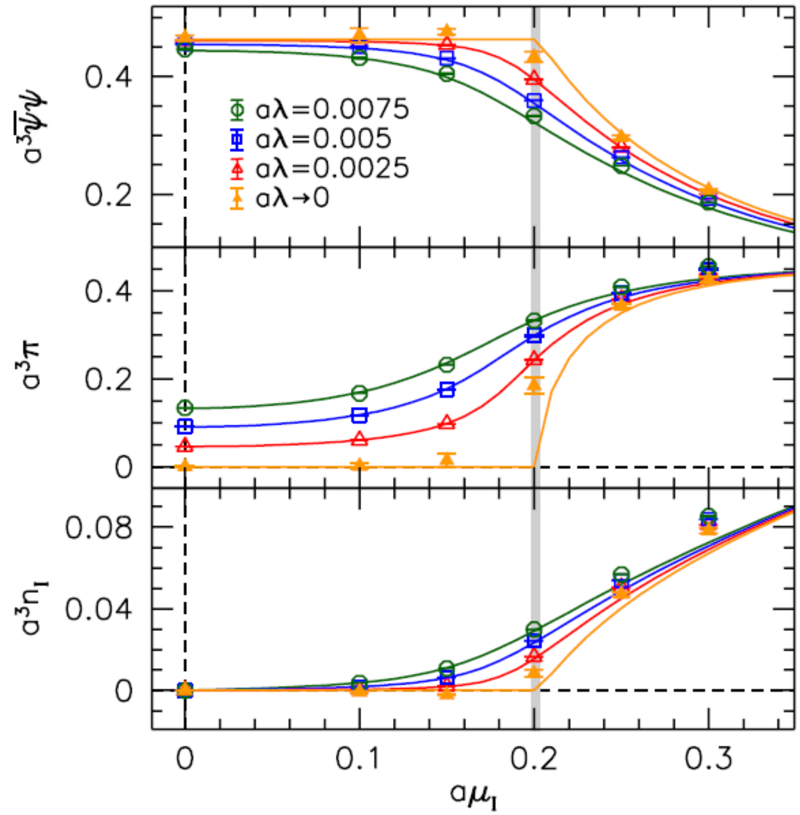
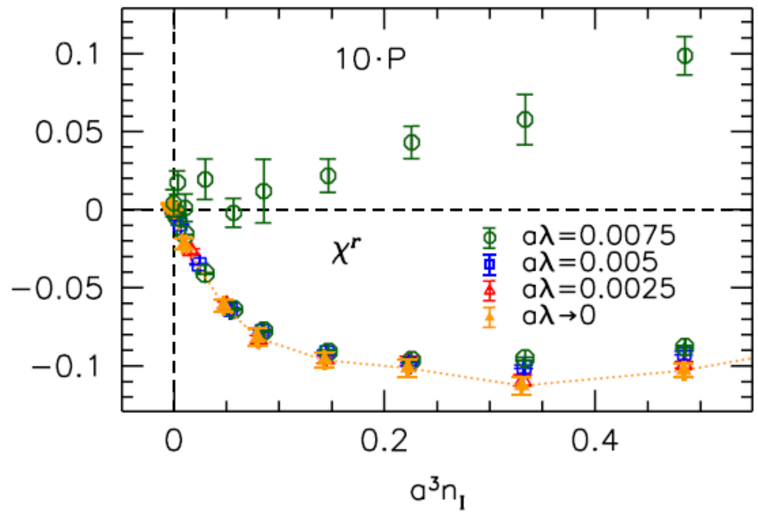
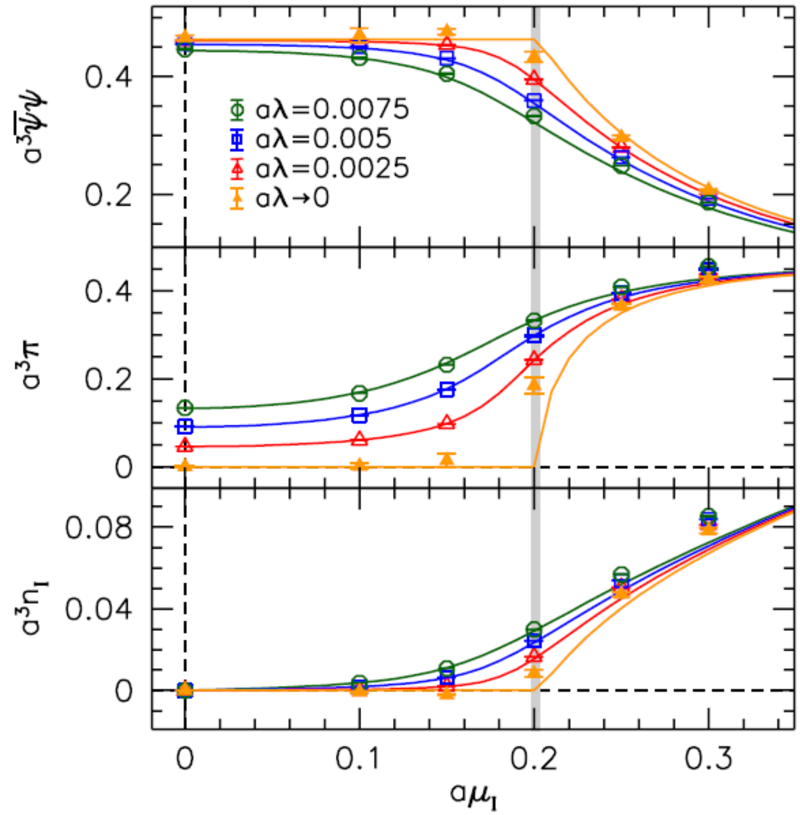
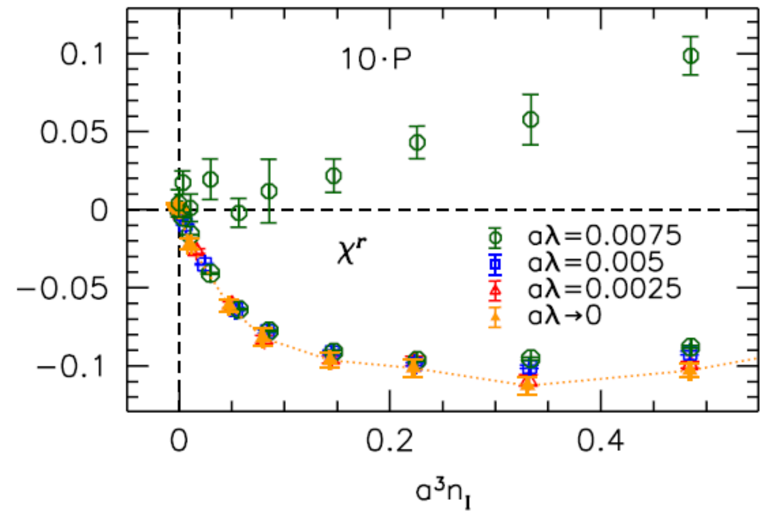
- Employing staggered fermions on 8^4 lattices with $a = 0.299(2)$ fm & lattice quark mass $ma = 0.025$, corresponding to $m_\pi \simeq$ 260 MeV, Endrődi (PRD '15) investigated the phase structure and obtained $a\mu_I^c \simeq 0.2$.





- $\lambda \to 0$ extrapolation in yellow, points (linear), line (chiral theory fit).
- Grey vertical band denotes $m_\pi/2$.
- Pion condensate & Isospin density become nonzero around $\mu_I^c \simeq m_\pi/2$, where chiral condensate drops rapidly as well.





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- Pion condensate & Isospin density become nonzero around $\mu_I^c \simeq m_\pi/2$, where chiral condensate drops rapidly as well. • Polyakov loop displayed in the upper half of the left panel shows deconfinement to occur there as well.

Introduction II : Nature of Probe

- High μ_I phase appears to have restored chiral symmetry and deconfinement. Leading candidate for χSB – topological excitations.

- Successful phenomenology built on Instanton-fermion couplings. (Schafer-Shuryak RMP '98, Diakonov hep-ph/9602375)

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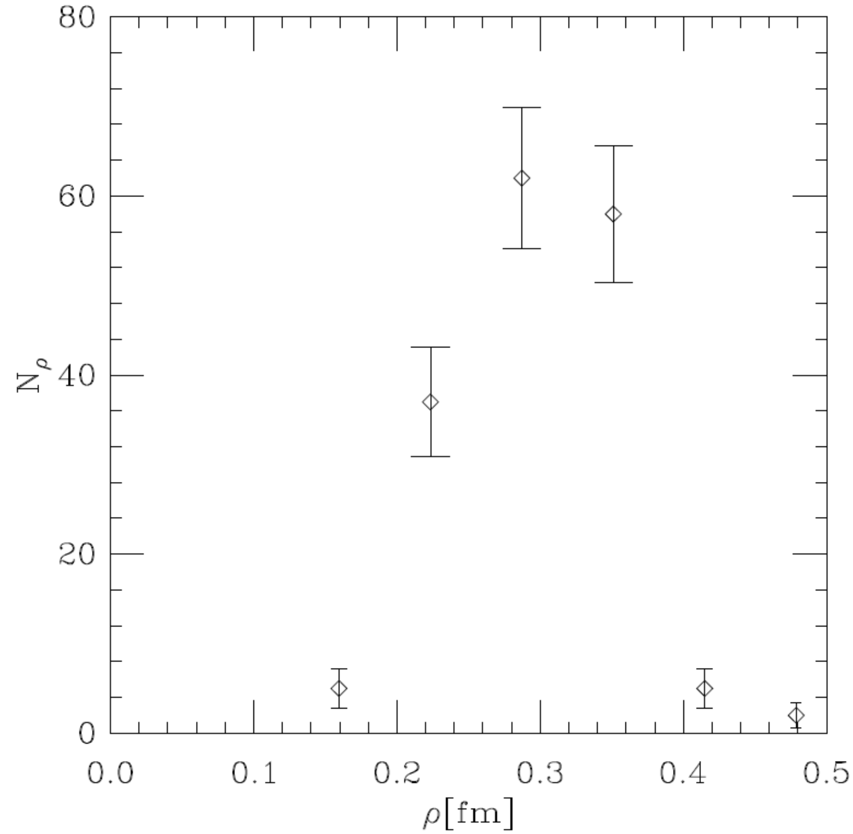
- LQCD simulations also support it : Instanton-distribution peaks at a radius $\rho = 0.3$ fm (DeGrand-Hasenfratz PRD '01).



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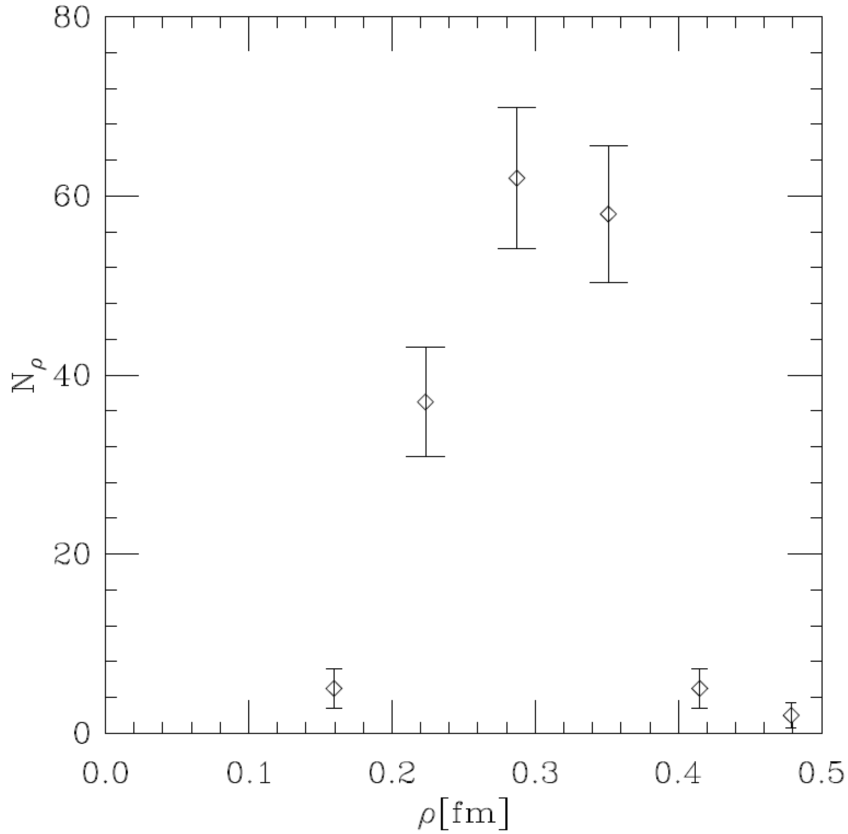

♠ Note that Overlap Dirac operator, which has *exact* chiral symmetry on the lattice as well as an index theorem, was used for the analysis above.

♣ The Overlap Dirac operator spectra has also been used to understand the nature of the high temperature phase.

♢ Number of low eigen modes do get depleted as $T \uparrow$. (Edwards-Heller-Kiskis-Narayanan, PRL '99, NPB (PS) '00, PRD '01; Gavai-Gupta-Lacaze, PRD '02)

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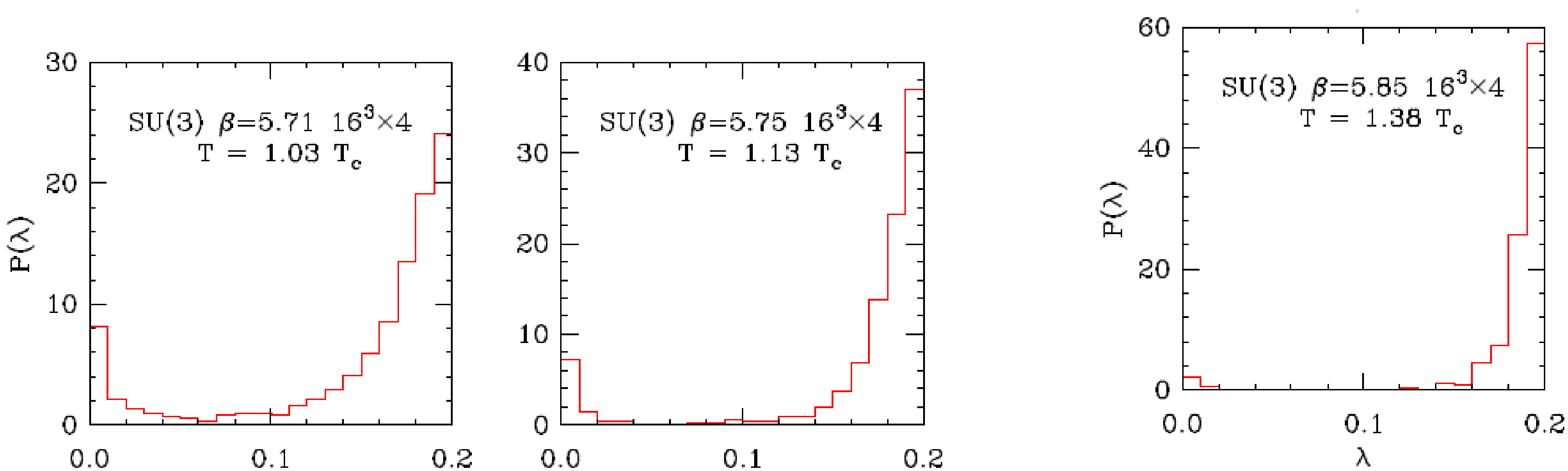
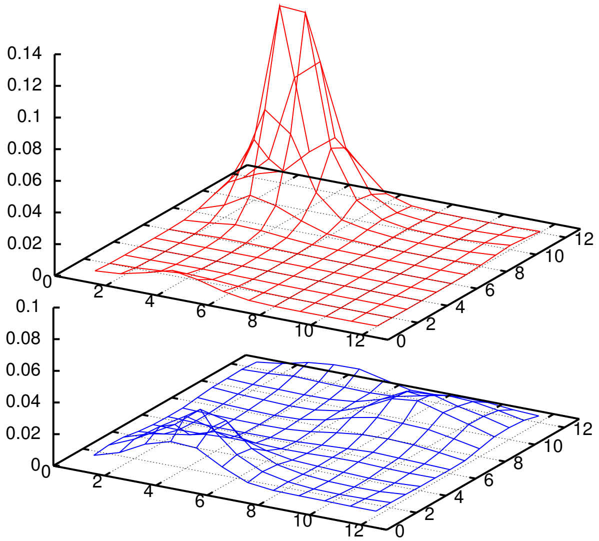


♢ Furthermore, a gap appears to separate the low modes from others.

Employing Exact Chiral Lattice Fermions

Overlap Dirac-Neuberger fermions possess satisfy the following correlator equalities in the chirally symmetric phase : $C_S(z) = -C_{PS}(z) \quad \text{and} \quad C_V(z) = C_{AV}(z)$.



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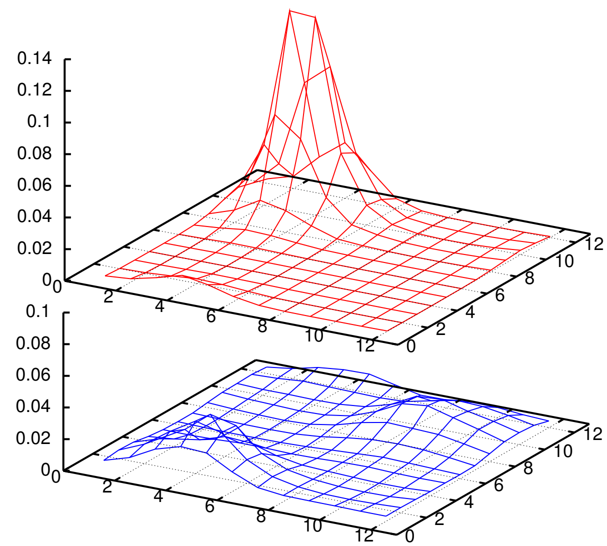

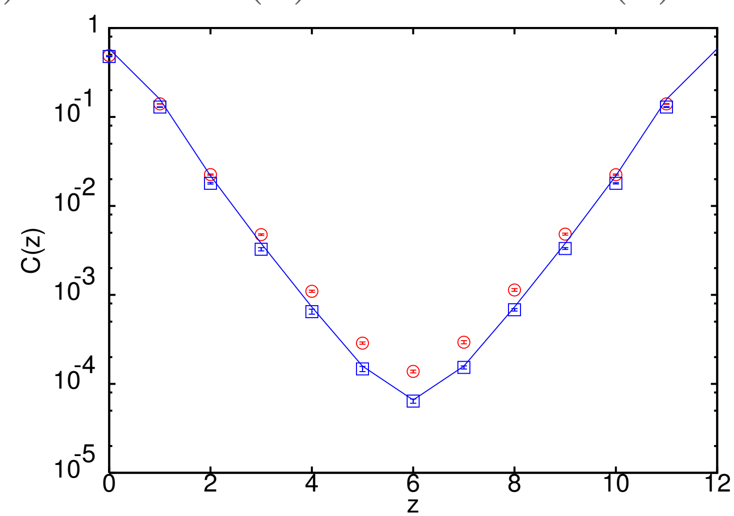

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♡ Vector and Axial vector correlators equal above T_c but Pseudoscalar and Scalar equal *only* without zero modes ($T = 1.5T_c$ above).

Our Results

- We employed the Arnoldi method to extract the eigenvalues of Overlap Dirac operator (defined on dynamical configurations with nonzero μ_I but without an explicit μ_I in the operator itself), demanding a residue $r = ||DX - \eta|| \leq 10^{-10}$.

- Extracted $\sim$500 eigenvalues from each configuration.

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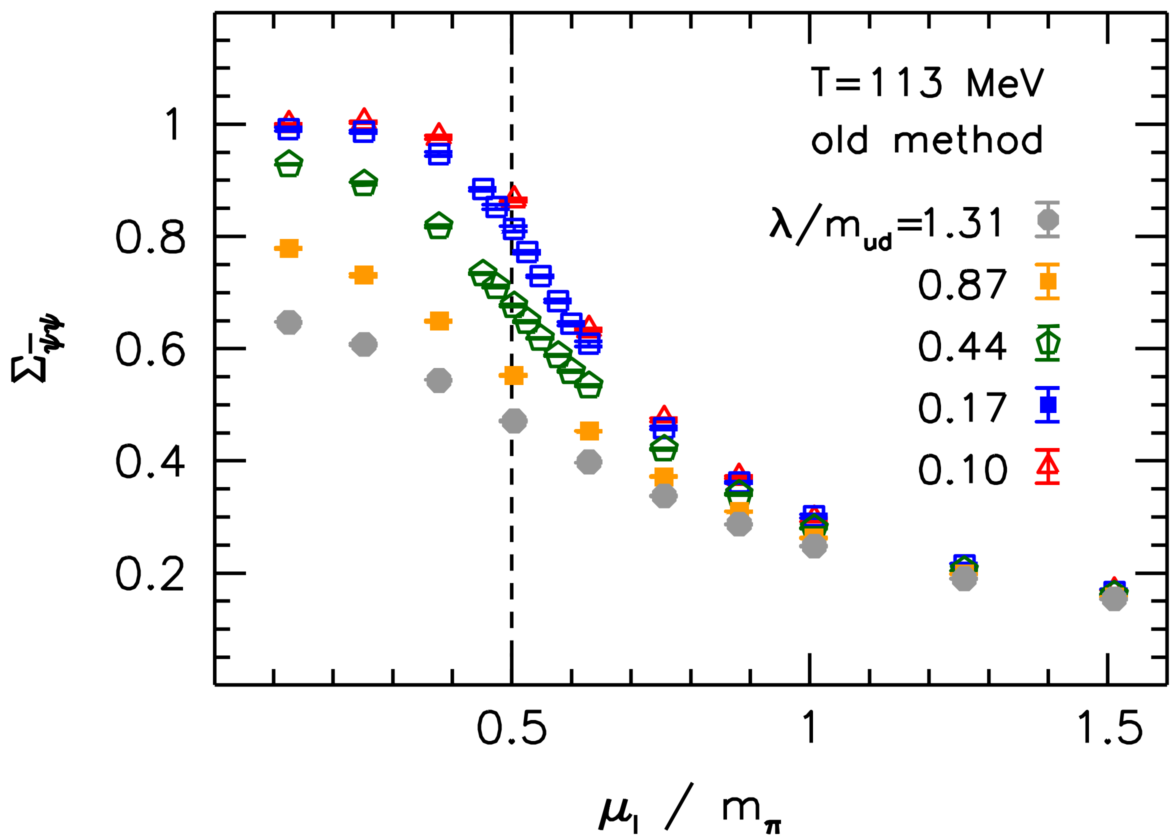
- Extracted ∼500 eigenvalues from each configuration.

- Used a larger $24^3 \times 6$ lattice and a Symanzik improved action with 2 stout steps. Quark mass was tuned to have the physical pion mass.

- $a\mu_I^c = 0.1$ there, which again corresponds to μ_I^c being $m_\pi/2$.

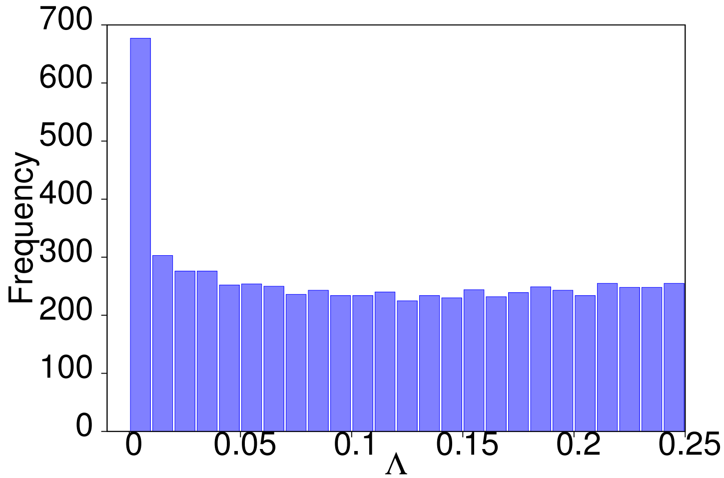
- Computations made at different values of μ_I/μ_I^c below and above the transition and two different λ — the isospin breaking parameter in the quark matrix.



(Figure from G. Endrődi)

$\lambda/m_{ud} = 0.11$

♡ We examined Overlap Dirac eigenmodes† for $\mu_I/\mu_I^c \simeq 0.5$ and 1.5, corresponding to $\mu_I/m_\pi = 0.25$ and 0.75 respectively.



♠ $\mu_I/\mu_I^c = 0.5$: Fairly uniform distribution with some low modes are seen.

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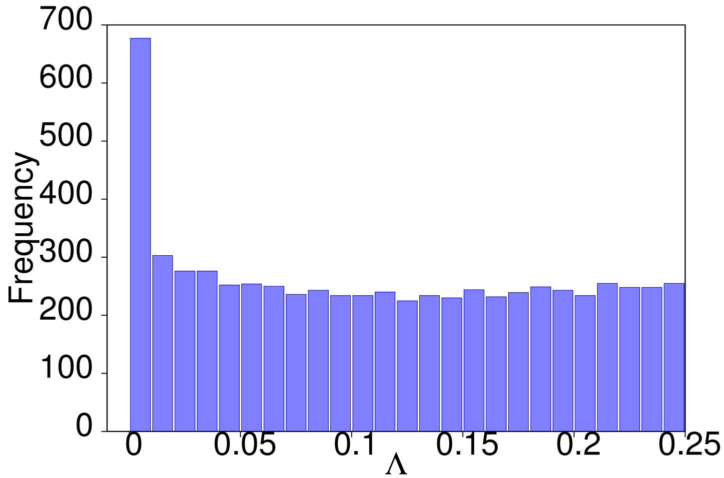

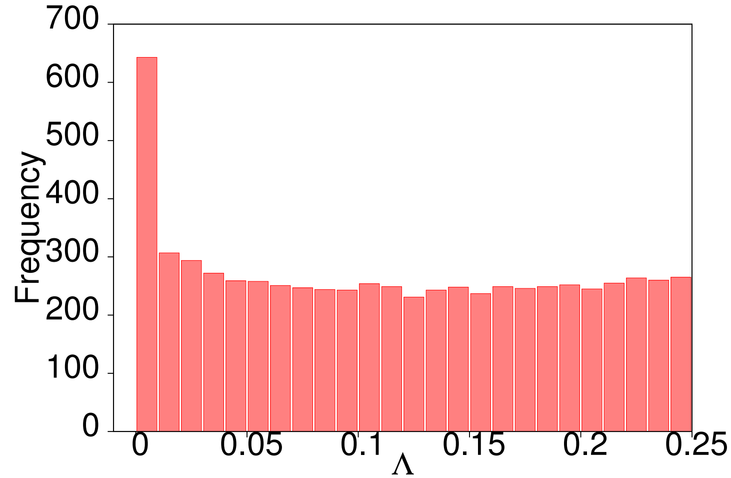

♠ $\mu_I/\mu_I^c = 0.5$: Fairly uniform distribution with some low modes are seen.

♡ $\mu_I/\mu_I^c = 1.5$: Surprisingly similar distribution as in the lower phase.

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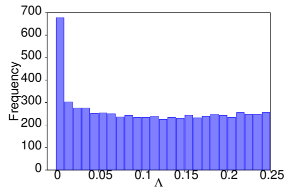
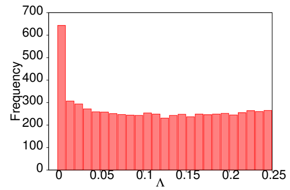
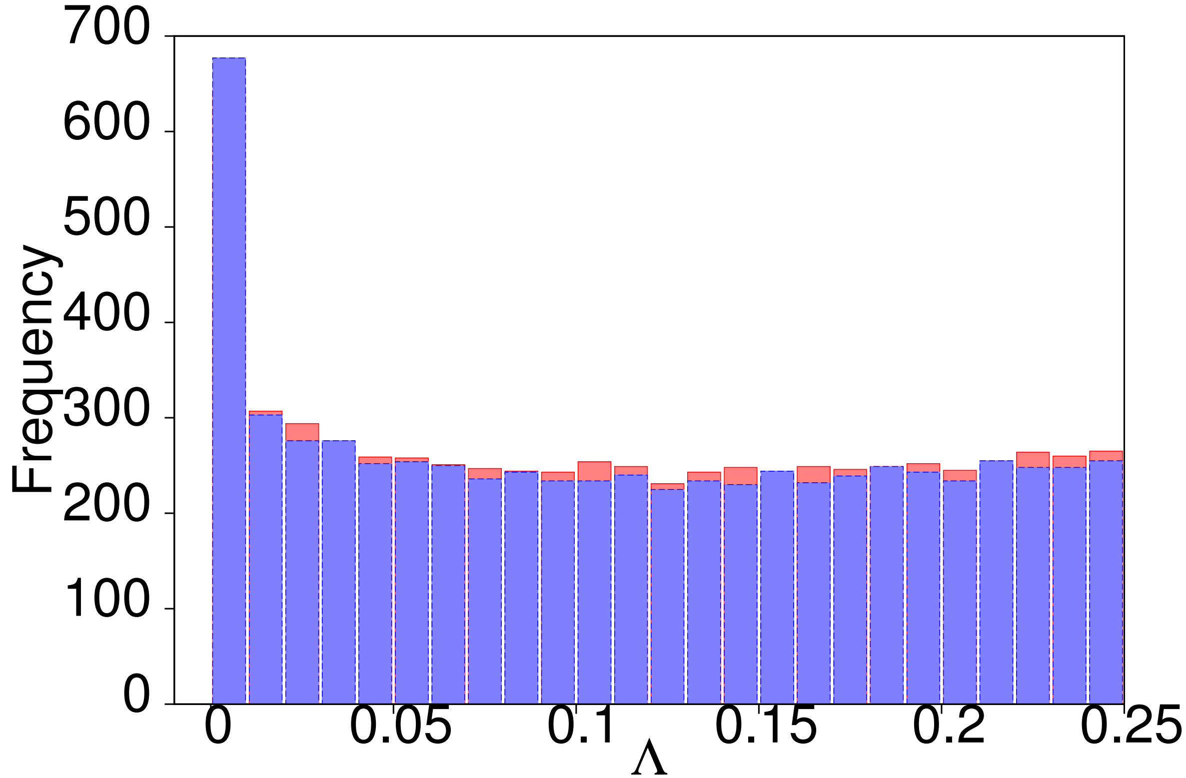
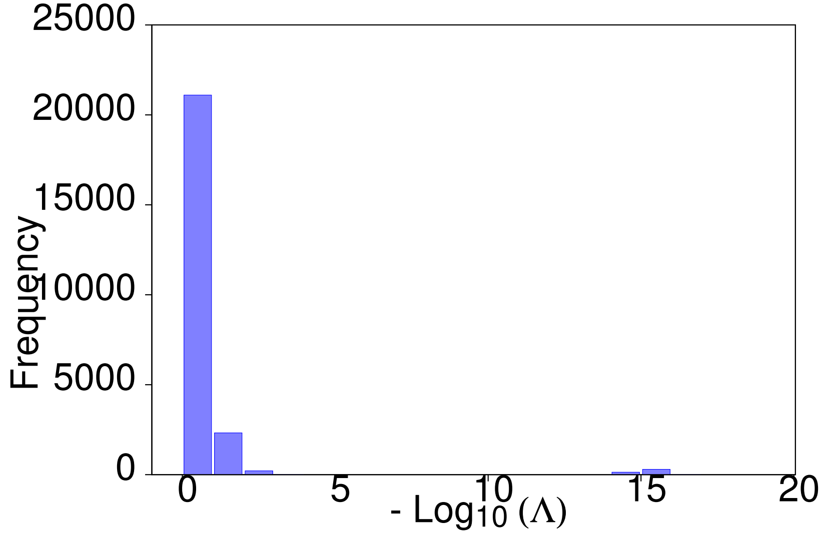






♡ As expected, the overlap is indeed significant. Alternatively, the surprise is confirmed to be not an illusion.

♡ Looking at the eigenvalue distribution on a log scale, one can easily identify the zero modes from the gap in the spectrum. Explicit chirality checks needed to confirm their nature.



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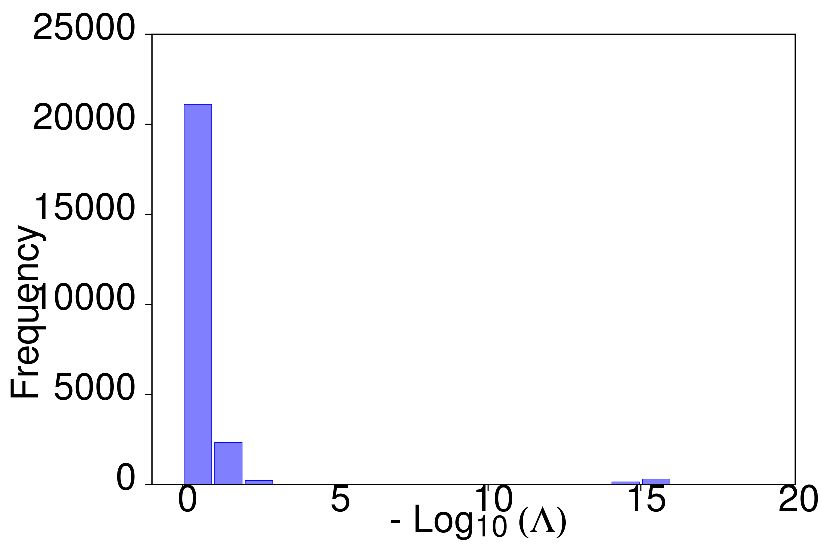

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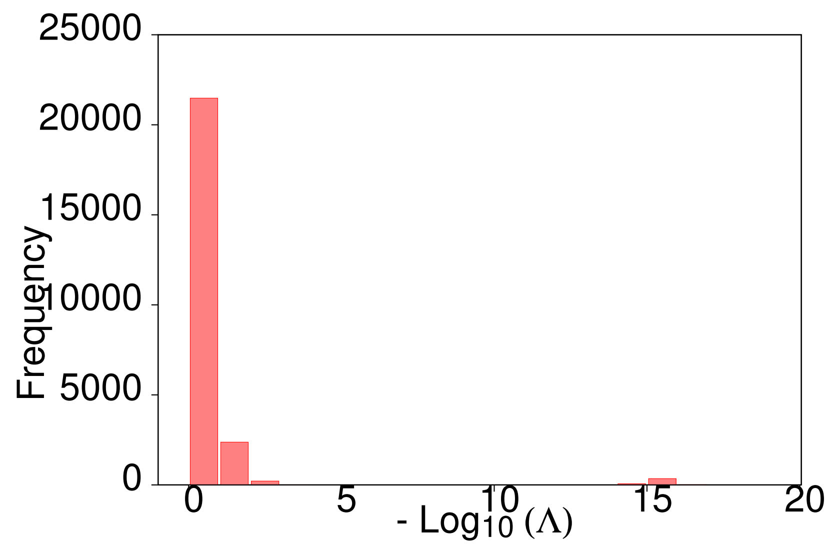
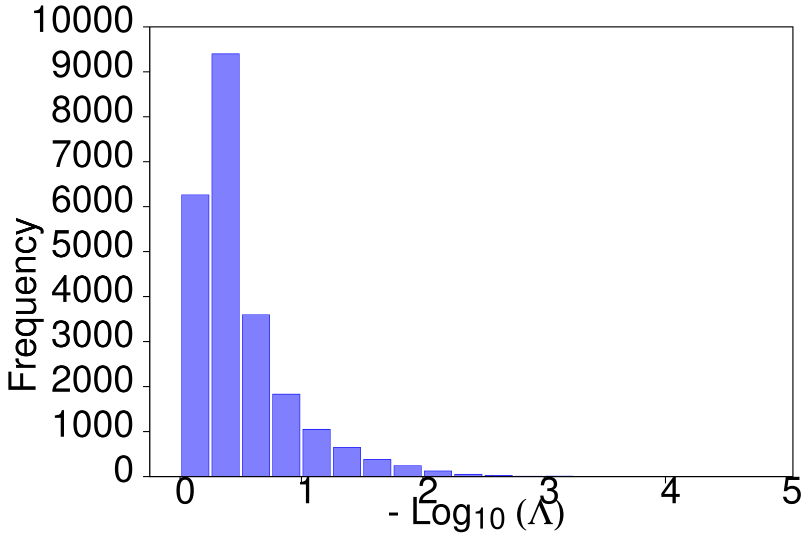


♡ $\mu_I/\mu_I^c = 1.5$: The number of zero modes still appear to be roughly the same.

♡ Zooming in on the eigenvalue distribution on the log scale to see if the near-zero modes have any difference which was missed.



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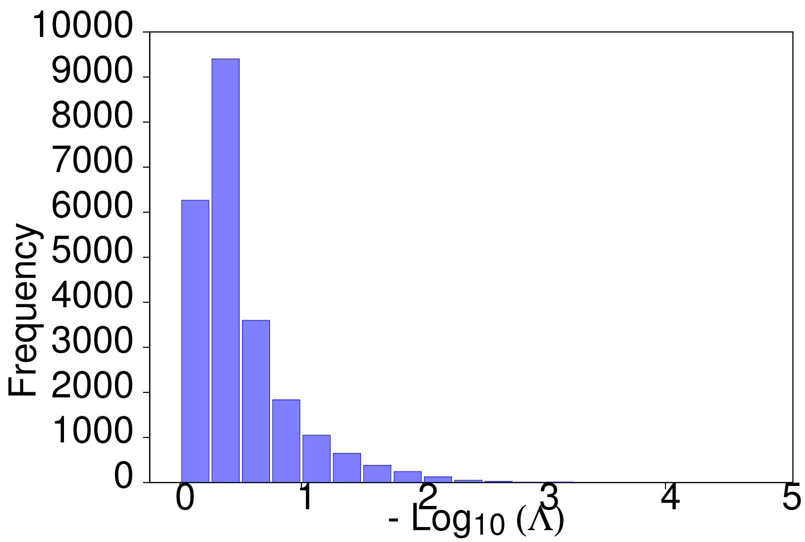
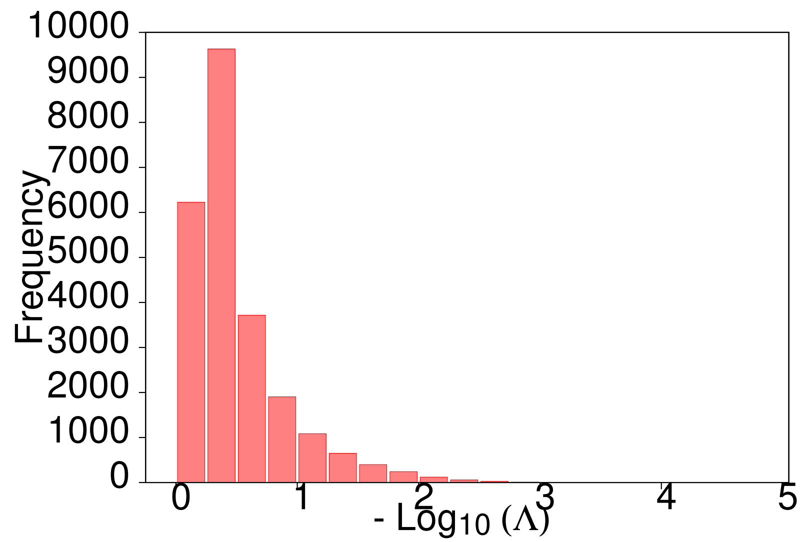
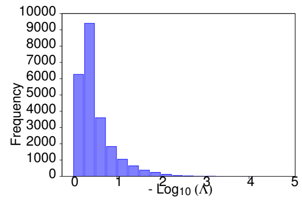
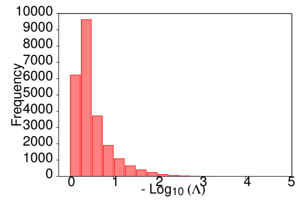


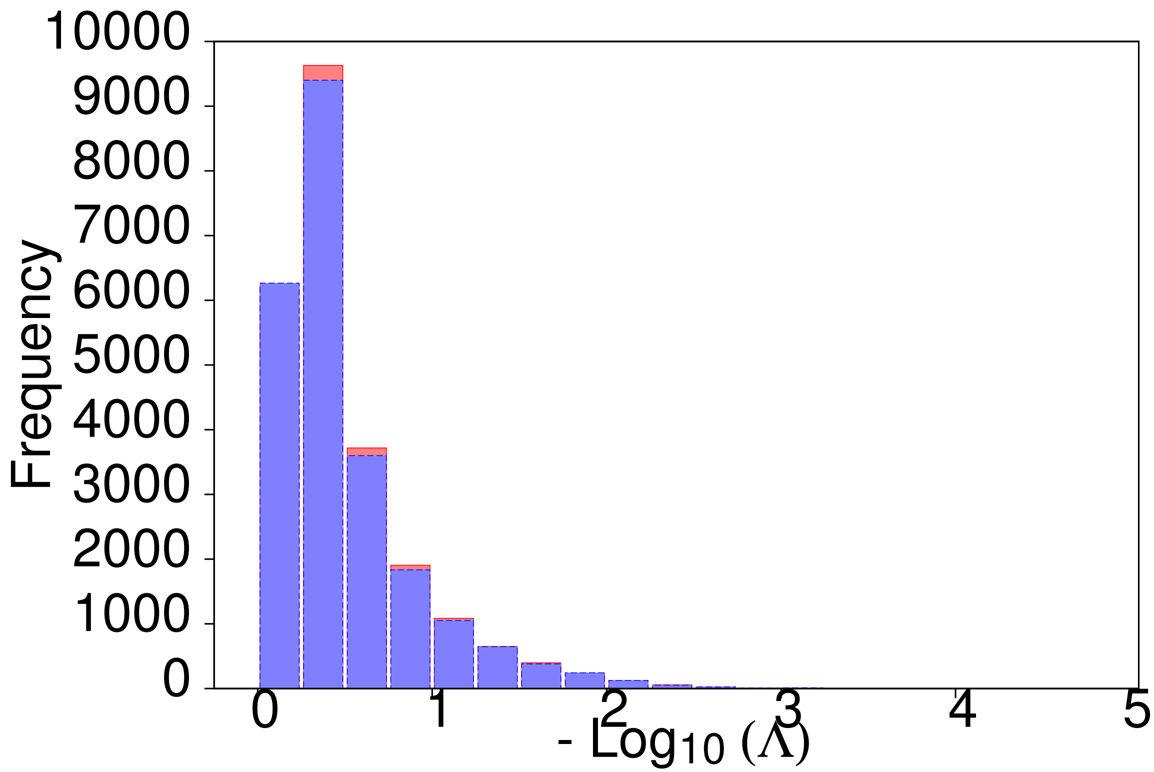


♠ $\mu_I/\mu_I^c = 0.5$: Nice smooth fall-off is seen.

♡ $\mu_I/\mu_I^c = 1.5$: Similarity in the distribution as in the lower phase still indicated.



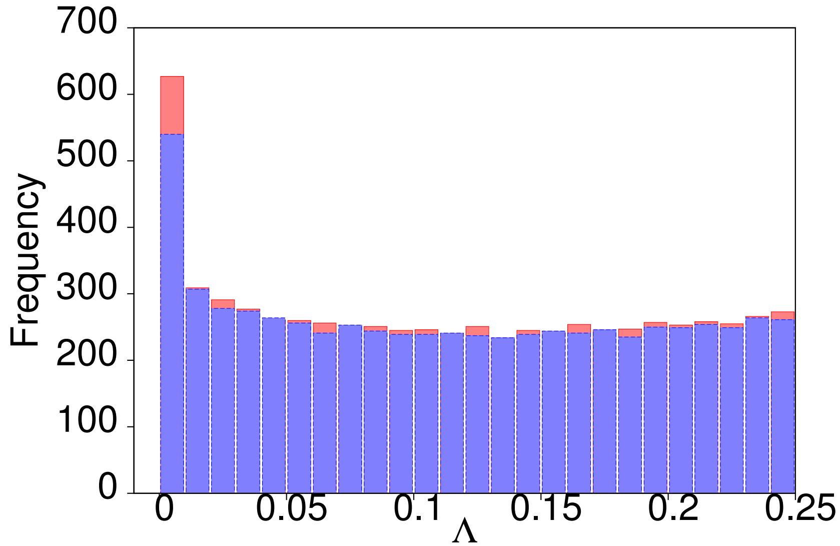




♡ No visible difference in the near-zero mode distributions.,

$$\lambda/m_{ud} = 0.44$$

♡ Again we examined Overlap Dirac eigenmodes for $\mu_I/\mu_I^c \simeq 0.5$ and 1.5, corresponding to $\mu_I/m_\pi = 0.25$ and 0.75 respectively.



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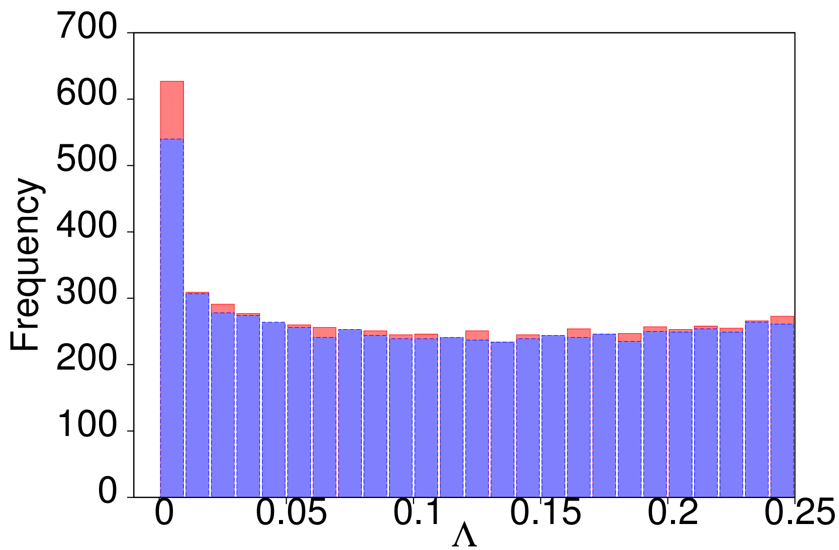

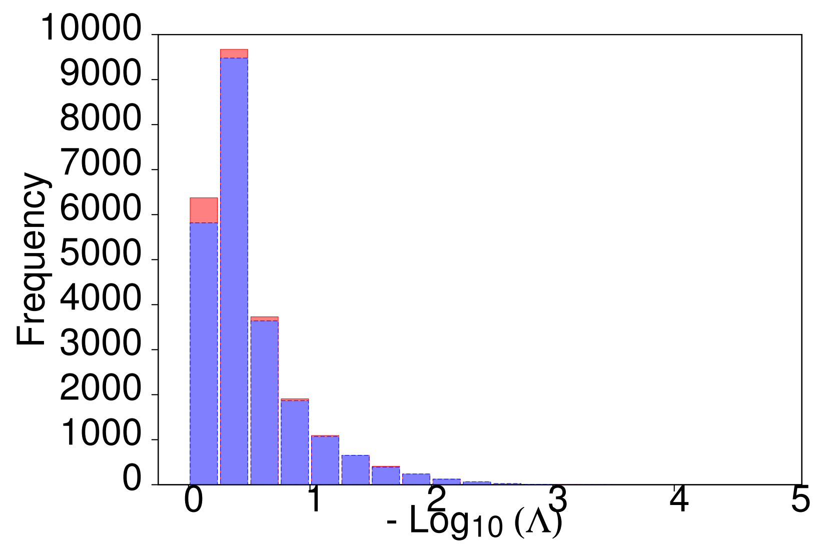

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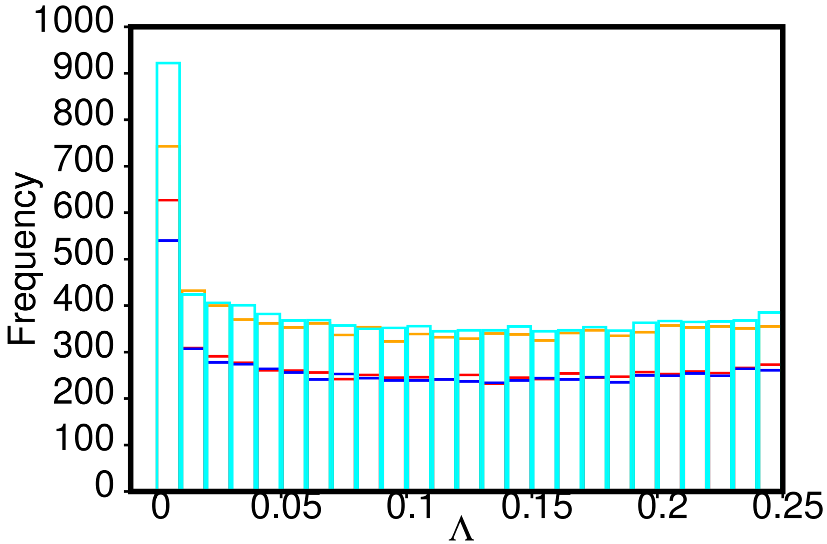

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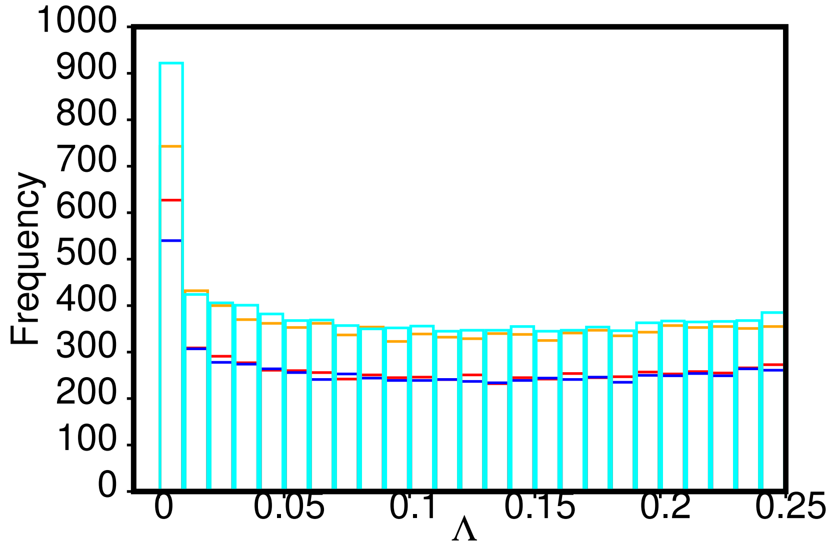

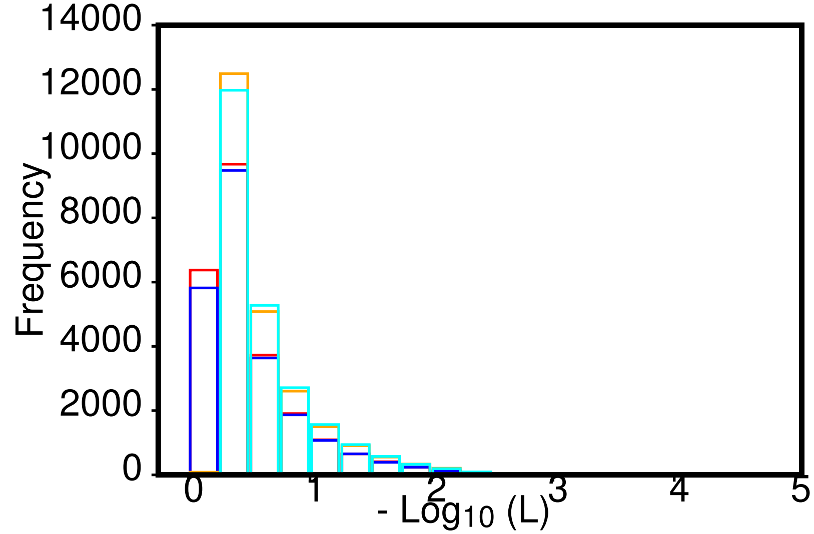

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What about Zero Modes?

♣ Nonzero modes are doubly degenerate for Overlap fermions as a result of the chiral symmetry.

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• For $T \neq 0$, Gavai-Gupta-Lacaze (PRD '02) found

T/T_c	N_{zero}
1.25	18
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2.0	1

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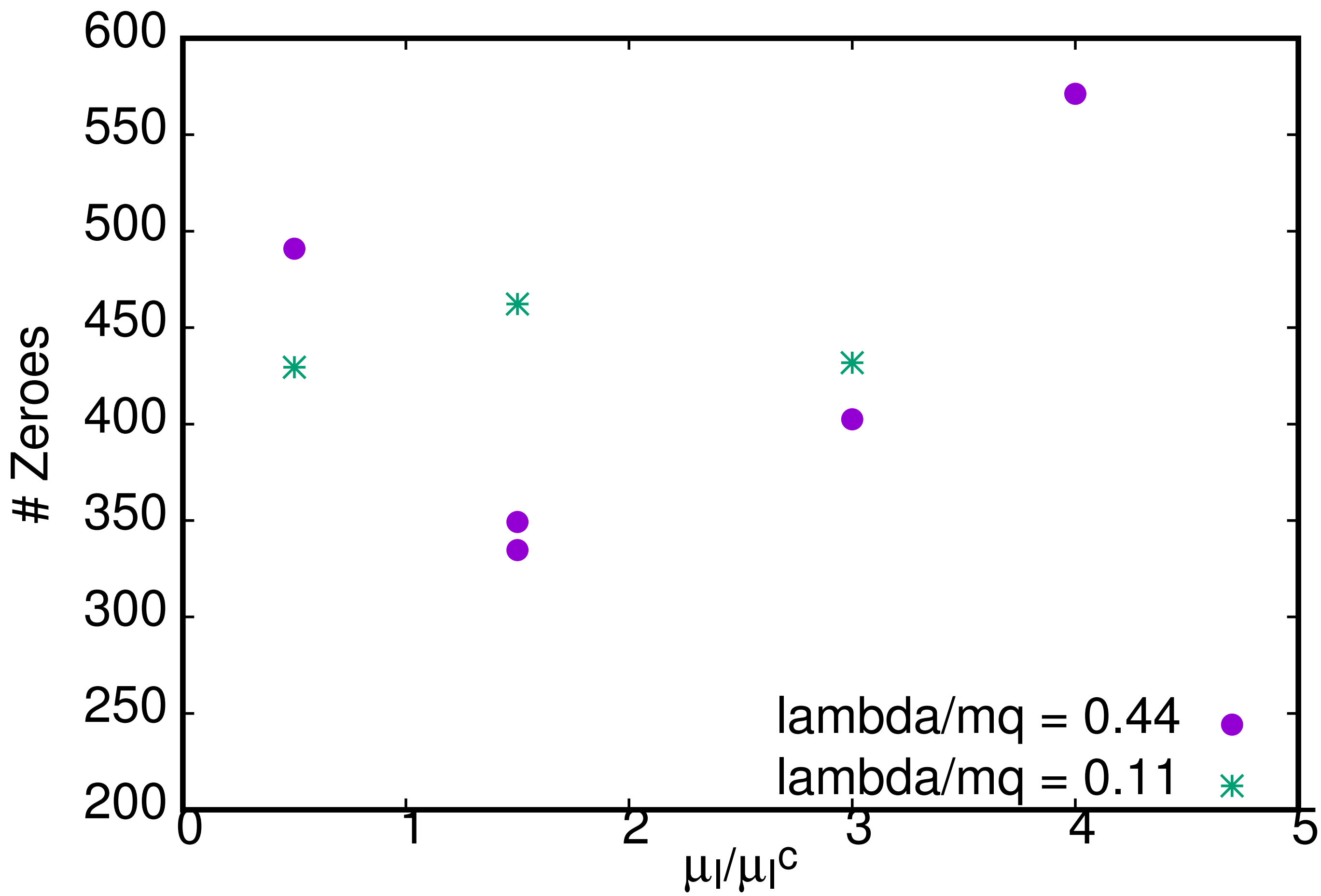
• For $\mu_I \neq 0$, we find for *same* number of configs (50) :

μ_I/μ_I^c	$N_{zero}^{0.11}$	$N_{zero}^{0.44}$
0.5	426	477
1.5	451	332
3.0	437	396
4.0	–	562

• No variation across μ_I^c for $\lambda/m_{ud} = 0.11$ & a mild dip for $\lambda/m_{ud} = 0.44$ (25% reduction at μ_I/μ_I^c=1.5)


600
550
500
450
400
350
300
250
200
0
1
2
3
4
5
Zeroes
μI/μIc
lambda/mq = 0.44
lambda/mq = 0.11

Summary

- We investigated the eigenvalue distributions for chirally exact Overlap Dirac operator for $\mu_I/\mu_I^c = 0.5$, 1.5, 3.0 & 4.0, *i. e.*, below and above the isospin phase transition. Very little variation as a function of μ_I at both λ.

- The distribution of zero and near-zero modes is nearly the same for both $\lambda/m_{ud} = 0.11$ and 0.44, with a possible dip of a 25 % reduction in the former near the phase transition and indications of rise with μ_I.

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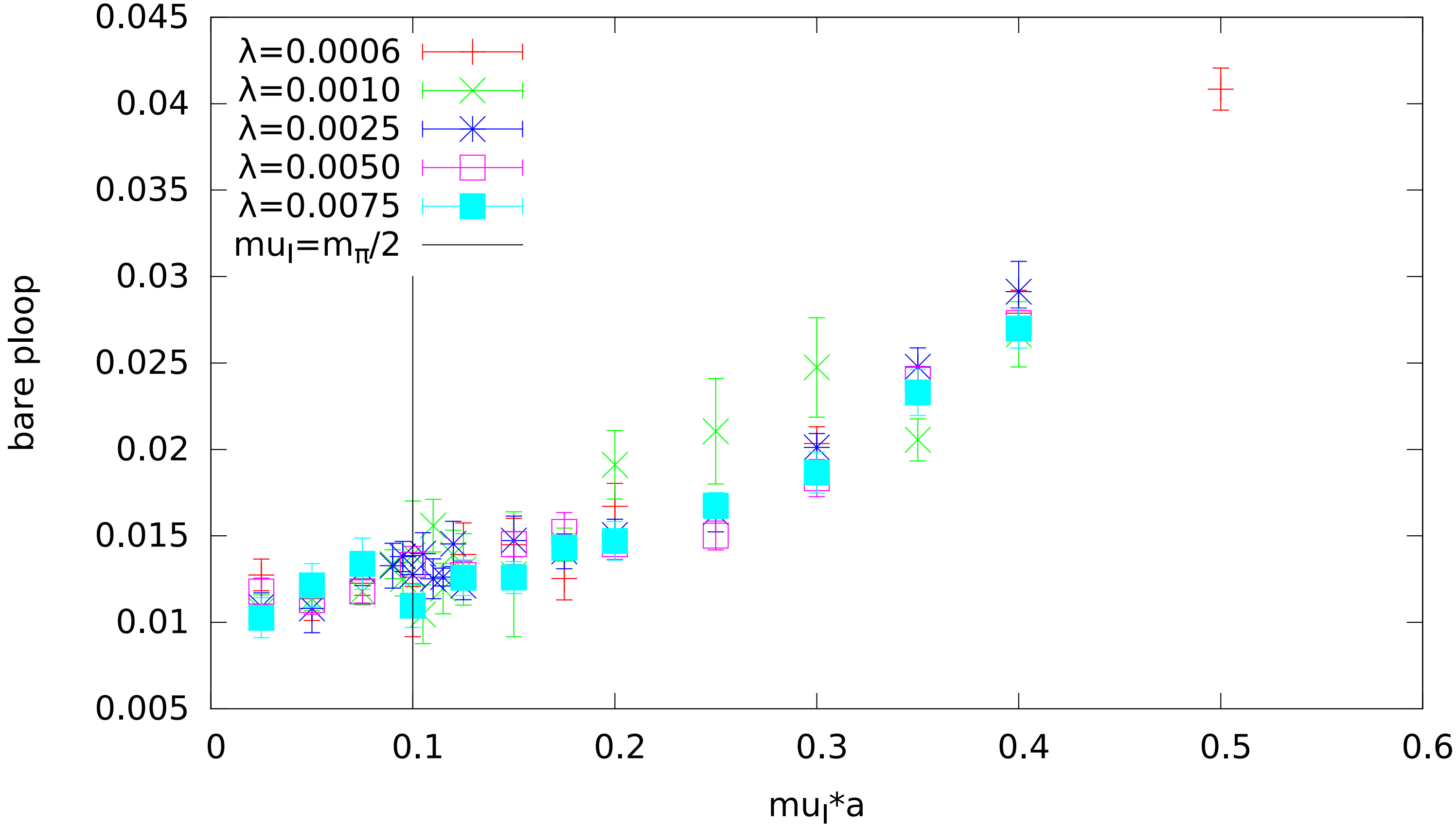
- This should be contrasted with the earlier $T \neq 0$ results, where too zero modes were present above the transition but decreased sharply as one moved away from the transition.

- Further investigations are going on to pin down the changes in the near-zero modes more quantitatively in an effort to understand the difference in T and μ_I directions.


0.045
0.04
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bare ploop
mu$_I$*a
λ=0.0006
λ=0.0010
λ=0.0025
λ=0.0050
λ=0.0075
mu$_I$=m$_\pi$/2