Leading isospin-breaking corrections to meson masses on the lattice

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OUTLINE

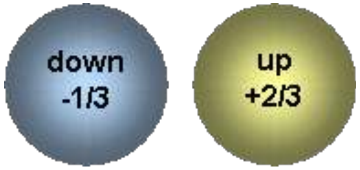
- Isospin breaking effects on the lattice (RM123 method)
- Results: • $M_{\pi+} - M_{\pi0}$ • $M_{K+} - M_{K0}$ • $(m_d - m_u)$ • ε_γ, $\varepsilon_{\pi0}$, ε_{K0} • $M_{D+} - M_{D0}$ • δM_{D_s+}

In collaboration with:

V. Lubicz, G. Martinelli, F. Sanfilippo, S. Simula, N. Tantalo, C. Tarantino

ISOSPIN BREAKING EFFECTS

Isospin symmetry is an almost exact property of the strong interactions



Isospin breaking effects are induced by:

$m_u \neq m_d$: $O[(m_d-m_u)/\Lambda_{QCD}] \approx 1/100$ "Strong"

$Q_u \neq Q_d$: $O(\alpha_{e.m.}) \approx 1/100$ "Electromagnetic"

Since electromagnetic interactions renormalize quark masses the two corrections are intrinsically related

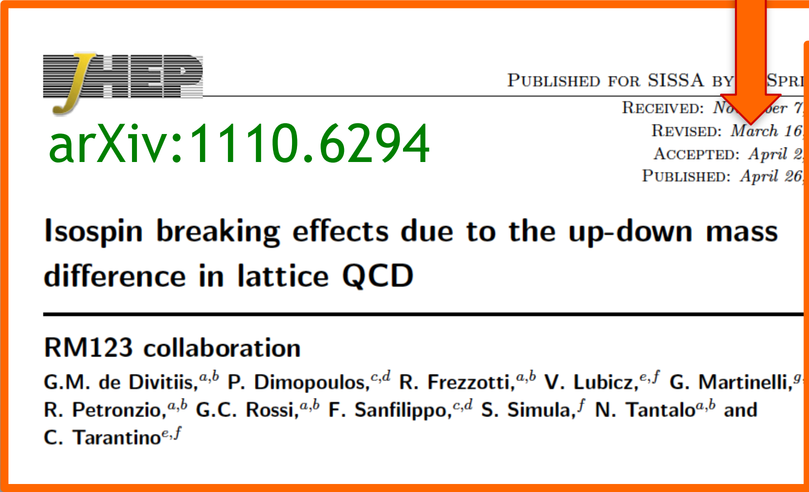
Though small, IB effects can play a very important role (quark masses, $M_n - M_p$, leptonic decay constants, vector form factor)

ISOSPIN BREAKING EFFECTS ON THE LATTICE

A strategy for Lattice QCD:

The isospin breaking part of the Lagrangian is treated as a perturbation

Expand in: $m_d - m_u$, α_{em}



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Isospin breaking effects due to the up-down mass difference in lattice QCD

RM123 collaboration

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Leading isospin breaking effects on the lattice

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(RM123 Collaboration) arXiv:1303.4896

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The (m_d-m_u) expansion

- Identify the isospin breaking term in the QCD action

$$S_m = \sum_x \left[m_u \bar{u}u + m_d \bar{d}d \right] = \sum_x \left[\frac{1}{2}\left(m_u + m_d\right)\left(\bar{u}u + \bar{d}d\right) - \frac{1}{2}\left(m_d - m_u\right)\left(\bar{u}u - \bar{d}d\right) \right] =$$

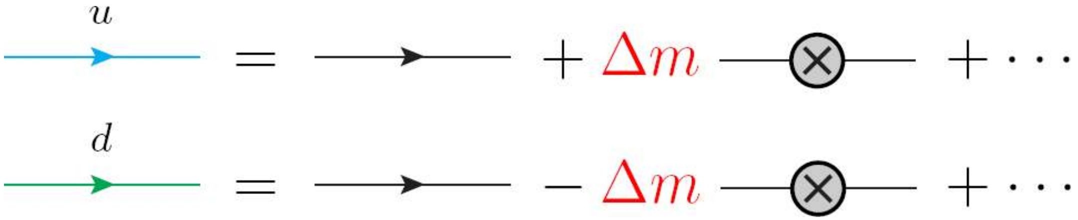
$$= \sum_x \left[m_{ud}\left(\bar{u}u + \bar{d}d\right) - \Delta m\left(\bar{u}u - \bar{d}d\right) \right] = S_0 - \Delta m \hat{S}$$

$$\boxed{\hat{S} = \sum_x \left(\bar{u}u - \bar{d}d\right)}$$

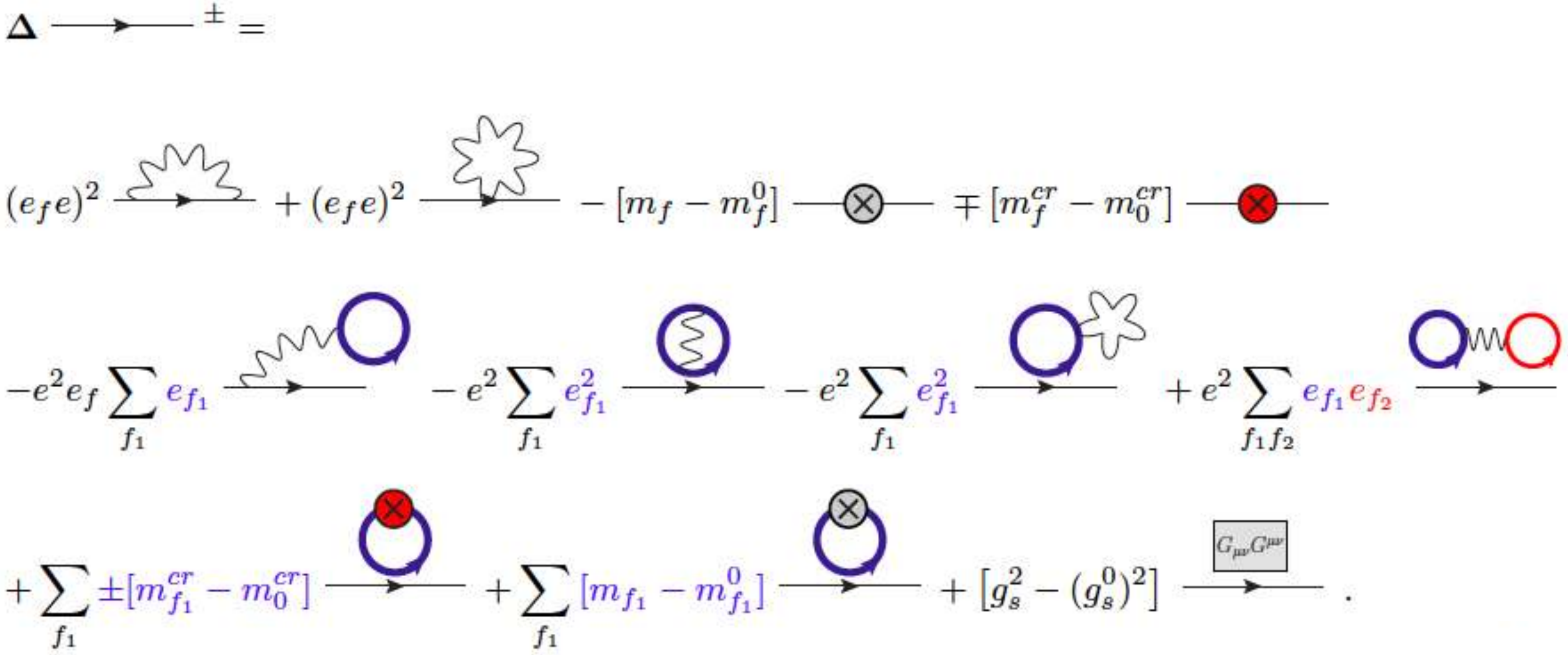
- Expand the functional integral in powers of Δm

$$\langle O \rangle = \frac{\int D\phi\, O\, e^{-S_0 + \Delta m \hat{S}}}{\int D\phi\, e^{-S_0 + \Delta m \hat{S}}} \overset{1st}{\simeq} \frac{\int D\phi\, O\, e^{-S_0}\left(1 + \Delta m \hat{S}\right)}{\int D\phi\, e^{-S_0}\left(1 + \Delta m \hat{S}\right)} \simeq \frac{\langle O \rangle_0 + \Delta m \langle O \hat{S} \rangle_0}{1 + \cancel{\Delta m \langle \hat{S} \rangle_0}}$$

Corrections to quark propagators at leading order in Δm:



The expansion for the quark propagator



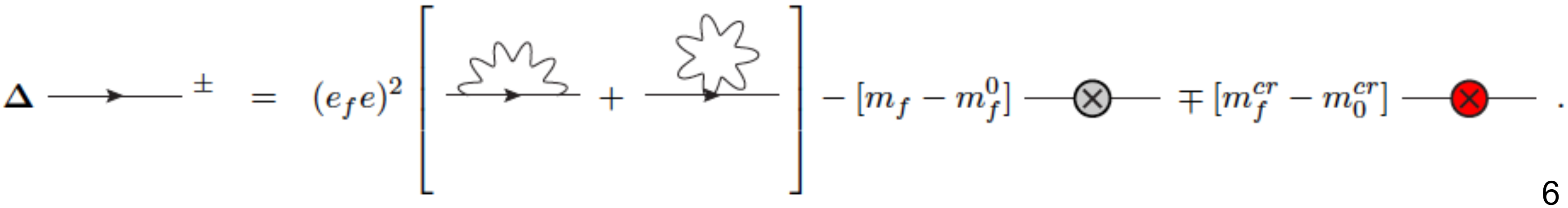
$$\Delta \longrightarrow^{\pm} =$$

$$(e_f e)^2 \quad + (e_f e)^2 \quad - [m_f - m_f^0] \quad \mp [m_f^{cr} - m_0^{cr}]$$

$$-e^2 e_f \sum_{f_1} e_{f_1} \quad - e^2 \sum_{f_1} e_{f_1}^2 \quad - e^2 \sum_{f_1} e_{f_1}^2 \quad + e^2 \sum_{f_1 f_2} e_{f_1} e_{f_2}$$

$$+ \sum_{f_1} \pm [m_{f_1}^{cr} - m_0^{cr}] \quad + \sum_{f_1} [m_{f_1} - m_{f_1}^0] \quad + [g_s^2 - (g_s^0)^2] \quad .$$

In the electro-quenched approximation:



$$\Delta \longrightarrow^{\pm} = (e_f e)^2 \left[\quad + \quad \right] - [m_f - m_f^0] \quad \mp [m_f^{cr} - m_0^{cr}] \quad .$$

Details of the lattice simulation

- We have used the gauge field configurations generated by ETMC, European Twisted Mass Collaboration, in the pure isosymmetric QCD theory with Nf=2+1+1 dynamical quarks

- Gluon action: Iwasaki

- Quark action: twisted mass at maximal twist
  (automatically O(a) improved)
  OS for s and c valence quarks

- 16 combinations of lattice spacings and pion masses with
  - 3 lattice spacings: a = 0.0885(36), 0.0815(30), 0.0619(18) fm
  - $M_\pi \geq 224$ MeV

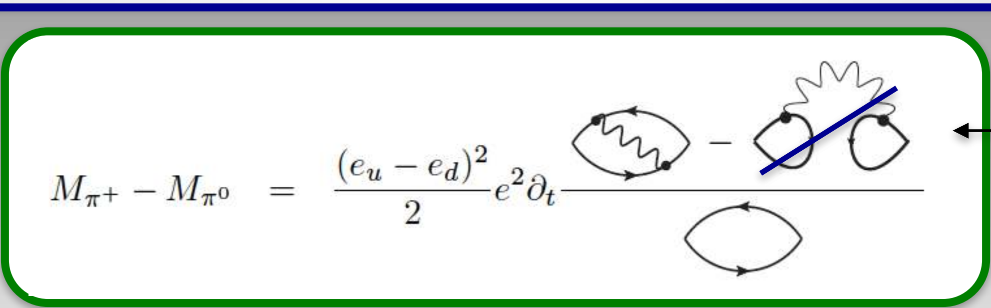
- Statistics of 80 ÷ 150 configurations per ensemble

PRD95(2017)114504

RESULTS

The charged-neutral pion mass splitting

The charged and neutral pion masses



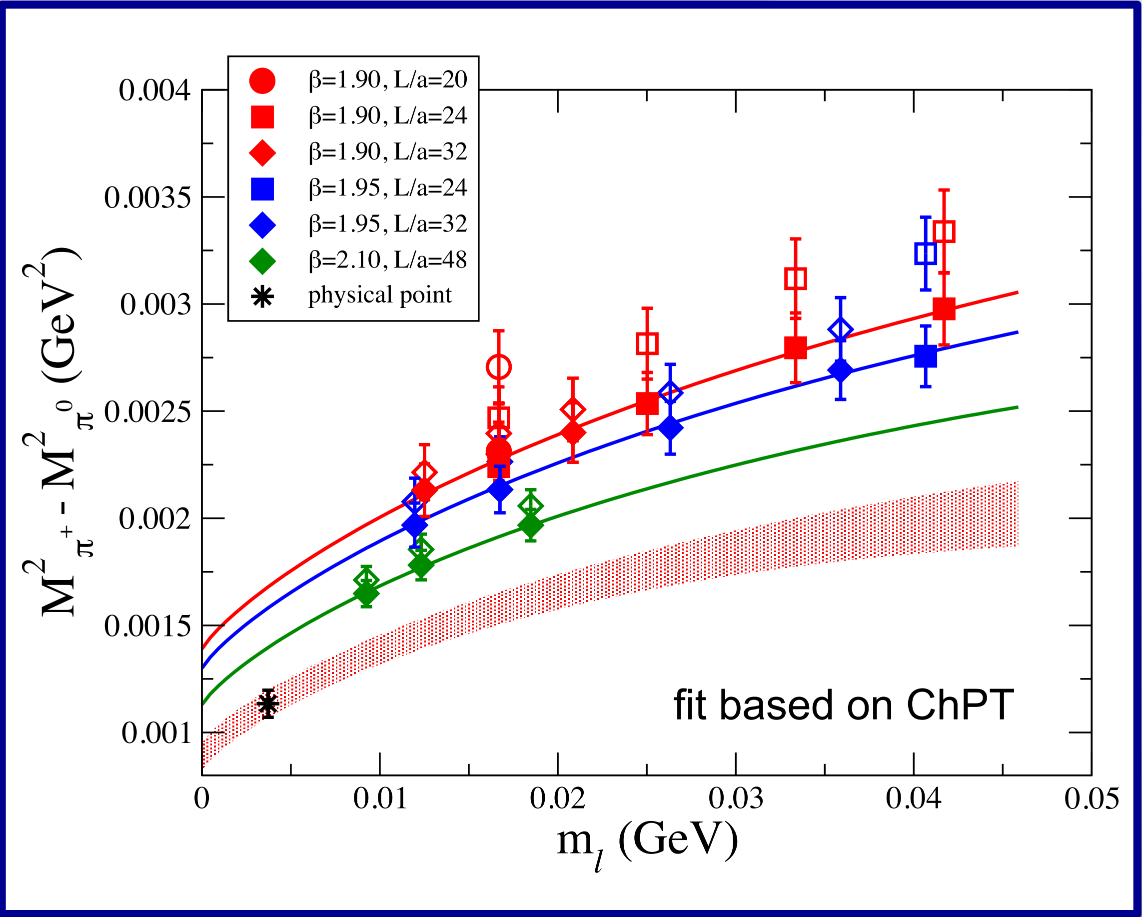
$O(\alpha_{em} m_{ud})$
statistically noisy

FSE

Empty markers: LO and NLO universal corrections subtracted (QED$_L$ adopted)

M. Hayakawa and S. Uno
arXiv:0804.2044 [hep-ph]

Full markers: NNLO SD corrections, of $O(1/L^3)$, subtracted



fit based on ChPT

The charged-neutral pion mass splitting

We obtain:

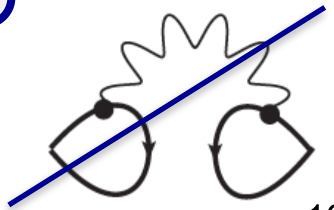
$$\boxed{\begin{aligned} M_{\pi^+} - M_{\pi^0} &= 4.21(23)_{stat}(13)_{syst}\ \text{MeV} \\ &= 4.21(26)\ \text{MeV} \end{aligned}}$$

where the errors are statistical and systematic (estimated from chiral and continuum extrapolations and FSE).

The result is in good agreement with the experimental determination

$$\boxed{\left[M_{\pi^+} - M_{\pi^0}\right]^{\exp} = 4.5936(5)\ \text{MeV}}$$

It suggests, a posteriori, that the effect of having neglected the disconnected contribution of $O(\alpha_{em} m_{ud})$ is small

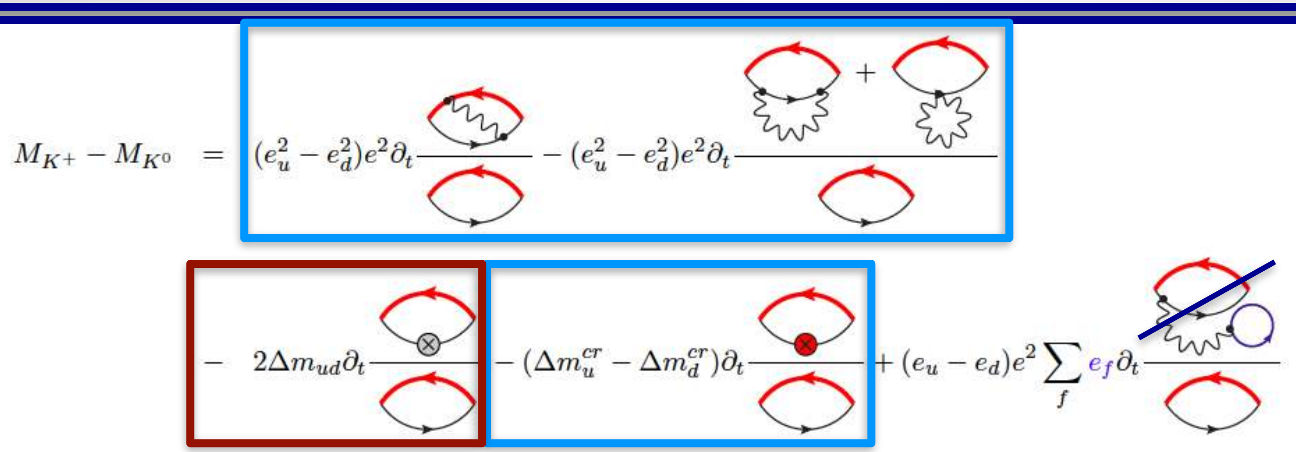


PRD95(2017)114504

RESULTS

The charged-neutral kaon mass splitting and $(m_d - m_u)$

The charged and neutral kaon masses

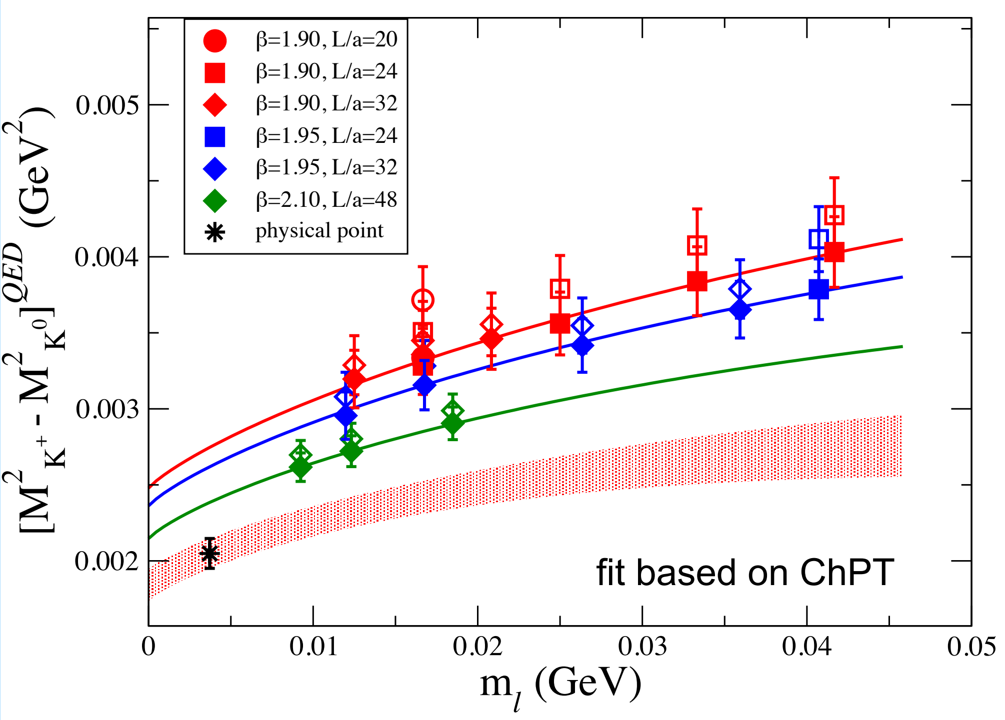


electro-quenched
approximation

Scheme: MSbar @ 2 GeV

QED →

∝QCD



Kaon mass splitting: results

$$\left[\mathrm{M}_{K^+} - \mathrm{M}_{K^0} \right]^{QED} = 2.07(15)\ \mathrm{MeV}$$

BMW Nf=2+1 (2016) $\varepsilon_\gamma = 0.74(18)$

$$\varepsilon_\gamma = 0.80(11)$$

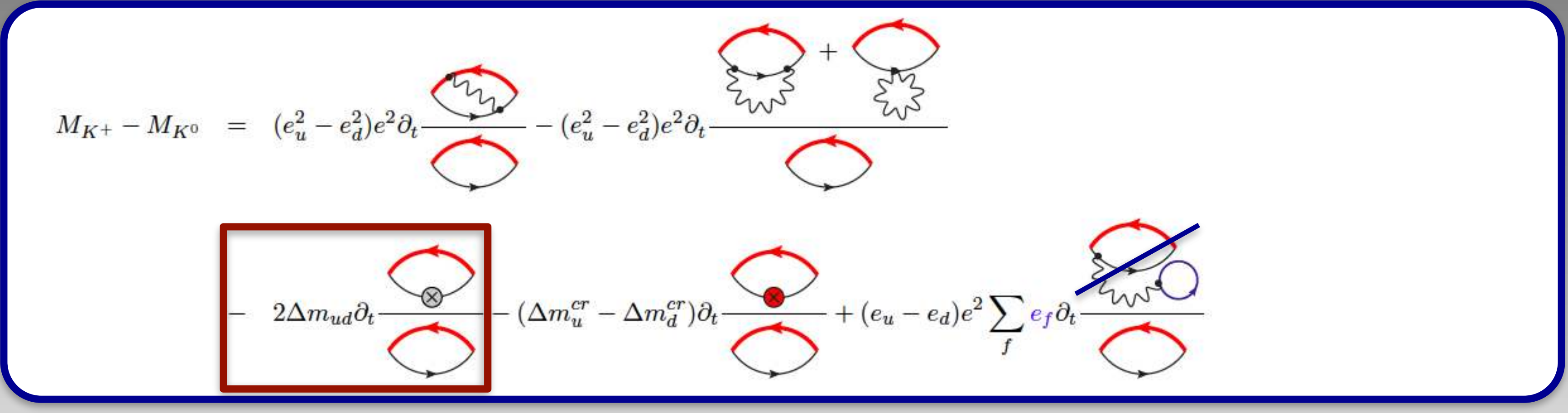
QCDSF/UKQCD Nf=3 (2015) $\varepsilon_\gamma = 0.50(6)$

FLAG (2016) $\varepsilon_\gamma = 0.7(3)$

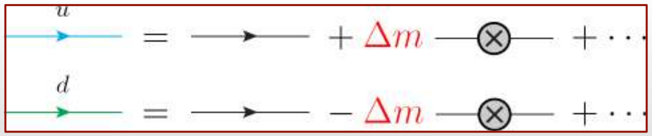
$$\epsilon_\gamma(\overline{\mathrm{MS}}, \mu) = \frac{\left[M_{K^+}^2 - M_{K^0}^2 \right]^{QED} (\overline{\mathrm{MS}}, \mu)}{M_{\pi^+}^2 - M_{\pi^0}^2} - 1$$

$$\left[\mathrm{M}_{K^+} - \mathrm{M}_{K^0} \right]^{\mathrm{exp}} = -3.934(20)\ \mathrm{MeV}$$

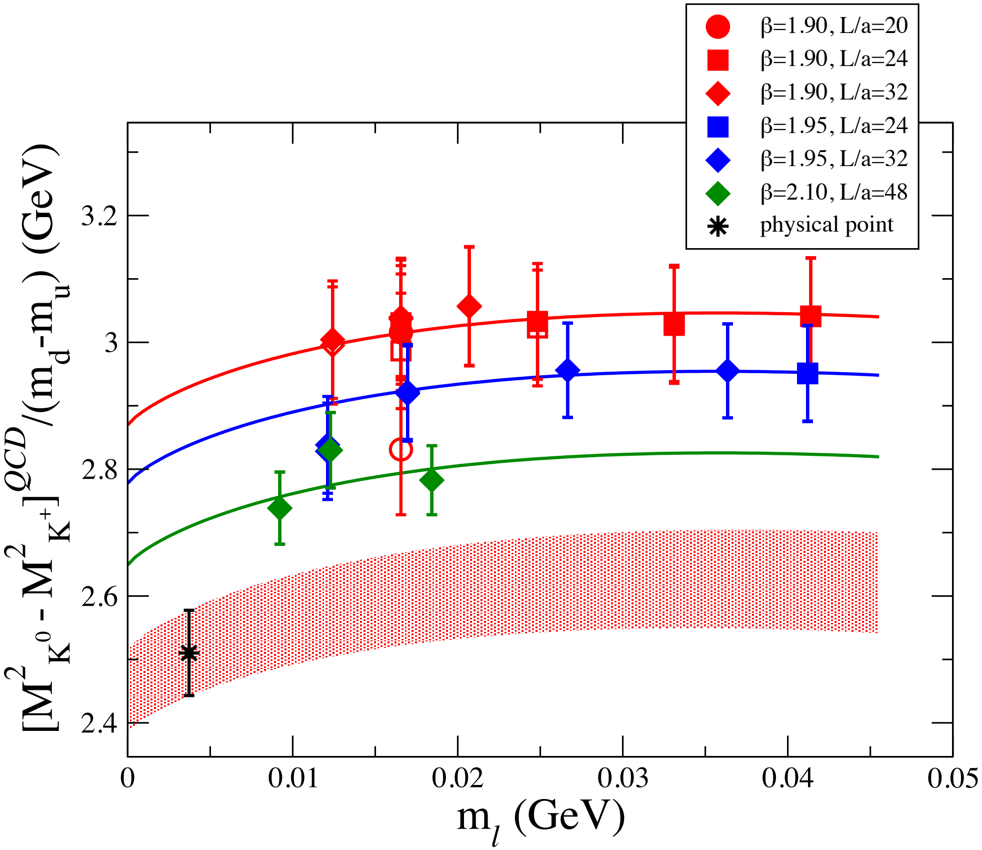
$$\left[\mathrm{M}_{K^+} - \mathrm{M}_{K^0} \right]^{QCD} = -6.00(15)\ \mathrm{MeV}$$



∝QCD



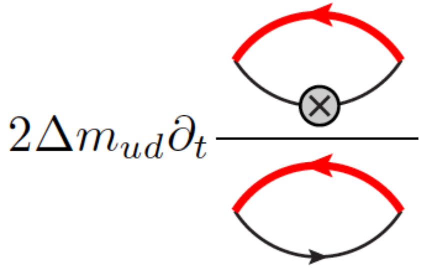
$$\hat{S} = \sum_x \left(\bar{u}u - \bar{d}d\right)$$



The up and down quark masses and related…

From:

$$\boxed{\frac{\left[M^2_{K^0} - M^2_{K^+} \right]^{QCD}}{m_d - m_u} = 2.51(18)\ \text{GeV}}$$



(all masses in the MSbar scheme at 2 GeV), one obtains:

$$(m_d - m_u) = 2.38(9)_{stat}(16)_{syst}\ \text{MeV}$$
$$= 2.38(18)\ \text{MeV}$$

$$m_u / m_d = 0.513(18)_{stat}(24)_{syst}$$
$$= 0.513(30)$$

$$\text{m}_{ud} = 3.70(17)\ \text{MeV}$$

$$m_u = 2.50(17)\ \text{MeV}$$
$$m_d = 4.88(20)\ \text{MeV}$$

$$\text{Nf} = 2+1 \begin{cases} \text{BMW (2016) } \text{m}_u = 2.27(9)\ \text{MeV} \\ \text{BMW (2016) } \text{m}_d = 4.67(9)\ \text{MeV} \end{cases}$$

Violation of Dashen's theorem for neutral mesons

Scheme: MSbar @ 2 GeV

$$\epsilon_{\pi^0} = \frac{\left[\delta M^2_{\pi^0}\right]^{QED}}{M^2_{\pi^+} - M^2_{\pi^0}}$$

$$\boxed{\varepsilon_{\pi^0} = 0.03(4)}$$

QCDSF/UKQCD Nf=3 (2015) $\varepsilon_{\pi^0} = 0.03(2)$

FLAG (2016) $\varepsilon_{\pi^0} = 0.07(7)$

$$\epsilon_{K^0} = \frac{\left[\delta M^2_{K^0}\right]^{QED}}{M^2_{\pi^+} - M^2_{\pi^0}}$$

$$\boxed{\varepsilon_{K^0} = 0.15(3)}$$

QCDSF/UKQCD Nf=3 (2015) $\varepsilon_{K^0} = 0.2(1)$

FLAG (2016) $\varepsilon_{K^0} = 0.3(3)$

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RESULTS

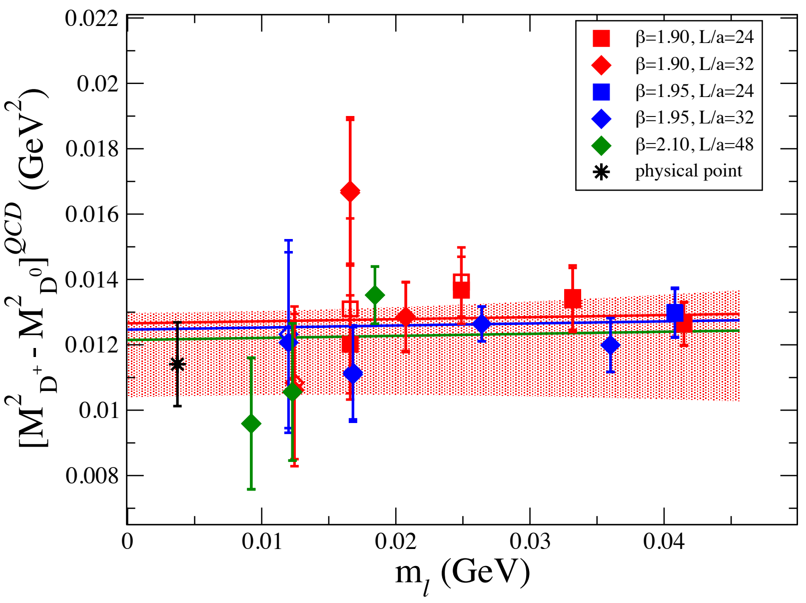
1st lattice results

The charged-neutral D meson mass splitting

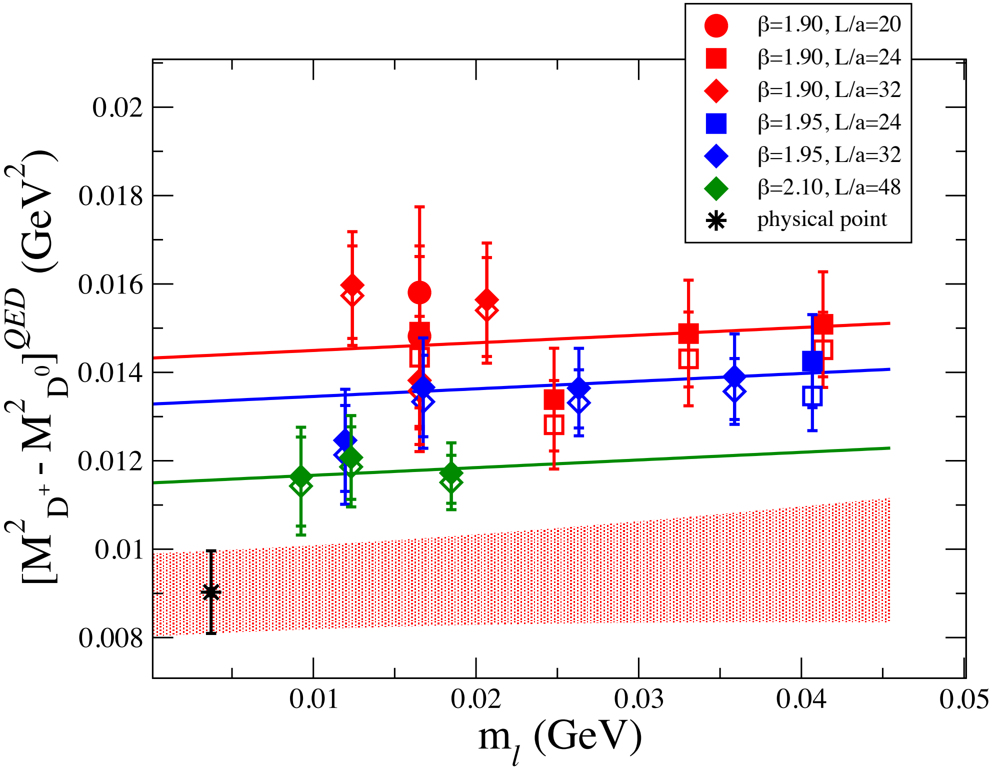
+

$\delta M_{D_s^+}$

D meson



$$\left[M_{D^+} - M_{D^0}\right]^{QCD} = 3.06(27)_{stat}(7)_{syst} \text{ MeV}$$

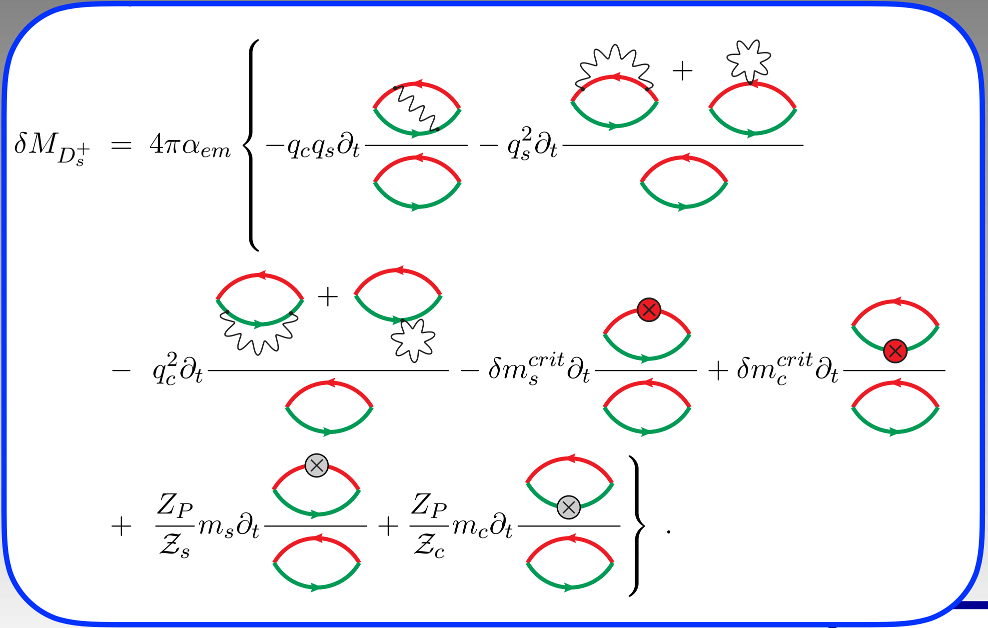


$$\left[M_{D^+} - M_{D^0}\right]^{QED} = 2.42(22)_{stat}(46)_{syst} \text{ MeV}$$

$$M_{D^+} - M_{D^0} = 5.47(53) \text{ MeV}$$

$$\left[M_{D^+} - M_{D^0}\right]^{exp} = 4.75(8) \text{ MeV}$$

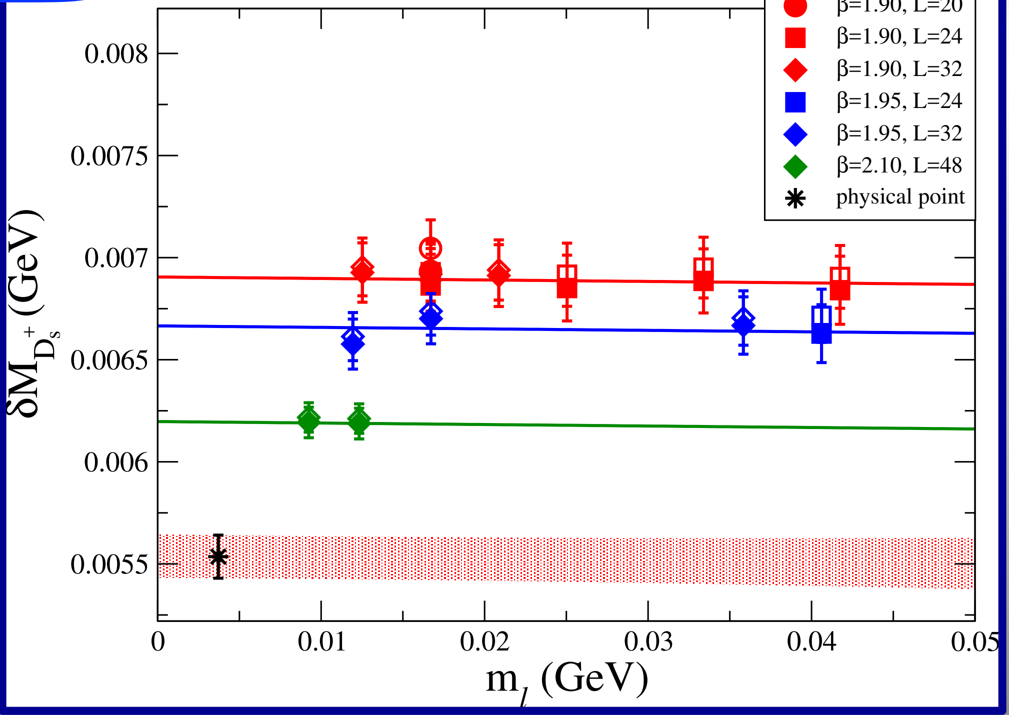
Ds meson



G. Martinelli and Y-C. Zhang PLB '83;
S. Aoki *et al.* PRD '98

$$\frac{1}{\mathcal{Z}_f}(\overline{\mathrm{MS}}, \mu) = \frac{q_f^2}{16\pi^2}\left[6\log(a\mu) - 22.596\right]$$

$$\delta M_{D_s^+} = 5.54(11)_{stat}(54)_{syst}\ \mathrm{MeV}$$



CONCLUSIONS

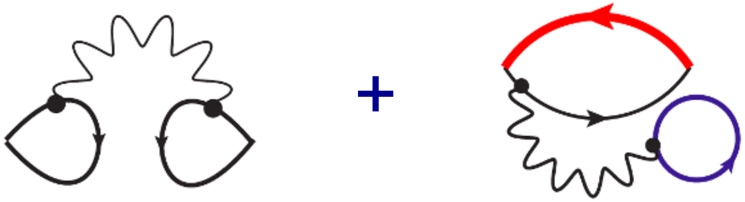
Several results

PRD95(2017)114504

arXiv:1704.06561 [hep-lat]

$$
\begin{aligned}
M_{\pi^+} - M_{\pi^0} &= 4.21\ (26)\ \mathrm{MeV} \quad [4.5936\ (5)\ \mathrm{MeV}]_{exp}\ , \\
[M_{K^+} - M_{K^0}]^{QED}\,(\overline{MS}, 2\ \mathrm{GeV}) &= 2.07\ (15)\ \mathrm{MeV}\ , \\
[M_{K^+} - M_{K^0}]^{QCD}\,(\overline{MS}, 2\ \mathrm{GeV}) &= -6.00\ (15)\ \mathrm{MeV}\ , \\
\\
(\widehat{m}_d - \widehat{m}_u)(\overline{MS}, 2\ \mathrm{GeV}) &= 2.38\ (18)\ \mathrm{MeV}\ , \\
\frac{\widehat{m}_u}{\widehat{m}_d}(\overline{MS}, 2\ \mathrm{GeV}) &= 0.513\ (30)\ , \\
\widehat{m}_u(\overline{MS}, 2\ \mathrm{GeV}) &= 2.50\ (17)\ \mathrm{MeV}\ , \\
\widehat{m}_d(\overline{MS}, 2\ \mathrm{GeV}) &= 4.88\ (20)\ \mathrm{MeV}\ , \\
\\
\epsilon_{\pi^0} &= 0.03\ (4)\ , \\
\epsilon_{\gamma}(\overline{MS}, 2\ \mathrm{GeV}) &= 0.80\ (11)\ , \\
\epsilon_{K^0}(\overline{MS}, 2\ \mathrm{GeV}) &= 0.15\ (3)\ , \\
\\
[M_{D^+} - M_{D^0}]^{QED}\,(\overline{MS}, 2\ \mathrm{GeV}) &= 2.42\ (51)\ \mathrm{MeV}\ , \\
[M_{D^+} - M_{D^0}]^{QCD}\,(\overline{MS}, 2\ \mathrm{GeV}) &= 3.06\ (27)\ \mathrm{MeV}\ , \\
M_{D^+} - M_{D^0} &= 5.47\ (53)\ \mathrm{MeV} \quad [4.75\ (8)\ \mathrm{MeV}]_{exp}\ , \\
\delta M_{D^+} + \delta M_{D^0} &= 8.2\ (9)\ \mathrm{MeV}\ , \\
\delta M_{D_s^+} &= 5.5\ (6)\ \mathrm{MeV}\ ,
\end{aligned}
$$

Future perspectives



ADDITIONAL

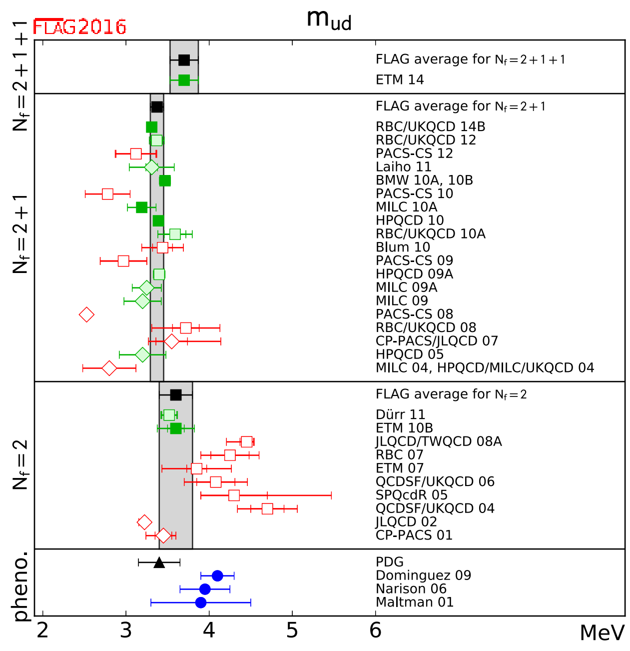
SLIDES

Theoretical motivations

Quark masses are fundamental parameters of the Standard Model

Lattice QCD in the isosymmetric limit ($m_u=m_d$, $Q_u=Q_d$) provide a determination of the average up-down quark mass:

$$m_{ud}=(m_u+m_d)/2$$



$m_{ud} = 3.70(17)$ MeV (Nf=2+1+1)

But the knowledge of m_u and m_d separately is important

- Accurate knowledge of quark masses is important for our understanding of flavor physics at the fundamental level

$m_u \simeq 2.5$ MeV $m_d \simeq 5$ MeV

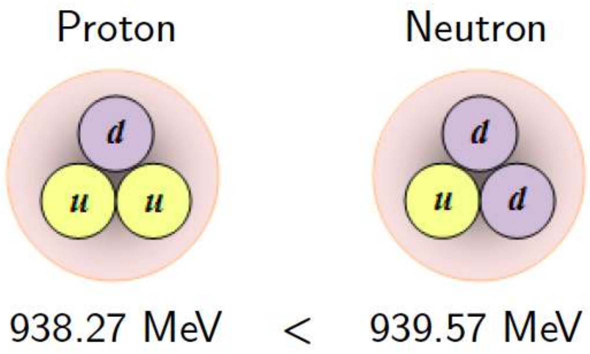
$m_c \simeq 1.2$ GeV $m_s \simeq 100$ MeV

$m_t \simeq 175$ GeV $m_b \simeq 4.3$ GeV

A remarkable relation
[Gatto, Sartori, Tonin]

$$\left(\frac{m_d}{m_s}\right)^{1/2} \simeq \left(\frac{m_u}{m_c}\right)^{1/4} \simeq V_{us} \simeq 0.22$$

- The actual values of the mass difference $m_d - m_u$ and quark charges Q_d, Q_u implies $M_n > M_p$ and guarantees



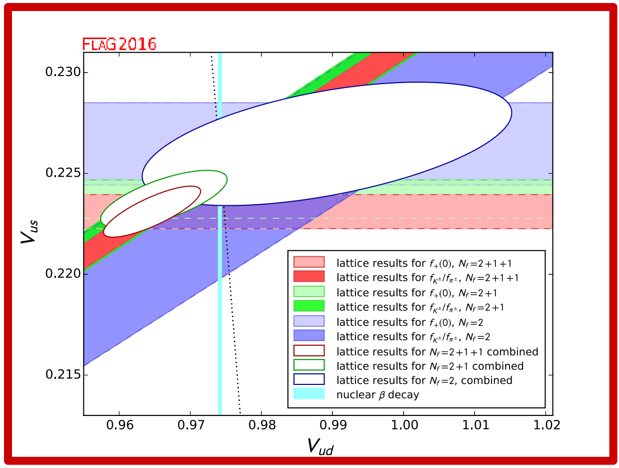
the stability of matter

M(n) – M(p) = 1.3 MeV = 0.14%

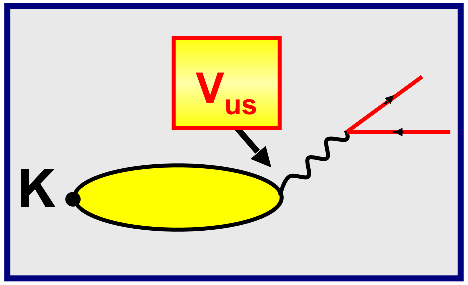
IB effects cannot be neglected at present in flavor physics phenomenology

Example: determination of Vus and the CKM first row unitarity test

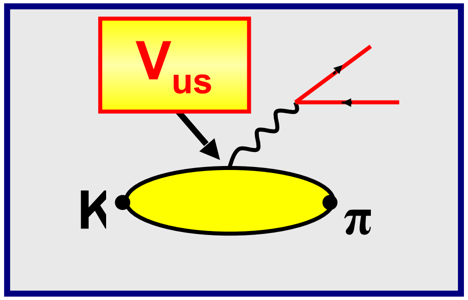
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$



$$\frac{\Gamma(K\to\mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi\to\mu\bar{\nu}_\mu(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2}\left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K\left(1-\frac{m_\mu^2}{m_K^2}\right)}{m_\pi\left(1-\frac{m_\mu^2}{m_\pi^2}\right)} \times 0.9930(35)$$

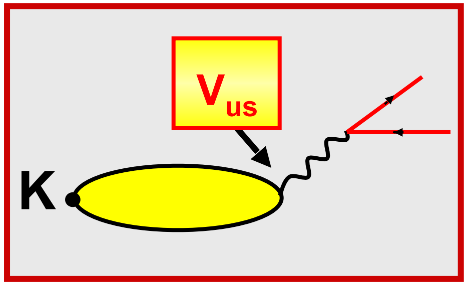


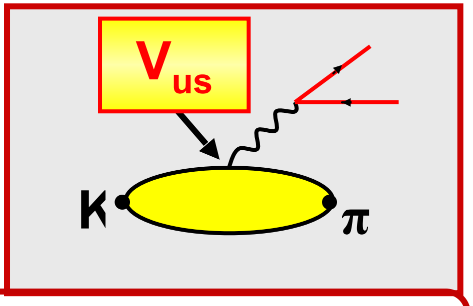
$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} S_{EW}[1+\Delta_{SU(2)}+2\Delta_{EM}] \times |V_{us}|^2 |f_+^{K\pi}(0)|^2$$



EXPERIMENTS

FlaviA net Kaon WG





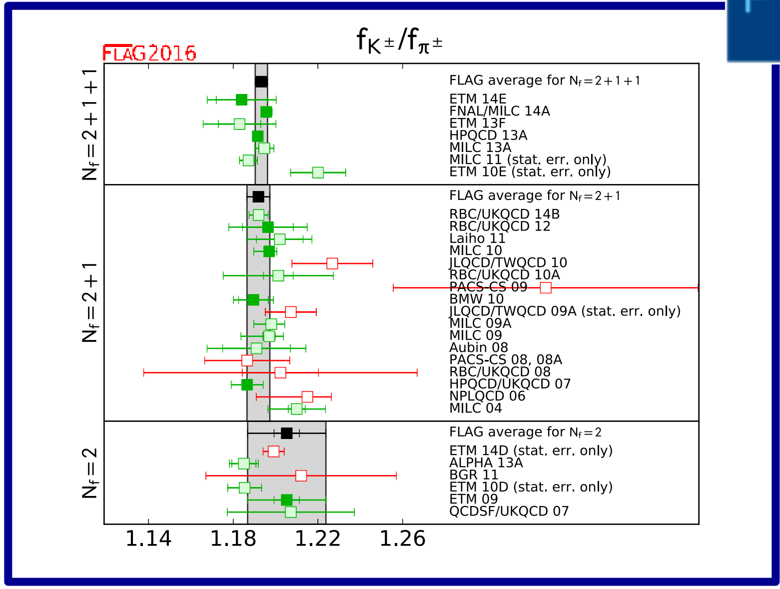
$$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.2758(5)$$

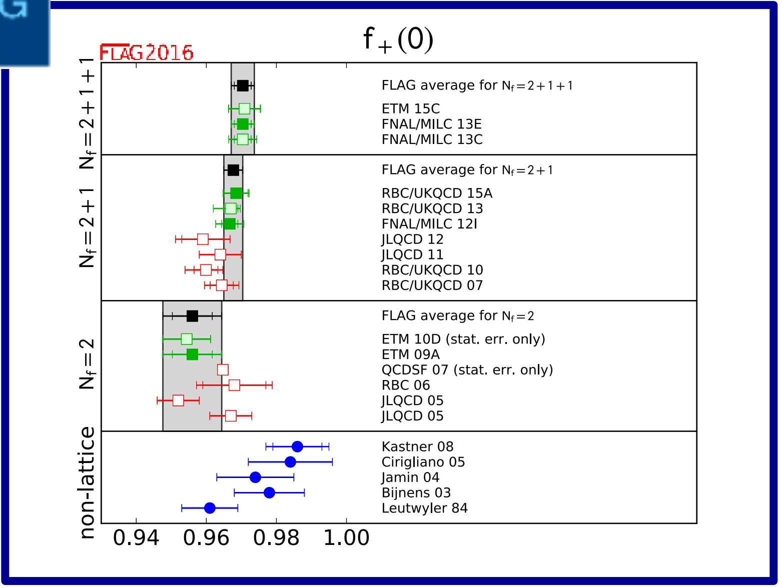
0.2%

$$|V_{us}| f_+(0) = 0.2163(5)$$

THEORY: LATTICE QCD

FLAG





$f_K/f_\pi = 1.193(3)$

0.3%

$f_+(0) = 0.9704(33)$

The leading isospin breaking expansion

- The QCD + QED action is written in terms of the full covariant derivative:

$$D^{+}_{\mu} q_f(x) = \left[E_\mu(x)\right]^{e_f} U_\mu(x)\, q_f(x+\hat{\mu}) - q_f(x)$$

QED

QCD

- Since $E_\mu(x) = e^{-ieA_\mu(x)} = 1 - ieA_\mu(x) - 1/2\; e^2 A^2_\mu(x) + \ldots$ the expansion of the lattice action up to $O(e^2)$ contains 2 contributions:

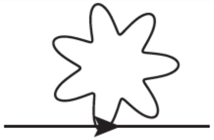
$$S_f = \sum_x \bar{q}_f(x) D_f[U,A]\, q_f(x) =$$

$$= S_f(e=0) + \sum_{x,\mu}\left[e_f e A_\mu(x) V^f_\mu(x) + \frac{(e_f e)^2}{2} A_\mu(x) A_\mu(x) T^f_\mu(x) + \ldots \right]$$

Both contributions are required for gauge invariance

$(e_f e)^2$

$(e_f e)^2$



The leading isospin breaking expansion

- Switching on the e.m. interactions requires the introduction of new counterterms which renormalize the couplings of the theory:

$$\vec{g}^0 = \left(0, g_s^0, m_u^0, m_d^0, m_s^0, \dots\right) \to \vec{g} = \left(e^2, g_s, m_u, m_d, m_s, \dots\right)$$

- For any observable, the leading isospin breaking expansion reads,

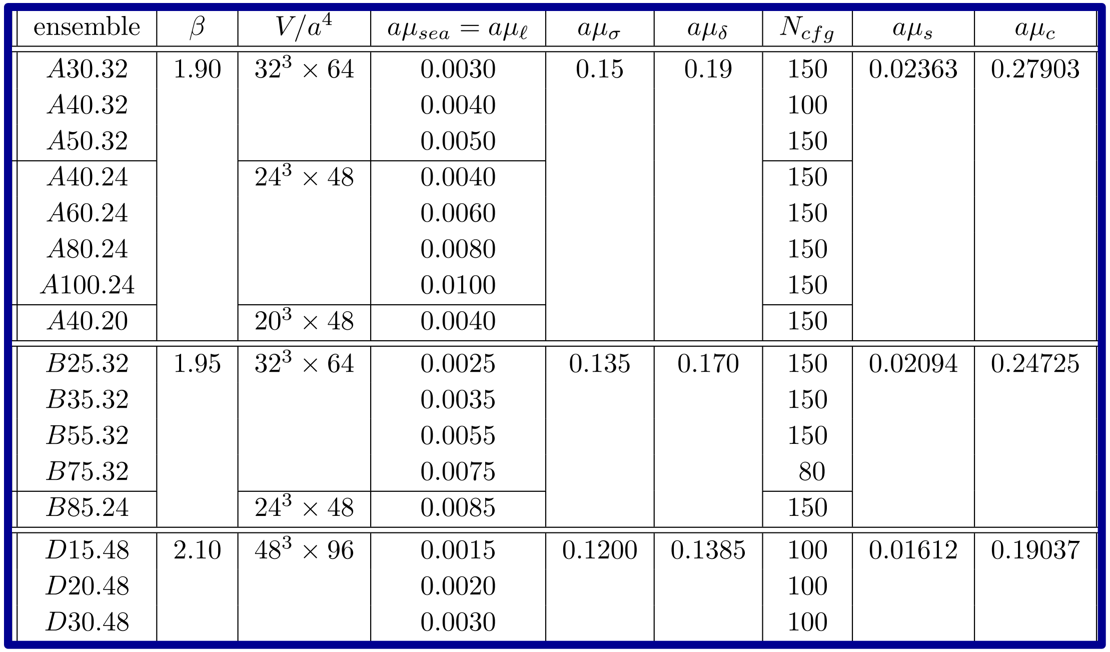
$$O(\vec{g}) = O(\vec{g}^0) + \left[e^2 \frac{\partial}{\partial e^2} + \left(g_s^2 - (g_s^0)^2\right) \frac{\partial}{\partial g_s^2} + \left(m_f - m_f^0\right) \frac{\partial}{\partial m_f} + \dots \right] O(\vec{g})\Big|_{\vec{g} = \vec{g}^0}$$

or, in terms of renormalized couplings,

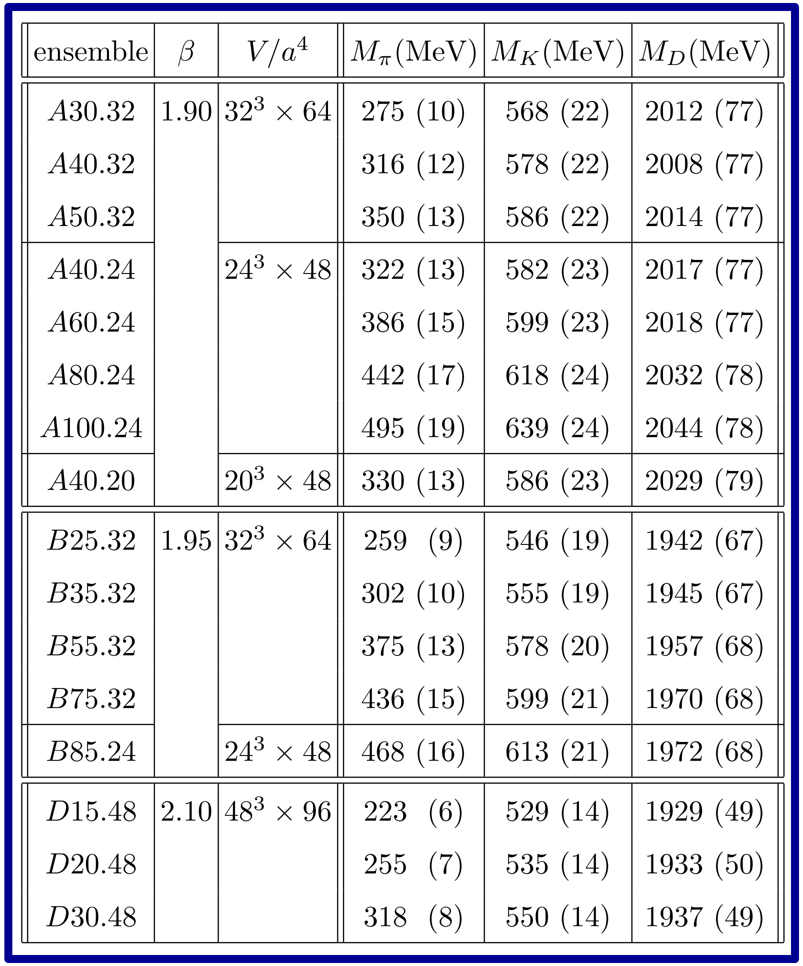
$$O(\hat{\vec{g}}) = O(\hat{\vec{g}}^0) + \left[\hat{e}^2 \frac{\partial}{\partial \hat{e}^2} + \left(\hat{g}_s^2 - \left(\frac{Z_{g_s}}{Z_{g_s}^0} g_s^0 \right)^2 \right) \frac{\partial}{\partial \hat{g}_s^2} + \left(m_f - \frac{Z_{m_f}}{Z_{m_f}^0} m_f^0 \right) \frac{\partial}{\partial \hat{m}_f} + \dots \right] O(\vec{g})\Big|_{\vec{g} = \vec{g}^0}$$

European Twisted Mass Collaboration

ETMC gauge ensembles
Nf=2+1+1



ensemble	β	V/a^4	$a\mu_{sea} = a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	N_{cfg}	$a\mu_s$	$a\mu_c$
$A30.32$	1.90	$32^3 \times 64$	0.0030	0.15	0.19	150	0.02363	0.27903
$A40.32$			0.0040			100		
$A50.32$			0.0050			150		
$A40.24$		$24^3 \times 48$	0.0040			150		
$A60.24$			0.0060			150		
$A80.24$			0.0080			150		
$A100.24$			0.0100			150		
$A40.20$		$20^3 \times 48$	0.0040			150		
$B25.32$	1.95	$32^3 \times 64$	0.0025	0.135	0.170	150	0.02094	0.24725
$B35.32$			0.0035			150		
$B55.32$			0.0055			150		
$B75.32$			0.0075			80		
$B85.24$		$24^3 \times 48$	0.0085			150		
$D15.48$	2.10	$48^3 \times 96$	0.0015	0.1200	0.1385	100	0.01612	0.19037
$D20.48$			0.0020			100		
$D30.48$			0.0030			100		



ensemble	β	V/a^4	M_π(MeV)	M_K(MeV)	M_D(MeV)
$A30.32$	1.90	$32^3 \times 64$	275 (10)	568 (22)	2012 (77)
$A40.32$			316 (12)	578 (22)	2008 (77)
$A50.32$			350 (13)	586 (22)	2014 (77)
$A40.24$		$24^3 \times 48$	322 (13)	582 (23)	2017 (77)
$A60.24$			386 (15)	599 (23)	2018 (77)
$A80.24$			442 (17)	618 (24)	2032 (78)
$A100.24$			495 (19)	639 (24)	2044 (78)
$A40.20$		$20^3 \times 48$	330 (13)	586 (23)	2029 (79)
$B25.32$	1.95	$32^3 \times 64$	259 (9)	546 (19)	1942 (67)
$B35.32$			302 (10)	555 (19)	1945 (67)
$B55.32$			375 (13)	578 (20)	1957 (68)
$B75.32$			436 (15)	599 (21)	1970 (68)
$B85.24$		$24^3 \times 48$	468 (16)	613 (21)	1972 (68)
$D15.48$	2.10	$48^3 \times 96$	223 (6)	529 (14)	1929 (49)
$D20.48$			255 (7)	535 (14)	1933 (50)
$D30.48$			318 (8)	550 (14)	1937 (49)

Finite size effects (FSE)

In pure QCD, the existence of a mass gap renders FSE exponentially small ~ $e^{-M_\pi \cdot L}$ (in most of the cases)

The QED photon is massless, the e.m. interactions are long ranged and FSE are only power suppressed.

With our regularization of the zero mode, FSE are expressed by:

$$M_{PS}^2(T,L) \underset{T,L\to+\infty}{\sim} M_{PS}^2 \left\{ 1 - q^2 \alpha_{em} \left[\frac{\kappa}{M_{PS}L} \left(1 + \frac{2}{M_{PS}L} \right) \right] \right\}$$

$$\boxed{\kappa = 2.837297(1)}$$

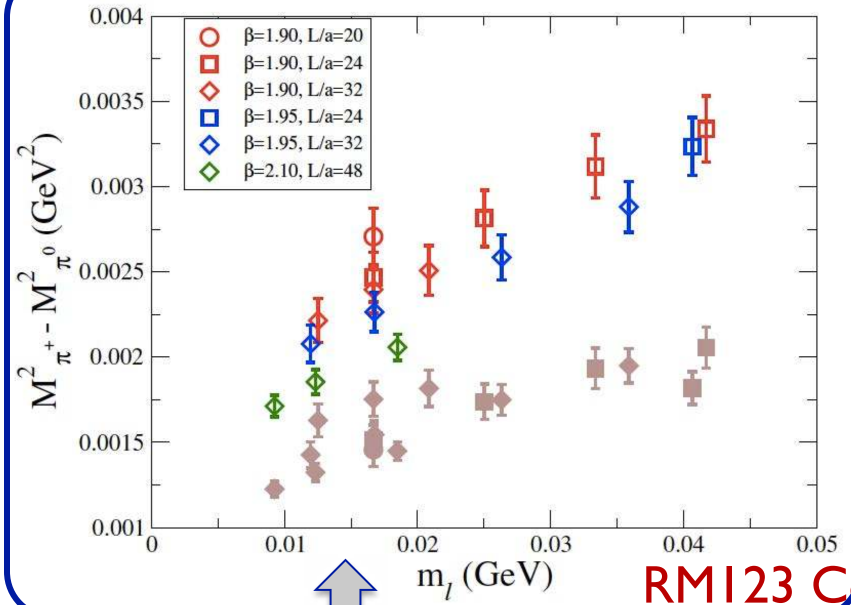
$$\boxed{\mathrm{QED}_L \;\; (\text{i.e. } \; A_\mu(k_0, \vec{k} = \vec{0}) \equiv 0 \text{ for all } k_0)}$$

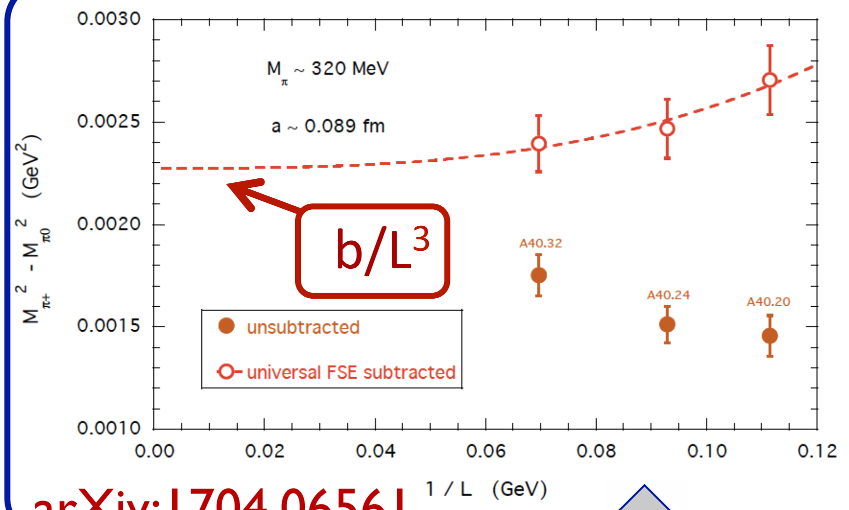
M. Hayakawa and S. Uno
arXiv:0804.2044 [hep-ph]

Sz. Borsanyi *et al.*
arXiv:1406.4088 [hep-ph]

The charged-neutral pion mass splitting

Since QED is long-ranged finite volume corrections are large and only power suppressed





RM123 Coll., arXiv:1704.06561

$$M_P^2(L) - M_P^2(\infty) = -\alpha_{em} Q^2 k \frac{2M_P}{L}\left(1 + \frac{1}{2M_P L}\right)$$

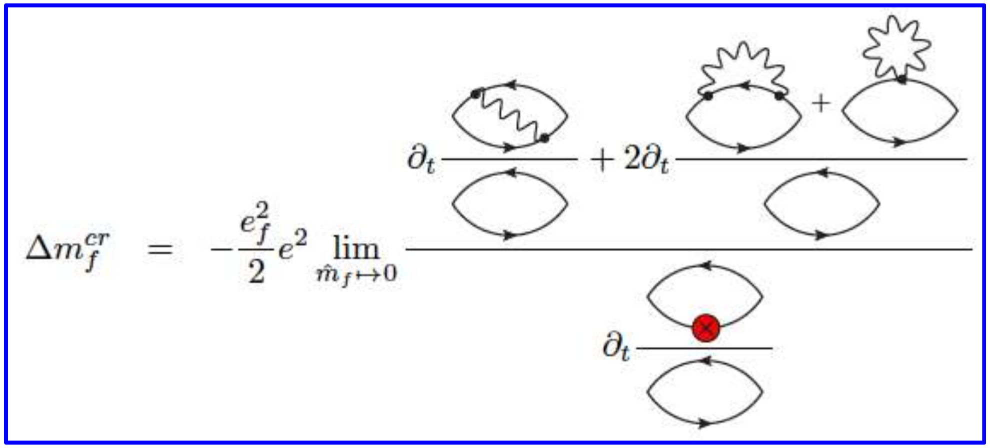
NNLO corrections, of O(1/L^3), fitted from the lattice data

The LO and NLO corrections are universal and known M.Hayakawa, S.Uno, 2008; Sz. Borsanyi et al., 2014

Tuning the critical mass

① The Dashen theorem: in the massless theory, the neutral pion and kaon are Goldstone bosons even in the presence of electromagnetic interactions:

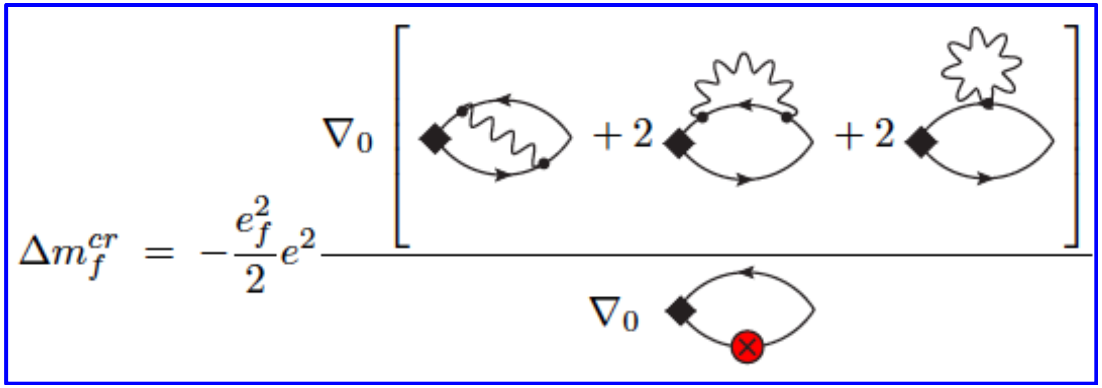
$$\lim_{m_f \to 0} M_{\pi^0} = \lim_{m_f \to 0} M_{K^0} = 0$$



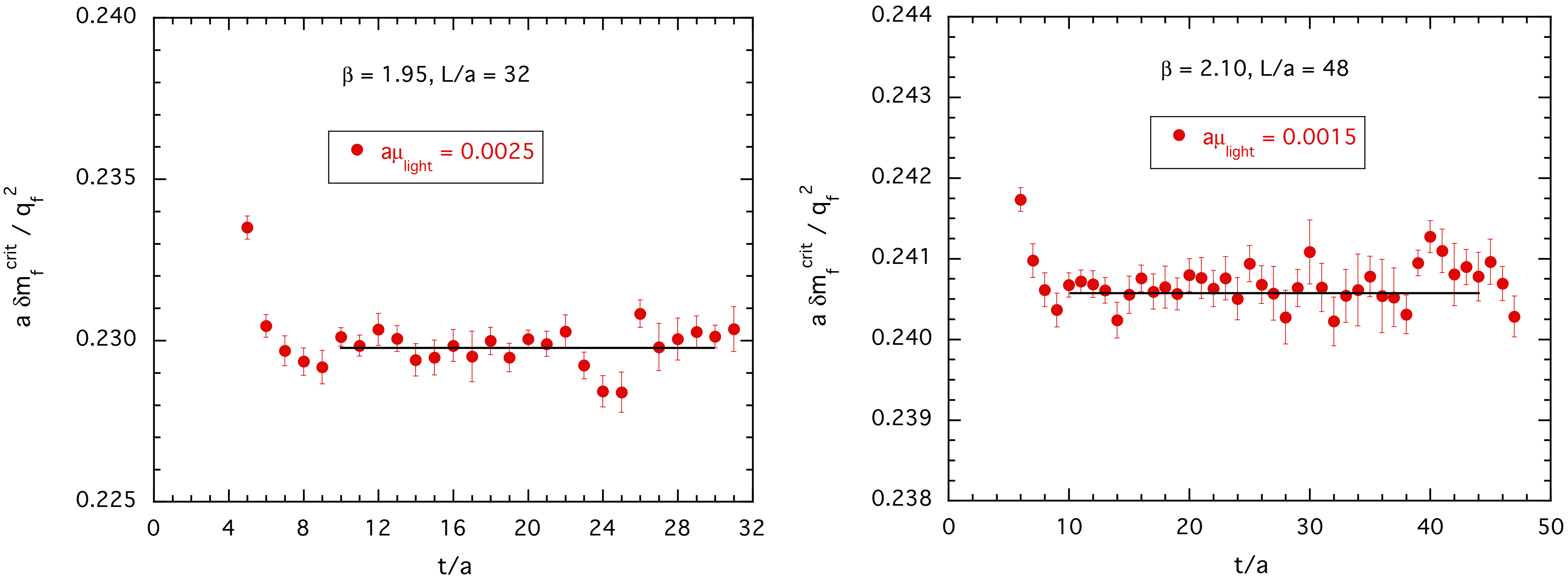
② With twisted mass fermions, one can extend the method used also in the isosymmetric QCD case, based on a specific Ward-Takahashi identity:

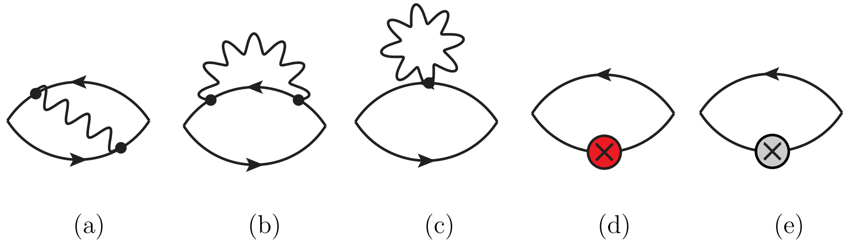
$$\nabla_\mu \left\langle V_\mu^1(x) P_5^2(0) \right\rangle = 0$$

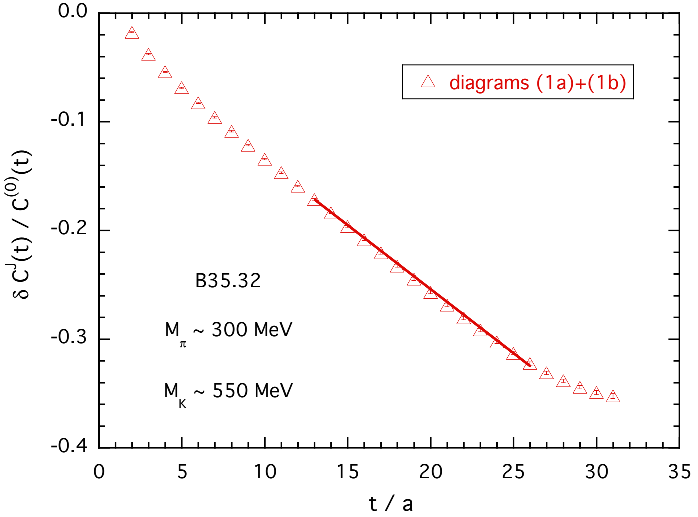
More precise: it does not require a chiral extrapolation



Plateaux for δm^{crit}




(a)
(b)
(c)
(d)
(e)


diagrams (1a)+(1b)
B35.32
M_π ~ 300 MeV
M_K ~ 550 MeV
$\delta C^J(t) / C^{(0)}(t)$
t / a
0.0
-0.1
-0.2
-0.3
-0.4
0
5
10
15
20
25
30
35

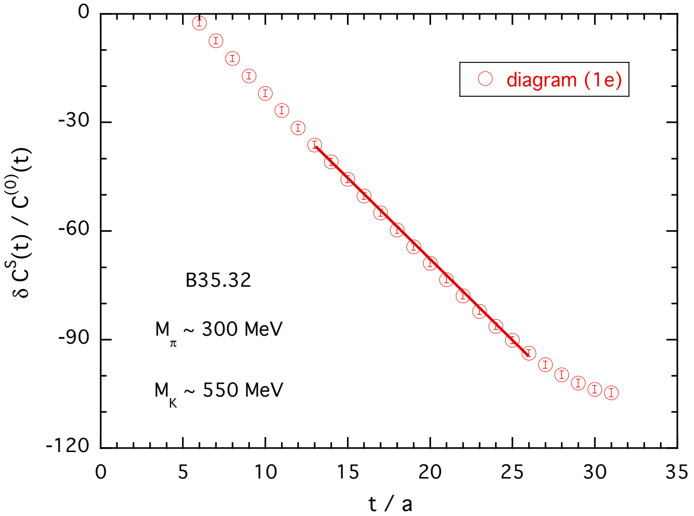
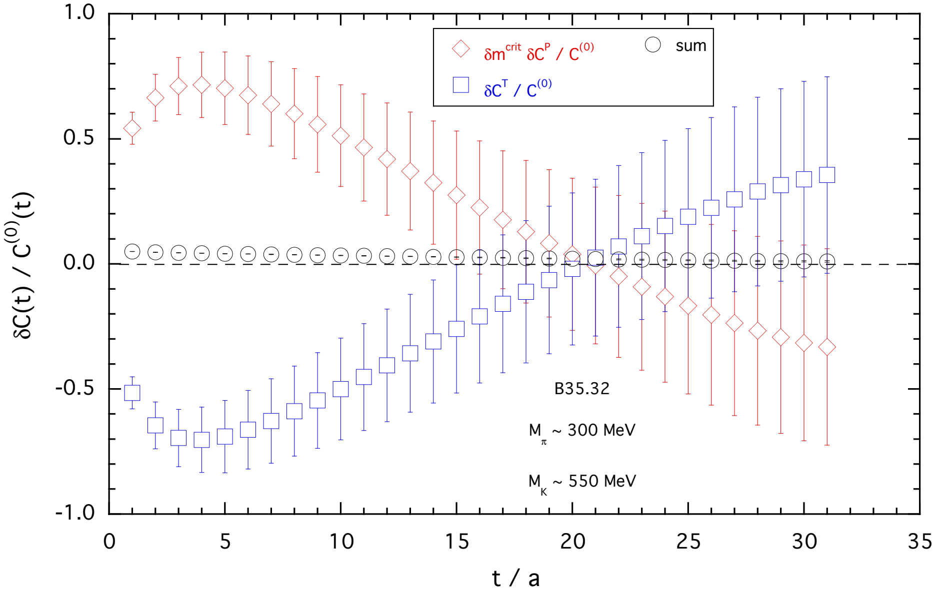
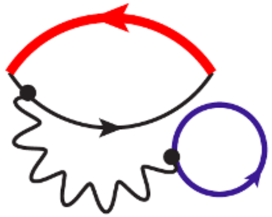


diagram (1e)
B35.32
M_π ~ 300 MeV
M_K ~ 550 MeV
$\delta C^S(t) / C^{(0)}(t)$
t / a
0
-30
-60
-90
-120
0
5
10
15
20
25
30
35


$\delta m^{crit}\ \delta C^P / C^{(0)}$
$\delta C^T / C^{(0)}$
sum
B35.32
M_π ~ 300 MeV
M_K ~ 550 MeV
$\delta C(t) / C^{(0)}(t)$
t / a
1.0
0.5
0.0
-0.5
-1.0
0
5
10
15
20
25
30
35

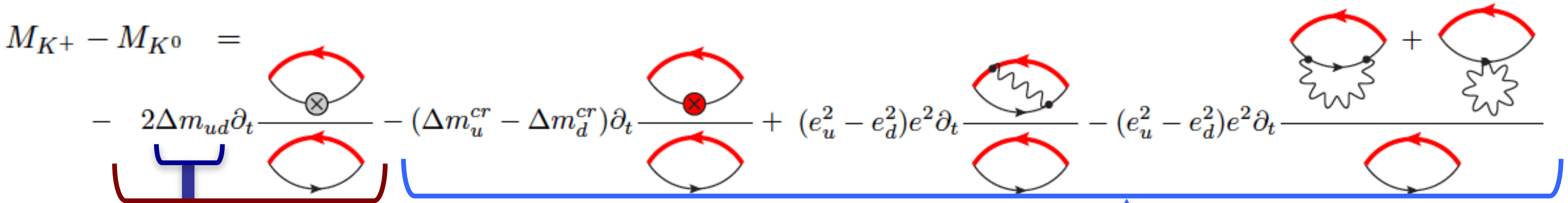
The charged/neutral kaon mass splitting

- We adopt the electro-quenched approximation:
  - To be addressed in a future calculation



= 0

- By choosing a specific renormalization scheme and scale for the quark masses, one can separate the QCD and QED contributions:



$$M_{K^+} - M_{K^0} = \; - \; \underbrace{2\Delta m_{ud}\partial_t}\; - (\Delta m_u^{cr} - \Delta m_d^{cr})\partial_t \; + \; (e_u^2 - e_d^2)e^2\partial_t \; - \; (e_u^2 - e_d^2)e^2\partial_t$$

∝ e^2, separately divergent

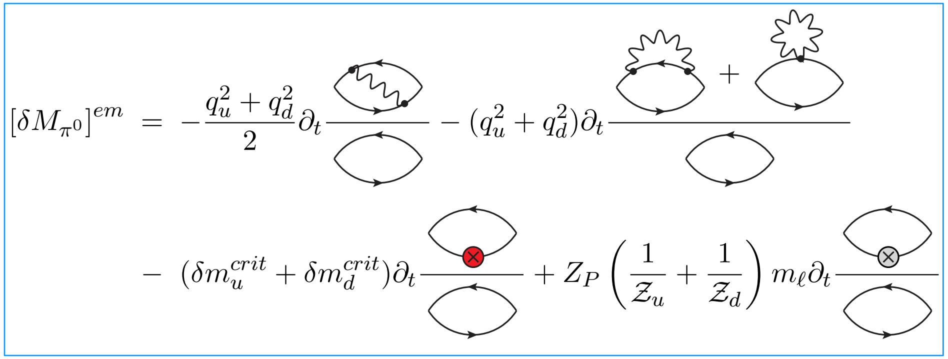
$$\Delta m_{ud} = \frac{1}{2}\left(\frac{\hat{m}_d}{Z_{m_d}} - \frac{\hat{m}_u}{Z_{m_u}}\right) = \frac{1}{2}\left(\frac{1}{Z_{m_d}} + \frac{1}{Z_{m_u}}\right)\Delta\hat{m}_{ud} + \overbrace{\frac{1}{2}\left(\frac{1}{Z_{m_d}} - \frac{1}{Z_{m_u}}\right)}\hat{m}_{ud}$$

S. Aoki *et al.*
arXiv:9802034 [hep-ph]

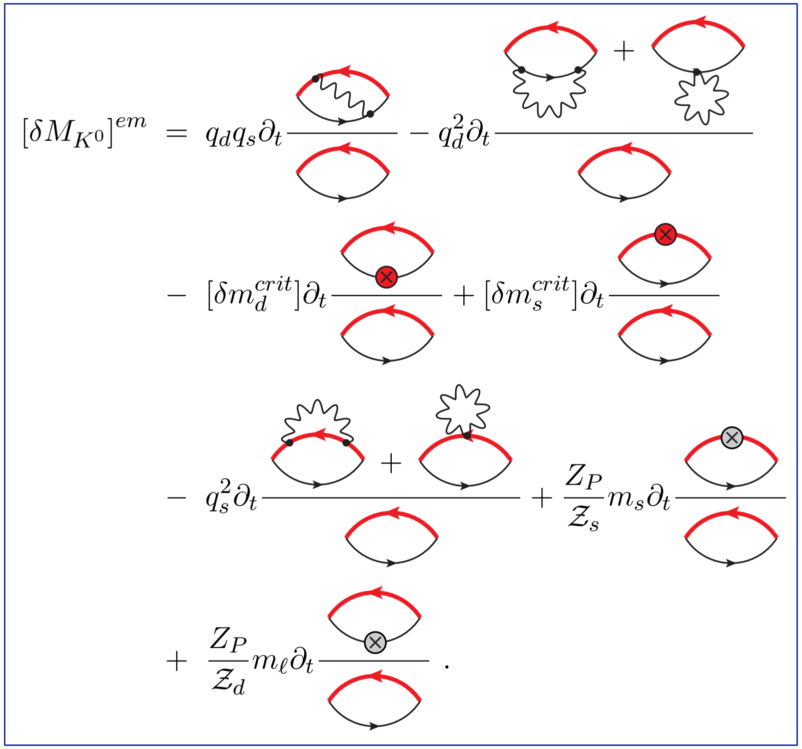
$$\boxed{\frac{1}{Z_{ud}}(\overline{\mathrm{MS}}, \mu) = \frac{e^2(e_d^2 - e_u^2)}{32\pi^2}\,[6\log(a\mu) - 22.596...]\, Z_P^0}$$

ε_{π^0} , ε_{K^0}

$$\boxed{\epsilon_{\pi^0} = \frac{e^2\left[\delta M^2_{\pi^0}\right]^{em}}{M^2_{\pi^+}-M^2_{\pi^0}}}$$



$$\boxed{\epsilon_{K^0} = \frac{e^2\left[\delta M^2_{K^0}\right]^{em}}{M^2_{\pi^+}-M^2_{\pi^0}}}$$



Flavour symmetry breaking parameters

$$R(\overline{MS}, 2\ \mathrm{GeV}) \equiv \frac{m_s - m_{ud}}{\widehat{m}_d - \widehat{m}_u}(\overline{MS}, 2\ \mathrm{GeV}) \ = \ 40.4\ (3.3)$$

$$Q(\overline{MS}, 2\ \mathrm{GeV}) \equiv \sqrt{\frac{m_s^2 - m_{ud}^2}{\widehat{m}_d^2 - \widehat{m}_u^2}}(\overline{MS}, 2\ \mathrm{GeV}) \ = \ 23.8\ (1.1)$$

BMW Nf=2+1 (2016) R = 38.20(1.95)

BMW Nf=2+1 (2016) $Q = 23.40(64)$

FLAG (2016) R = 35.6(5.1)

FLAG (2016) $Q = 22.2(1.6)$

Comparison with other approaches/results

- Other lattice studies of QCD + QED have been /are being performed.

- They are based on the "standard" approach: QED is introduced directly in the Monte Carlo simulation, like QCD.

- Advantages of our approach:

- The small parameters Δm and e are factorized in the expansion
- No need to generate new gauge configurations
- IB is introduced only where needed (no large overall FSE due to e.m.)
- Specific diagrammatic contributions can be easily isolated.

  E.g. separation between strong and e.m. IB effects

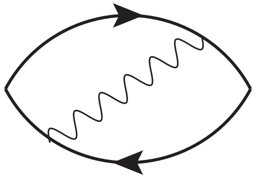
- Disadvantages:

- More vertices and correlations functions to be computed

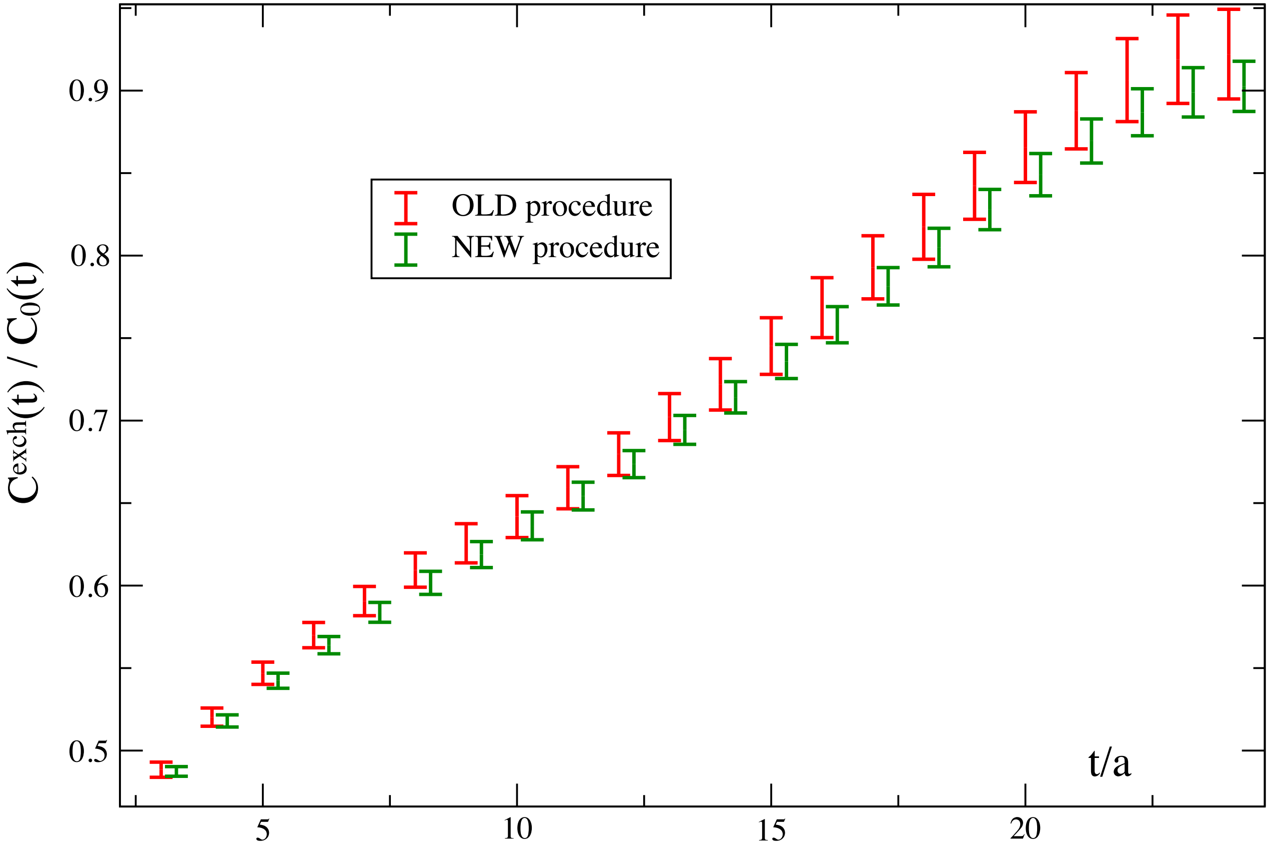
stochastic evaluation of the photon propagator

courtesy of
F. Sanfilippo

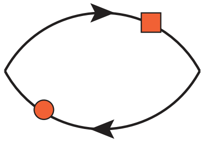
“exchange” diagram



JHEP 04 (2012) 124
PRD 87 (2013) 114505



OLD procedure

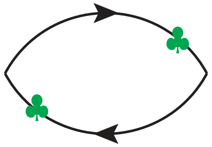


● $= \eta(x)$

■ $= \varphi(x) = \Delta_\gamma(x-y)\eta(y)$

this work

NEW procedure



♣ $= \rho(x) = \int dk e^{ikx} \sqrt{\Delta_\gamma(k)}\tilde{\eta}(k)$

* computation of ϱ(x) (via FFT) is expensive as the one of φ(x)
* the new procedure requires 1 inversion less
* noise is reduced, in particular for the exchange diagram