Chiral extrapolation of the hadronic vacuum polarization contribution to the muon anomalous magnetic moment

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Hadronic vacuum polarization contribution to muon anomalous magnetic moment:

expression: $$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dQ^2}{Q^2}\, w(Q^2)[\Pi(Q^2) - \Pi(0)]$$ (Blum, '03)

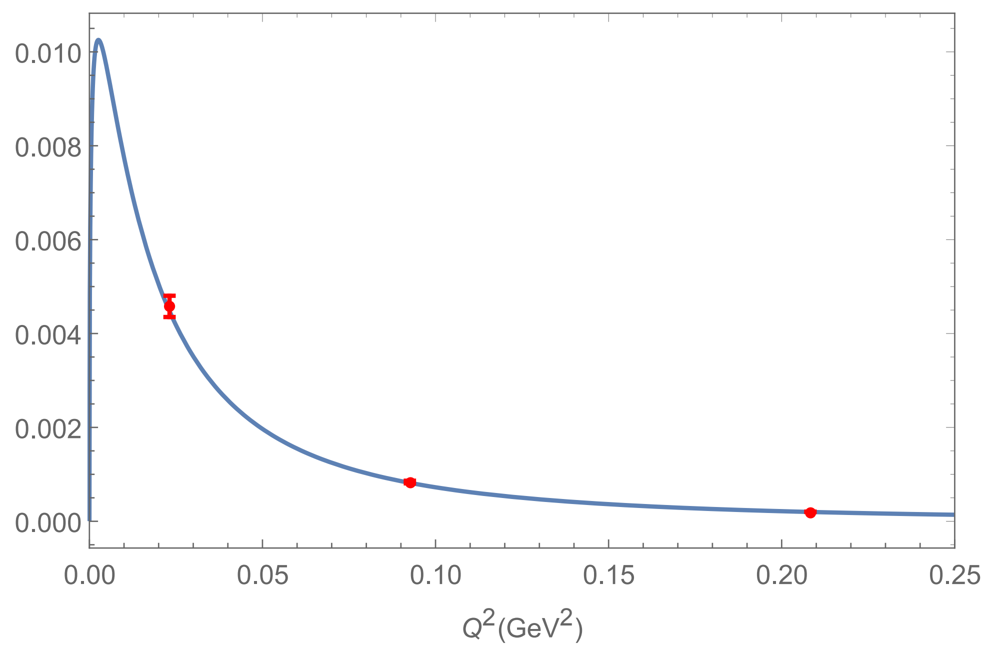
with $w(Q^2)$ a known weight function, and $\Pi(Q^2)$ the HVP obtained from

$$\Pi_{\mu\nu}(Q) = \left(\delta_{\mu\nu}Q^2 - Q_\mu Q_\nu\right)\Pi(Q^2)$$

integrand looks like

new statistics '15

DDS, AMA (Blum et al., '13)



Aim: computation to better than 1%:

- Need very good, *model-independent* low- Q^2 representation (or, equivalently, large-t representation)
- Finite-volume effects: $\sim 5\%$ even at $m_\pi L = 4$?
- Disconnected contribution and isospin breaking
- Strange and charm contributions, 4 (3) dynamical flavors
- Pion mass dependence of $a_\mu^{\rm HVP}$ -- this talk

ETMC trick (Feng et al. 2011)

Improve expression for $a_\mu^{\rm HVP}$:

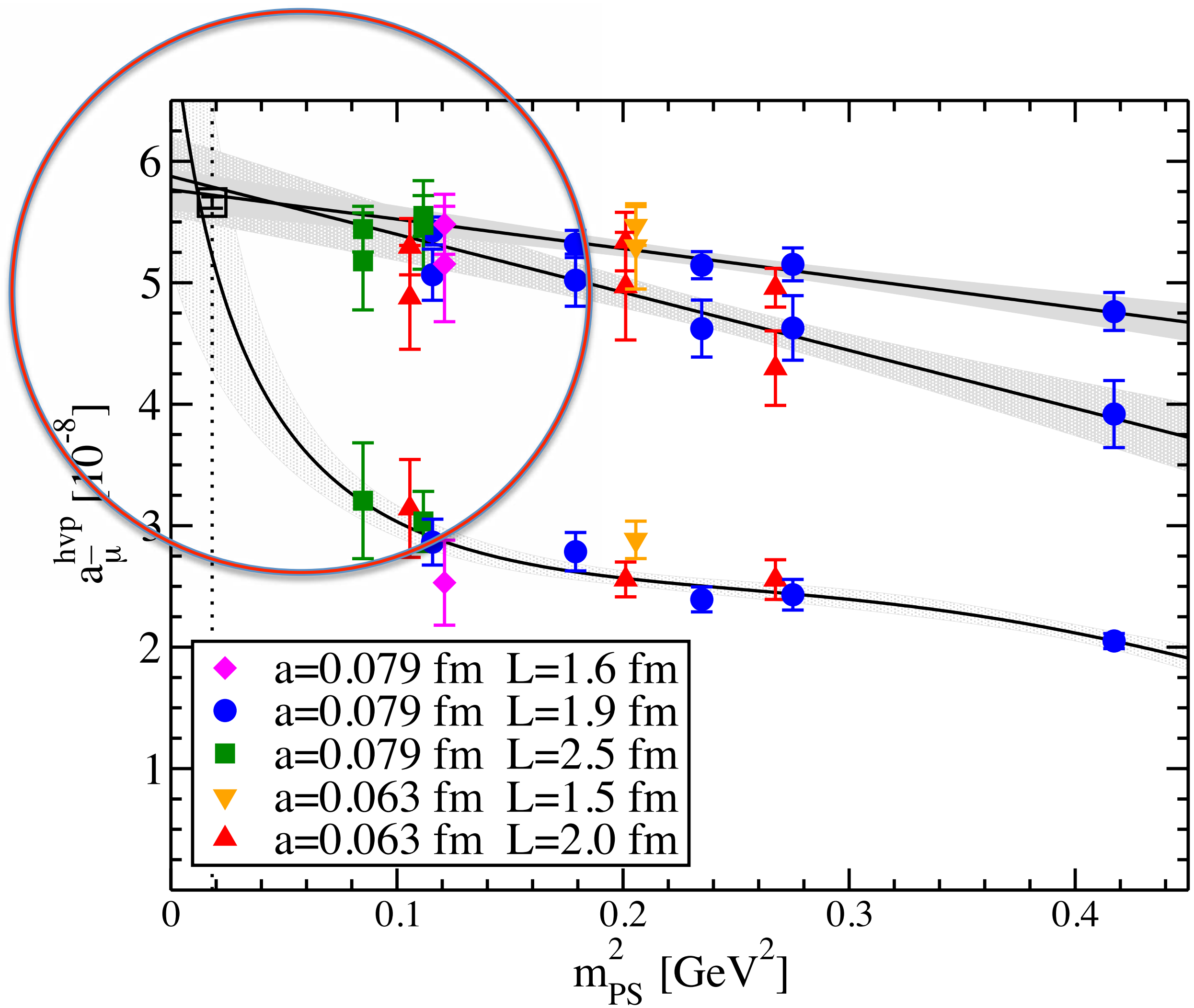
from
$$a_\mu^{\rm HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dQ^2}{Q^2}\, w(Q^2)[\Pi(Q^2) - \Pi(0)]$$

to
$$a_\mu^{\rm HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \frac{dQ^2}{Q^2}\, w(Q^2) \left[\Pi\left(\frac{m^2_{\rho,{\rm latt}}}{m^2_\rho}\, Q^2\right) - \Pi(0)\right]$$

VMD:
$$\Pi(Q^2) = \frac{f_\rho^2 m_\rho^2}{Q^2 + m_\rho^2} + \Pi_{\rm PT}(Q^2) \qquad \Rightarrow$$

$$\Pi_{\rm latt}\left(\frac{m^2_{\rho,{\rm latt}}}{m^2_\rho}\, Q^2\right) = \frac{f_\rho^2 m^2_{\rho,{\rm latt}}}{\frac{m^2_{\rho,{\rm latt}}}{m^2_\rho}\, Q^2 + m^2_{\rho,{\rm latt}}} + \cdots = \frac{f_\rho^2 m_\rho^2}{Q^2 + m_\rho^2} + \dots$$

HPQCD variant: take out lowest-order pion loop first, and put it back at the physical pion mass at the end



Dependence of a_μ^{HVP} on m_π^2. Lower curve: “uncorrected”

Upper curve: “corrected”

Chiral perturbation theory

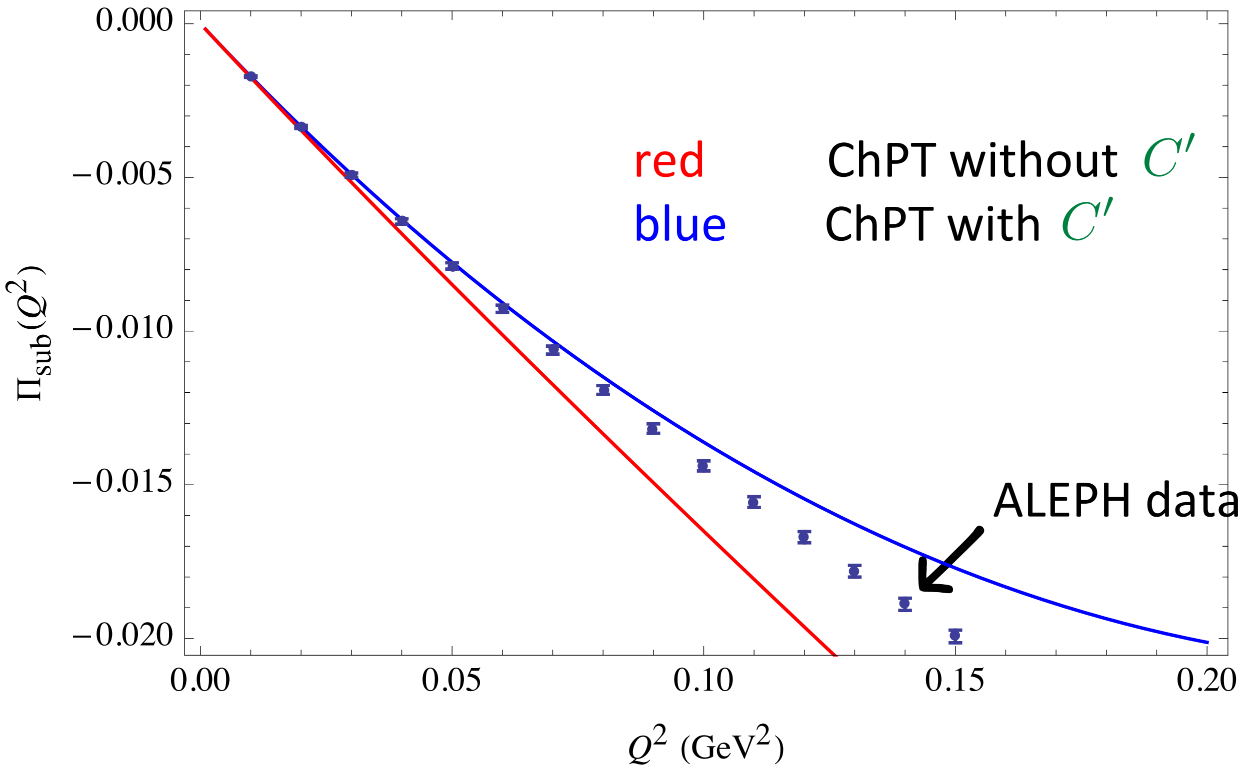
Amoros, Bijnens and Talavera (2000):

$$
\begin{aligned}
\Pi^{I=1}(Q^2) - \Pi^{I=1}(0) = & - 4(F(Q^2) - F(0)) \\
& - \frac{4Q^2}{f_\pi^2} F^2(Q^2) + \frac{16Q^2}{f_\pi^2} L_9 \, F(Q^2) + 8C_{93}Q^2 \\
& + C'(Q^2)^2
\end{aligned}
$$

- $F(Q^2)$ known function (two-pion and two kaon cuts)
- $L_9 = 0.00593(43)$ NLO LEC (Bijnens and Talavera (2002))
- $C_{93} = -0.01536(44)\ \mathrm{GeV}^{-2}$ NNLO LEC (new, from ALEPH tau-decay data)
- $C' = 0.289\ \mathrm{GeV}^{-4}$ additional analytic NNNLO term (from ALEPH data)

(all at $0.77\ \mathrm{GeV}$)

Chiral perturbation theory



$$\tilde{a}_\mu \equiv a_\mu^{I=1}(Q^2_{\rm max} = 0.1) = 9.73 \times 10^{-8} \qquad \text{(ChPT)}$$

$$= 9.81 \times 10^{-8} \qquad \text{(ALEPH data)}$$

$$a_\mu^{I=1}(Q^2_{\rm max} = \infty) = 11.95 \times 10^{-8} \qquad \text{(22\% larger)}$$

Model pion mass dependence of $\tilde{a}_\mu \equiv a_\mu^{I=1}(Q^2 = 0.1\ \mathrm{GeV}^2)$:

- From MILC (HISQ):

$$
\begin{aligned}
m_\pi &= 223,\ 262,\ 313,\ 382,\ 440\ \mathrm{MeV} \\
m_K &= 514,\ 523,\ 537,\ 558,\ 582\ \mathrm{MeV} \\
f_\pi &= 98,\ 101,\ 104,\ 109,\ 114\ \mathrm{MeV} \\
m_\rho &= 826,\ 836,\ 859,\ 894,\ 929\ \mathrm{MeV}
\end{aligned}
$$

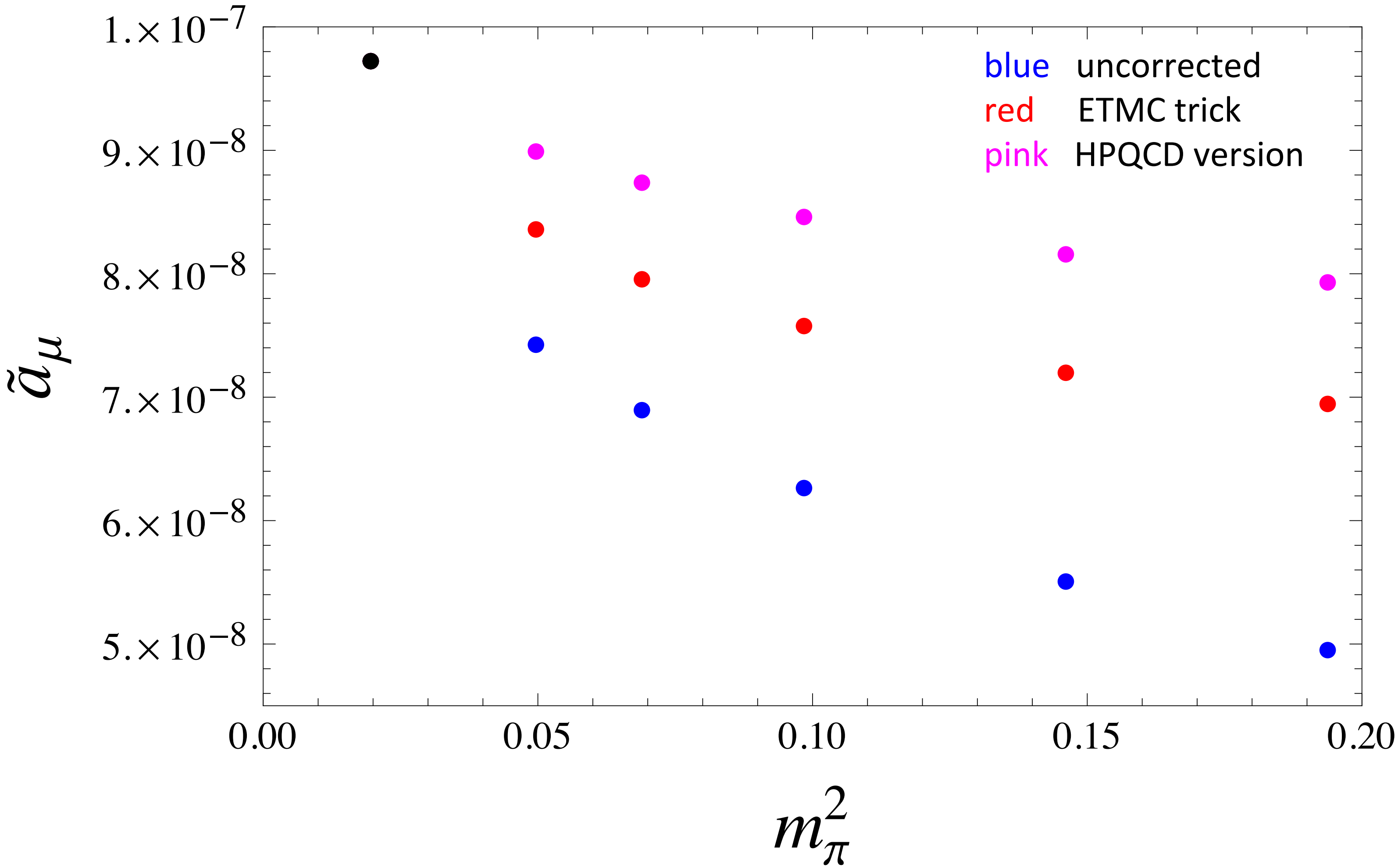
- $C_{93} \sim 1/m_\rho^2$ hence replace $C_{93} \to C_{93,\mathrm{latt}}^{\mathrm{eff}} = C_{93}\,\frac{m_\rho^2}{m_{\rho,\mathrm{latt}}^2}$

  $C' \sim 1/m_\rho^4$

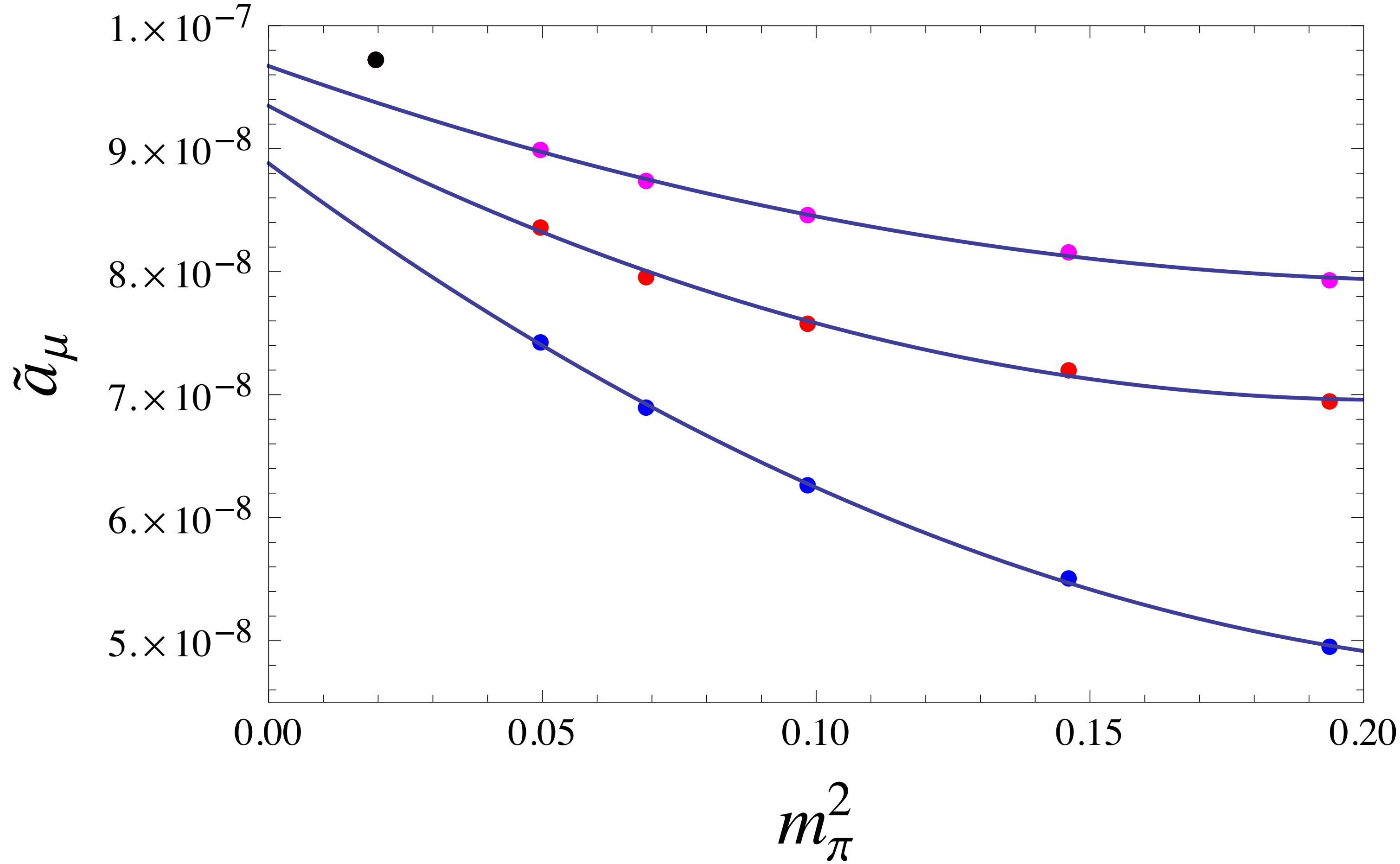
$$C' \to C'^{\mathrm{eff}}_{\mathrm{latt}} = C'\,\frac{m_\rho^4}{m_{\rho,\mathrm{latt}}^4}$$

(takes residual quark-mass dependence at higher orders into account; clearly visible in ALEPH data)

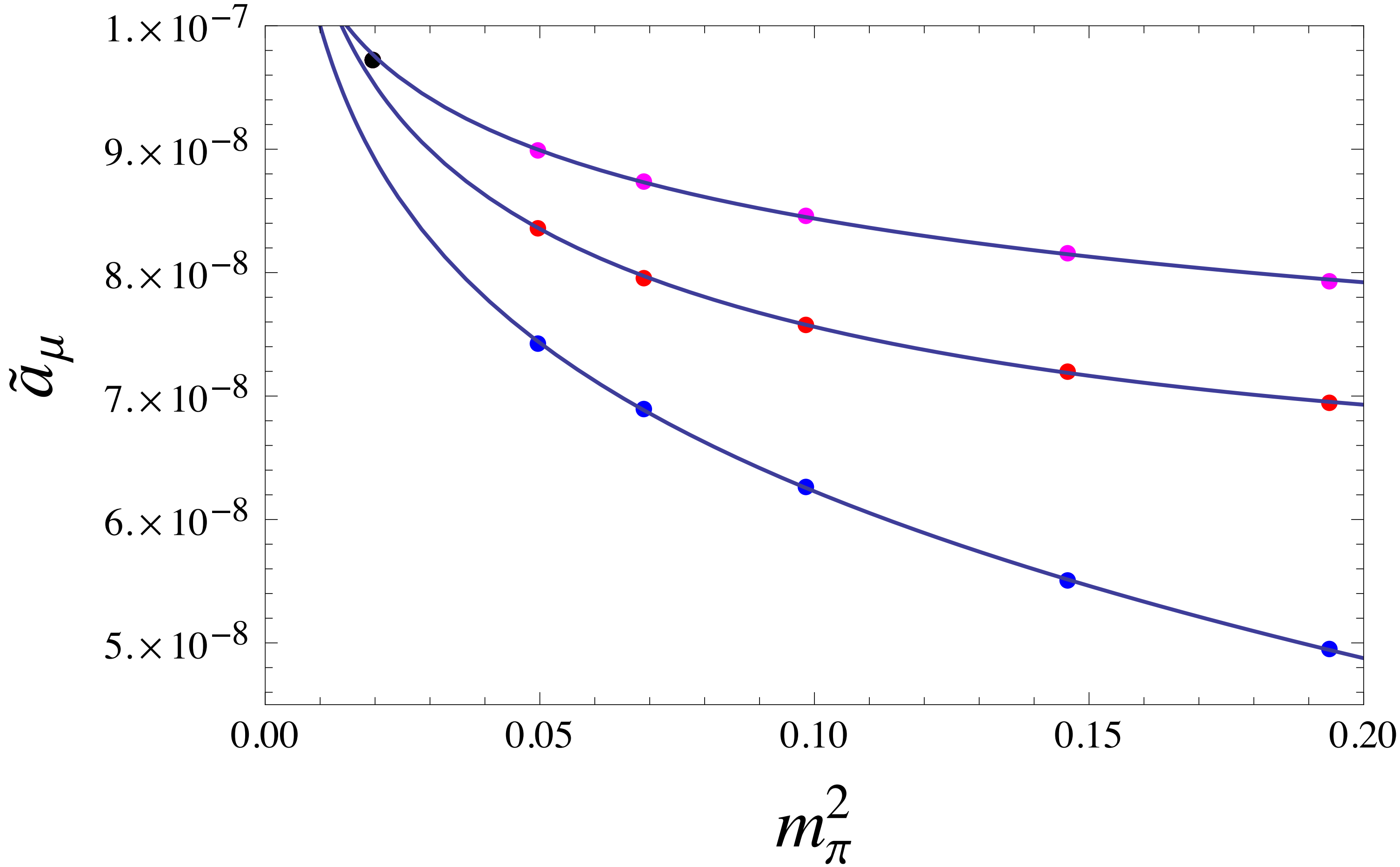
$$\tilde{a}_\mu \equiv a_\mu^{I=1}(Q^2 = 0.1\ \mathrm{GeV}^2)$$



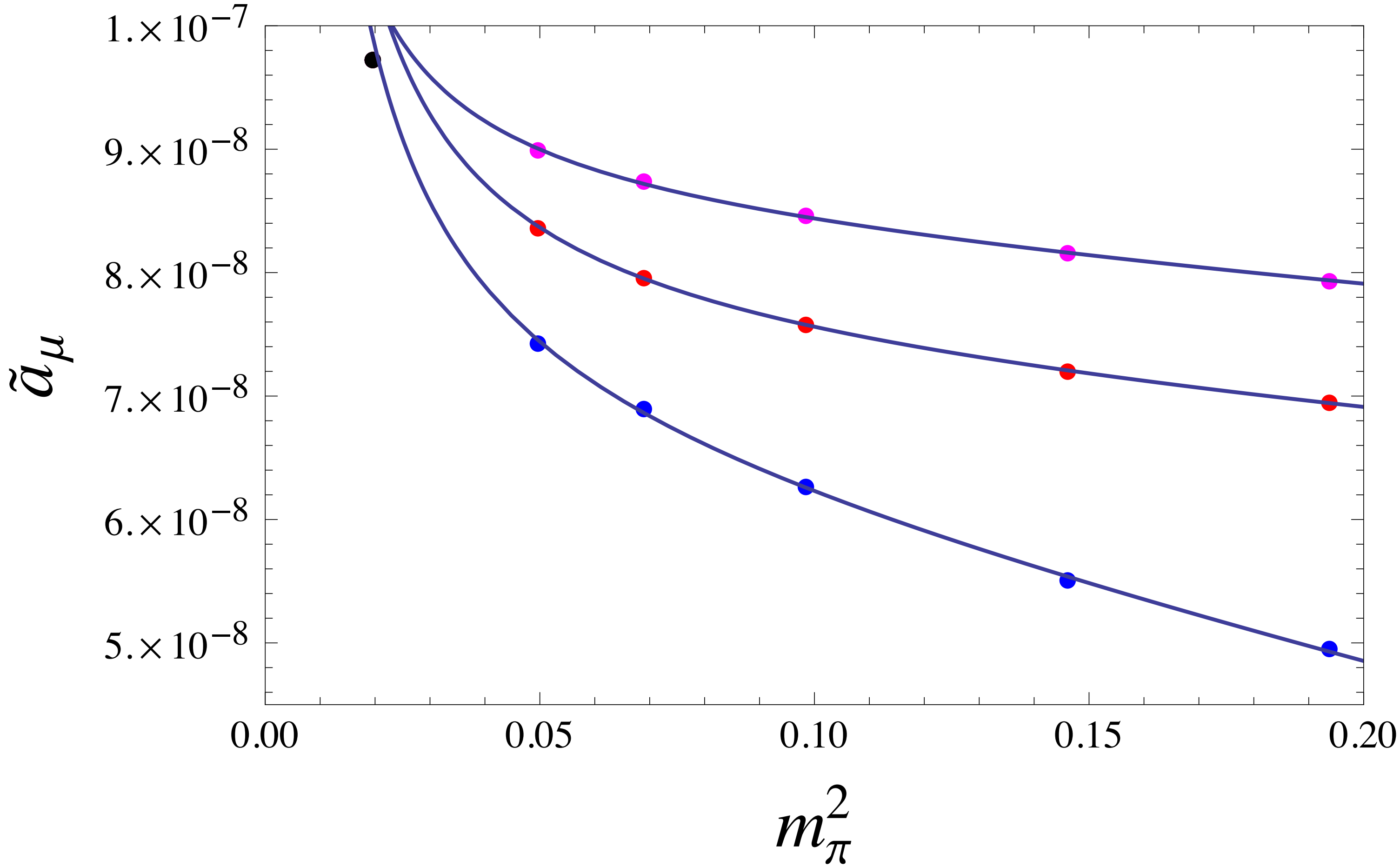
quadratic fit: $am_\pi^4 + bm_\pi^2 + c$



logarithmic fit: $a \log(m_\pi^2/m_{\pi,\mathrm{phys}}^2) + b m_\pi^2 + c$



linear inv. fit: $a/m_\pi^2 + bm_\pi^2 + c$



Observations about fits:

- log fit much better than quadratic fit (same number of parameters)

			ETMC trick	HPQCD
relative errors --	quadratic:	15%	8%	4%
	logarithmic:	8%	2%	0.4%
	linear inv:	2%	8%	6%

- however, can predict coefficient of logarithm (for $m_\pi \to 0$) :

  theory: $-\alpha^2/(12\pi^2) = -4.5 \times 10^{-7}$

  fits: -1.5×10^{-8} to -0.8×10^{-8}

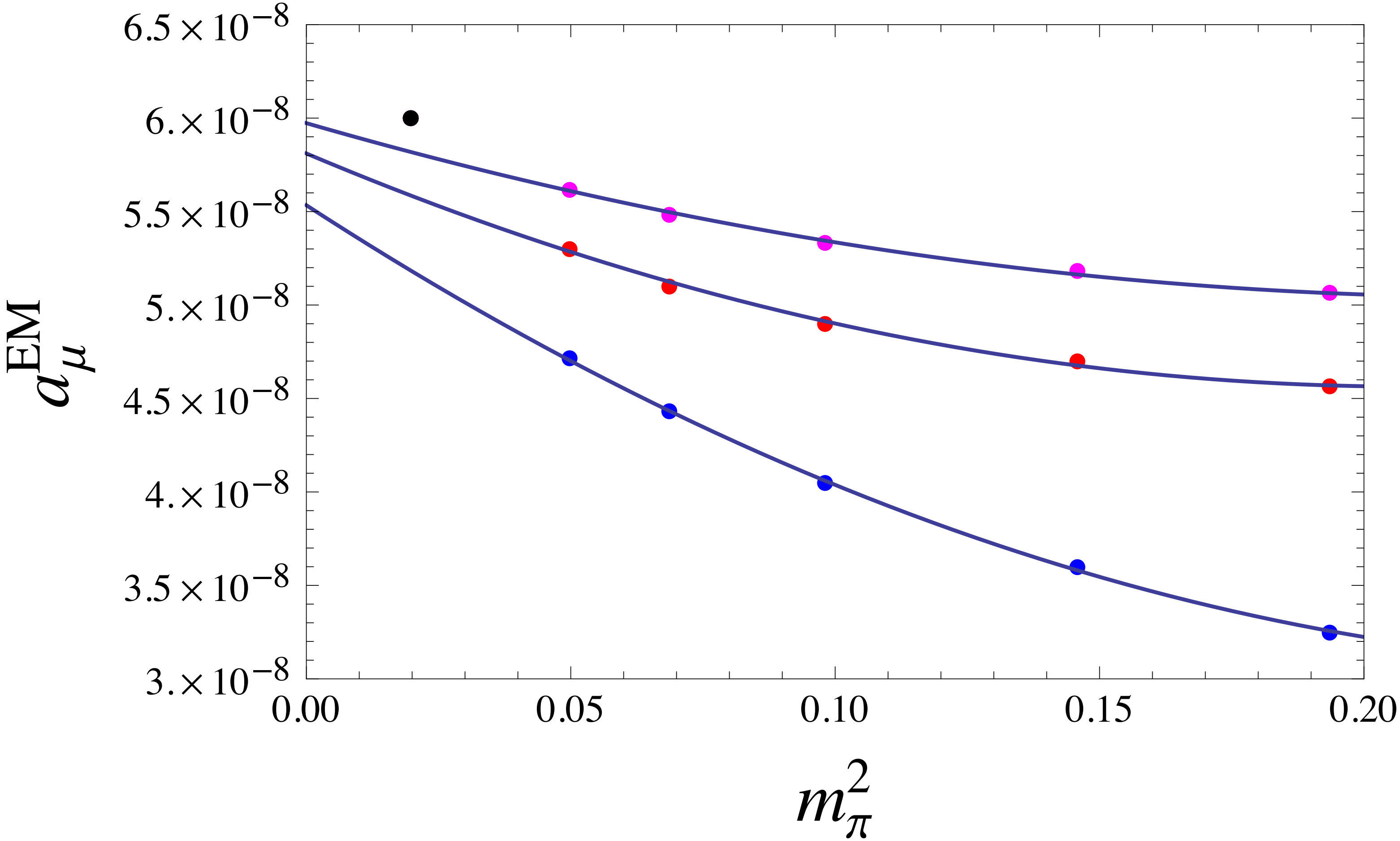
  $\Rightarrow$ all fits just models (fit with log fixed at theory value is disaster)

  $\Rightarrow$ need to estimate errors from comparing fits, e.g. quadratic and log

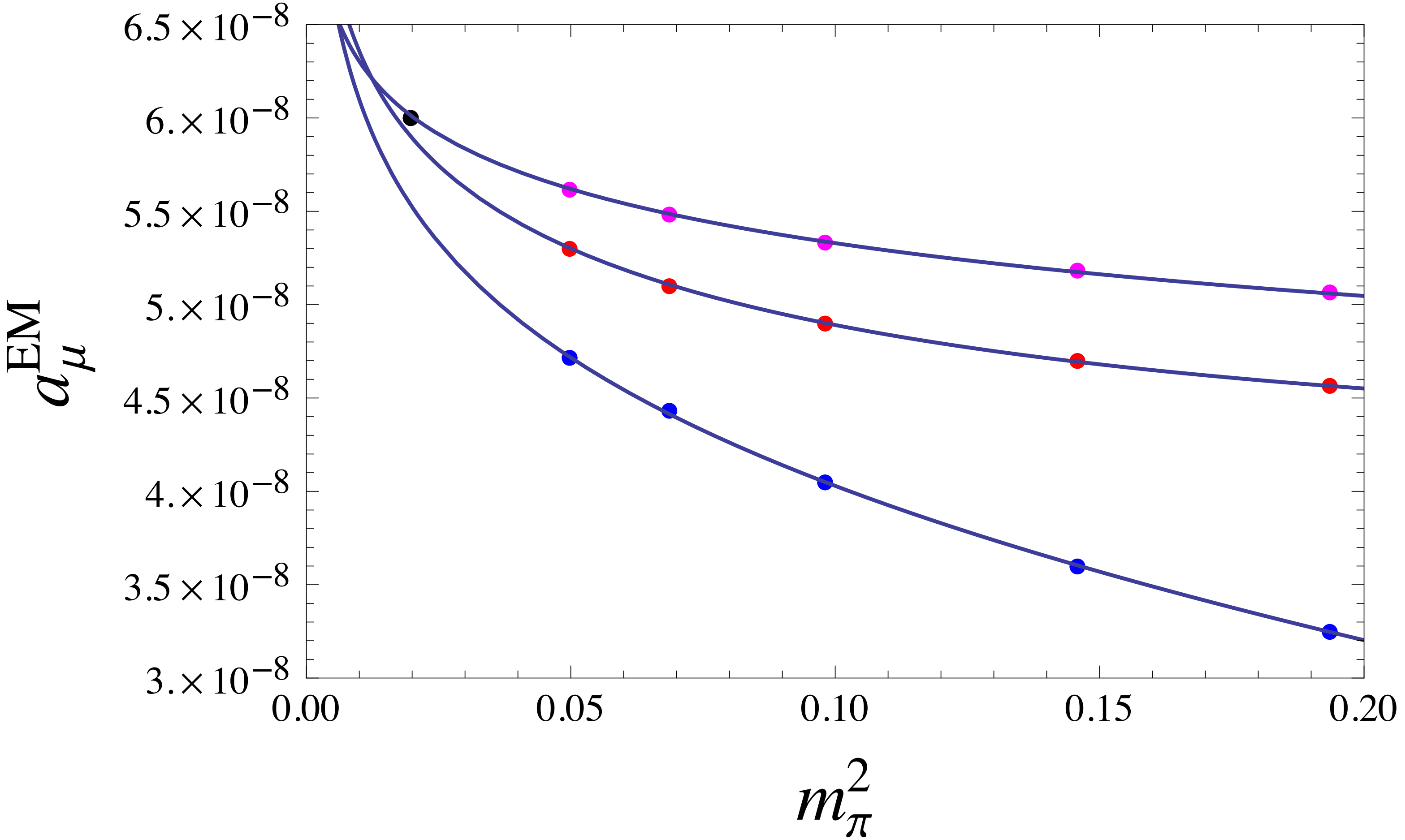
- need to add integral from $0.1\ \mathrm{GeV}^2$ to ∞ ; unlikely to change lessons (no qualitative change if we integrate up to $0.2\ \mathrm{GeV}^2$)

Same game with EM current:

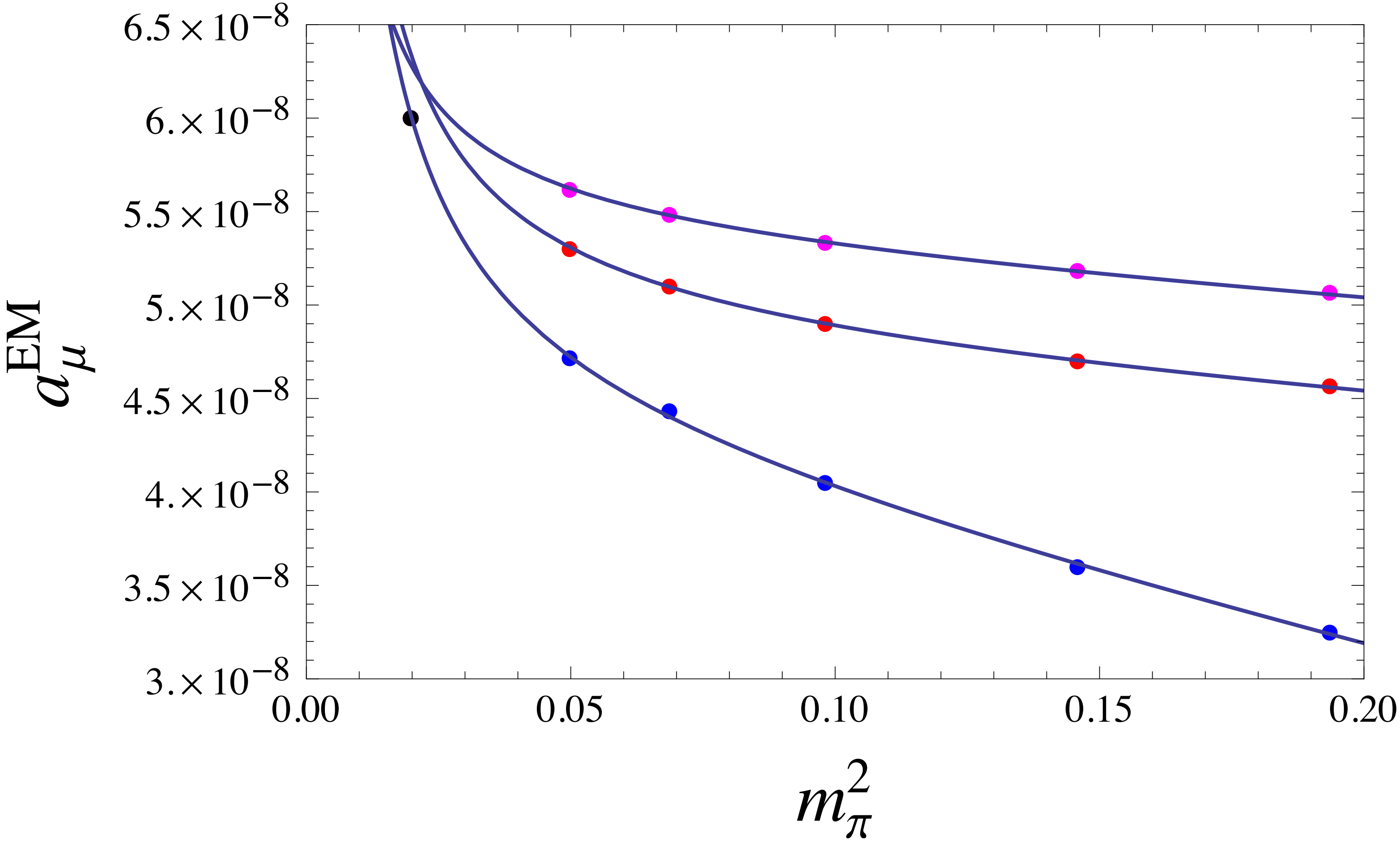
quadratic fit: $am_\pi^4 + bm_\pi^2 + c$



logarithmic fit: $a\log(m_\pi^2/m_{\pi,\mathrm{phys}}^2) + bm_\pi^2 + c$



linear inv. fit: $a/m_\pi^2 + bm_\pi^2 + c$



Observations about fits (EM case):

- log fit much better than quadratic fit (same number of parameters)

		ETMC trick	HPQCD
relative errors -- quadratic:	14%	7%	3%
logarithmic:	8%	2%	0.3%
linear inv:	0.7%	6%	5%

- again, can predict coefficient of logarithm:

  theory: $-\alpha^2/(24\pi^2) = -2.2 \times 10^{-8}$

  fits: -0.8×10^{-8} to -0.4×10^{-8}

  $\Rightarrow$ all fits just models (fit with log fixed at theory value is disaster)

  $\Rightarrow$ need to estimate errors from comparing fits, e.g. quadratic and log

- need to add integral from $0.1\ \mathrm{GeV}^2$ to ∞ ; unlikely to change lessons (no qualitative change if we integrate up to $0.2\ \mathrm{GeV}^2$)

Conclusion

- a_μ^{HVP} is very sensitive to the pion mass; long extrapolation from $m_\pi = 220\ \mathrm{MeV}$
- ETMC/HPQCD trick helps, but is not sufficient for <1% accuracy
- 2-pion cut important, in addition to pion-mass dependence of m_ρ
- No “easy” chiral extrapolation to physical pion mass:
  need lattice computations at values well below $200\ \mathrm{MeV}$
  with sufficient accuracy for sub-percent error in a_μ^{HVP}