

# $\bar{B} \rightarrow D^* \ell \bar{\nu}$ at non-zero recoil

Alejandro Vaquero

University of Utah

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On behalf of the Fermilab/MILC collaborations, with:

Carleton DeTar, University of Utah

Daping Du,

Aida El-Khadra, University of Illinois

Andreas Kronfeld, FNAL

John Laiho, University of Syracuse

Ruth Van de Water, FNAL

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- Available data and simulations
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    - Ratio  $X_V$  and  $h_V$
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# Introduction and motivation

- Precision test of the standard model, looking into new physics
- Weak decays and CKM matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Tension between current inclusive and exclusive  $V_{cb}$  determinations

$$|V_{cb}|_{inc} = (42.2 \pm 0.8) \times 10^{-3}, \quad |V_{cb}|_{exc} = (39.2 \pm 0.7) \times 10^{-3}$$

PDG 2016

- Forthcoming experiments (LHCb, Belle-II) aim to reduce the uncertainty in the determination of the CKM matrix elements
- Imperative to reduce errors coming from lattice determination

# Form factors and the $|V_{cb}|$ CKM matrix element

- Form factors

$$\frac{\langle D^*(p_{D^*}, \epsilon^\nu) | \mathcal{V}^\mu | \bar{B}(p_B) \rangle}{2\sqrt{m_B m_{D^*}}} = \frac{1}{2} \epsilon^{\nu*} \epsilon^{\mu\nu}_{\rho\sigma} v_B^\rho v_{D^*}^\sigma h_V(w)$$

$$\frac{\langle D^*(p_{D^*}, \epsilon^\nu) | \mathcal{A}^\mu | \bar{B}(p_B) \rangle}{2\sqrt{m_B m_{D^*}}} = \frac{i}{2} \epsilon^{\nu*} [g^{\mu\nu} (1+w) h_{A_1}(w) - v_B^\nu (v_B^\mu h_{A_2}(w) + v_{D^*}^\mu h_{A_3}(w))]$$

- From the differential decay rate and the form factors (encoded in  $\mathcal{F}(w)$ ) we can extract  $V_{cb}$
- Zero recoil suppression limit experimental measurements

$$\frac{d\Gamma}{dw} = \frac{G_F^2 M_B^5}{4\pi^3} r^3 (1-r^2) |\eta_{EW}|^2 |V_{cb}|^2 (w^2-1)^{\frac{1}{2}} \chi(w) |\mathcal{F}(w)|^2$$

$$r = \frac{M_{D^*}}{M_B}, \quad w = v_{D^*} \cdot v_B = \frac{E_{D^*}}{M_{D^*}}$$

# Form factors and the $|V_{cb}|$ CKM matrix element

$$\frac{d\Gamma}{dw} = \frac{G_F^2 M_B^5}{4\pi^3} r^3 (1-r^2) |\eta_{EW}|^2 |V_{cb}|^2 (w^2 - 1)^{\frac{1}{2}} \chi(w) |\mathcal{F}(w)|^2$$

- Kinematic factors

$$\left. \begin{aligned} t^2(w) &= \frac{1-2wr+r^2}{(1-r)^2} \\ \lambda(w) &= \frac{1+\frac{4w}{w+1}t^2(w)}{12} \end{aligned} \right\} \longrightarrow \chi(w) = (1+w)^2 \lambda(w),$$

- Definition of  $\mathcal{F}(w)$

$$\mathcal{F}(w) = h_{A_1}(w) \sqrt{\frac{H_0^2(w) + H_+^2(w) + H_-^2(w)}{\lambda(w)}},$$

# Form factors and the $|V_{cb}|$ CKM matrix element

- HQET helicity amplitudes

$$H_0(w) = \frac{w-r-X_3(w)-rX_2(w)}{1-r}, \quad \text{longitudinal polarization}$$

$$H_{\pm}(w) = t(w) (1 \mp X_V(w)), \quad \text{transversal polarization}$$

where

$$X_V(w) = \sqrt{\frac{w-1}{w+1}} \frac{h_V(w)}{h_{A_1}(w)},$$

$$X_2(w) = (w-1) \frac{h_{A_2}(w)}{h_{A_1}(w)}, \quad X_3(w) = (w-1) \frac{h_{A_3}(w)}{h_{A_1}(w)}$$

# Form factors and the $|V_{cb}|$ CKM matrix element

- Playing with the polarization/momentum of the  $D^*$  we can calculate the different helicity form factors

$$\langle D^*(p_\perp) | A_1 | \bar{B}(0) \rangle = (1+w) h_{A_1}(w)$$

Double ratio

$$|R_{A_1}(w)|^2 = \frac{\langle D^*(p_\perp) | A_1 | \bar{B}(0) \rangle \langle \bar{B}(0) | A_1 | D^*(p_\perp) \rangle}{\langle D^*(0) | V_0 | D^*(0) \rangle \langle \bar{B}(0) | V_0 | \bar{B}(0) \rangle} = \left( \frac{1+w}{2} \right)^2 |h_{A_1}(w)|^2$$

Axial form factors:

$$R_0(p) = \frac{\langle D^*(p_\parallel) | A_4 | \bar{B}(0) \rangle}{\langle D^*(p_\perp) | A_1 | \bar{B}(0) \rangle} = \frac{\sqrt{w^2-1} (1 - h_{A_2}(w) + w h_{A_3}(w))}{(1+w) h_{A_1}(w)}$$

$$R_1(p) = \frac{\langle D^*(p_\parallel) | A_1 | \bar{B}(0) \rangle}{\langle D^*(p_\perp) | A_1 | \bar{B}(0) \rangle} = w - \frac{(w^2-1) h_{A_3}(w)}{(1+w) h_{A_1}(w)}$$

# Form factors and the $|V_{cb}|$ CKM matrix element

Vector form factor:

$$X_V(p) = \frac{\langle D^*(p_\perp) | V_1 | \bar{B}(0) \rangle}{\langle D^*(p_\perp) | A_1 | \bar{B}(0) \rangle} = \sqrt{\frac{w-1}{w+1}} \frac{h_V(w)}{h_{A_1}(w)}$$

Recoil parameter:

$$w^2 = 1 + v_{D^*}^2$$

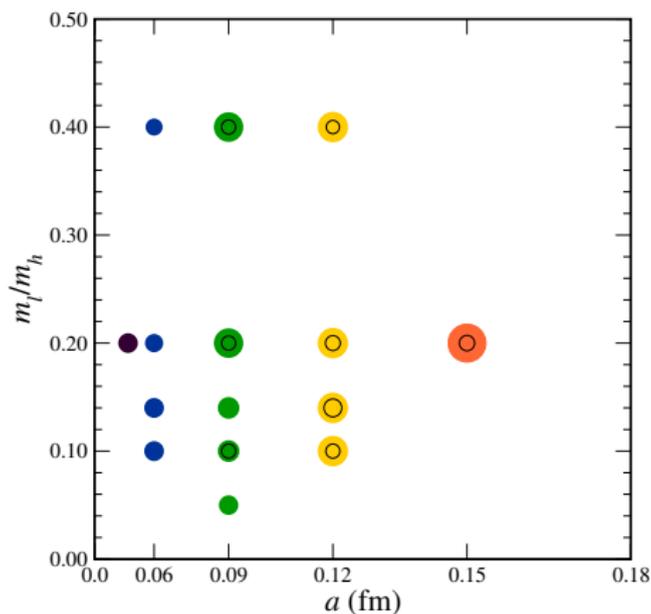
It is measured as a dynamical quantity with the ratio

$$X_f(p) = \frac{\langle D^*(p) | \mathbf{V} | D^*(0) \rangle}{\langle D^*(p) | V_4 | D^*(0) \rangle} = \frac{\mathbf{v}_{D^*}}{w+1}$$

From here

$$w(p) = \frac{1 + \mathbf{x}_f^2}{1 - \mathbf{x}_f^2}$$

# Available ensembles



- $N_f = 2 + 1$  staggered asqtad sea quarks
- Size of the point proportional to the statistics (min 2372, max 15072)

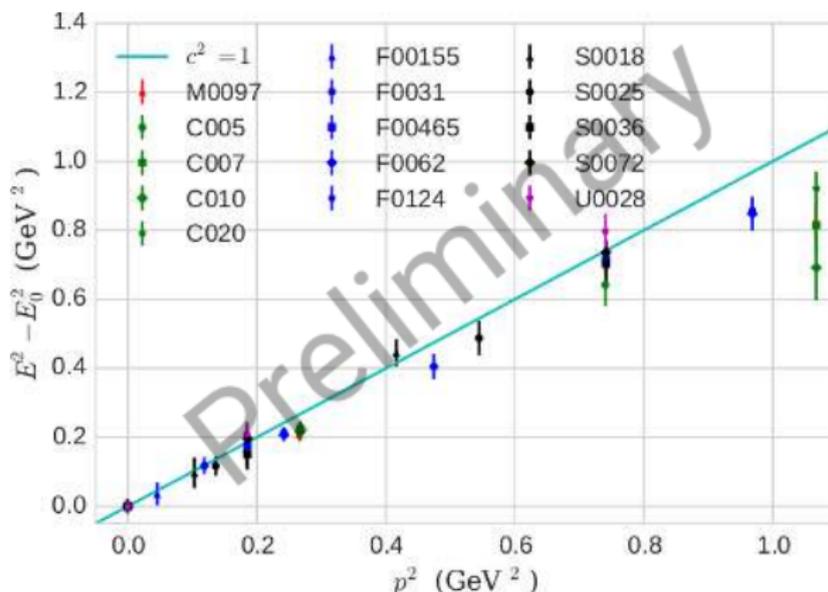
# Analysis: two-point functions

- Light spectator quark modeled with the asqtad action
- Heavy quarks modeled with the Fermilab action
- Smeared  $1S$  at source, sink, both or none
- Joint fits:
  - Three different smearings  $\rightarrow$  point,  $1S$ , symmetric average point- $1S$  (3 corr.)
  - If  $p \neq 0$ , two different momenta  $p_{\parallel}$  and  $p_{\perp}$  (6 corr.)
- Fit ranges:  $t_{Min}$  the same for all the ensembles,  $t_{Max}$  depends on statistics
  - $t_{Min}$  the same for all the ensembles
  - $t_{Max}$  depends on statistics and the size of the correlation matrix

Ensemble	$a$ (fm)	Zero momentum (3 corr.)		Non-zero momentum (6 corr.)	
		$t_{Min}^D / t_{Min}^B$	$t_{Min}^D / t_{Min}^B$ in (fm)	$t_{Min}$	$t_{Min}$ in (fm)
Medium coarse	0.150	9 / 5	1.350 / 0.750	7	1.050
Coarse	0.120	12 / 6	1.440 / 0.720	9	1.020
Fine	0.090	16 / 8	1.440 / 0.720	11	0.990
Superfine	0.060	24 / 12	1.440 / 0.720	17	1.020
Ultrafine	0.045	31 / 16	1.395 / 0.720	23	1.035

# Analysis: two-point functions

- Try 1 + 1, 2 + 2 and 3 + 3 fits and check stability against  $t_{Min}$ , kept 2 + 2
- Momenta available  $|p| = 0, 1, 2$  in lattice units
- Dispersion relation and speed of light



# Analysis: three-point functions

- Joint fit of all the available smearings
  - Parent meson  $\bar{B}$  always smeared
  - Daughter meson  $D^*$  could be 1S or unsmeared
- Oscillating states removed with an average over different sinks

$$\bar{R}(t, T) = \frac{1}{2}R(t, T) + \frac{1}{4}R(t, T + 1) + \frac{1}{4}R(t + 1, T + 1)$$

- In general, fit taking into account first excited state to

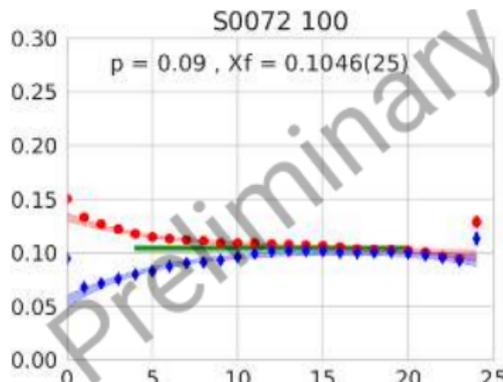
$$R(t) = r \left( 1 + Ae^{-\Delta E_{M_0}t} + Be^{-\Delta E_{M_T}(T-t)} \right)$$

- Tight priors (data) for  $\Delta E$  coming from the two-point fits
- Loose priors for  $r$ ,  $A$  and  $B$
- The  $Z$  factors were applied per jackknife bin
- For each ratio, common fit ranges for all the ensembles

# Results: $X_f$ and the recoil parameter $w$

- Ratio

$$\frac{\langle D_{s_1}^*(p_\perp) | \mathbf{V} | D_{1S}^*(0) \rangle}{\langle D_{s_1}^*(p_\perp) | V_4 | D_{1S}^*(0) \rangle} = \mathbf{x}_f, \quad \rightarrow \quad w = \frac{1 + \mathbf{x}_f^2}{1 - \mathbf{x}_f^2}$$



- Meson at source smeared (blue) and unsmeared (red)
- Meson at sink always smeared, set  $B = 0$  in the fit ansatz

$$R(t) = r \left( 1 + Ae^{-\Delta E_{M_0} t} + Be^{-\Delta E_{M_T} (T-t)} \right)$$

# Results: double ratio $R_{A_1}$ and $h_{A_1}$

- Excited states at source and sink coming from  $D^*$
- Missing  $\langle \bar{B}(0) | A_1 | D^*(p_\perp) \rangle$ , used T-reversal
- Compare double ratio against single ratio

Double ratio

$$\frac{\langle D_{s_1}^*(p_\perp) | A_1 | \bar{B}_{1S}(0) \rangle \langle \bar{B}_{1S}(0) | A_1 | D_{s_2}^*(p_\perp) \rangle}{\langle D_{s_3}^*(0) | V_0 | D_{1S}^*(0) \rangle \langle \bar{B}_{1S}(0) | V_0 | \bar{B}_{1S}(0) \rangle} =$$
$$\sqrt{\frac{Z_{s_1}(p_\perp) Z_{s_2}(p_\perp)}{Z_{1S}(0) Z_{s_3}(0)} \frac{m_{D^*}}{E_{D^*}} e^{-(E_{D^*} - m_{D^*})T} \left(\frac{1+w}{2}\right)^2} |h_{A_1}(w)|^2$$

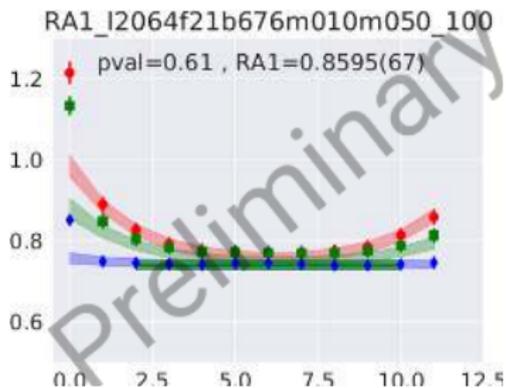
Single ratio

$$\frac{\langle D_{1S}^*(p_\perp) | A_1 | \bar{B}_{1S}(0) \rangle}{\langle D_{1S}^*(0) | A_1 | \bar{B}_{1S}(0) \rangle} = \sqrt{\frac{Z_{1S}(p_\perp)}{Z_{1S}(0)} \frac{m_{D^*}}{E_{D^*}} e^{-(E_{D^*} - m_{D^*})t} \frac{1+w}{2}} \frac{h_{A_1}(w)}{h_{A_1}(0)}$$

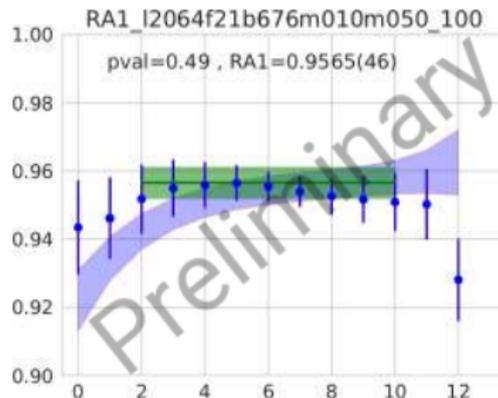
# Results: double ratio $R_{A_1}$ and $h_{A_1}$

- Compare double ratio against single ratio
- Blue: smeared. Red: unsmeared. Green: mix smeared-unsmeared.

Double ratio

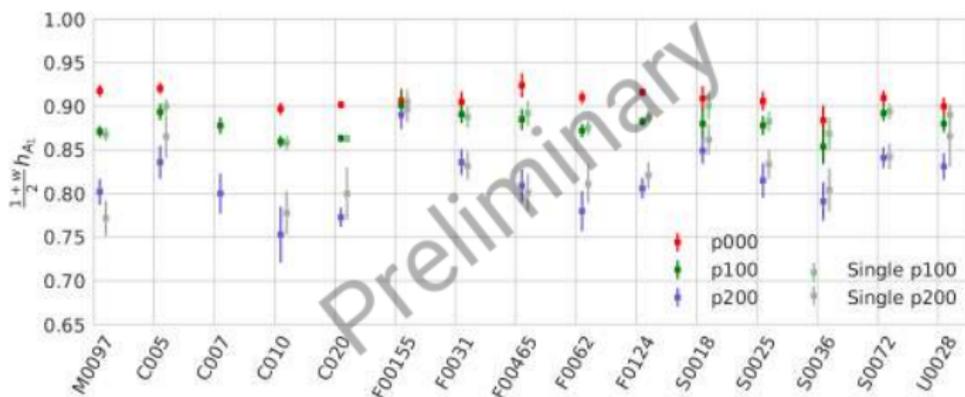


Single ratio



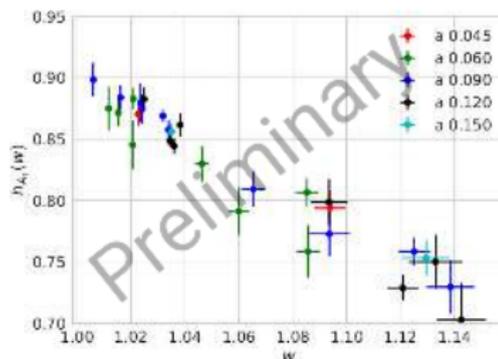
# Results: double ratio $R_{A_1}$ and $h_{A_1}$

- Consistent single-double ratio

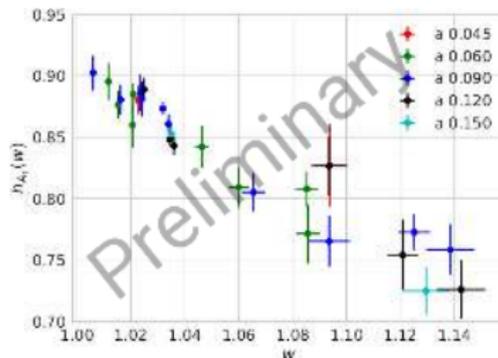


# Results: double ratio $R_{A_1}$ and $h_{A_1}$

## Double ratio



## Single ratio



# Results: ratios $R_0$ , $R_1$ and $h_{A_2}$ , $h_{A_3}$

- Ratios

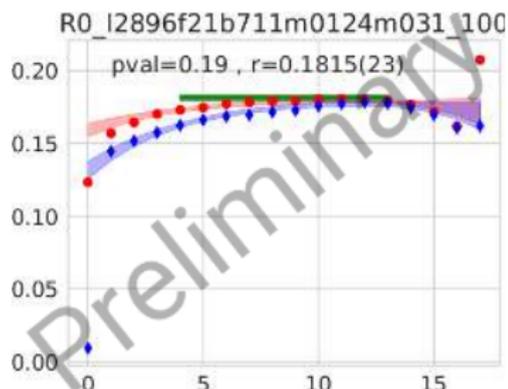
$$R_0(p) = \frac{\langle D^*(p_{\parallel}) | A_4 | \bar{B}(0) \rangle}{\langle D^*(p_{\perp}) | A_1 | \bar{B}(0) \rangle} = \sqrt{\frac{Z(p_{\parallel})}{Z(p_{\perp})}} \frac{\sqrt{w^2 - 1} (1 - h_{A_2}(w) + w h_{A_3}(w))}{(1 + w) h_{A_1}(w)}$$

$$R_1(p) = \frac{\langle D^*(p_{\parallel}) | A_1 | \bar{B}(0) \rangle}{\langle D^*(p_{\perp}) | A_1 | \bar{B}(0) \rangle} = \sqrt{\frac{Z(p_{\parallel})}{Z(p_{\perp})}} \left( w - \frac{(w^2 - 1) h_{A_3}(w)}{(1 + w) h_{A_1}(w)} \right)$$

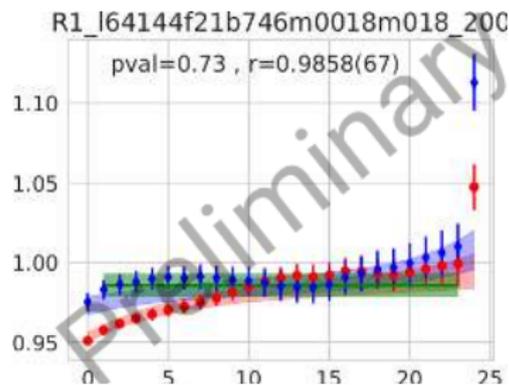
- Excited states at source from  $D^*$  and at sink from  $\bar{B}$

# Results: ratios $R_0$ , $R_1$ and $h_{A_2}$ , $h_{A_3}$

## Fit for $R_0$

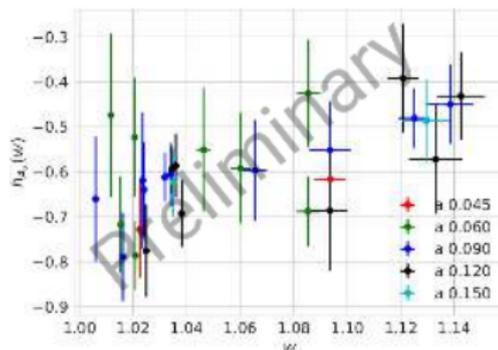


## Fit for $R_1$

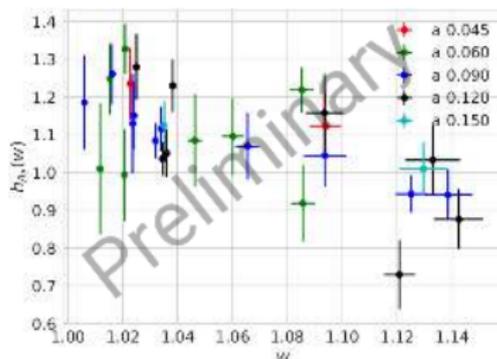


# Results: ratios $R_0$ , $R_1$ and $h_{A_2}$ , $h_{A_3}$

$h_{A_2}(w)$



$h_{A_3}(w)$

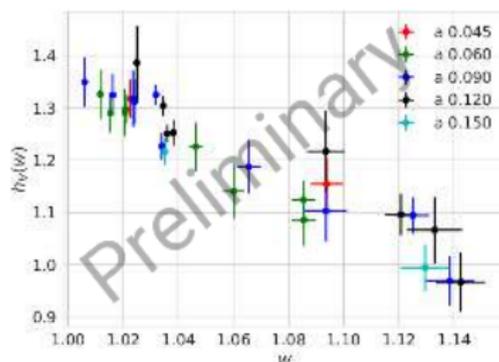
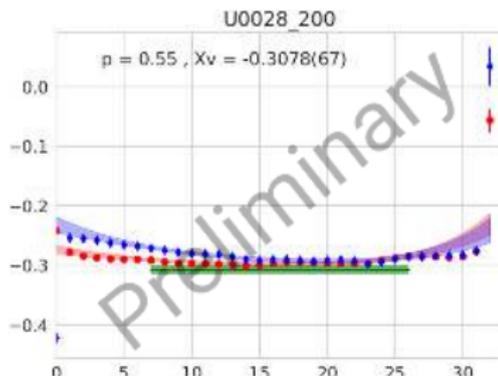


# Results: $X_V$ and $h_V$

- Ratio

$$\frac{\langle D^*(p_\perp) | V_1 | \bar{B}(0) \rangle}{\langle D^*(p_\perp) | A_1 | \bar{B}(0) \rangle} = \sqrt{\frac{w-1}{w+1}} \frac{h_V(w)}{h_{A_1}(w)}$$

- Blue: smeared daughter meson. Red: unsmeared daughter meson.



# Summary and future work

- The CKM matrix elements are a good place to look for new physics
- As experiments improve, we *must* reduce our uncertainty in order to validate the SM/rule out BSM scenarios
- Data at zero and small recoil much necessary to complete experimental data
- This ongoing analysis is an important piece in the puzzle, but much to be done yet
  - Improve fits and reduce errors
  - HQET and chiral fits
  - Continuum limit
  - z-expansion
  - Combined fit with experimental data
  - ...

Thank you for your attention