Three-point gluonic Green's functions: Low-momentum behaviour and the QCD running coupling

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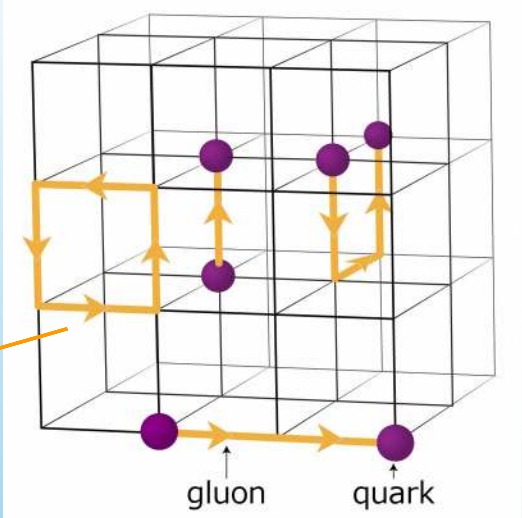
The vertex and the three-gluon Green's function

$$\mathcal{G}^{abc}_{\alpha\mu\nu}(q,r,p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle = f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p),$$

$$\tilde{A}^a_\mu(q) = \frac{1}{2}\mathrm{Tr}\sum_x A_\mu(x+\hat{\mu}/2)\exp[iq\cdot(x+\hat{\mu}/2)]\lambda^a$$

$$A_\mu(x+\hat{\mu}/2) = \frac{U_\mu(x)-U^\dagger_\mu(x)}{2iag_0} - \frac{1}{3}\mathrm{Tr}\frac{U_\mu(x)-U^\dagger_\mu(x)}{2iag_0}$$



Tree-level Symanzik gauge action

$$S_g = \frac{\beta}{3}\sum_x\left\{b_0\sum_{\substack{\mu,\nu=1\\1\le\mu<\nu}}^{4}\left[1-\mathrm{Re\,Tr}\,(U^{1\times1}_{x,\mu,\nu})\right] + b_1\sum_{\substack{\mu,\nu=1\\\mu\neq\nu}}^{4}\left[1-\mathrm{Re\,Tr}(U^{1\times2}_{x,\mu,\nu})\right]\right\}$$

The gauge fields are to be nonperturbatively obtained from lattice QCD simulations and applied then to get the gluon Green's functions

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Symmetric configuration:

$q^2 = r^2 = p^2$ and $q\cdot r = q\cdot p = r\cdot p = -q^2/2$;

$$\mathcal{G}_{\alpha\mu\nu}(q,r,p) = g\Gamma_{\alpha'\mu'\nu'}(q,r,p)\Delta_{\alpha'\alpha}(q)\Delta_{\mu'\mu}(r)\Delta_{\nu'\nu}(p),$$

$$G_{\alpha\mu\nu}(q,r,p) = T^{sym}(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + S^{sym}(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)$$

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$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q)A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu/q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q\cdot\mathcal{G} = r\cdot\mathcal{G} = p\cdot\mathcal{G} = 0$.

In Landau gauge and for particular kinematical configurations, transversality and Bose symmetry make possible a simple tensorial decomposition of the gluon Green's function

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$$W_{\alpha\mu\nu} = \lambda^{tree}_{\alpha\mu\nu} + \lambda^{S}_{\alpha\mu\nu}/2$$

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Asymmetric configuration:

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MOM renormalization prescription:

$$\Delta_R(q^2;q^2) = Z_A^{-1}(q^2)\,\Delta(q^2) = 1/q^2,$$
$$T_R^{\rm sym}(q^2;q^2) = Z_A^{-3/2}(q^2)\,T^{\rm sym}(q^2) = g_R^{\rm sym}(q^2)/q^6.$$

$$g^{\rm sym}(q^2) = q^3 \frac{T^{\rm sym}(q^2)}{[\Delta(q^2)]^{3/2}} = q^3 \frac{T_R^{\rm sym}(q^2;\mu^2)}{[\Delta_R(q^2;\mu^2)]^{3/2}}.$$

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After the required projection and the appropriate renormalization, one can define a QCD coupling from the Green's functions, and relate it to the 1PI vertex form factor, in both symmetric...

The vertex and the three-gluon Green's function

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$$g^{\text{asym}}(r^2) = r^3 \frac{T^{\text{asym}}(r^2)}{[\Delta(r^2)]^{1/2}\Delta(0)} = r^3 \frac{T_R^{\text{asym}}(r^2;\mu^2)}{[\Delta_R(r^2;\mu^2)]^{1/2}\Delta_R(0;\mu^2)}$$

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$$g^{asym}(\mu^2)\,\Gamma^{asym}_{T,R}(q^2;\mu^2) = \frac{g^{asym}(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

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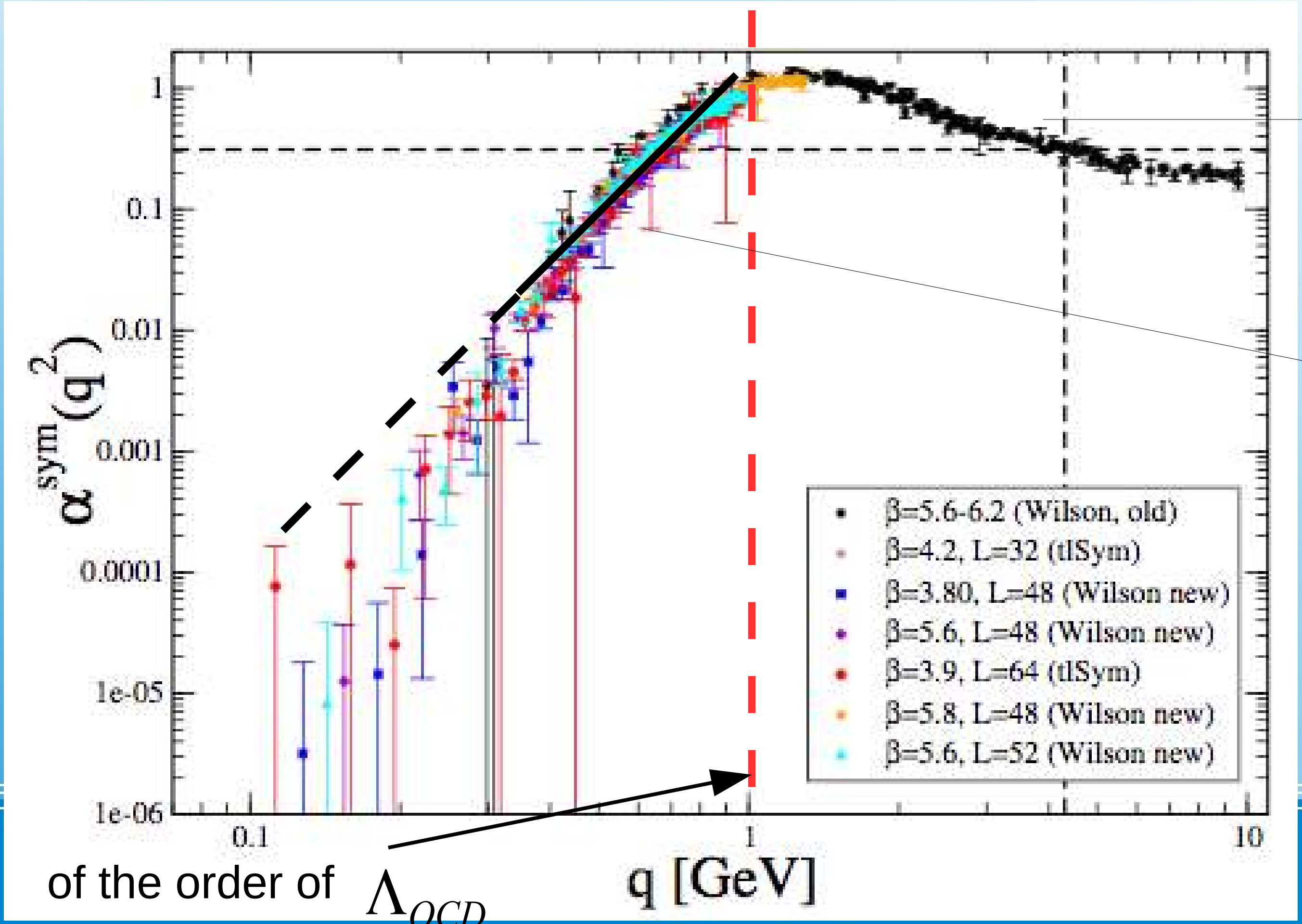
After the required projection and the appropriate renormalization, one can define a QCD coupling from the Green's functions, and relate it to the 1PI vertex form factor, in both symmetric and asymmetric kinematical configurations.

The vertex and the three-gluon Green's function

Let's first focus on the symmetric coupling:

(Spoiler: next talk by F. De Soto!!!)

$$\alpha^{sym}(q^2) = \frac{\left(g^{sym}(q^2)\right)^2}{4\pi} = \frac{q^6}{4\pi}\frac{\left[T^{sym}(q^2)\right]^2}{\left[\Delta(q^2)\right]^3}$$



Logarithmic running accounted for by perturbation theory

Can we somehow interpret this feature?

Next talk by De Soto

Two domains, wherein very different running behaviors appear to dominate each, lie separated by a momentum scale of the order of Λ_{QCD}

The vertex and the three-gluon Green's function

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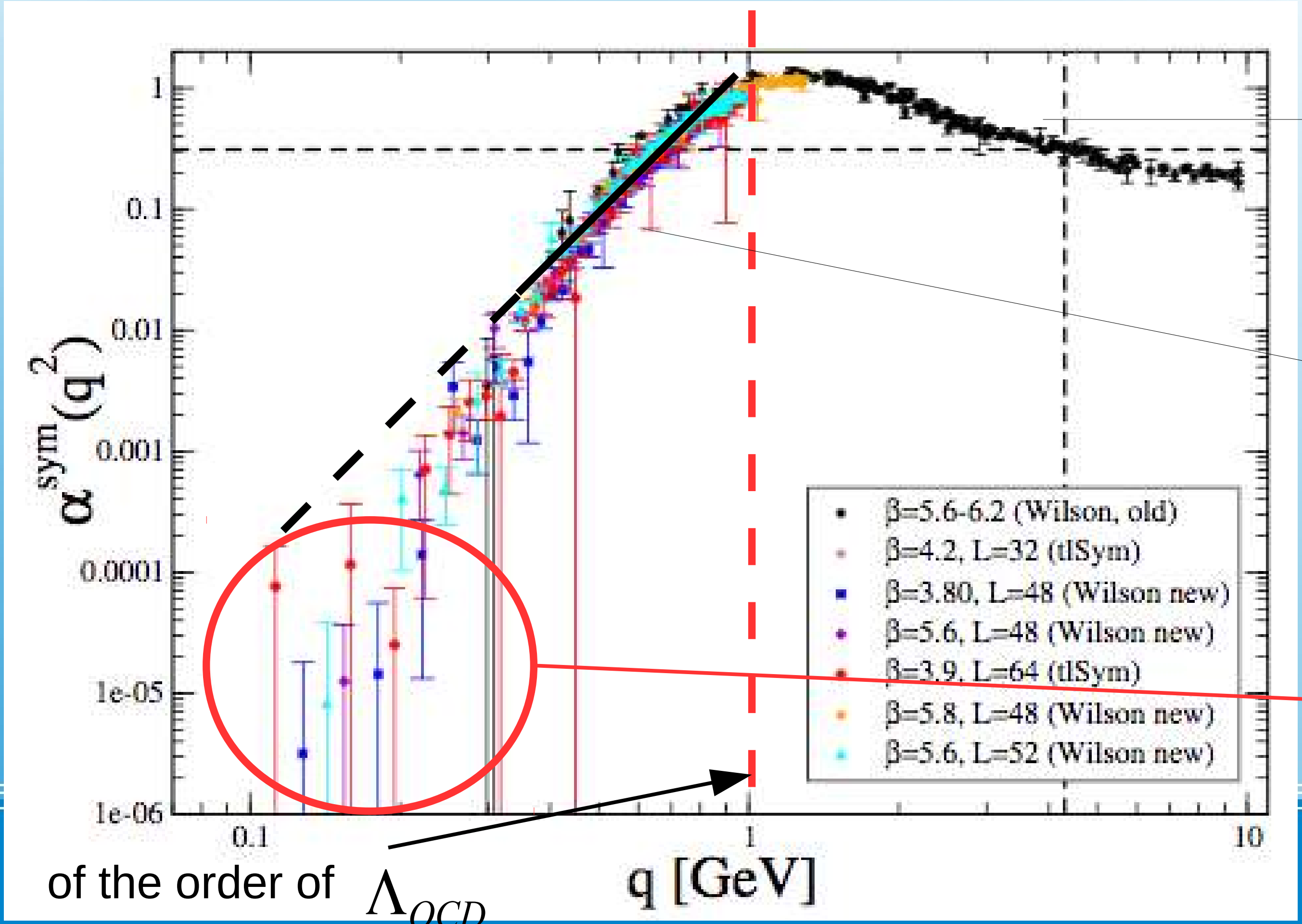

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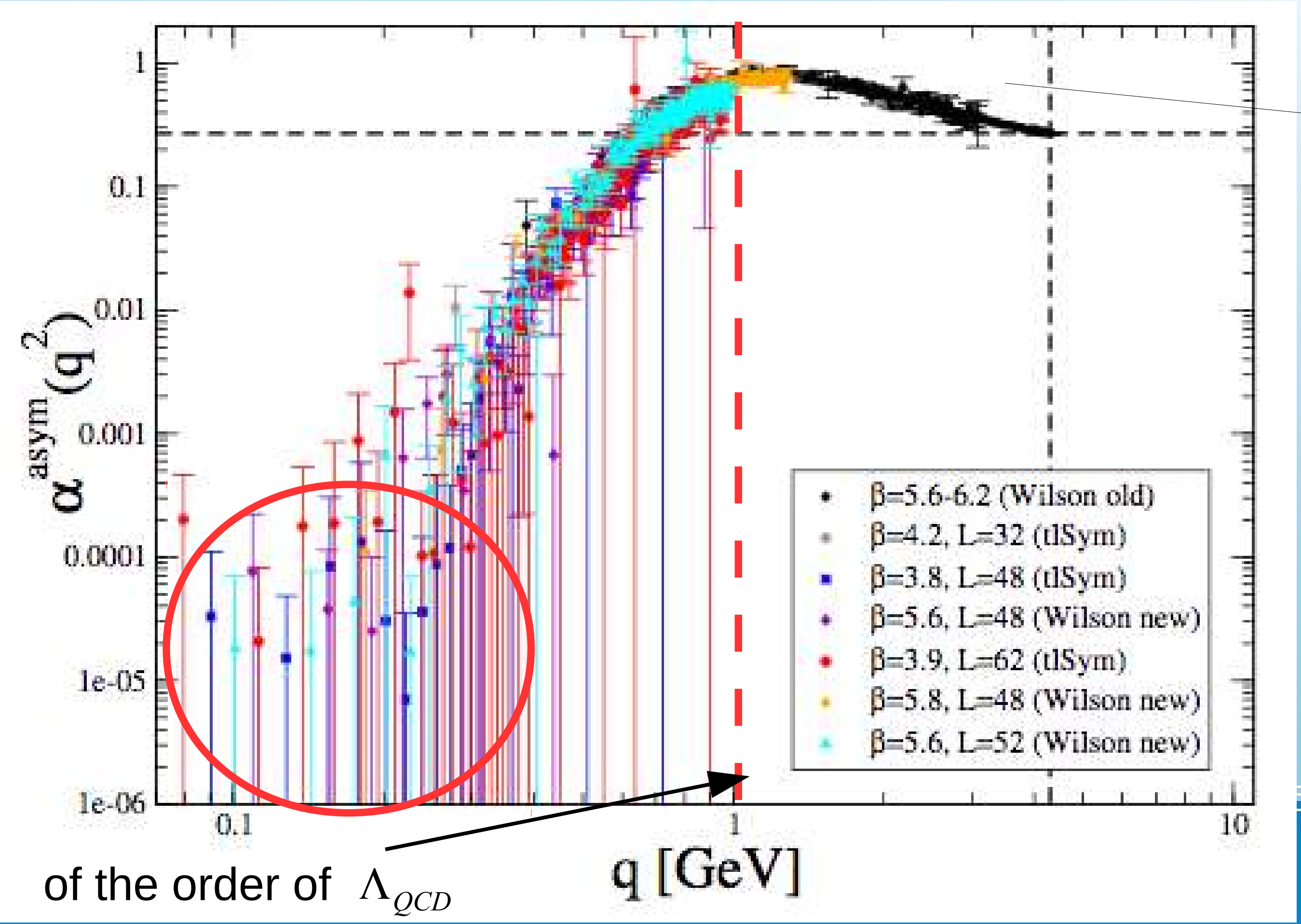
… and what about this?

Two domains, wherein very different running behaviors appear to dominate each, lie separated by a momentum scale of the order of Λ_{QCD}

The vertex and the three-gluon Green's function

… but also the symmetric coupling:

$$\alpha^{asym}(q^2) = \frac{\left(g^{asym}(q^2)\right)^2}{4\pi} = \frac{q^6}{4\pi}\frac{\left[T^{asym}(q^2)\right]^2}{\left[\Delta(0)\right]^2\Delta(q^2)}$$



Logarithmic running accounted for by perturbation theory

Two domains, wherein very different running behaviors appear to dominate each, lie separated by a momentum scale of the order of Λ_{QCD}

The vertex and the three-gluon Green's function

Many different set-up parameters, two different gauge lattice actions...

β	4.2	3.8	3.9	5.8	5.6	5.6
N	32	48	64	48	48	52
$(\text{Volume})^{1/4}$ [fm]	4.45	13.7	15.6	6.72	11.3	12.3
confs.	420	1050	2000	960	1920	1790
action	tlSym	tlSym	tlSym	Wilson	Wilson	Wilson

… and a very good scaling!!!

The zero-crossing of the three-gluon vertex

$$g^i(\mu^2)\Gamma^i_{T,R}(q^2;\mu^2) = \frac{g^i(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

$i = sym, asym.$

$$g^{sym}(q^2) = q^3 \frac{T^{sym}(q^2)}{[\Delta(q^2)]^{3/2}}$$

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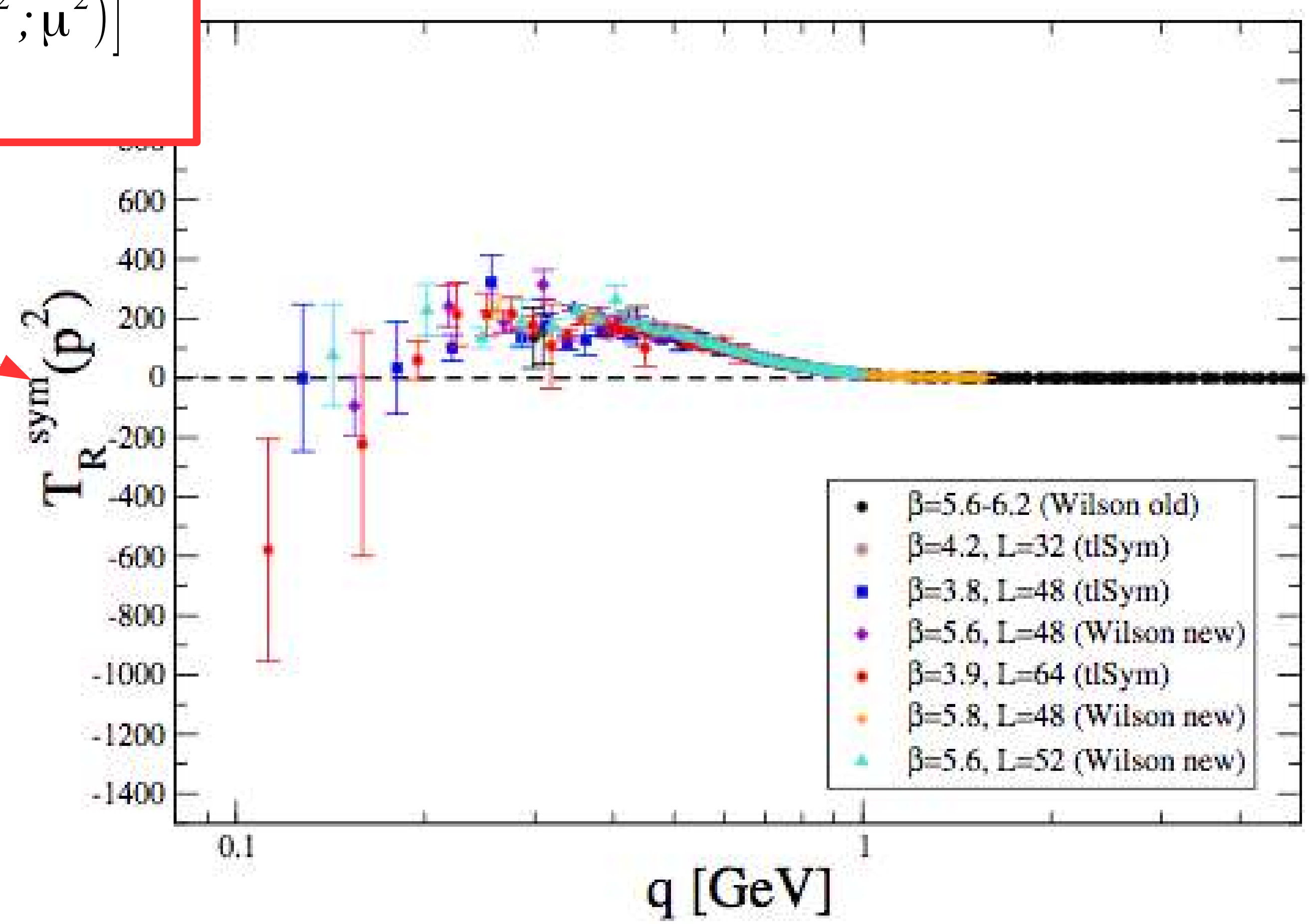

Let's then focus (again) on the symmetric case: the form factor appears to change its sign at very deep IR momenta and show then a zero-crossing. This appears to happen below ~0.2 GeV.

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008
A. Blum et al.; PRD89(2014)061703
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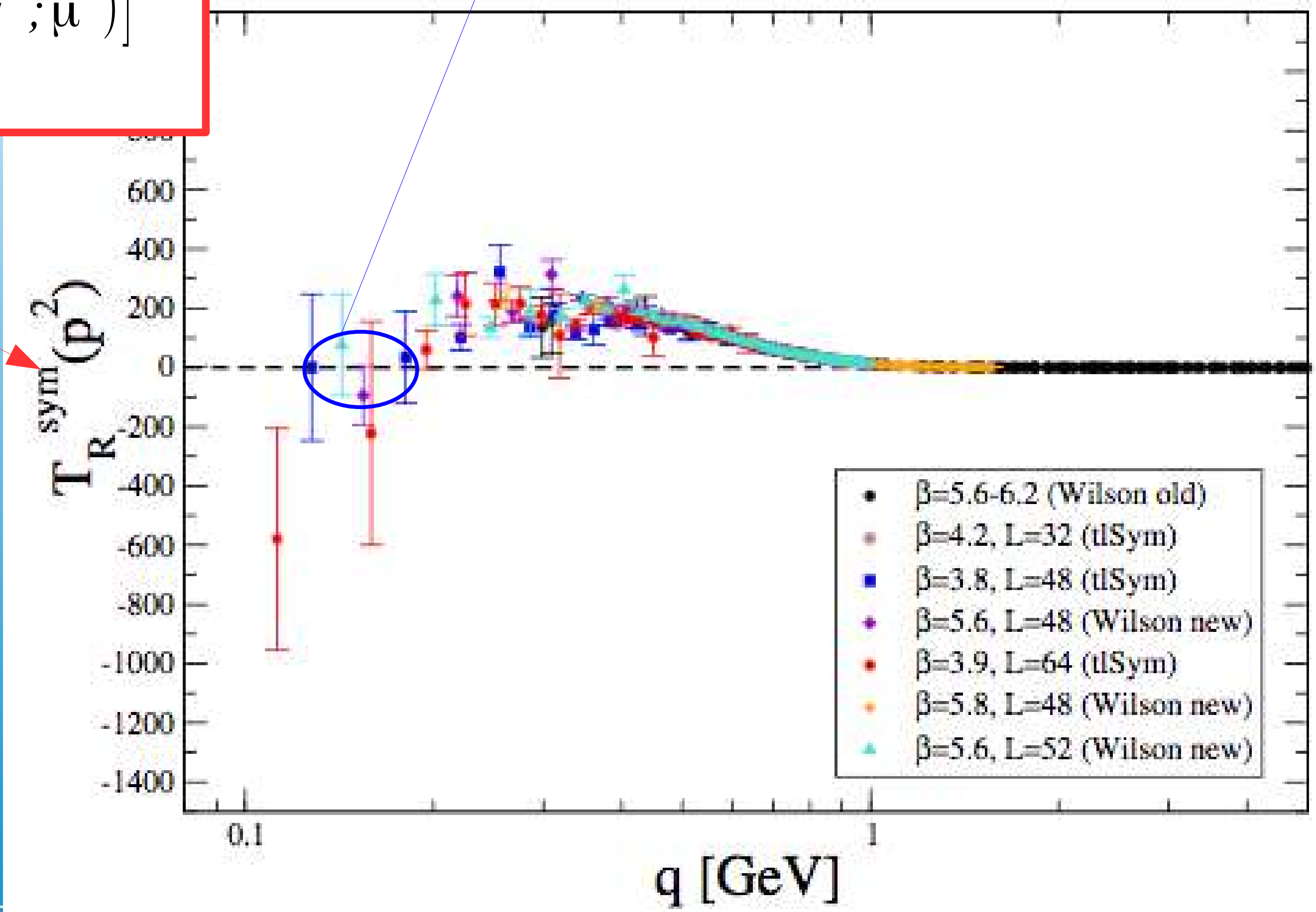

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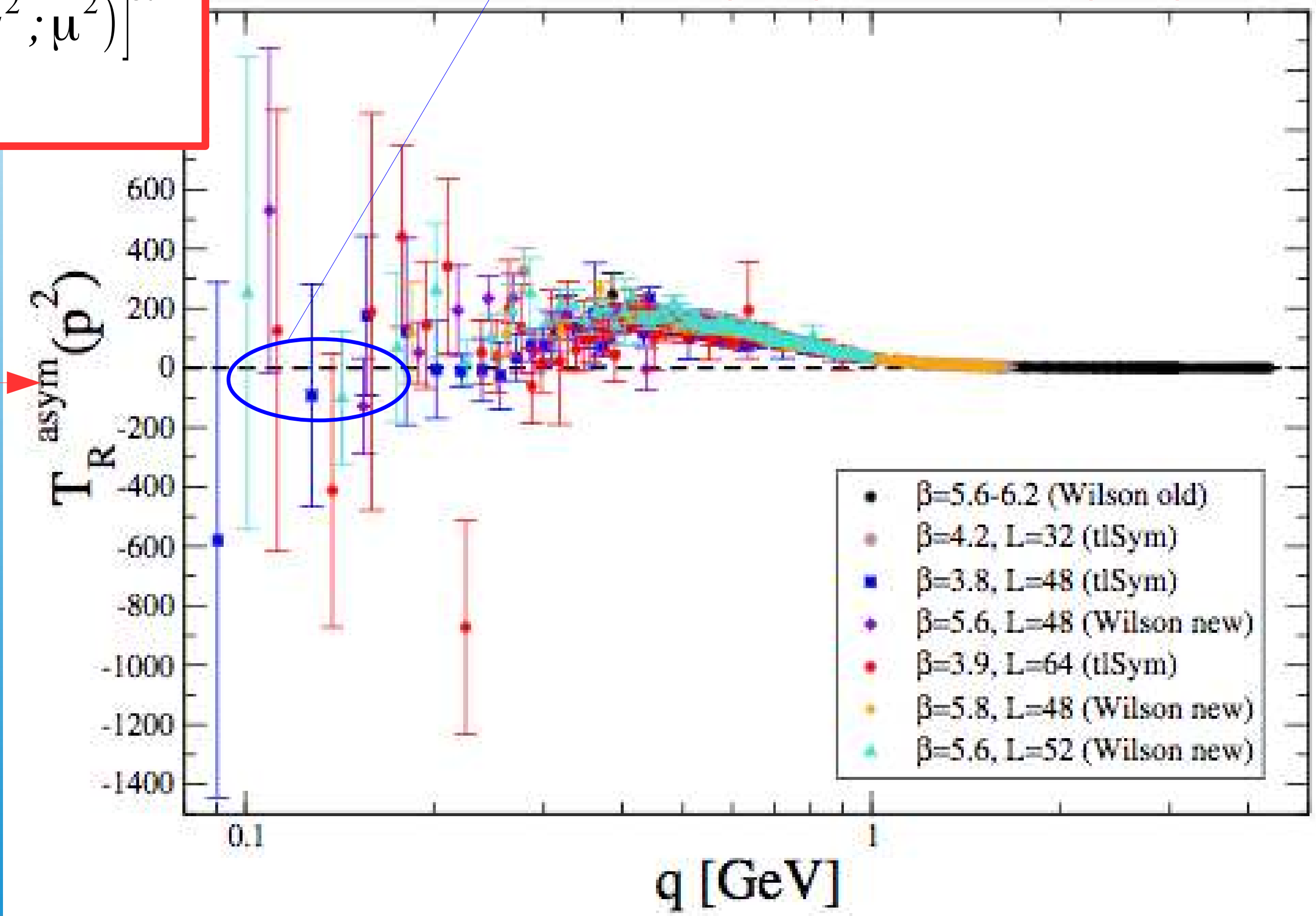
Zero-crossing?



Let's consider now the asymmetric case: the results are much noisier (surely because of the zero-momentum gluon field in the correlation function), although there appear to be strong indications for the happening of the zero-crossing.

The zero-crossing of the three-gluon vertex

$$g^{i}(\mu^2)\Gamma^{i}_{T,R}(q^2;\mu^2)=\frac{g^{i}(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

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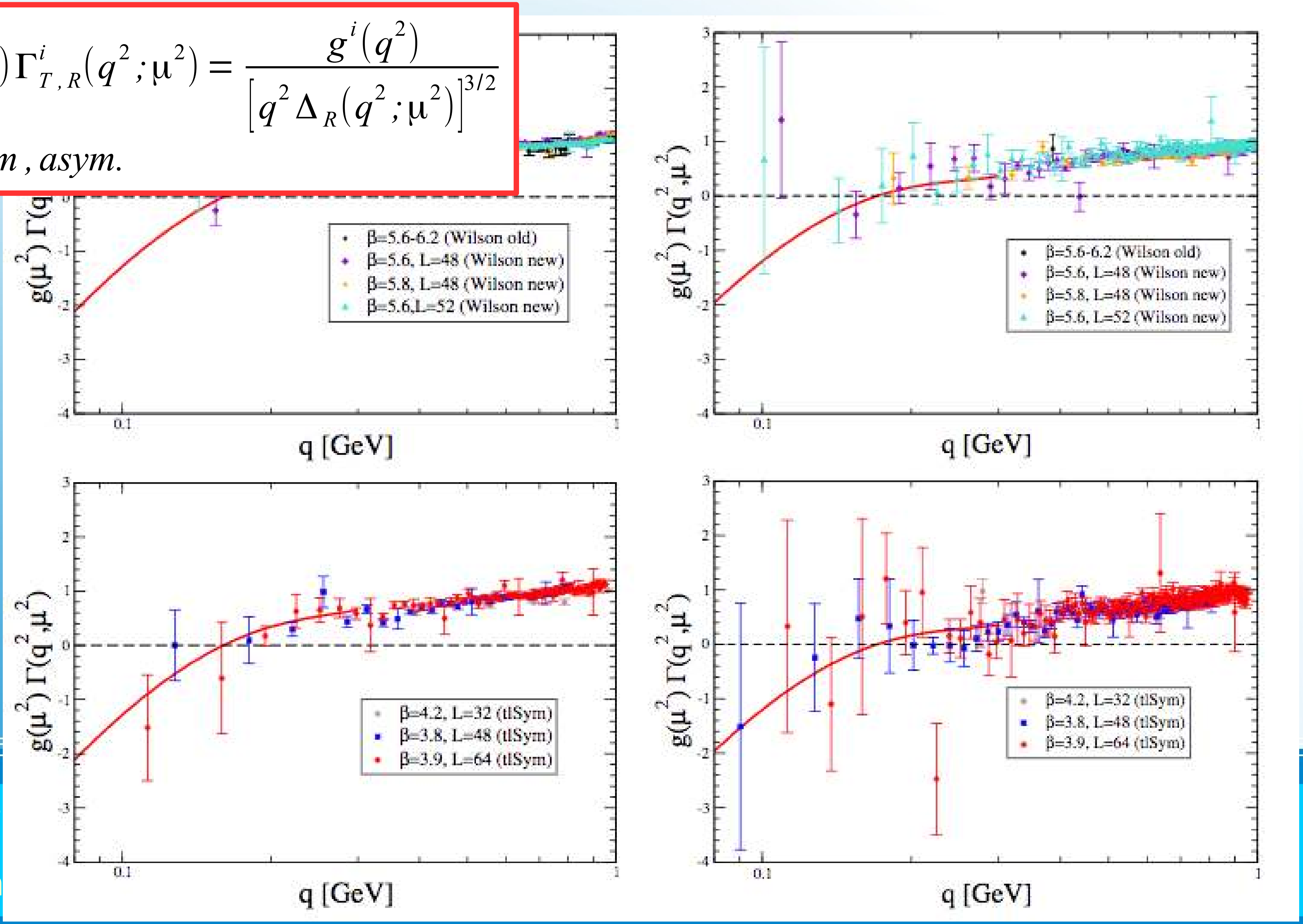

After le
crossin
asymm

The zero-crossing of the three-gluon vertex

$g^i(\mu^2$

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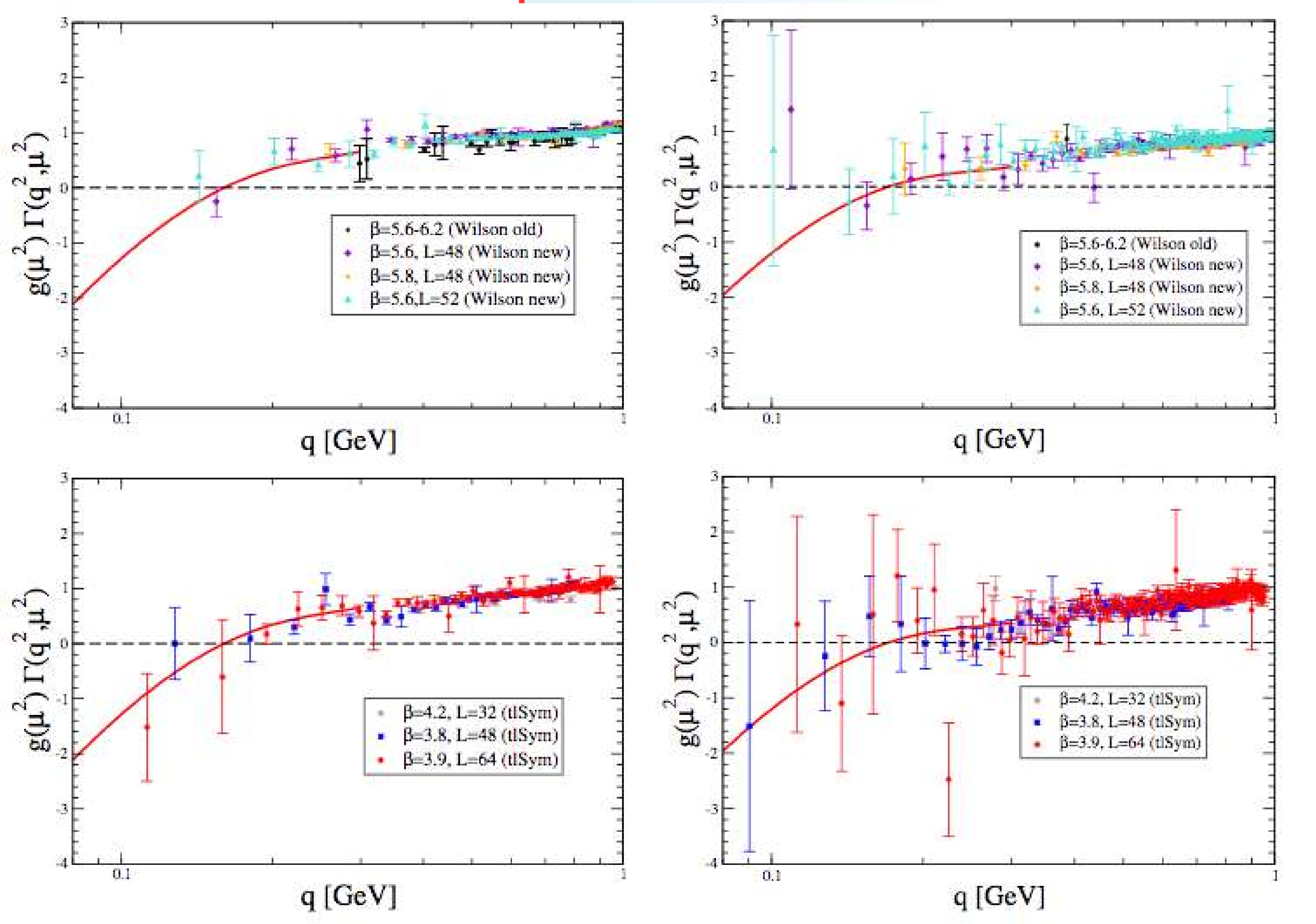

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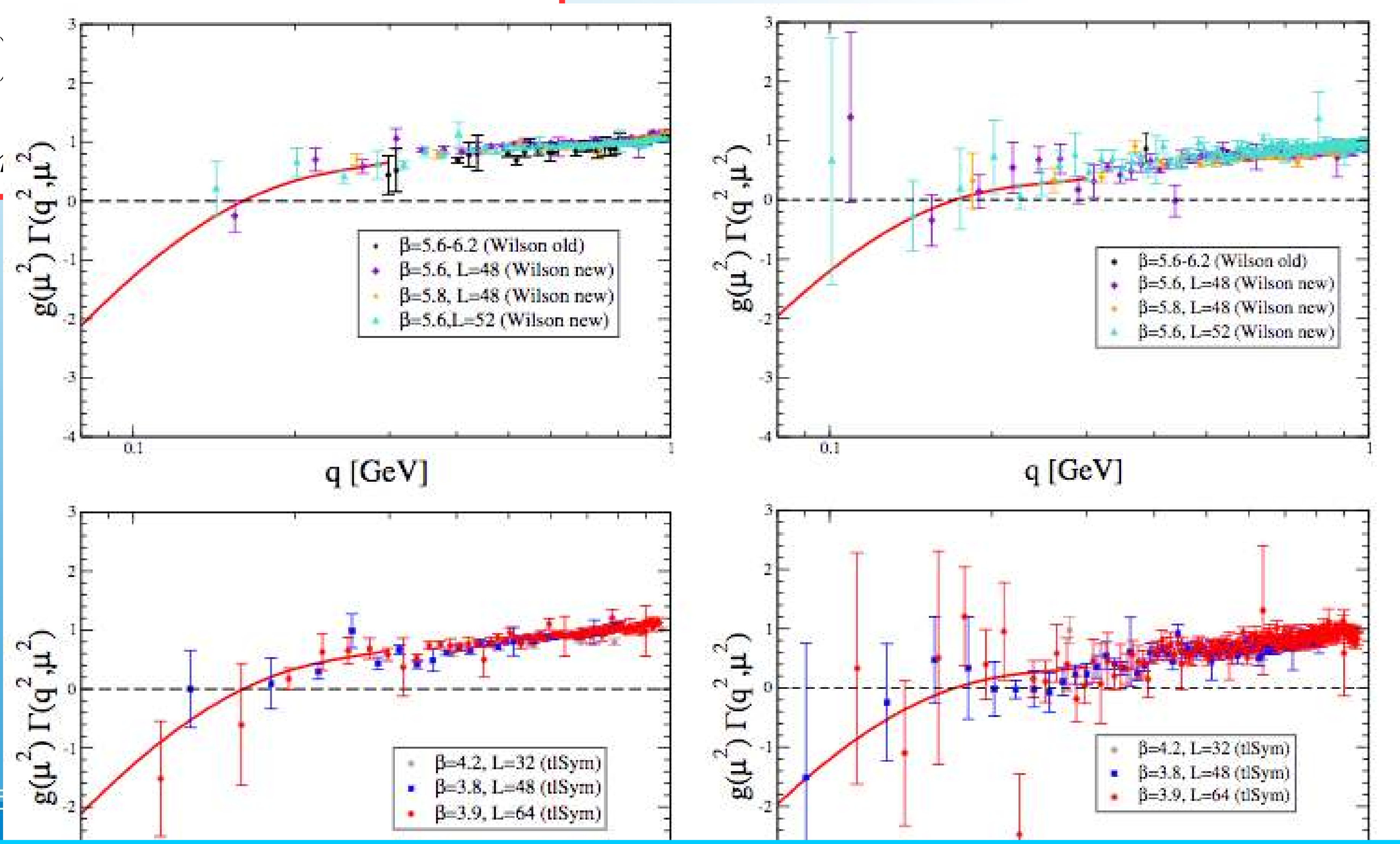
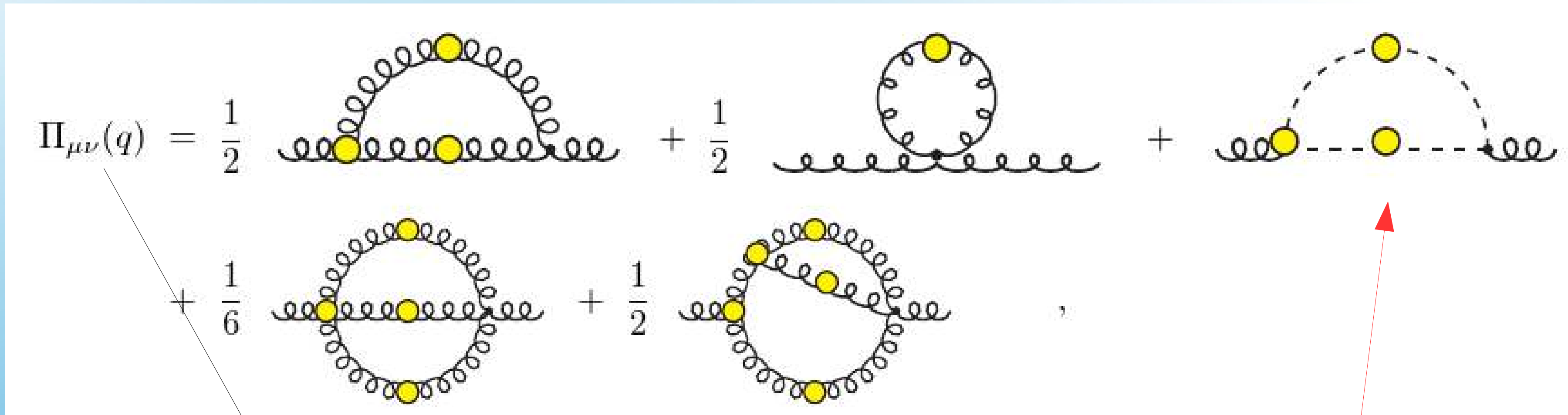


After leg amputation, the 1PI form factor for the tree-level tensor shows clearly the zero-crossing. Tthe trend is the same for both Wilson and tlSym actions and symmetric and asymmetric configurations.

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008

DSE-based explanation:



$$\Delta_R^{-1}(q^2;\mu^2) \underset{q^2\to 0}{=} q^2\left[a + b\log\frac{q^2+m^2}{\mu^2} + c\log\frac{q^2}{\mu^2}\right] + m^2,$$

$$\Pi_c(q^2) = \frac{g^2 C_A}{6} q^2 F(q^2) \int_k \frac{F(k^2)}{k^2(k+q)^2},$$

In PT-BFM truncation:

$$\Gamma_{T,R}^{i,(B)}(p^2;\mu^2) \underset{p^2/\mu^2 \ll 1}{\simeq} F_R(0;\mu^2)\frac{\partial}{\partial p^2}\Delta_R^{-1}(p^2;\mu^2) + \ldots$$

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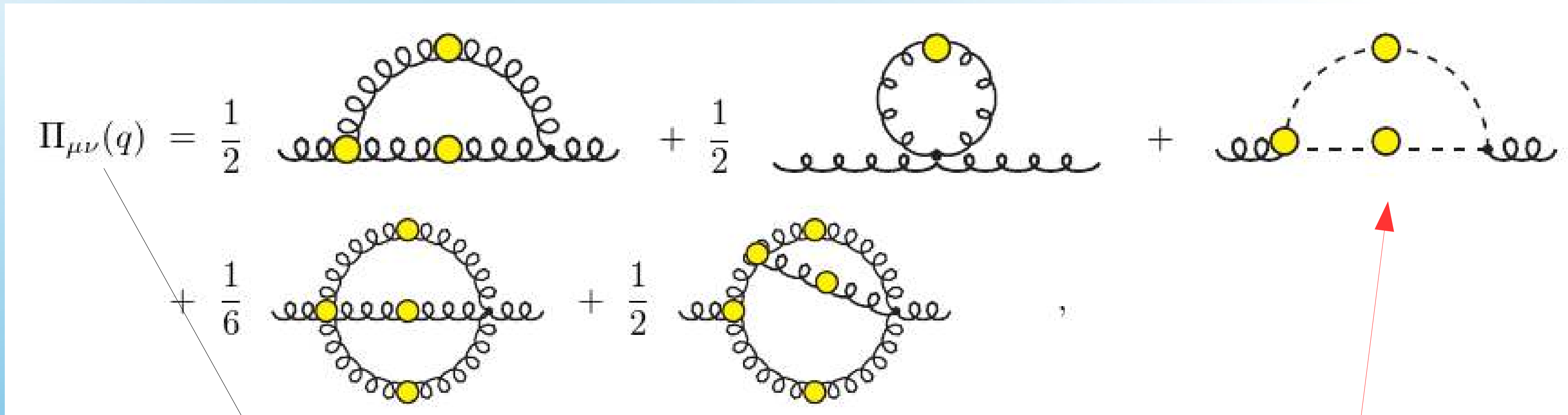

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In PT-BFM truncation:

$$\Gamma^{i,(B)}_{T,R}(p^2;\mu^2) \underset{p^2/\mu^2 \ll 1}{\simeq} F_R(0;\mu^2)\left(a + b\ln\frac{m^2}{\mu^2} + c\right) + c\, F_R(0;\mu^2)\ln\frac{p^2}{\mu^2} + \ldots$$

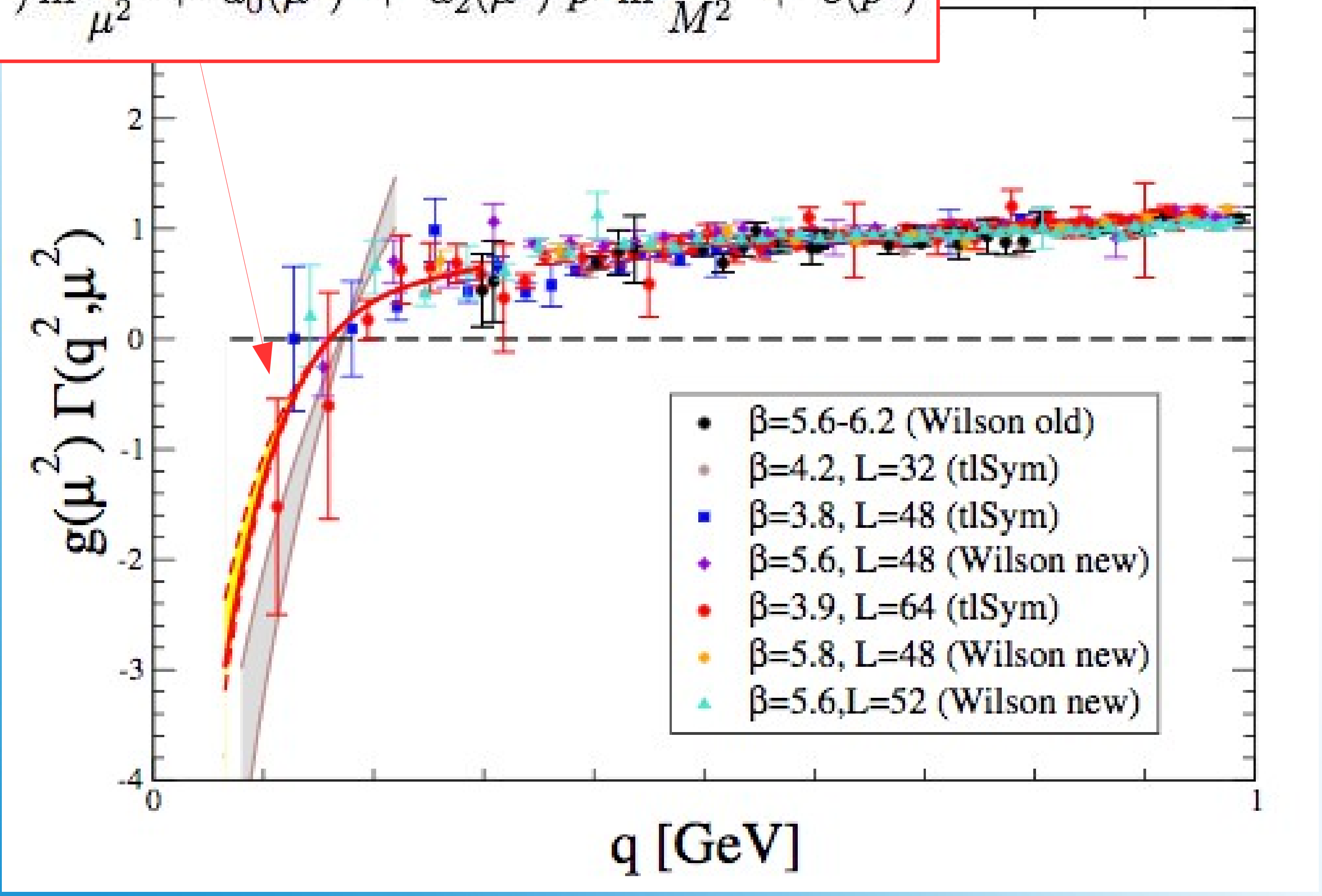
A logarithmic divergent contribution at vanishing momentum, pulling down the 1PI form factor and generating a zero crossing, can be understood with a DSE analysis.

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008
Ph. Boucaud et al.; PRD95(2017)114503

$$g^i_R(\mu^2)\Gamma^i_R(p^2;\mu^2) = a^i_{ln}(\mu^2)\ln\frac{p^2}{\mu^2} + a^i_0(\mu^2) + a^i_2(\mu^2)\,p^2\ln\frac{p^2}{M^2} + o(p^2)$$

i = symmetric



We can thus perform a fit, only over a deep IR domain, of our data to the DSE-based formula and describe the behaviour of the 1PI form factor.

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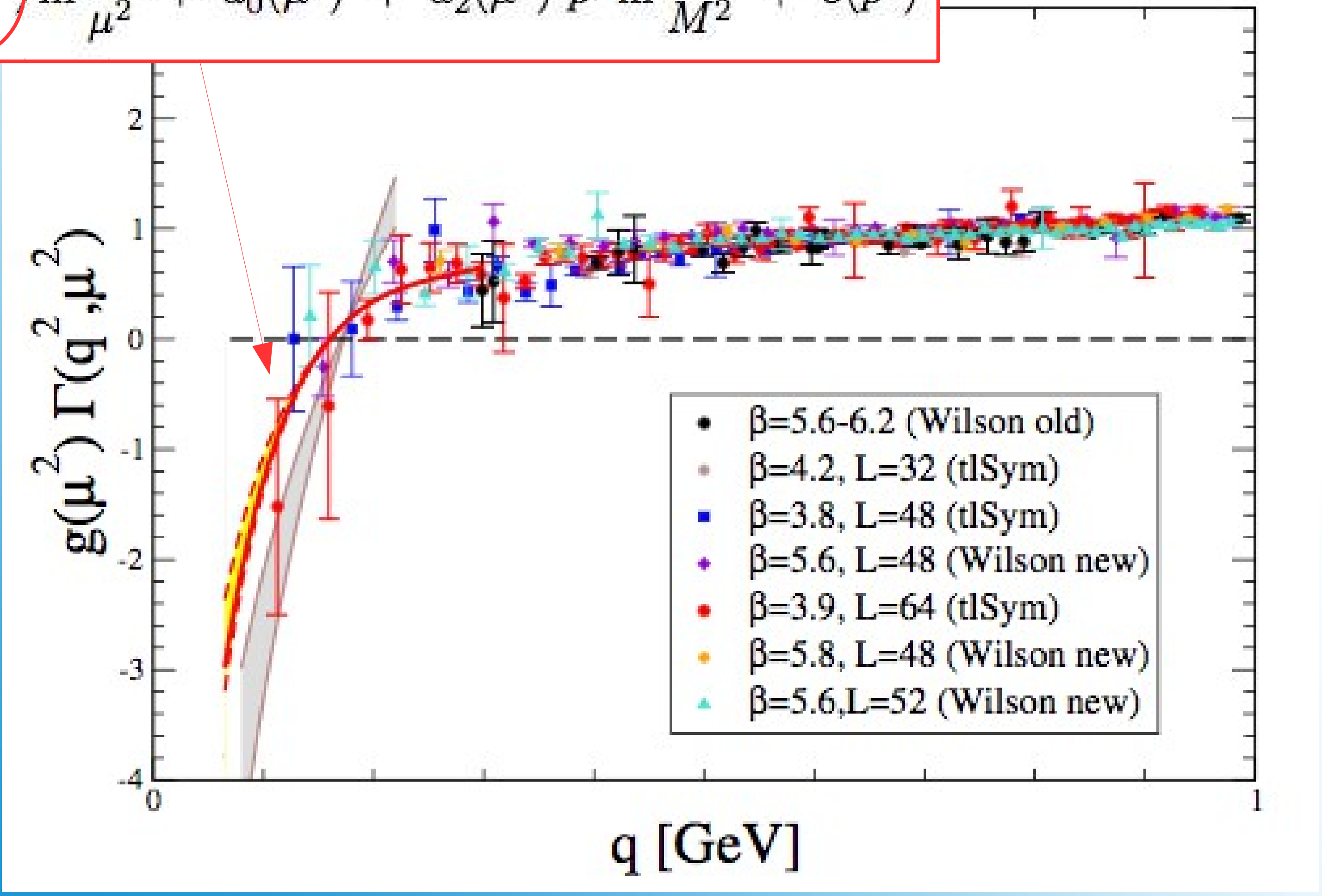
Consistent with direct large-volume lattice evaluations of the gluon and ghost two-point Green functions.



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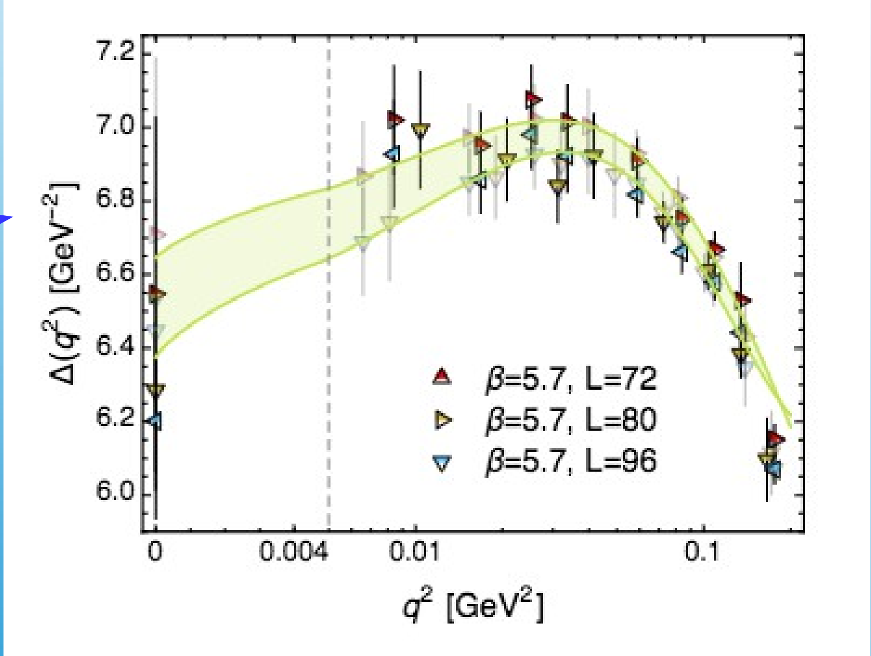

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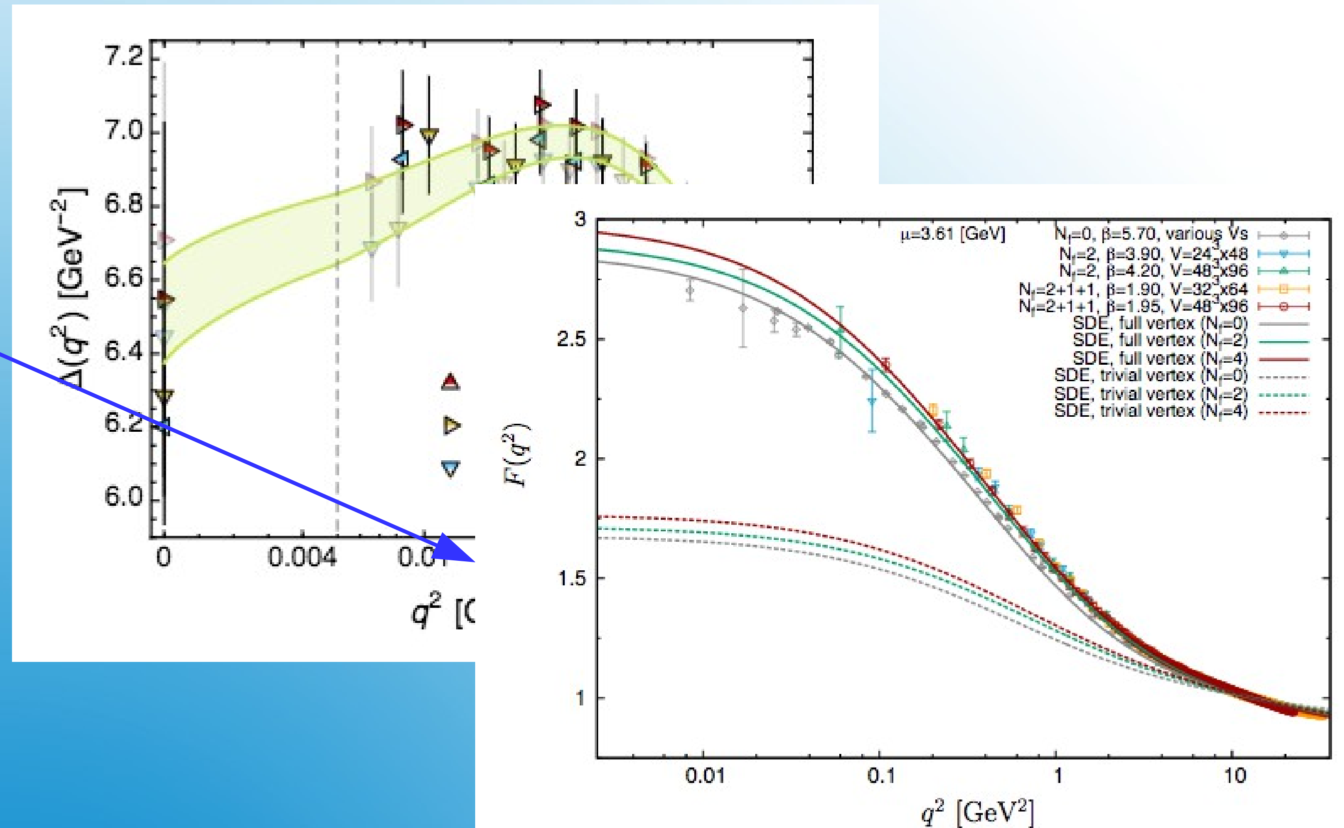

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i = asymmetric

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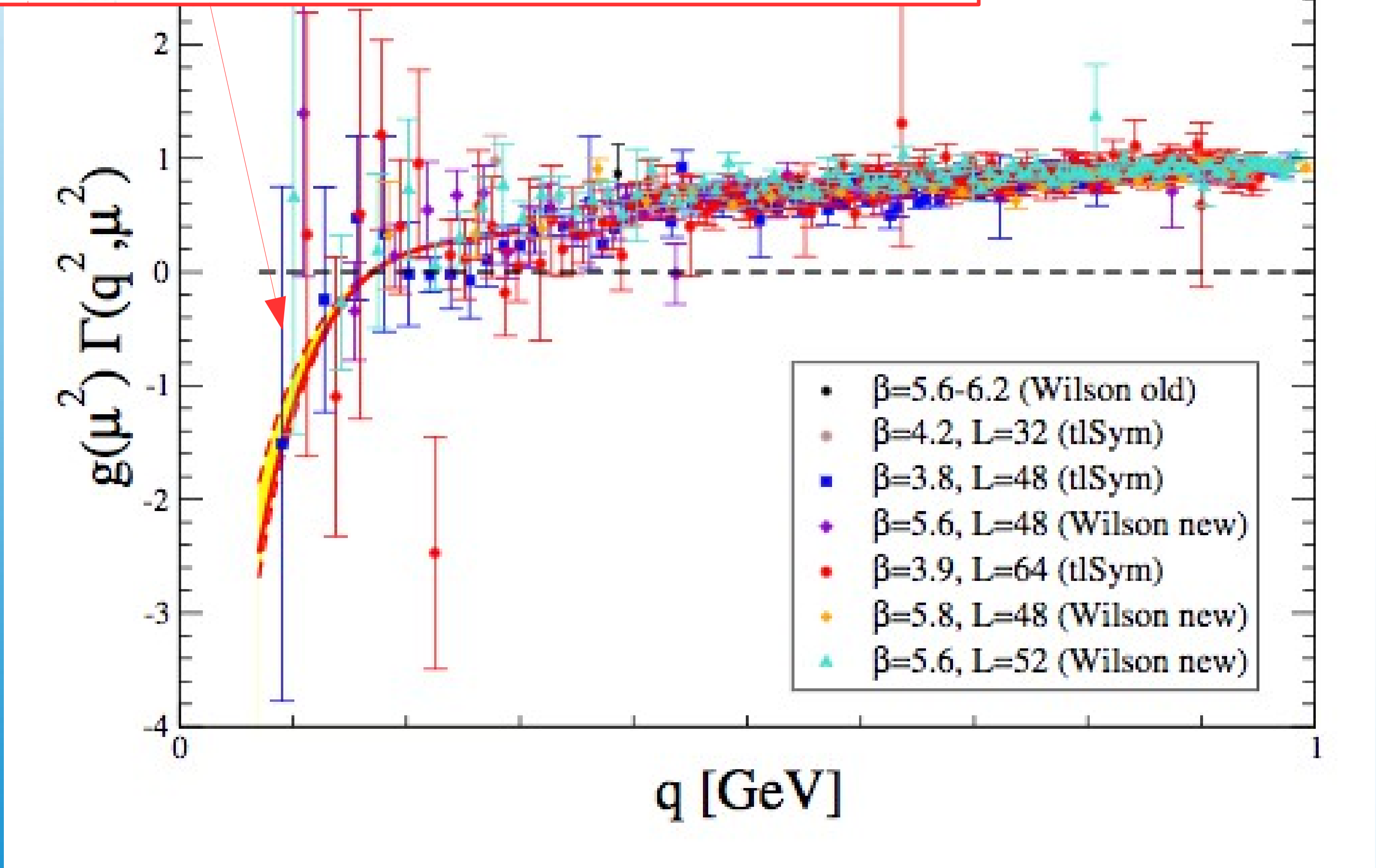

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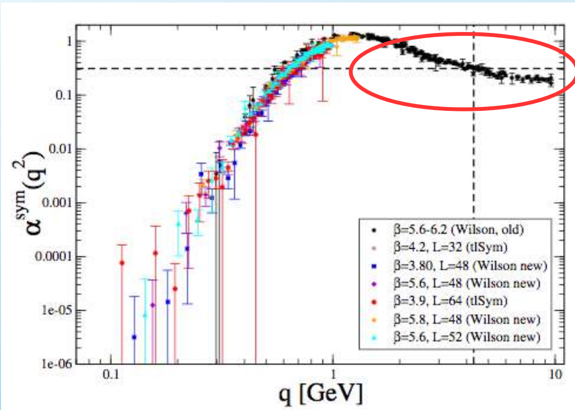
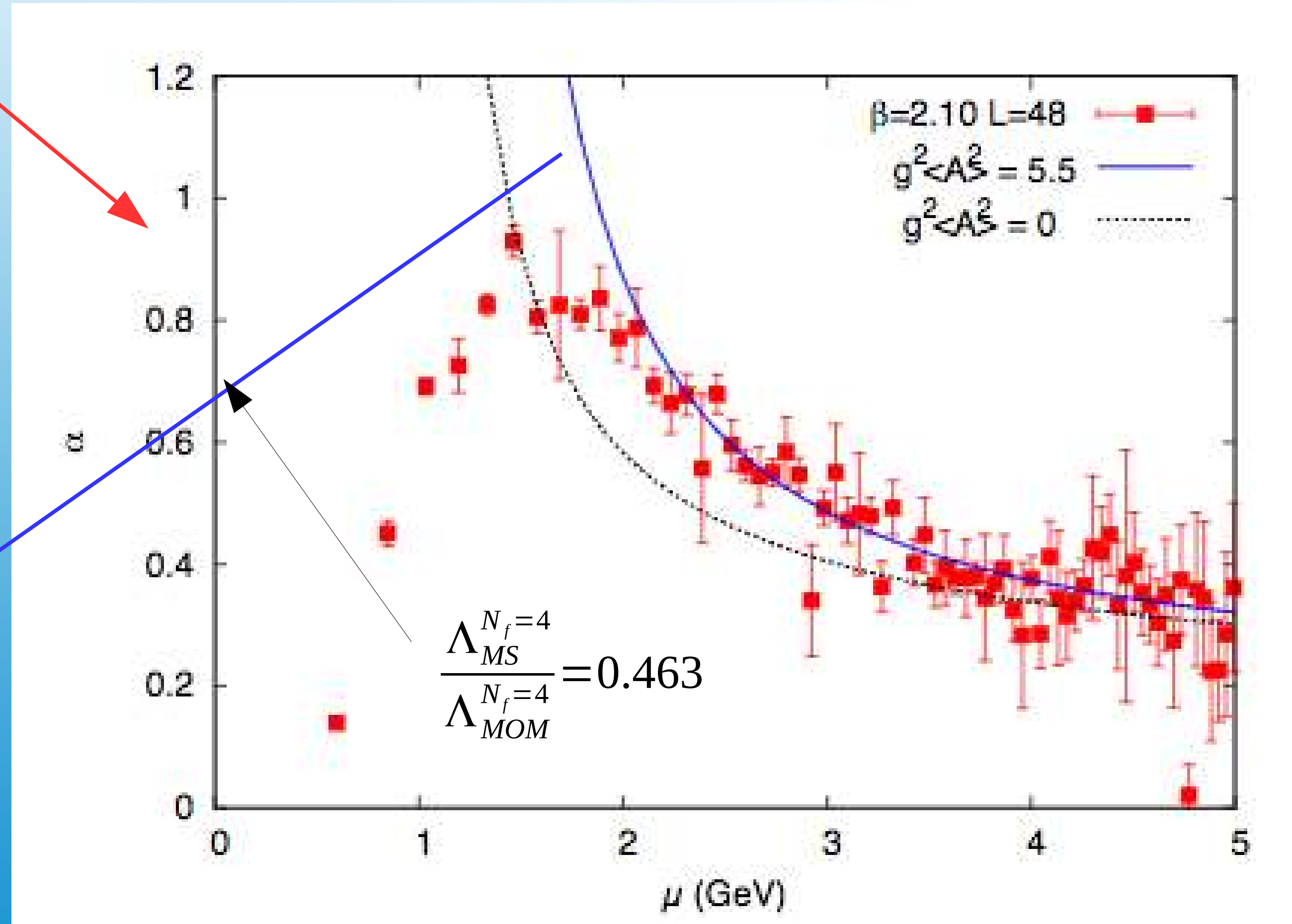
The three-gluon running coupling:



ETMC Nf=2+1+1



$$\Lambda_{MS}^{N_f=4}=314\,MeV$$

A final remark on some work in progress: the UV domain gives direct access to the strong running coupling in a particular scheme that can be properly translated to MS. Combining different Green's functions, a reliable prediction can be obtained!!!

Conclusions:

Next talk by F. De Soto!

- The three-gluon Green's function shows a feature at very low-momentum not fitting in the multi-instanton picture: the zero-crossing which can be explained as a soft quantum effect induced by the contribution of unprotected (by a mass) ghost-loops.

- The strong running coupling can be computed from the three-gluon Green's function and, combined with other Green's functions, may allow for a reliable prediction of Λ_{MS} .