Parton Distributions in the LHC Era

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Plan

1. PDFs from fitting experimental data
2. progress with quasi-PDF
3. other methods
4. parallel sessions [today - all afternoon]

1. PDF from DIS

$$k^\mu = k'^\mu + q^\mu$$
$$Q^2 = -q^2$$
$$\nu = p \cdot q, \quad x = Q^2/(2\nu)$$

$$\boxed{d\sigma = \frac{d^3k'}{|\mathbf{k}'|} \, \frac{1}{2s(Q^2 - m_N^2)} \, L^{\mu\nu}(k,k') W_{\mu\nu}(p,q)}$$

$$\begin{aligned} W_{\mu\nu}(p,q) &= \frac{1}{4\pi} \sum_X \langle p | j_\mu(0)^\dagger | X \rangle \langle X | j_\nu(0) | p \rangle (2\pi)^4 \delta\left(p_X - p - q\right) \\ &= \frac{1}{4\pi} \int d^4y \; e^{iq\cdot y} \; \langle p | j_\mu(y)^\dagger j_\nu(0) | p \rangle \end{aligned}$$

Structure functions and PDFs

$$W_{\mu\nu}(p,q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(Q^2,\nu) + \\ + \left(p_\mu - q_\mu \frac{\nu}{q^2}\right)\left(p_\nu - q_\nu \frac{\nu}{q^2}\right) F_2(Q^2,\nu)/\nu$$

Factorization:

$$\boxed{\begin{aligned} F_2(x,Q^2) &= x \sum_i \int \frac{dz}{z}\, C_i(z)\, f_i\left(\frac{x}{z},\mu^2\right) \\ &= x \sum_{\mathbf{i}} C_i(x) \otimes \mathbf{f_i}(x,\mu^2) \end{aligned}}$$

DGLAP evolution

- dependence on the factorization scale

$$\mu^2 \frac{d}{d\mu^2} f_i(x, \mu^2) = \sum_j P_{ij}\left(x, \alpha_s(\mu^2)\right) \otimes f_j(x, \mu^2)$$

- $P_{ij}\left(x, \alpha_s(\mu^2)\right)$: perturbative Altarelli-Parisi splitting functions

  $\hookrightarrow$ LO, NLO, NNLO PDFs

- Solution of the evolution equation:

$$\boxed{f_i(x, \mu^2) = \sum_j \Gamma_{ij}(x, \alpha_s, \alpha_s^0) \otimes f_j(x, \mu_0^2)}$$

PDFs from data: the non-singlet case

Simple example:

$$F_2^{\mathrm{NS}}(x,Q^2) = F_2^{\mathrm{p}}(x,Q^2) - F_2^{\mathrm{d}}(x,Q^2)$$
$$= C_{\mathrm{NS}}(y,Q^2) \otimes f_{\mathrm{NS}}\left(\frac{x}{y},Q^2\right)$$

where

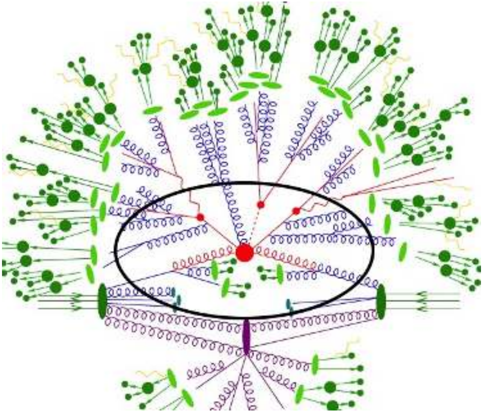
$$f_{\mathrm{NS}}(x,Q^2) = \left[\left(u(x,Q^2)+\bar{u}(x,Q^2)\right) - \left(d(x,Q^2)+\bar{d}(x,Q^2)\right)\right]$$

combining the evolution and the coefficient function

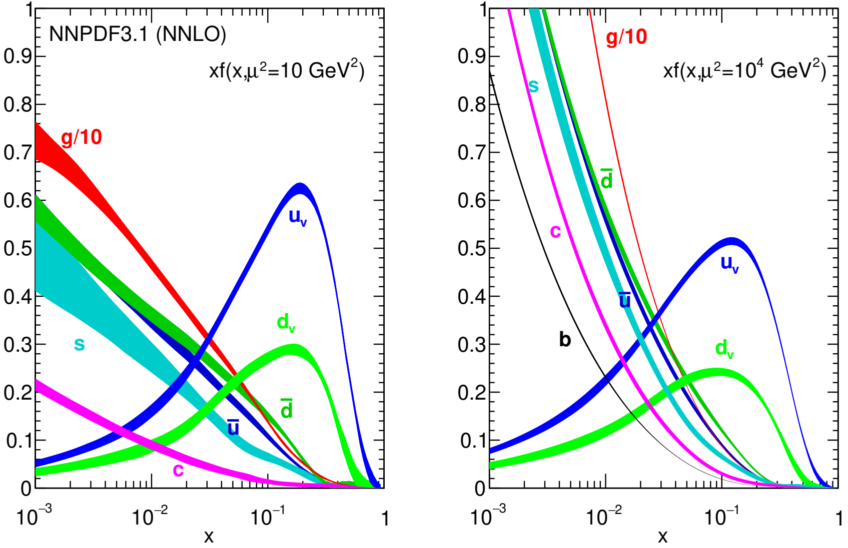
$$F_2^{\mathrm{NS}}(x,Q^2) = \int_x^1 \frac{dy}{y}\, K_{\mathrm{NS}}\left(y,\alpha_s\left(Q^2\right),\alpha_s\left(Q_0^2\right)\right)\, f_{\mathrm{NS}}\left(\frac{x}{y},Q_0^2\right)$$

Collider processes

$$\sigma(H_1 H_2 \to X) = \sum_{i,j} \int dx_1 dx_2 \; f_{i/H_1}(x_1, \mu^2) f_{j/H_2}(x_2, \mu^2) \times \\ \times \hat{\sigma}_{ij\to X}(x_1 x_2 s, \mu^2, \mu_R^2)$$

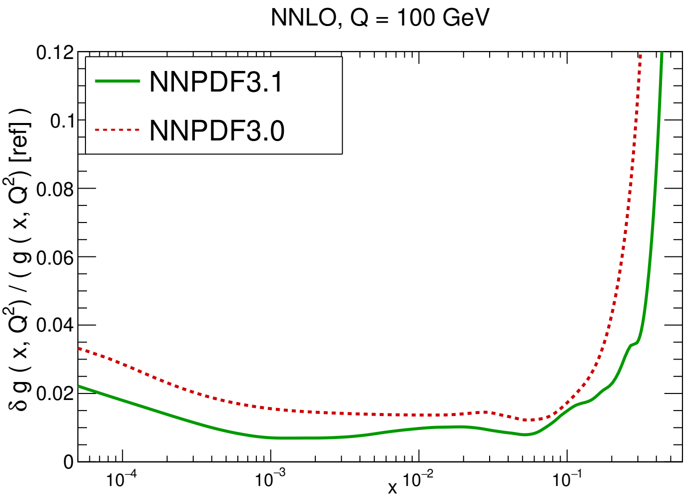


NNPDF3.1

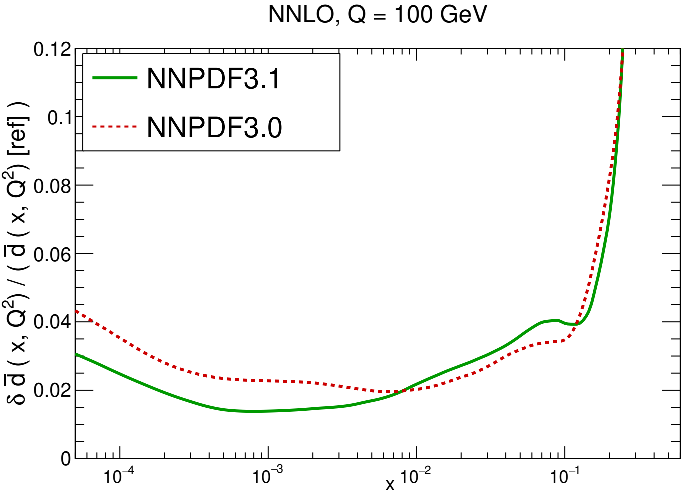


NNPDF3.1 [1706.00428]

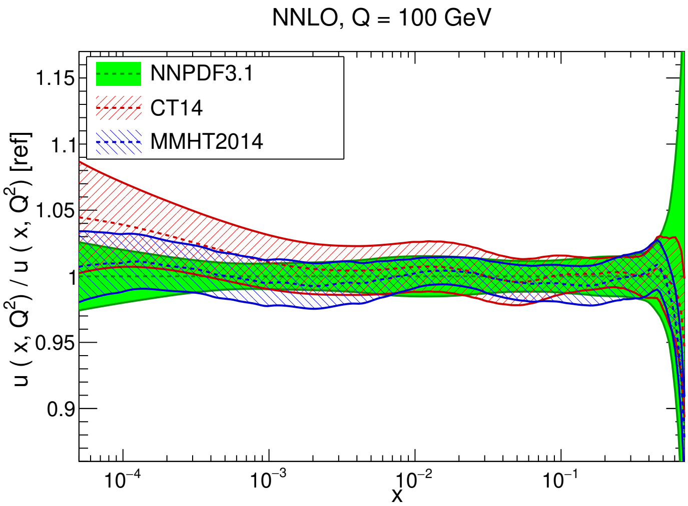
Gluon distribution - 1



Quark distribution

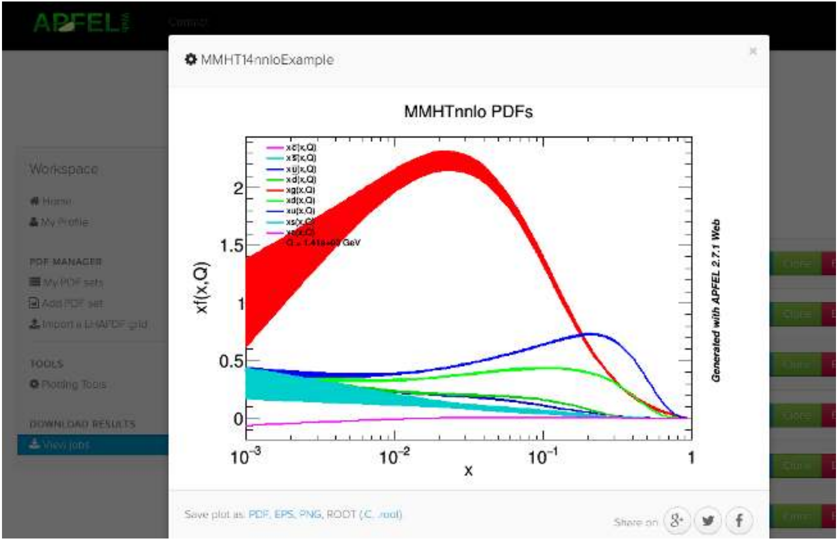


Global fits



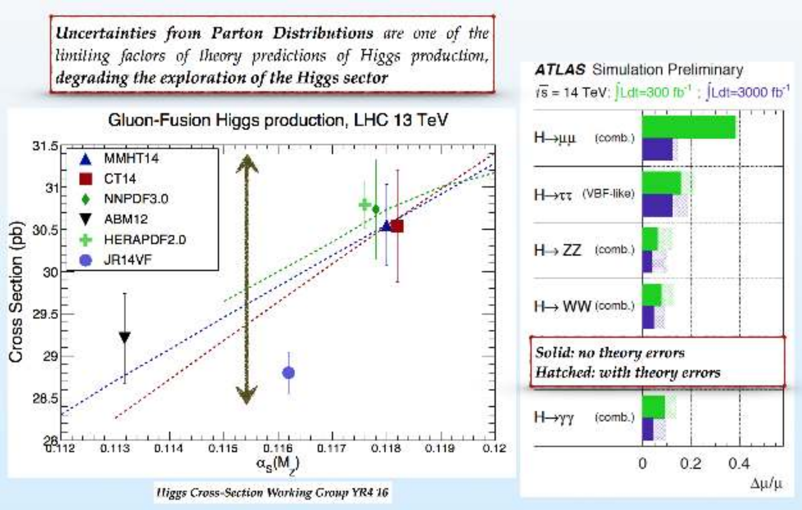
NNLO, Q = 100 GeV
NNPDF3.1
CT14
MMHT2014
u (x, Q^2) / u (x, Q^2) [ref]
1.15
1.1
1.05
1
0.95
0.9
10^{-4}
10^{-3}
10^{-2}
10^{-1}
x

LHAPDF distributions ABM, CT, HERAPDF, JR, MMHT, NNPDF



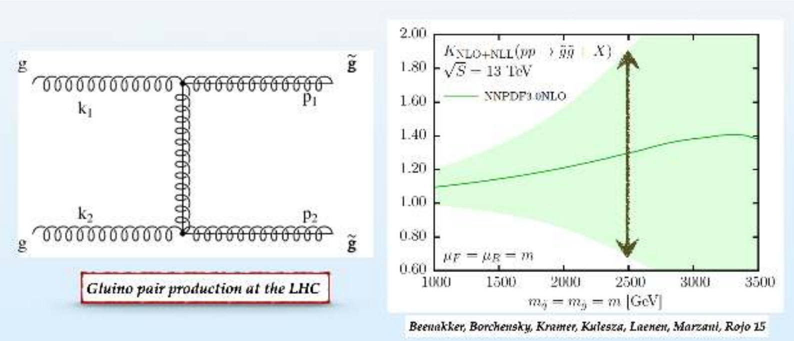
[http://apfel.mi.infn.it]

PDF uncertainties for Higgs physics



[plot courtesy of J Rojo]

PDF uncertainties for new physics



Beenakker et al [1510.00375]

Summary

- central-x region: error at % level at $Q = 100\ \mathrm{GeV}$
- LHC data improve the fits
- LHC Run-2 not included in global fits yet - stat error will improve
- current global fits are robust
- reliable uncertainties needed for precision physics & discovery
- large-x, small-x regions still have large errors
- target for lattice simulations

2. PDFs and Quasi-PDFs

$$\boxed{\mathcal{M}_i(\zeta, P) = \langle P|\bar{\psi}(\zeta)\Gamma_i \, \mathrm{P}\exp\left(-ig\int_0^\zeta d\eta \, A(\eta)\right)\psi(0)|P\rangle}$$

light-cone PDF – $\zeta = (0, y^-, \vec{0}_\perp)$:

$$f(x, \mu) = \int \frac{dy^-}{4\pi} \, e^{-i(xP^+)y^-} \, \mathcal{M}^+(y^-, P^+)$$

quasi-PDF, time-independent quantity – $\zeta = (0, 0, 0, z)$:

$$q(x, \mu, M_N, P_z) = \int \frac{dz}{4\pi} \, e^{-i(xP_z)z} \, \mathcal{M}^z(z, P_z) \stackrel{P_z \to \infty}{\longrightarrow} f(x, \mu)$$

From quasi-PDFs to PDFs

Extracting PDFs from lattice simulations:

- renormalization of the lattice operator
  - ◇ RI/MOM prescription
  - ◇ matching to $\overline{\mathrm{MS}}$
  - ◇ trace operators and power divergencies
- Euclidean to Minkowski space
- factorization theorem for the renormalized quasi-PDF
- gradient flow

Lattice perturbation theory

An instructive example (31/5/17) [1705.11193]

- $O_i(z) = \bar{\psi}(z)\Gamma_i W(z,0)\psi(0)$
- renormalization pattern: $O^Y_{R,i}(z) = Z^{X,Y}_{ij}(z) O^X_j(z)$
- one-loop computations: $Z = 1 + g^2 C_F/(16\pi^2) Z^{(1)} + \ldots$

$$Z^{\mathrm{LR},\overline{\mathrm{MS}}} = 1 + \frac{g^2 C_F}{16\pi^2}\left(\ldots + e_2 \frac{|z|}{a} + \ldots - 3\log(a^2\mu^2)\right)$$

$$Z_{12}^{\mathrm{LR},\overline{\mathrm{MS}}} = 0 + \frac{g^2 C_F}{16\pi^2}\left(\ldots\right)$$

- one-loop matching:

$$\mathcal{C}^{\overline{\mathrm{MS}},\mathrm{RI}} = (Z^{\mathrm{LR},\overline{\mathrm{MS}}})^{-1} \cdot (Z^{\mathrm{LR},\mathrm{RI}}) = (Z^{\mathrm{DR},\overline{\mathrm{MS}}})^{-1} \cdot (Z^{\mathrm{DR},\mathrm{RI}})$$

RI/MOM prescription

renormalization condition:

$$O_{R,i}(z) = Z_{ij}(z) O_j(z)$$
$$\Lambda(p,z) = S(p)^{-1} \left\langle O_i(z) \psi(-p) \bar{\psi}(p) \right\rangle S(p)^{-1}$$
$$Z_q^{-1} \frac{1}{12} \mathrm{Tr} \left[\Lambda_{R,i}(p,z) (\Lambda_j^{\mathrm{tree}}(p,z))^{-1} \right] \Big|_{p^2=\mu^2} = \delta_{ij}$$

linear divergence from the Wilson line:

$$Z(z) = \mathcal{Z}(z) e^{\delta m |z|/a - c|z|}$$

automatically subtracted in this framework

ETMC (1/6/17) [1706.00265], J-W Chen et al (5/6/17) [1706.01295]

Euclidean/Minkowski

matrix element at $t = 0$ is agnostic about space-time signature

$$\mathcal{M}_z(z, P_z) = \langle P_z | O_z(z) | P_z \rangle$$

computed from different correlators in Euclidean and Minkowski

$$\langle N(\tau', P_z) O_z(z) N(\tau, P_z) \rangle = \mathcal{Z}\mathcal{Z}' \mathcal{M}_z(z, P_z) e^{-\omega_P(\tau' - \tau)} + \dots$$

$$\int d^D y \, d^D y' \, e^{i(P(y'-y))} \langle T N(y') O_z(z) N(y) \rangle \sim$$

$$\sim \frac{i\mathcal{Z}'}{P^2 - m^2} \mathcal{M}_z(z, P_z) \frac{i\mathcal{Z}}{P^2 - m^2}$$

Carlson et al [1702.05775], Briceno et al [1703.06072]

Factorization/matching

define a UV finite quasi-PDF - independent of the lattice action used

$$q(x, \mu, M_N, P_z) = \int \frac{dz}{4\pi}\, e^{-i(xP_z)z}\, \mathcal{M}_R^z(z, \mu, P_z)$$
$$= \lim_{a\to 0} \int \frac{dz}{4\pi}\, e^{-i(xP_z)z}\, Z(z, \mu a)\mathcal{M}^z(z, P_z, a)$$

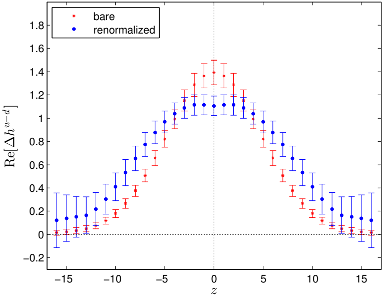
factorization/matching

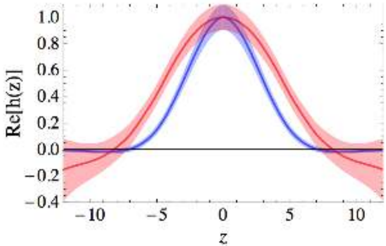
$$q(x, \mu, M_N, P_z) = C_Q\left(x, \frac{P_z}{\mu}\right) \otimes f(x, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{P_z^2}, \frac{M_N^2}{P_z^2}\right)$$

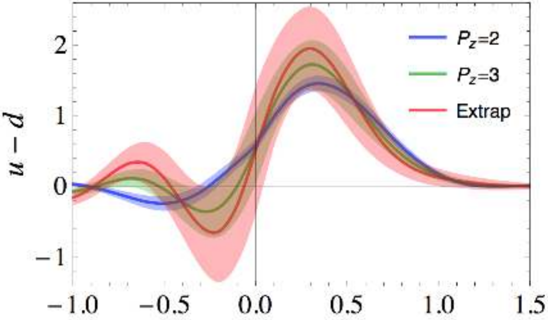
$$\Lambda_{\rm QCD}, M_N \ll P_z \ll \mu, a^{-1}$$

Ji, Ji et al, Ma & Qiu, Ishikawa et al

First results







Trace operators

Mellin transform:

$$A(n) = \int dx\, x^{n-1} q(x, P_z)$$

given by matrix elements of local operators:

$$\langle P|\phi(0)\partial^{\nu_1} \ldots \partial^{\nu_n}\phi(0)|P\rangle$$

mixing under renormalization with trace operators

$$A(n)|_{\mathrm{trace}} = 1/(a^2 P_z^2)$$

Are there any divergences left after inverse Mellin transform?

Rossi & Testa (14/6/17) [1706.04428]

Gradient flow

$$b^s\left(n,\sqrt{\tau}P_z,\frac{\Lambda_{\mathrm{QCD}}}{P_z},\frac{M_N}{P_z}\right)=\int dx\,x^{n-1}q^s\left(x,\sqrt{\tau}P_z,\frac{\Lambda_{\mathrm{QCD}}}{P_z},\frac{M_N}{P_z}\right)$$

$$=C_{\mathrm{GF}}(n,\sqrt{\tau}\mu,\sqrt{\tau}P_z)a(n,\mu)+\mathcal{O}\left(\sqrt{\tau}\Lambda_{\mathrm{QCD}},\frac{\Lambda^2_{\mathrm{QCD}}}{P_z^2},\frac{M_N^2}{P_z^2}\right)$$

$$\Lambda_{\mathrm{QCD}},M_N\ll P_z\ll 1/\sqrt{\tau}$$

$$\boxed{q^s\left(x,\sqrt{\tau}P_z,\frac{\Lambda_{\mathrm{QCD}}}{P_z},\frac{M_N}{P_z}\right)=\int\frac{d\xi}{\xi}\,\tilde{Z}\left(\frac{x}{\xi},\sqrt{\tau}\mu,\sqrt{\tau}P_z\right)f(\xi,\mu)}$$

Monahan & Orginos [1612.01584]

3. Auxiliary field method

$$\langle \zeta(z+\lambda n)\bar{\zeta}(z)\rangle_\zeta = \theta(\lambda) W(z+\lambda n, z) e^{-m\lambda}$$

Bilinear $\phi(z) = \bar{\zeta}(z)\psi(z)$

$$O_i(z) = \langle \bar{\phi}(z)\Gamma_i\phi(0)\rangle_\zeta$$

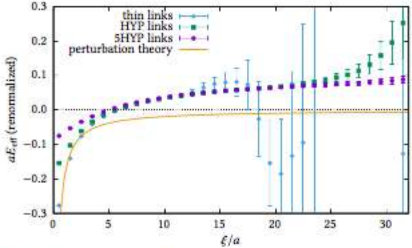
In Landau gauge, compute

$$S_\zeta(\xi) = \langle W(x+\xi n, x)\rangle_{\mathrm{QCD}}$$

and take its effective energy

$$E_{\mathrm{eff}}(\xi) \equiv -\frac{d}{d\xi}\log \mathrm{Tr}\, S_\zeta(\xi),$$

which renormalizes as $E^R_{\mathrm{eff}}(\xi) = m + E_{\mathrm{eff}}(\xi)$.

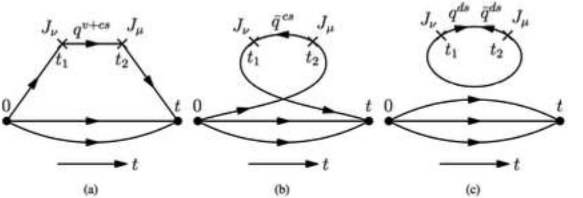


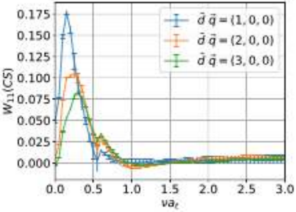
Match thin links with 1-loop PT at small ξ.
Then match thin with smeared at larger ξ.

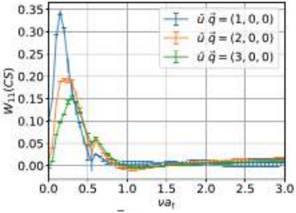
[Green, Jansen, Steffens - poster]

Four-point functions

$$G = \sum_{\vec{x}_f} e^{-i\vec{p}\cdot\vec{x}_f} \sum_{\vec{x}_2 \vec{x}_1} e^{-i\vec{q}\cdot(\vec{x}_2-\vec{x}_1)} \langle \chi_N(\vec{x}_f, t_f) J_\mu(\vec{x}_2, t_2) J_\nu(\vec{x}_1, t_1) \bar{\chi}_N(\vec{0}, t_0) \rangle.$$







[K-F Liu - poster]

Feynman-Hellman

Simulate with modified action: $\mathcal{L} \mapsto \mathcal{L} + \lambda \mathcal{J}_3$

$$T_{\mu\nu}(p,q) = \frac{1}{4\pi} \int d^4y \; e^{iq\cdot y} \; \langle p | T j_\mu(x)^\dagger j_\nu(0) | p \rangle$$

$$T_{33}(p,q) = -2E_\lambda(p,q) \; \frac{\partial^2}{\partial\lambda^2} E_\lambda(p,q) \Big|_{\lambda=0}$$

Compute $t_i = T_{33}(\omega_i)$, two recipes to extract PDFs:

$$t_i = 4\omega_i^{2j} \mu_j$$

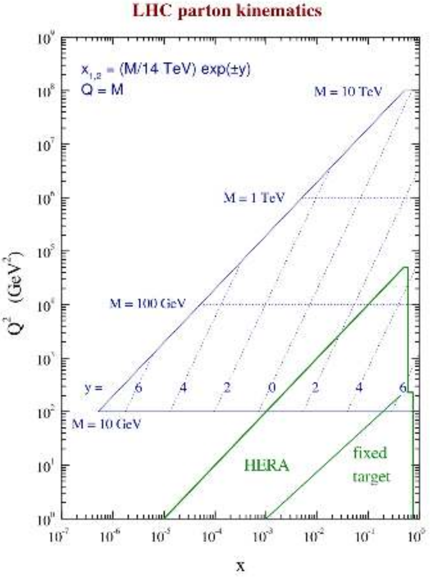
$$t_i = \epsilon \sum_j K_{ij} f_j$$

Chambers et al [1703.01153]

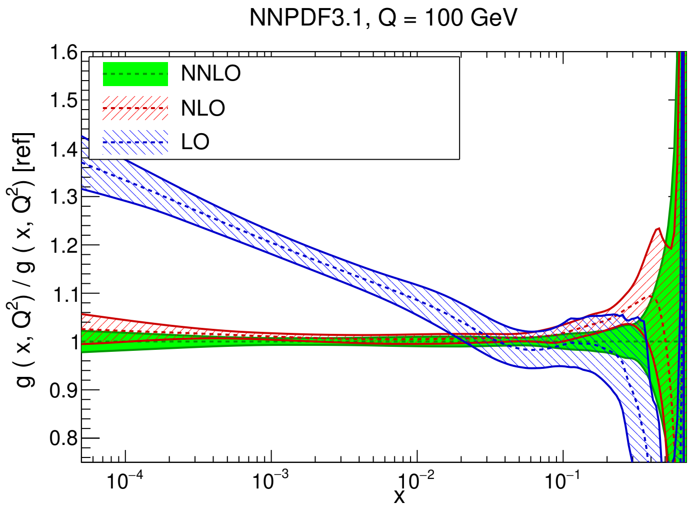
Outlook

- precision determinations of α_s and PDFs needed for LHC
- kinematic regions not constrained by data - opportunities for lattice studies?
- quasi-PDF: renormalization and matching to MS scheme - issues to be clarified
- what is actually being computed?
- other methods are being explored — Orginos et al (18/6/17) [1706.05373]
- aim for % precision in order to have an impact on phenomenology

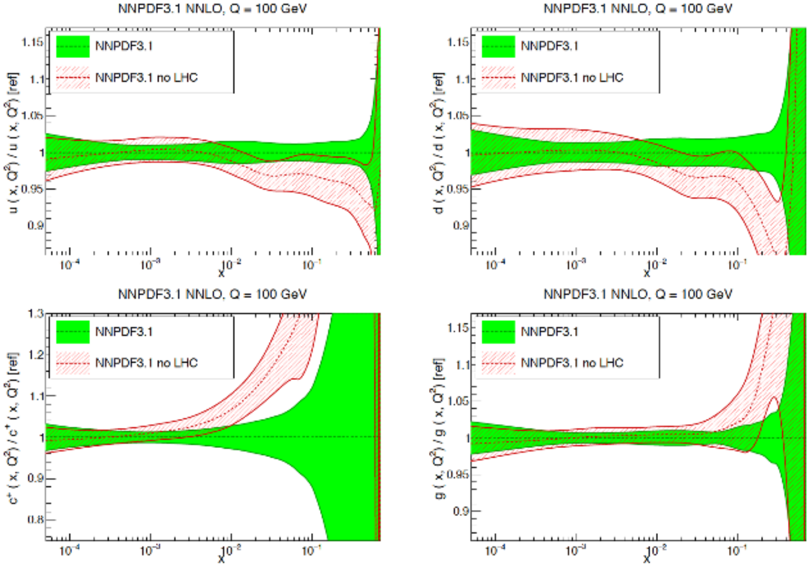
Kinematical range



Gluon distribution - 2



Impact of LHC data



Moments

Mellin transform:

$$a(n, \mu) = \int dx \, x^{n-1} f(x, \mu)$$

$a(n, \mu)$ related to the matrix elements of local operators:

$$\langle P | \mathcal{O}^{\{\nu_1 \ldots \nu_n\}} | P \rangle = 2a(n, \mu) P^{\nu_1} \ldots P^{\nu_n} - \mathrm{traces}$$

twist-2 operators

$$\mathcal{O}^{\{\nu_1 \ldots \nu_n\}} = \bar{\psi}(0) \gamma^{\nu_1} D^{\nu_2} \ldots D^{\nu_n} \psi(0)$$