Order parameters and color-flavor center symmetry in QCD

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Motivation

Defining order parameters in QCD is notoriously hard!

$$\langle \mathrm{tr}\,\Omega\rangle = \langle \mathrm{tr}\,\mathcal{P}\,e^{i\int_0^L dx_4 A_4}\rangle\,, \qquad \langle \sum_a \bar{q}_a q_a\rangle$$

These are not order parameters in QCD: no exact center or chiral symmetries.

Common lore: No proper order parameters in QCD.*

Our work: they do exist when d = gcd(n_F, N) > 1 and quarks have identical masses, due to "color-flavor-center" (CFC) symmetry. Useful to map T, μ, m, n_F/N phase diagram!

* which are non-trivial at zero baryon density.

CFC Symmetry

No standard center symmetry with periodic quark BCs, so we take flavor-center-symmetric BCs on spatial x_1:

$$q_a(x_1{+}L) = \mathcal{U}^{ab}\, q_b(x_1)$$

$$\mathcal{U} = \nu^{0 \text{ or } 1/2}\,\mathrm{diag}(1,\nu,\cdots,\nu^{n_f-1})\,, \quad \nu = e^{2\pi i/n_f}$$

Z_d CFC symmetry: color-center transformations together with cyclic flavor permutations keep path integral invariant.

Order parameters: Polyakov loops and local operators

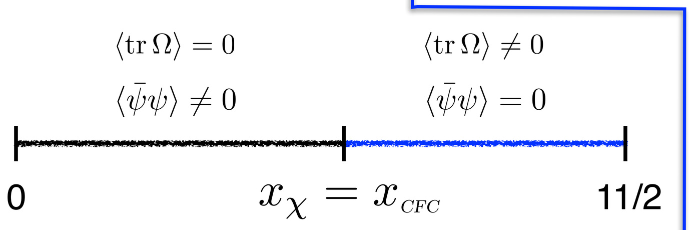
$$\langle \mathrm{tr}\,\Omega\rangle = \langle \mathrm{tr}\,\mathcal{P}\,e^{i\int_0^L dx_1 A_1}\rangle\,, \quad \mathcal{O}_\Gamma^{(p)} = \sum_{a=1}^{n_f} \nu^{ap}\,\bar{q}_a \Gamma q_a$$

$$\mathrm{tr}\,\Omega^p \;\to\; e^{2\pi i p/d}\,\mathrm{tr}\,\Omega^p\,, \quad \mathcal{O}_\Gamma^{(p)} \;\to\; \nu^{-n_f p/d}\,\mathcal{O}_\Gamma^{(p)}$$

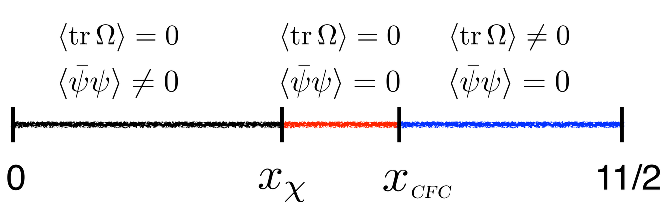
See reference note for history.

Conformal Window

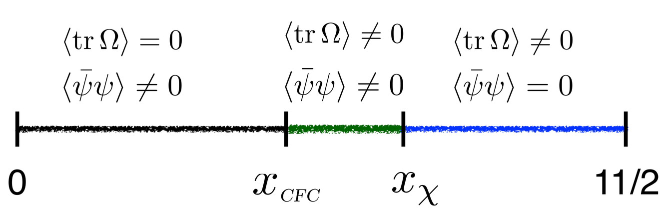
CFC symmetry is spontaneously broken within the conformal window of QCD.



Have shown it in Veneziano large N limit with x = n_F/N fixed and x → 11/2 from below.



There are CFC symmetry restoring phase transitions as function of T, m with fixed x.



Vary x with m = μ = T = 0

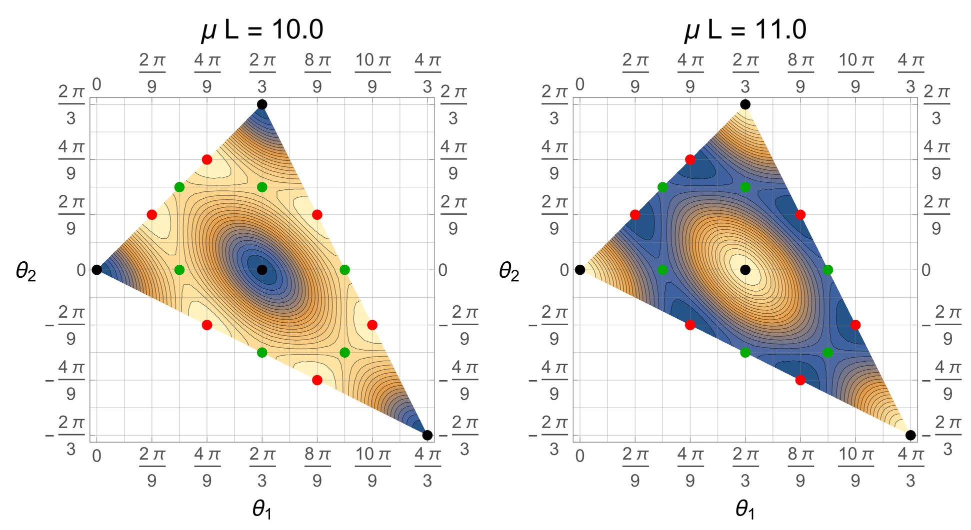
$$V_{\rm eff} = -\frac{2}{\pi^2 L^4}\sum_{n=1}^{\infty}\frac{1}{n^4}\left(|\mathrm{tr}\,\Omega^n|^2 - 1\right) + \frac{2}{\pi^2 L^4 n_f^3}\sum_{n=1}^{\infty}\frac{(\pm 1)^n}{n^4}\left(\mathrm{tr}\,\Omega^{n_f n} + \mathrm{h.c.}\right)$$

N = n_F = 3 Dense QCD

Take, μ >> Λ, m, with T << μ. Make L μ very large.

$$V_{\rm eff}(\Omega) \approx \frac{(\pm 1)^{n_f} T \mu\, e^{-n_f \pi L T}}{n_f \pi L^2}\,\sin(n_f \mu L)\,(\mathrm{tr}\,\Omega^{n_f} + \mathrm{h.c.})$$

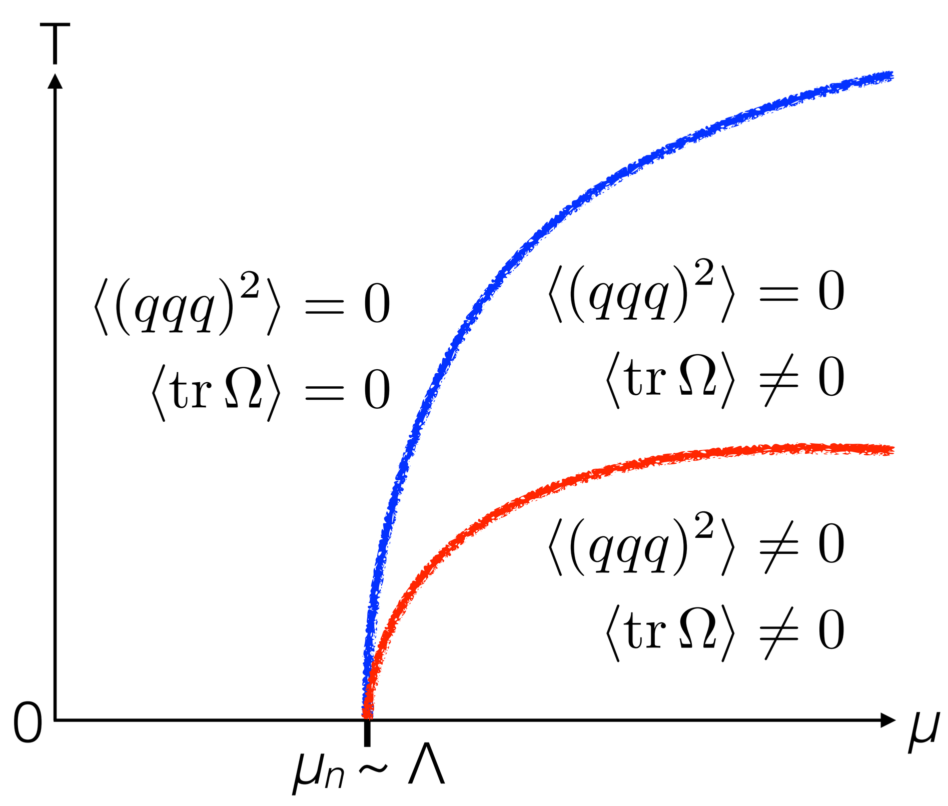
Global minima cycle between Z_3-breaking values. Infinite number of quantum phase transitions as μL→∞!



R^4 limit is a multiphase point!

T-μ Phase Diagram

$m \neq 0$ $SU(3)_V$ symmetric QCD



Outlook

QCD with d = gcd(N, n_F)>1 and common quark masses has a Z_d CFC symmetry. Its realization depends on the values of m, T, μ, and n_F/N.

Interesting implications for conformal window studies and dense quark matter.

CFC has **local** $SU(3)_V$-breaking order parameters. How do they behave in the cold dense quark matter in the R^4 limit?

$SU(3)_V$ breaking effects need to be explored.

Other applications?

Reference comment: appearance of symmetry with BCs of this type explored by Kouno, Sakai, Makiyama, Tokunaga, Sasaki, Yahiro, Misumi, Kashiwa, Iritani, Etou, Takahashi, Hirakida, 2012 - now. Our interpretation of the physics is different.