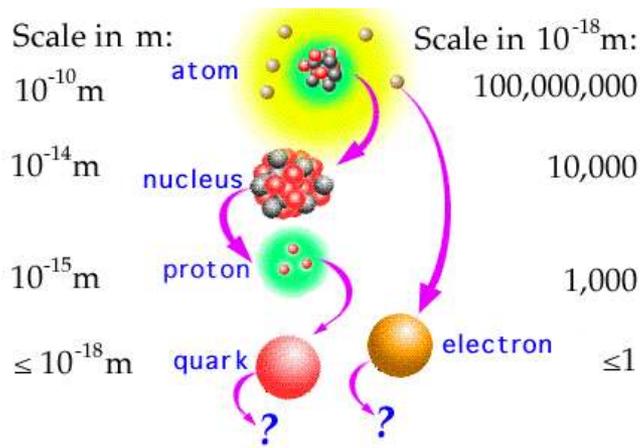# Isovector and Isoscalar Charges of the Nucleon

Rajan Gupta

Laboratory Fellow

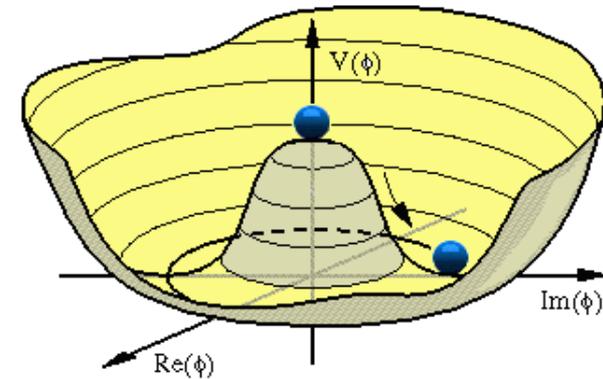**Theoretical Division**

Los Alamos National Laboratory, USA

Lattice 2017, Granada, 19 June 2017

Abstract

We present results for the isovector and isoscalar charges of the nucleon. The 2+1+1-flavor calculations were done using MILC HISQ ensembles with three lattice spacings a=0.12, 0.09, 0.06 fm and three pion masses approximately 310, 220, 130 MeV. The 2+1-flavor clover calculations were done on four ensembles with approximately 300 and 190 MeV pion masses.

Work done in collaboration with

PNDME collaboration
(Clover-on-HISQ)

- Tanmoy Bhattacharya
- Vincenzo Cirigliano
- Yong-Chull Jang
- Huey-Wen Lin
- Boram Yoon

Bhattacharya et al, PRD85 (2012) 054512
Bhattacharya et al, PRD89 (2014) 094502
Bhattacharya et al, PRD92 (2015) 114026
Bhattacharya et al, PRL 115 (2015) 212002
Bhattacharya et al, PRD92 (2015) 094511
Bhattacharya et al, PRD94 (2016) 054508
Gupta et al, arXiv:1705:06834

NME collaboration
(Clover-on-Clover)

- Tanmoy Bhattacharya
- Vincenzo Cirigliano
- Jeremy Green
- Yong-Chull Jang
- Bálint Joó
- Huey-Wen Lin
- Kostas Orginos
- David Richards
- Sergey Syritsyn
- Frank Winter
- Boram Yoon

Yoon et al., PRD D93 (2016) 114506
Yoon el al., PRD D95 (2017) 074508
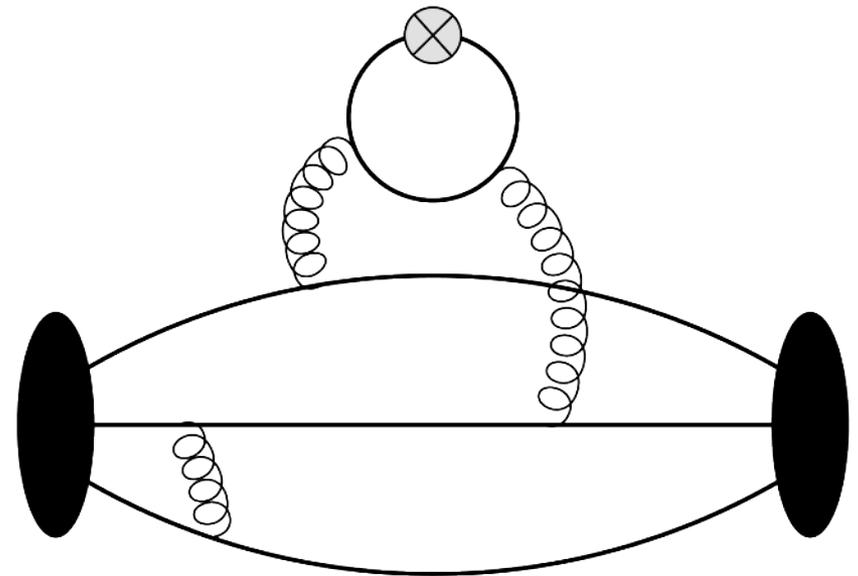
Acknowledgement: Computing Resources

If we can extract the matrix elements of quark bilinear operators within the nucleon state by calculating the "connected" and "disconnected" correlation functions with high precision, we can address a number of physics questions.
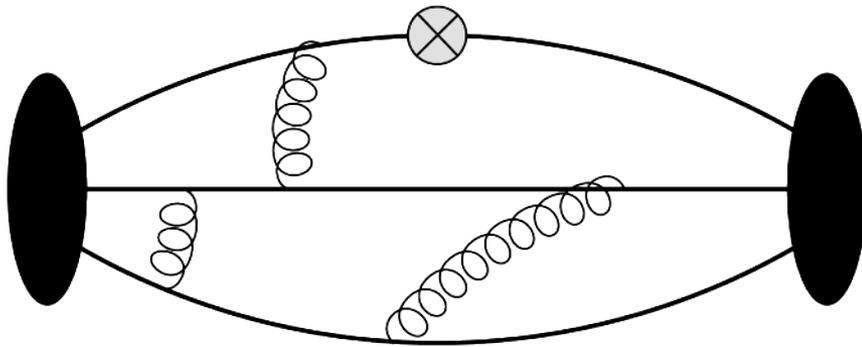
Connected

Disconnected

Phenomenology requires a number of matrix elements within nucleon states

- Isovector charges g_A, g_S, g_T $\qquad \langle p | \bar{u} \Gamma d | n \rangle$

- Axial vector form factors $\qquad \langle p(q) | \bar{u} \gamma_\mu \gamma_5 d(q) | n(0) \rangle$

- Vector form factors $\qquad \langle p(q) | \bar{u} \gamma_\mu d(q) | n(0) \rangle$

- Flavor diagonal matrix elements $\qquad \langle p | \bar{q} q | p \rangle$

- Quark EDM and quark chromo EDM

- Generalized Parton Distribution Functions

Simplest quantities to calculate
are the isovector charges

$$g_A, \; g_S, \; g_T$$

Challenges to obtaining high precision results for matrix elements within nucleon states

– **High Statistics: O(1,000,000) measurements**

– **Demonstrating control over all Systematic Errors:**

   • Non-perturbative renormalization of bilinear operators (RI_{smom} scheme)

   • Contamination from excited states

   ➢ Finite volume effects

   ➢ Chiral extrapolation to physical m_u and m_d (simulate at physical point)

   ➢ Extrapolation to the continuum limit (lattice spacing $a \rightarrow 0$)

Perform simulations on ensembles with multiple values of
➢ Lattice size $M_\pi L \rightarrow \infty$
➢ Light quark masses $\rightarrow$ physical m_u and m_d
➢ Lattice spacing $a \rightarrow 0$

Toolkit

- Multigrid Dirac invertor $\rightarrow$ propagator $S_F = D^{-1}\eta$

- Truncated solver method with bias correction (AMA)

- Coherent source sequential propagator

- Deflation + hierarchical probing (for disconnected)

- 3-5 values of t_{sep} with smeared sources

- 2-state (3-state fit) to multiple values of t_{sep}

- Non-perturbative methods for renormalization constants

- Combined extrapolation in $a,\ M_\pi,\ M_\pi L$

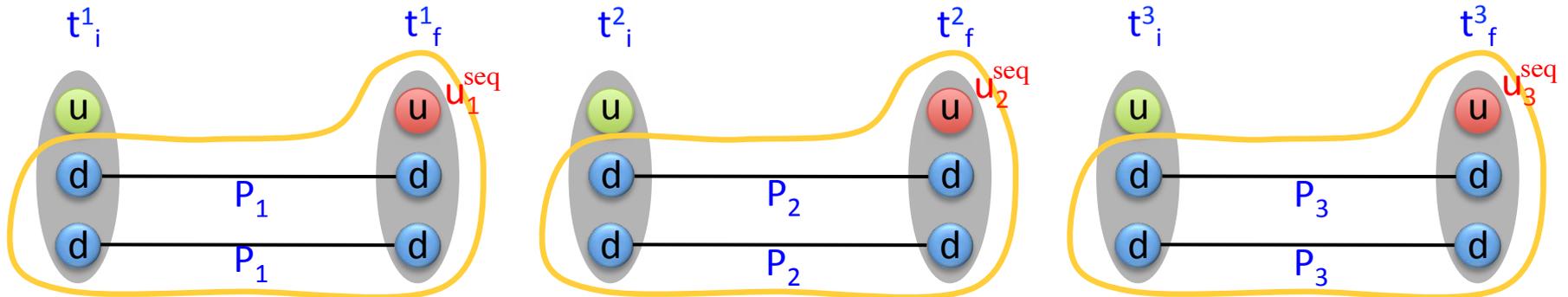- Variation of results with extrapolation Ansatz

Truncated solver + bias correction (AMA)

$$C^{AMA} = \frac{1}{N_{LP}} \sum_{i=1}^{N_{LP}} C_{LP}(x_i^{LP}) + \frac{1}{N_{HP}} \sum_{i=1}^{N_{HP}} \{C_{HP}(x_i^{HP}) - C_{LP}(x_i^{HP})\}$$
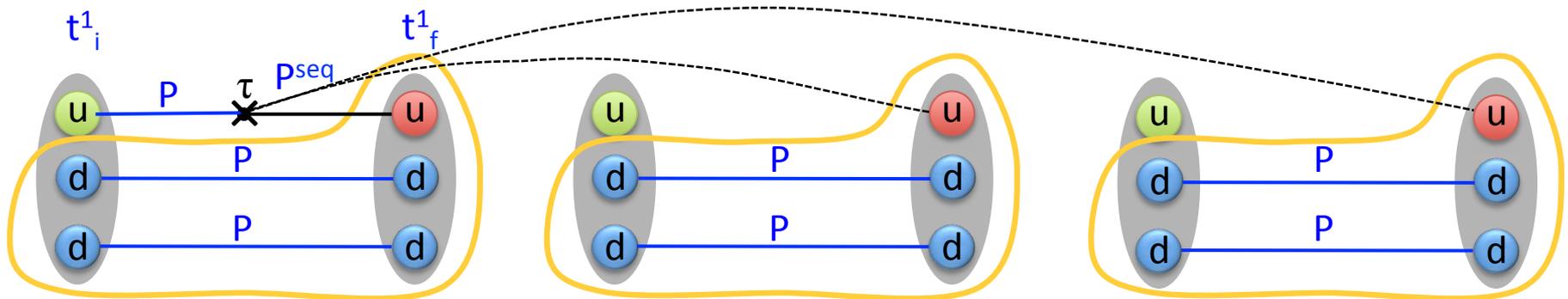
- Use multigrid inverter with
  - $r_{LP} = 10^{-3}$
  - $r_{HP} = 10^{-7} - 10^{-10}$
  - $N_{LP} = 64-160$, $N_{HP} = 3-5$ per configuration
- The bias term is negligible (~1% of the error)
- The AMA error is <15% larger than LP
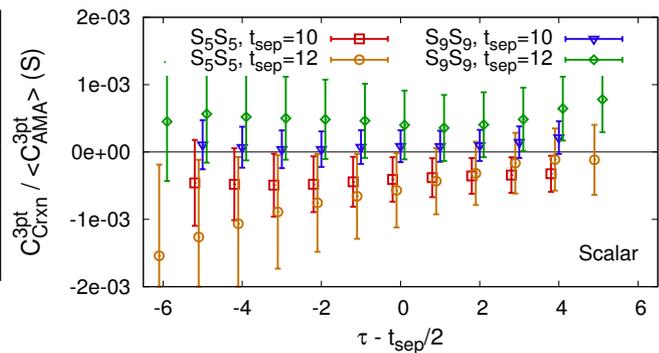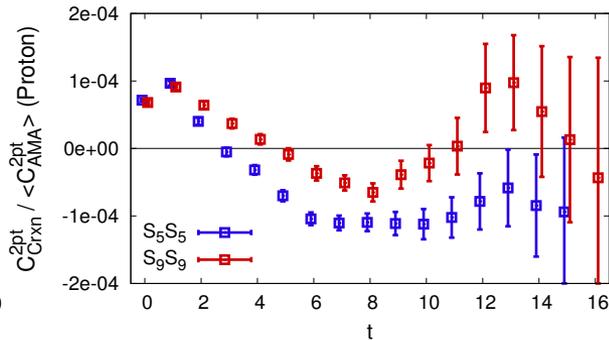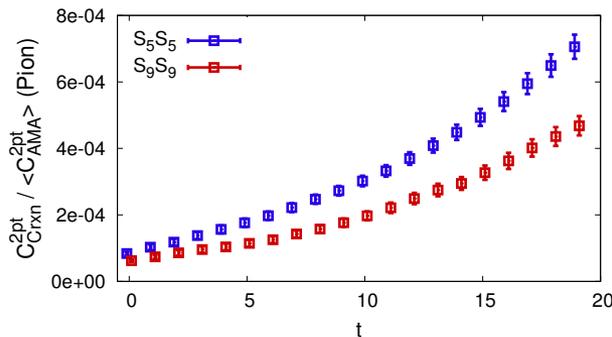
Coherent Source Sequential Propagator

3 measurements being done in a single computer job

Bias = 0 after gauge integral: Increase in Variance ???

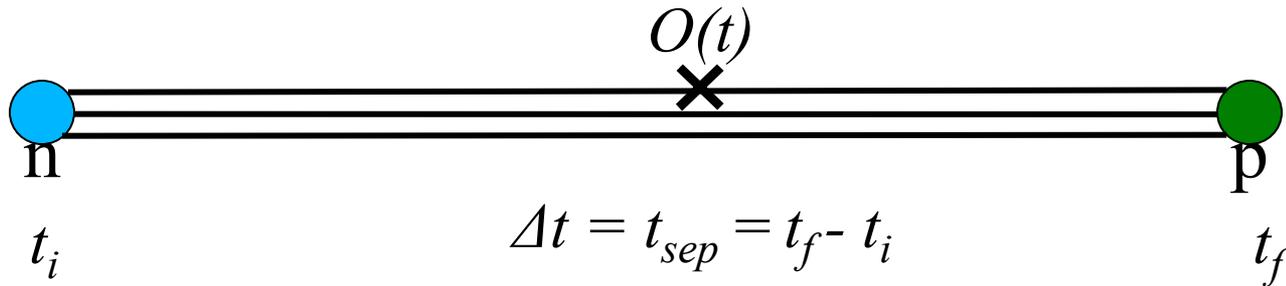# Controlling excited-state contamination: n-state fit

$$\Gamma^2(t) = |A_0|^2 \, e^{-M_0 t} + |A_1|^2 \, e^{-M_1 t} + |A_2|^2 \, e^{-M_2 t} + |A_3|^2 \, e^{-M_3 t} + \ldots$$

$$\Gamma^3(t, \Delta t) = |A_0|^2 \langle 0|O|0 \rangle \, e^{-M_0 \Delta t} + |A_1|^2 \langle 1|O|1 \rangle e^{-M_1 \Delta t} +$$

$$A_0 A_1^* \langle 0|O|1 \rangle e^{-M_0 \Delta t} e^{-\Delta M(\Delta t - t)} + A_0^* A_1 \langle 1|O|0 \rangle e^{-\Delta M t} e^{-M_0 \Delta t} + \ldots$$

$M_0, M_1, \ldots$ masses of the ground & excited states
$A_0, A_1, \ldots$ corresponding amplitudes

$O(t)$

n ——— p

$\Delta t = t_{sep} = t_f - t_i$

t_i t_f

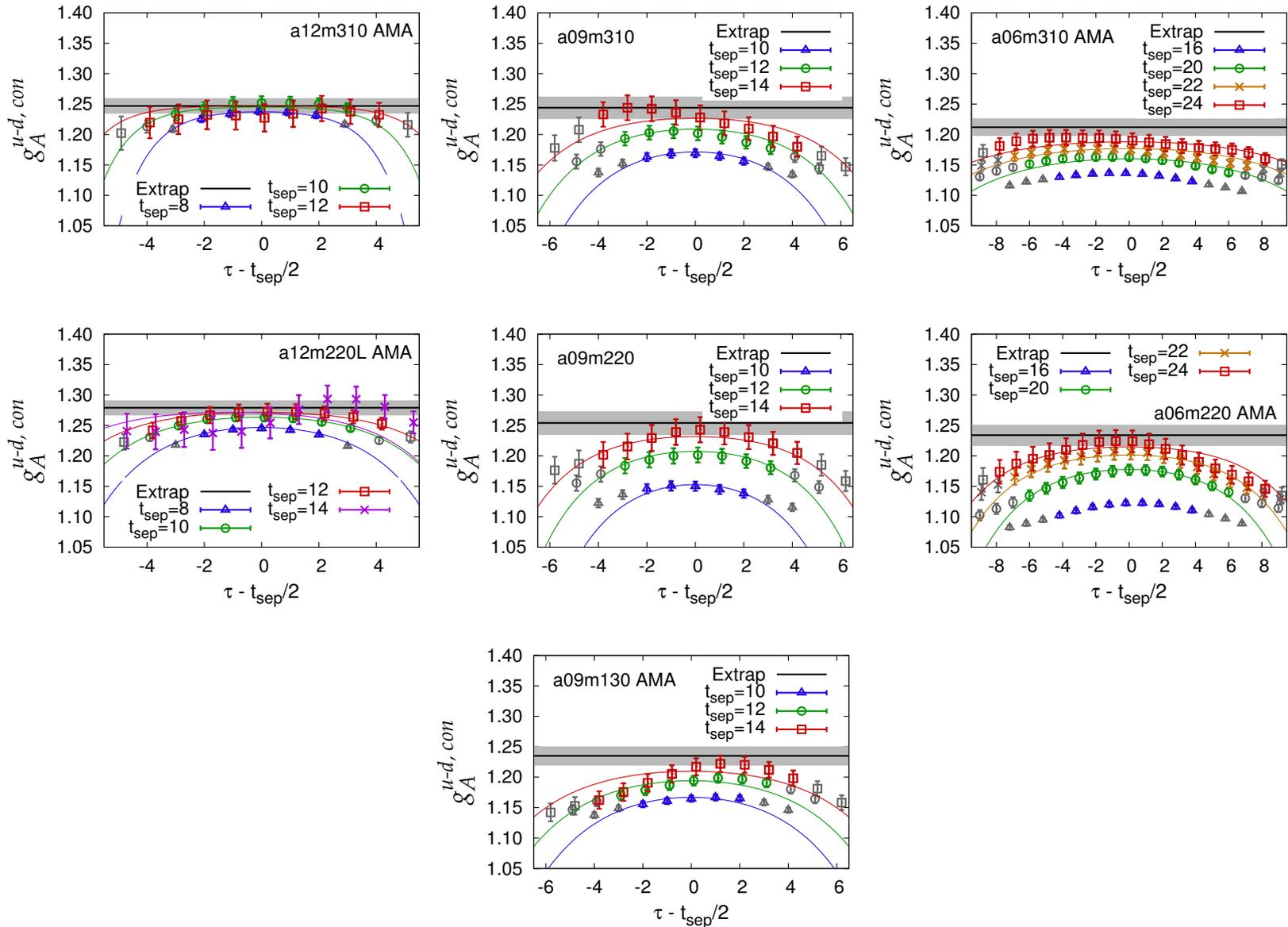Make a simultaneous fit to data at multiple $\Delta t = t_{sep} = t_f - t_i$

Controlling excited-state contamination

- Reduce A_n/A_0 in an n-state fit

  - Tune source smearing size σ

  - Tune the interpolating operator

  - Variational method

- Generate data at multiple values of t_{sep}

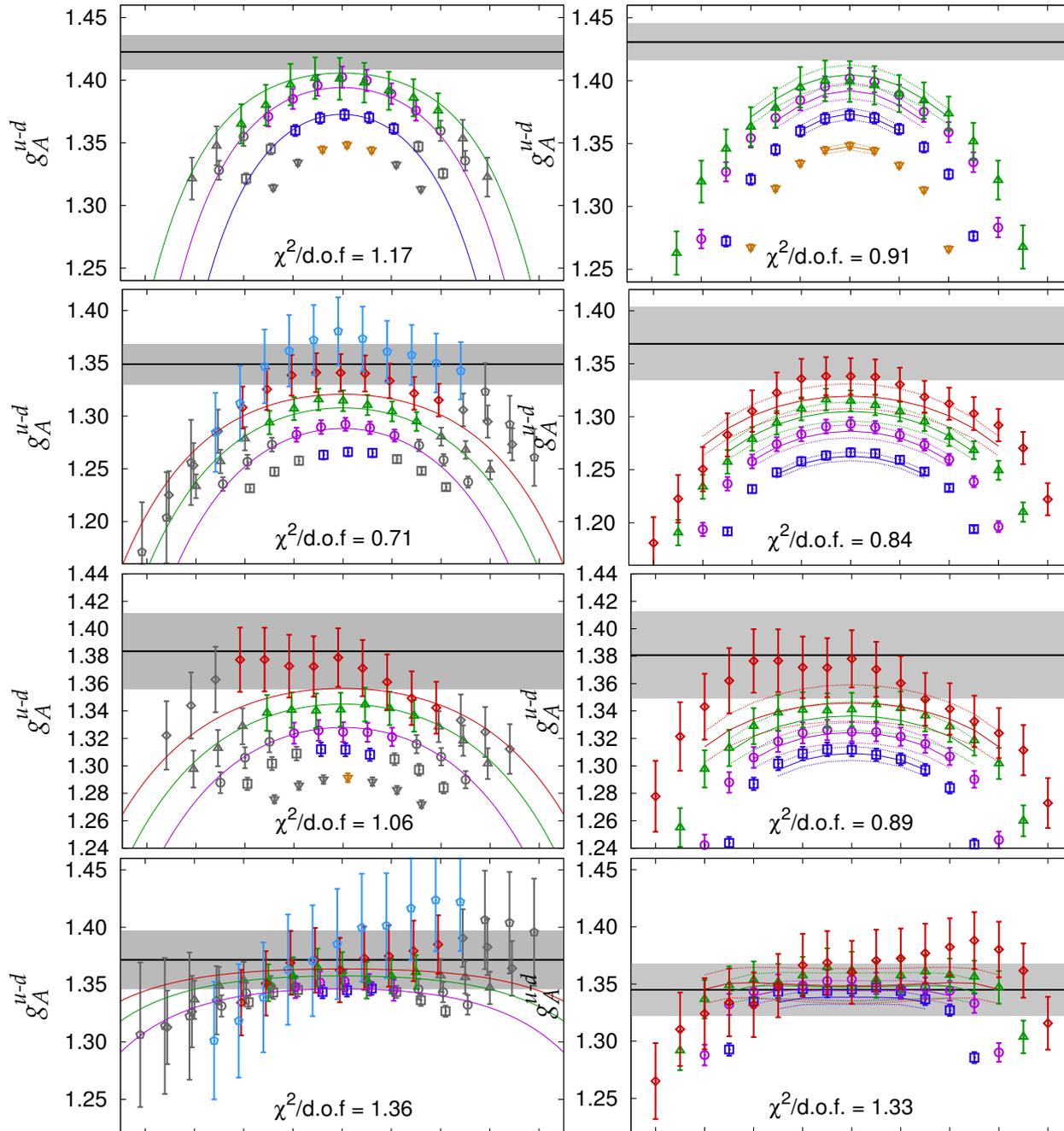- $2 \rightarrow 3 \rightarrow \ldots$ state fits to data at multiple values of t_{sep}

Yoon et al, PRD D93 (2016) 114506
Yoon el al, PRD D95 (2017) 074508

g_A: Excited State Contamination

Data and fits on 7 clover-on-HISQ ensembles: Bhattacharya et al, PRD94 (2016) 054508

14

ESC

Clover on Clover

Yoon et al., PRD D93
(2016) 114506

Analyzing lattice data $\Omega(a, M_\pi, M_\pi L)$:
Extrapolations in $a, M_\pi^2, M_\pi L$

We use lowest order corrections when fitting lattice data w.r.t.

- Lattice spacing: a

- Dependence on light quark mass: $m_q \sim M_\pi^2$

- Finite volume: $M_\pi L$

$$g_{A,T}(a, M_\pi, L) = g + A\,a + B\,M_\pi^2 + C\,M_\pi^2 e^{-M_\pi L} + \ldots$$

$$g_S(a, M_\pi, L) = g + A\,a + B\,M_\pi + C\,M_\pi e^{-M_\pi L} + \ldots$$

2+1+1 flavor HISQ lattices from MILC

M$_s$ tuned to its physical value using M$_{ss}$

a (fm)		m$_l$/m$_s$	Volume	M$_\pi$ L	M$_\pi$ (MeV)	# Configs	$\bar{H}$P	AMA
0.12	□	0.2	$24^3 \times 64$	4.55	310	1013	8,104	64,832
0.12	△	0.1	$24^3 \times 64$	3.29	225	1000	24,000	
0.12	◆	0.1	$32^3 \times 64$	4.38	228	958	7,664	
0.12	▽	0.1	$40^3 \times 64$	5.49	228	1010	8,080	68,680
0.09	□	0.2	$32^3 \times 96$	4.51	313	881	7,048	
0.09	◆	0.1	$48^3 \times 96$	4.79	226	890	7,120	
0.09	○	0.037	$64^3 \times 96$	3.90	138	883	7,064	56,512
0.06	□	0.2	$48^3 \times 144$	4.52	320	1000	8,000	64,000
0.06	◆	0.1	$64^3 \times 144$	4.41	235	650	2,600	41,600

2+1 flavor Clover lattices (Jlab/W&M)

M_s tuned to its physical value using $(2M_{K^+}^2 - M_{\pi^0}^2)/M_{\Omega^-}^2$

a fm	M_π MeV	Lattice Volume	$M_\pi L$	t_{sep}	Smearing σ	# of Configs	HP Src.	LP Src
0.114	316	$32^3 \times 96$	5.85	8,10,12,14	5	1000	4000	128,480
0.081	312	$32^3 \times 64$	4.11	10,12,14,16,18	5	1005	3,015	96,480
0.081	312	$32^3 \times 64$	4.11	8,10,12,14,16	7	1005	3,015	96,480
0.081	312	$32^3 \times 64$	4.11	10,12,14,16,18	9	1005	3,015	96,480
0.081	312	$32^3 \times 64$	4.11	12	V357, V579	443	0,1329	42,528
0.079	192	$48^3 \times 96$	3.7	8,10,12,14,16	7	629	2,516	80,512
0.079	198	$64^3 \times 128$	5.08	8,10,12,14,16	7	467	2,335	74,720

Simultaneous extrapolation in $a, M_\pi^2, M_\pi L$

9 clover-on-HISQ ensembles: Bhattacharya et al, PRD94 (2016) 054508
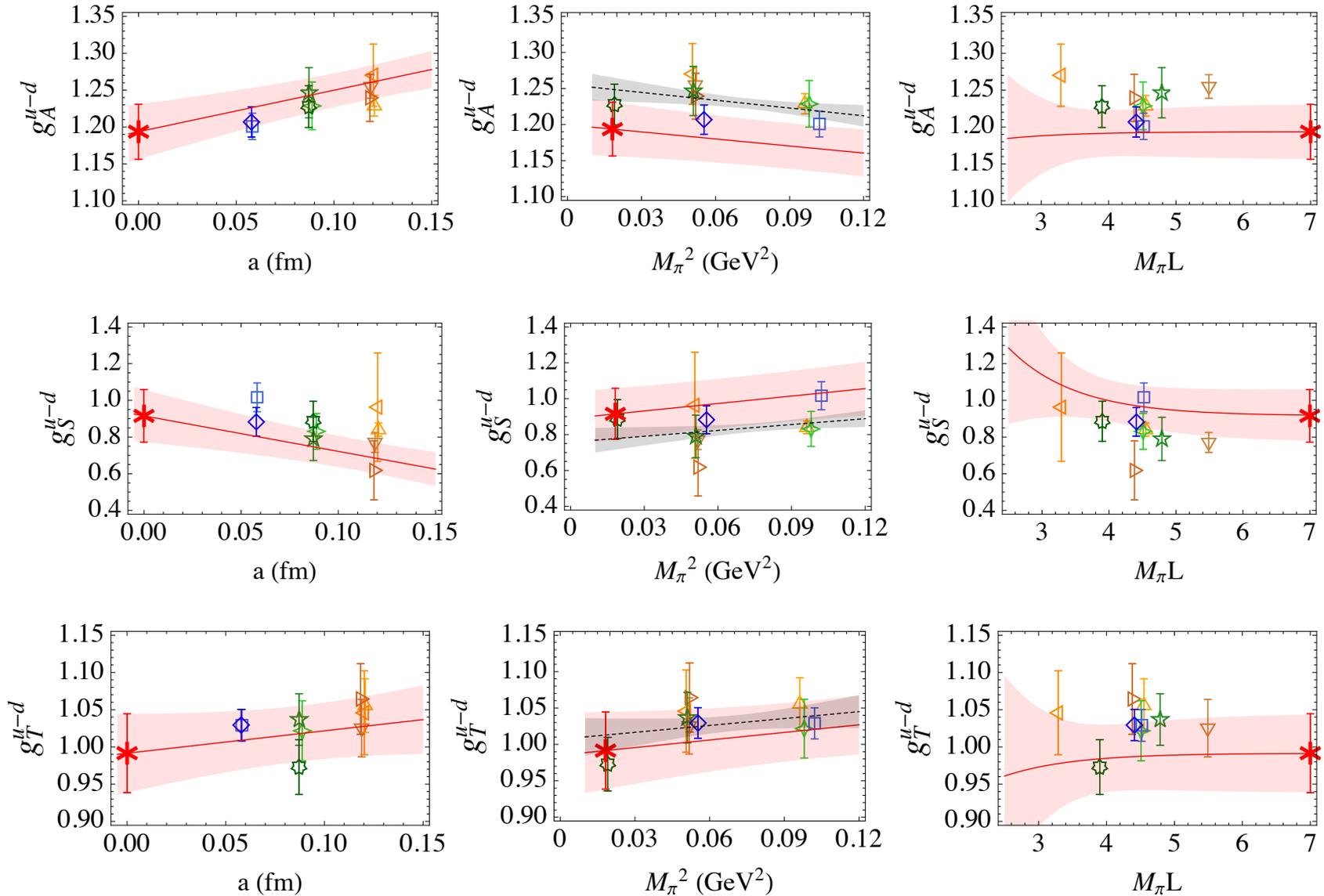
Results on isovector charges of the proton (clover-on-HISQ)

(Bhattacharya et al, PRD94 (2016) 054508)

Isovector charges

***	$g_T = 0.987(51)$
**	$g_A = 1.195(33)$
**	$g_S = 0.97(12)$

Flavor diagonal charges

$$g_T^u = 0.792(42)$$
$$g_T^d = -0.194(14)$$

2-state fits to 2-point and 3-point functions

Constraints on $[\varepsilon_S, \varepsilon_T]$: β-decay versus LHC

- LHC: (u+d → e+ν) look for events with an electron and missing energy at high transverse mass

- low-energy experiments + lattice with $\delta g_S/g_S \sim 10\%$

LHC

CURRENT CONSTRAINTS

Low-energy: $g_{S,T}$ from quark model

Low-energy: $g_{S,T}$ from lattice

LHC: $\sqrt{s} = 8$ TeV L = 20, fb^{-1}

PROSPECTIVE CONSTRAINTS

Low-energy: $g_{S,T}$ from quark model

Low-energy: $g_{S,T}$ from lattice

LHC: $\sqrt{s} = 14$ TeV L = 10, 300 fb^{-1}
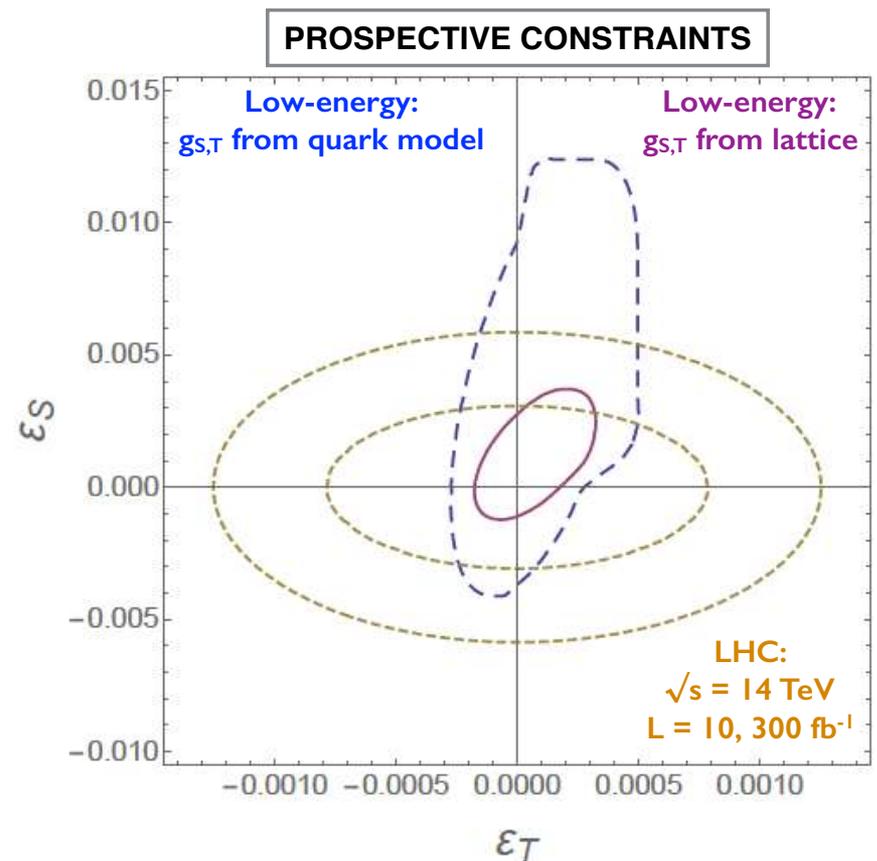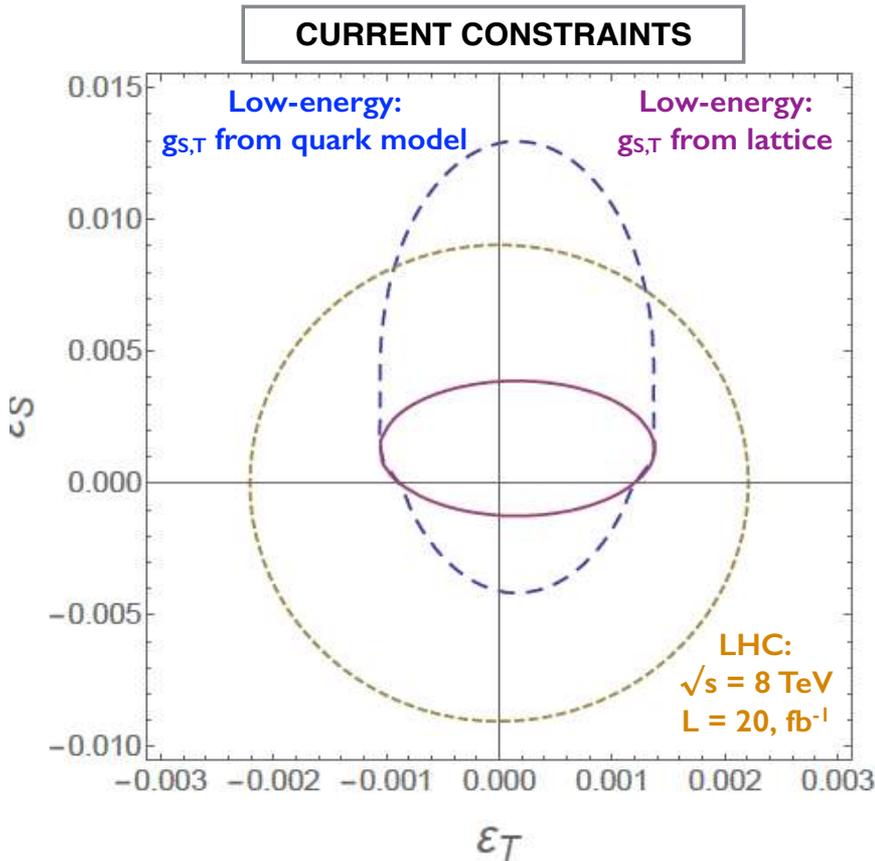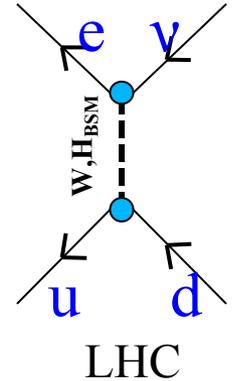
Update: 2+1+1 flavor HISQ lattices from MILC

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0.09	□	0.2	$32^3 \times 96$	4.51	313	881		42,176
0.09	◆	0.1	$48^3 \times 96$	4.79	226	890		53,312
0.09	○	0.037	$64^3 \times 96$	3.90	138	883	7,064	56,512
0.06	□	0.2	$48^3 \times 144$	4.52	320	1000	8,000	64,000
0.06	◆	0.1	$64^3 \times 144$	4.41	235	650	2,600	41,600
0.06	○	0.037	$96^3 \times 192$	3.7	135	322	1,610	51,520

2017: New results on isovector charges

PNDME (Preliminary):

- Better smearing on a09
- Added a06m135 ensemble
- All ensembles with AMA
- 4-state fits to 2-point functions
- 3-state fits to 3-point functions
- Covariant error matrix
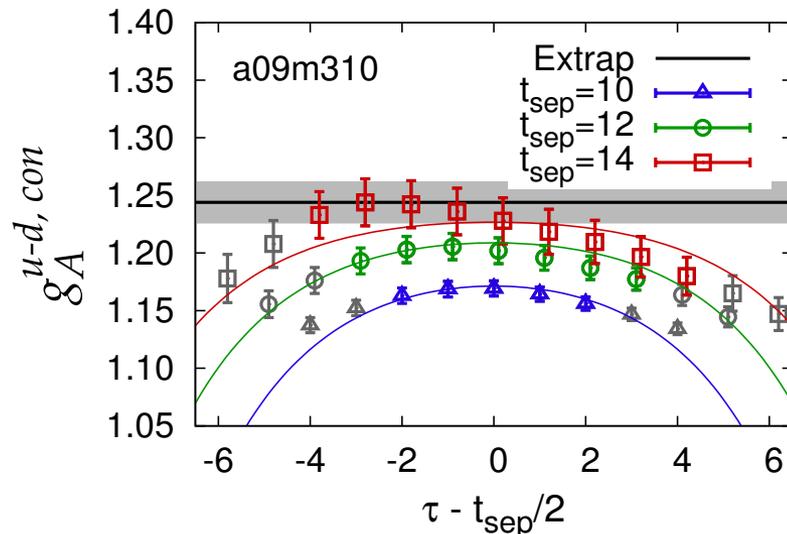
CalLAT (DWF-on-HISQ)

Multistate fit to
$$\frac{C_3(\tau + 1)}{C_2(\tau + 1)} - \frac{C_3(\tau)}{C_2(\tau)}$$
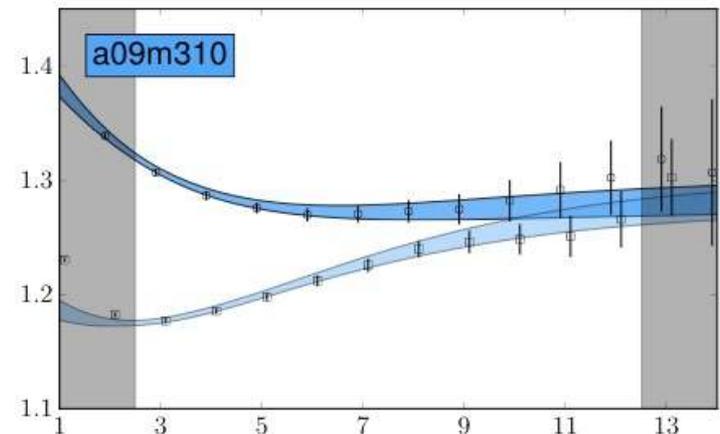
$$g_A = 1.278(21)(26)$$

Multistate fits: CalLat versus PNDME
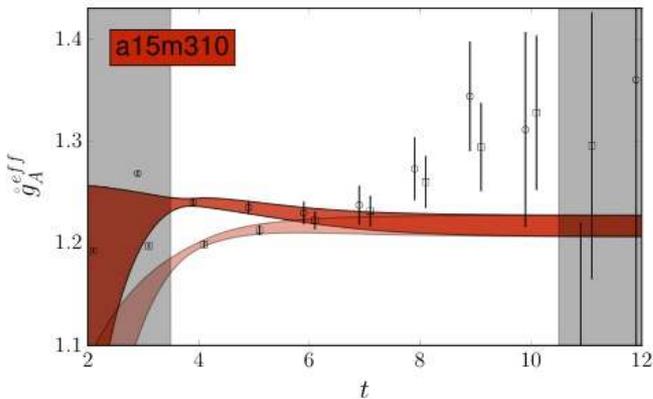
$4 \le t \le 6$
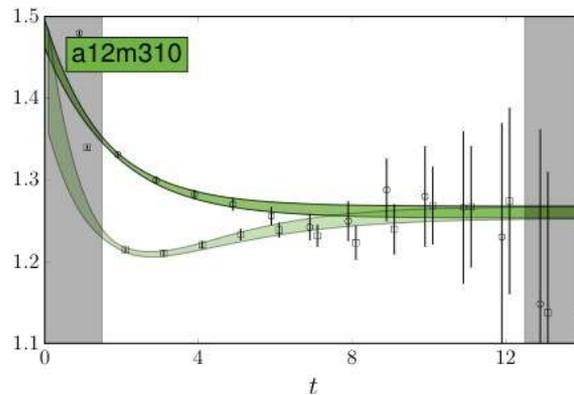
$2 \le t \le 7$
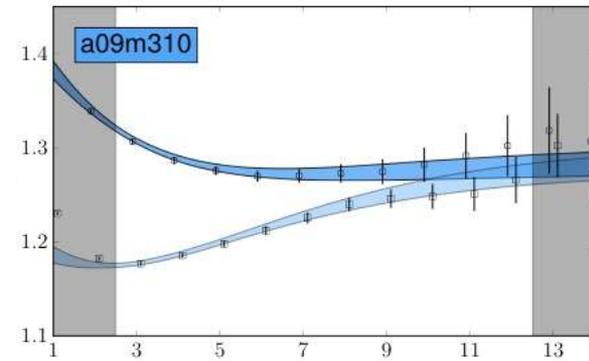
$3 \le t \le 10$

$t_{sep} = 8, 10, 12$

$t_{sep} = 10, 12, 14$
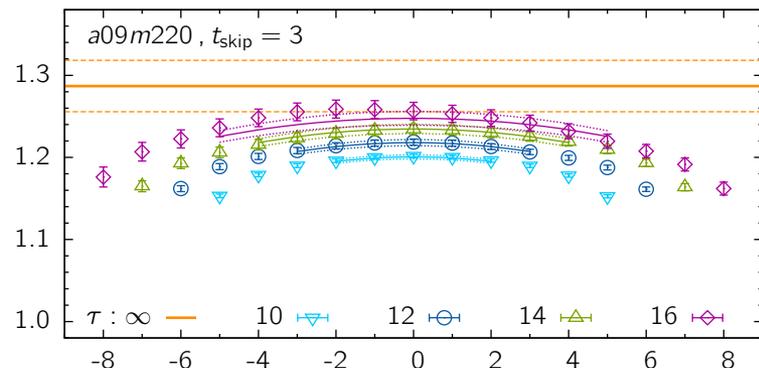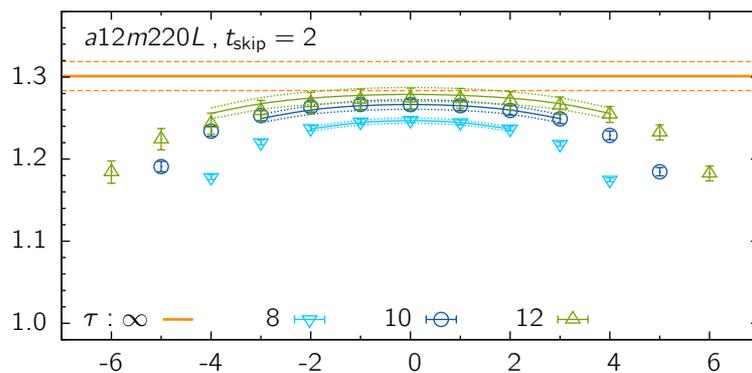
CalLat: Gain in signal comes mainly from including much smaller t_{sep} in fit for extracting g_A

g_A: Simultaneous extrapolation in a, M_π^2, $M_\pi L$
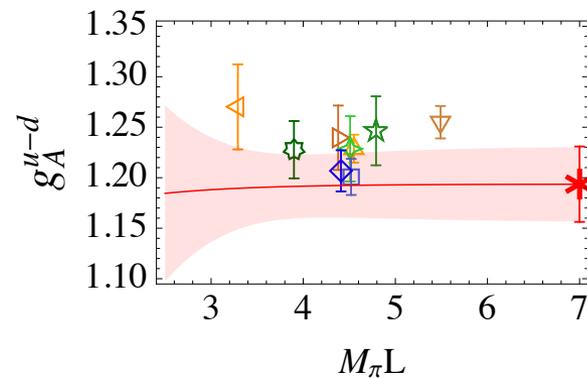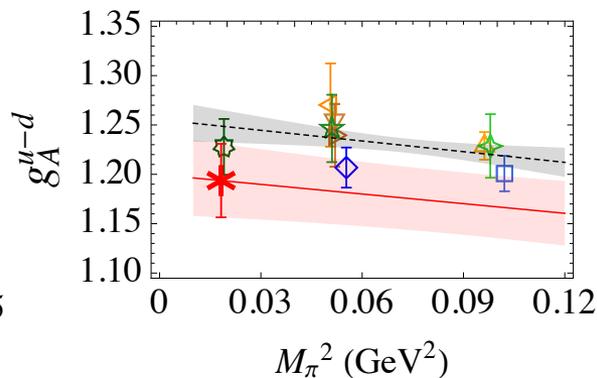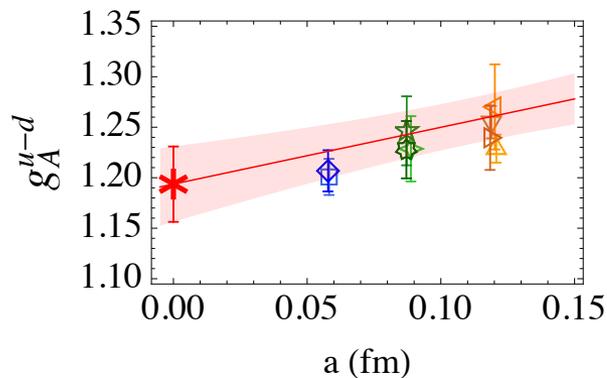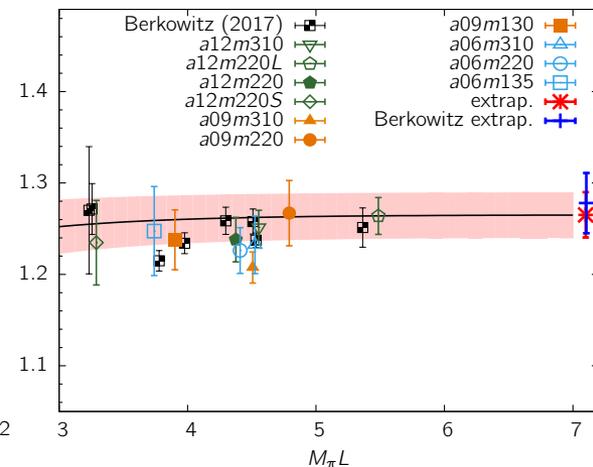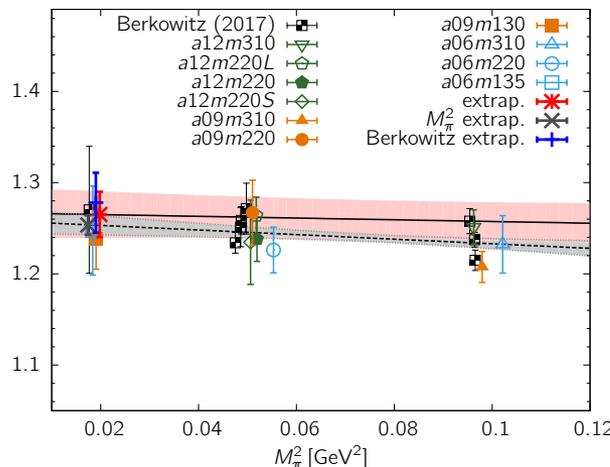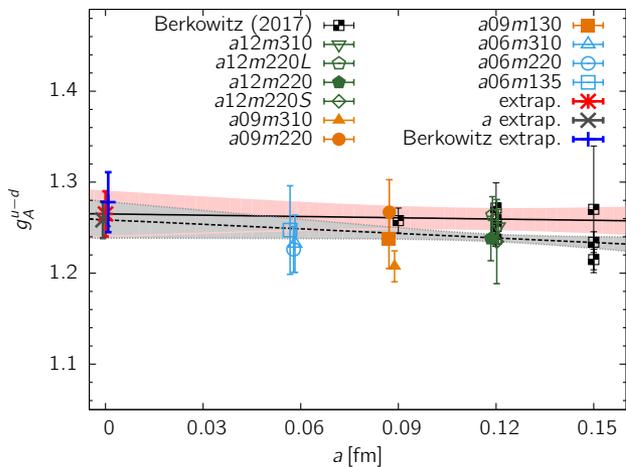
↓ 2017

PNDME 2017:   CalLat: PNDME+ CalLat:

$g_A = 1.223(39)$ $1.278(21)(26)$ $g_A = 1.265(25)$

10 clover-on-HISQ ensembles: PNDME   8 DWF-on-HISQ ensembles: CalLAT   25

g_S: Simultaneous extrapolation in $a, M_\pi^2, M_\pi L$

$\downarrow$ 2017

$$g_S = 0.97(12) \implies 1.03(14)$$

PNDME (2017): 10 clover-on-HISQ ensembles
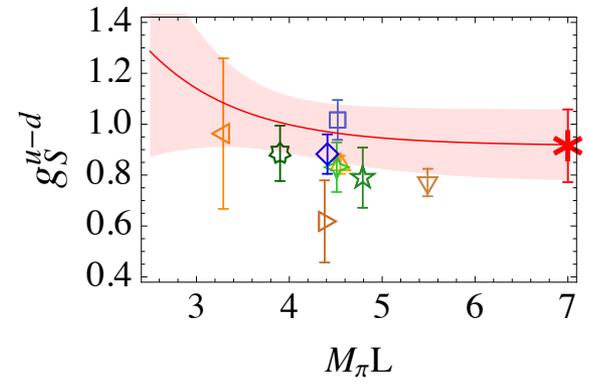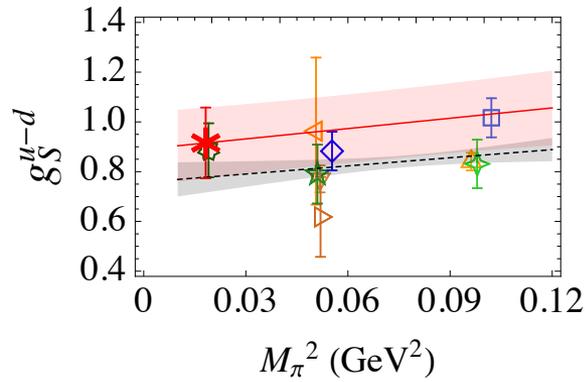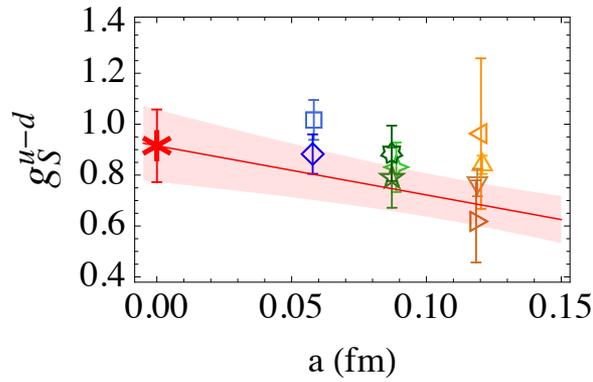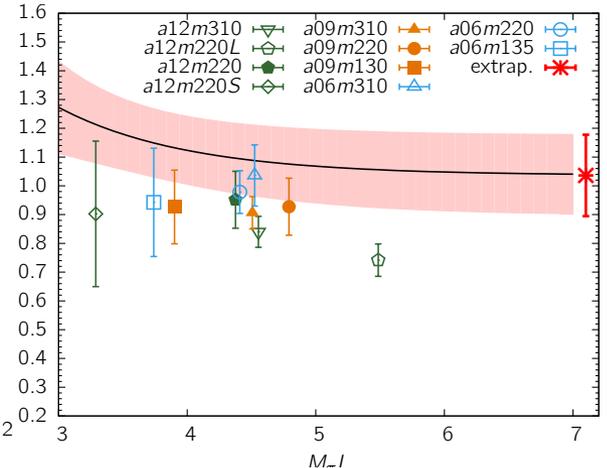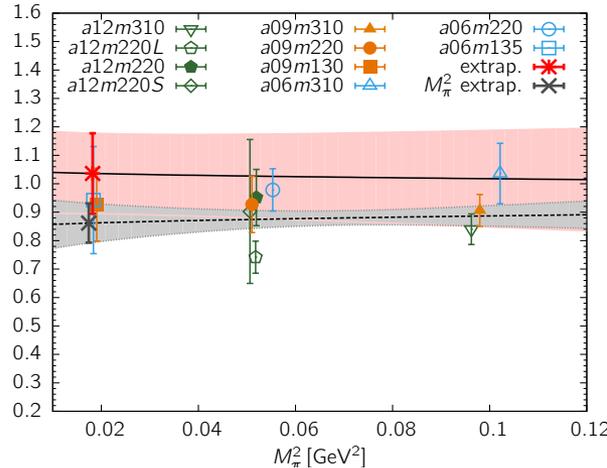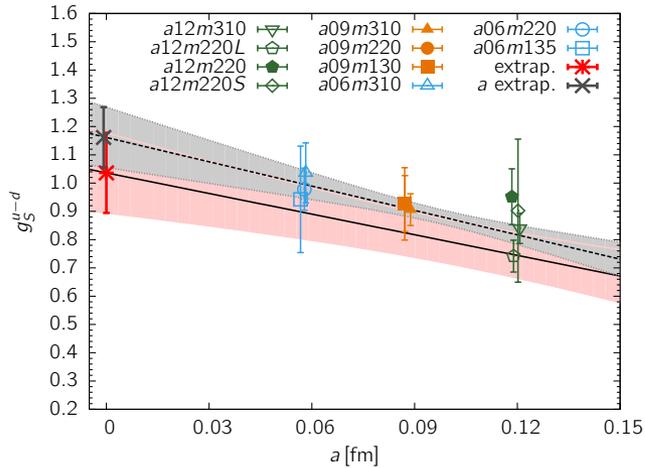
g_T: Simultaneous extrapolation in $a, M_\pi^2, M_\pi L$
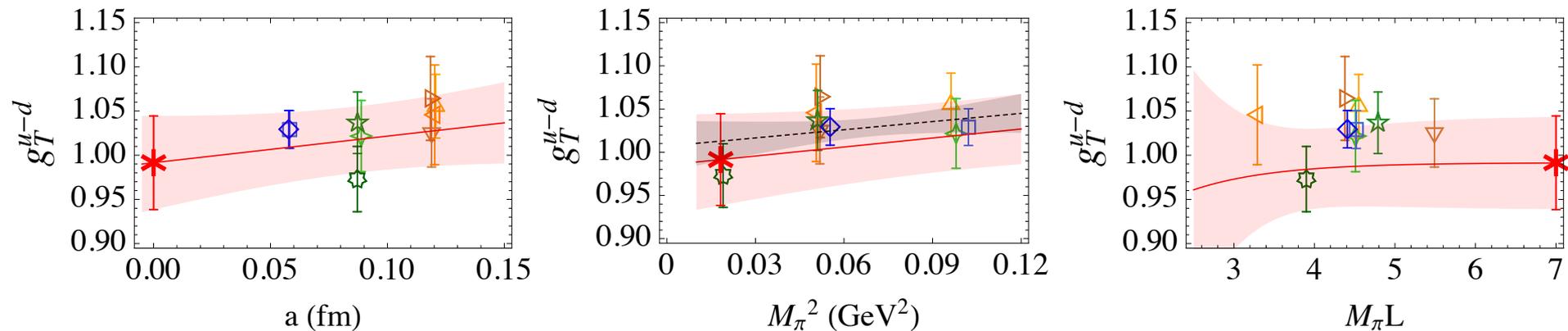
↓ 2017

$$g_T = 0.987(51) \implies 1.016(43)$$

PNDME (2017): 10 clover-on-HISQ ensembles
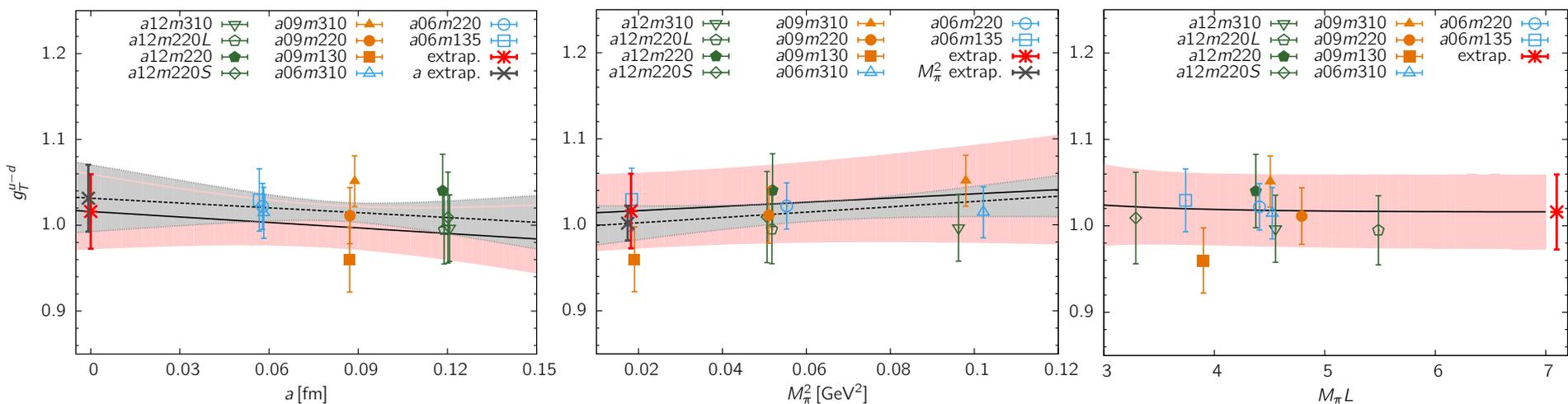
Physics probed by flavor diagonal charges

$$g^q_\Gamma = <N|q\Gamma q|N>$$

- Axial charges g_A
  - *Contribution of quarks to the nucleon spin ($\Sigma = g_A^u + g_A^d + g_A^s$)*
  - *Spin dependent interactions of dark matter*
- Scalar charges g_S
  - *Dependence of nucleon mass on the quark masses ($\sigma_{\pi N} = m_l <N| uu+dd|N>$; $\sigma_s = m_s <N| ss|N>$)*
  - *Coupling of dark matter to nucleons (needed for direct detection of dark matter via scattering off nucleons)*
  - *Constraining BSM theories: bounds on effective scalar interactions (e.g. Higgs mediated lepton flavor violation)*
- Tensor charges g_T
  - *Contributions of quarks to the neutron EDM*
  - *Constraining BSM theories: bounds on effective tensor interactions*
  - *Transversity*
- Chromo EDM
  - *Constraining BSM via their CP violating interactions, e.g., Higgs CP odd couplings*

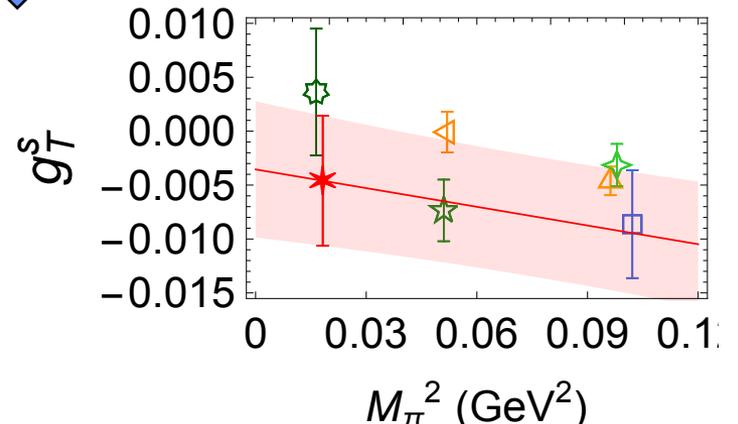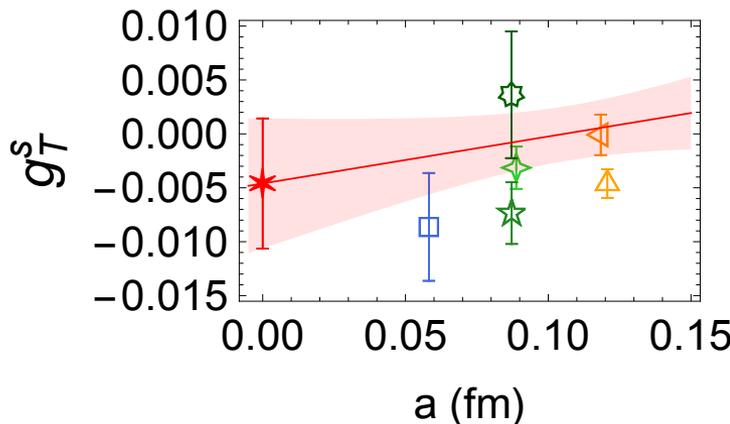# Evolution of the tensor charge g_T^s

Phys Rev D92 (2015) 094511

$g_T^s = 0.008(9)$ contributed the dominant error to qEDM
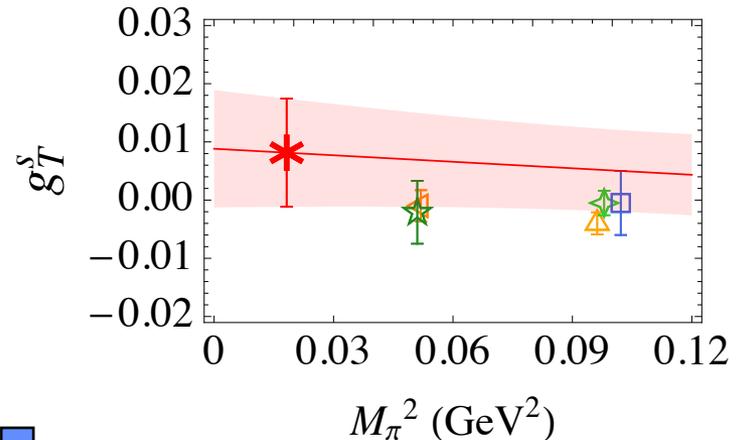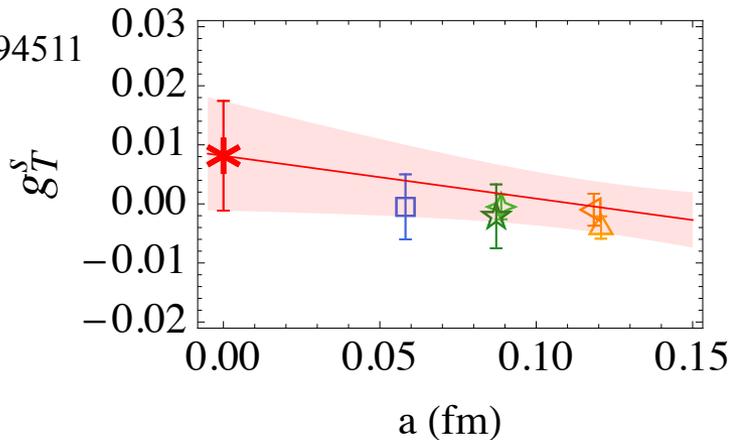
Improved analysis + new point at a09m130

$g_T^s = -0.005(6)$

Need to increase statistics in the a06m310 (blue square) and the new a09m130 (green hexagon) data

Continuum and chiral extrapolations for g^s are becoming stable
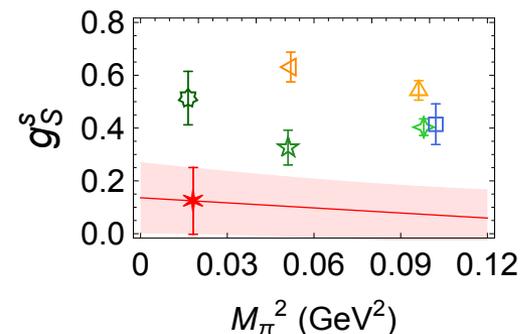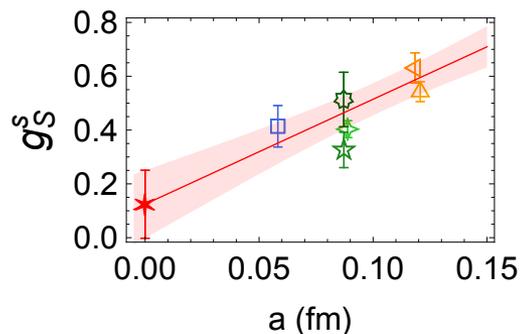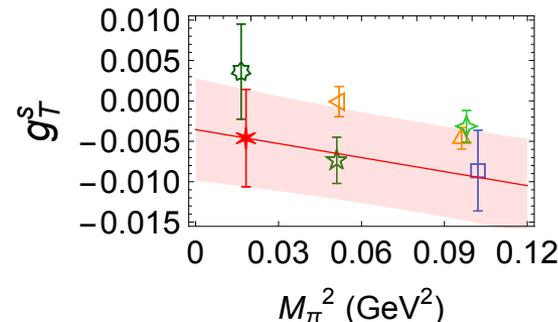
Ensembles

- △ *a12m310*
- ▷ *a12m220*
- ✦ *a09m310*
- ★ *a09m220*
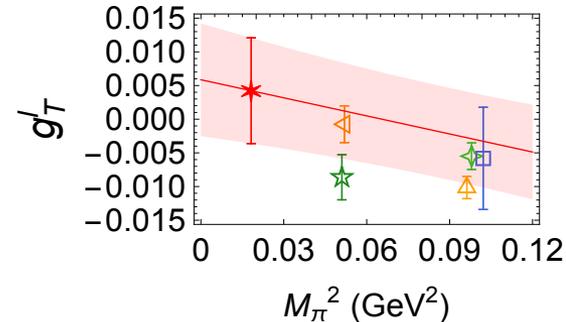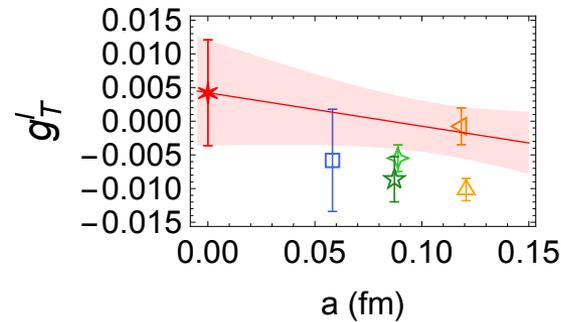- ✶ *a09m130*
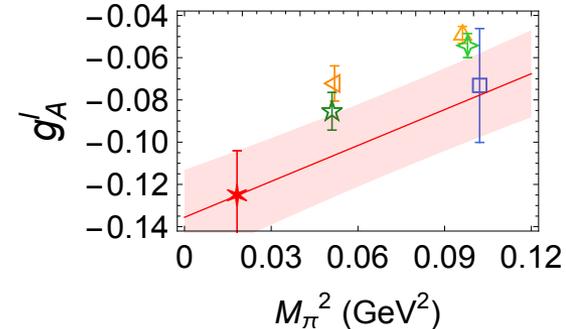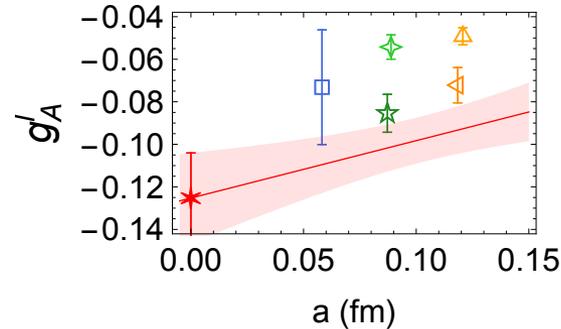- ☐ *a06m310*

Need more points and smaller errors
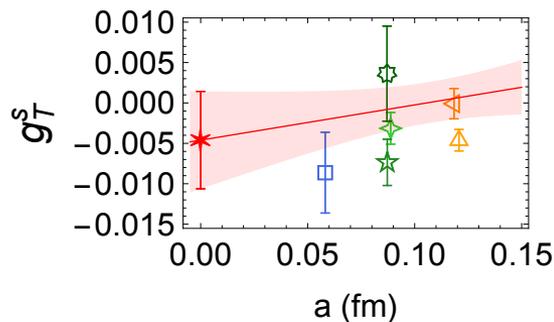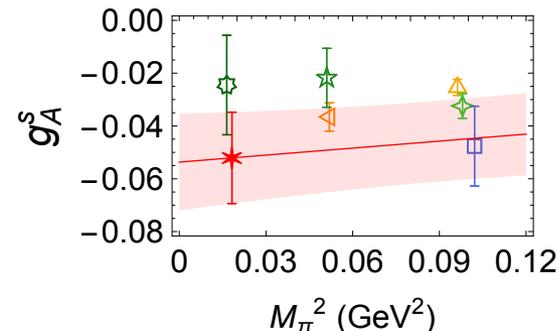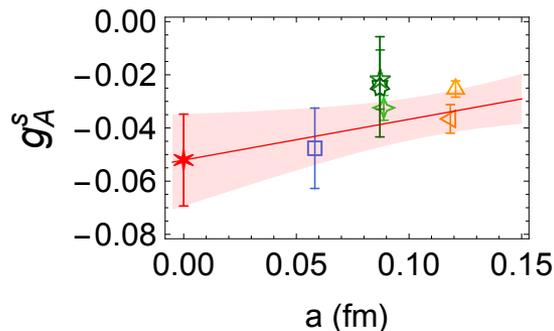
Disconnected light quark contribution

The a09m220 point (green diamond) is new

Extrapolation $a \to 0$ leveraged by noisy a06m310 (blue square) data.

Need to increase statistics.
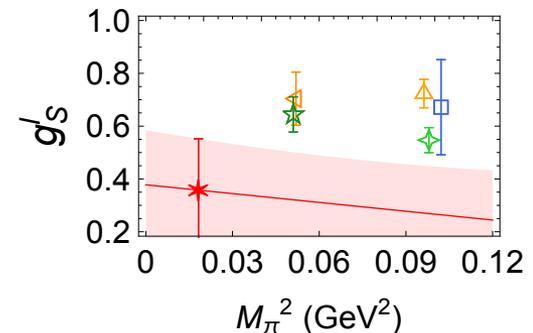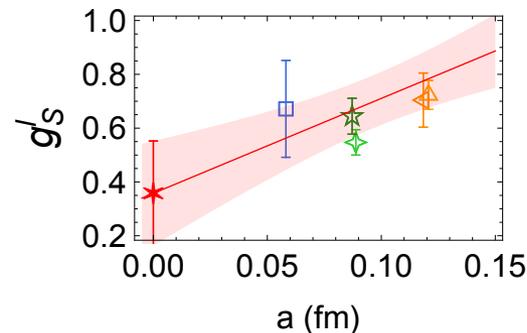
Goal: Provide first analysis of continuum and chiral extrapolations for g^s and tighten estimates of g^l

	Ensemble	L Size	N^l_{conf}	N^l_{src}	N^s_{conf}	N^s_{src}
△	*a12m310*	$24^3 \times 64$	1013	5000	1013	1500
▷	*a12m220*	$32^3 \times 64$	958	11000	958	4000
☆	*a09m310*	$32^3 \times 96$	1081	4000	1081	2000
★	*a09m220*	$48^3 \times 96$	712	8000	200	8000
☆	**a09m130**	**$64^3 \times 96$**			**877**	**10000**
☐	*a06m310*	$48^3 \times 144$	100	10000	200	5000
ToDo	*a06m220*	$64^3 \times 144$				

Improvements:
- Statistics: N_{conf}
- N_{src}: 128 noise vectors × 64 Hadamard vectors
- Full 4-D Sources