Tetraquark candidate $Z_c(3900)$ from coupled-channel scattering

-- how to extract hadronic interactions? --

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HAL QCD (Hadrons to Atomic nuclei from Lattice QCD)

S. Aoki, T. Aoyama, D. Kawai, T. Miyamoto, K. Sasaki (YITP, Kyoto Univ.)

T. Doi, T. M. Doi, S. Gongyo, T. Hatsuda, T. Iritani (RIKEN)

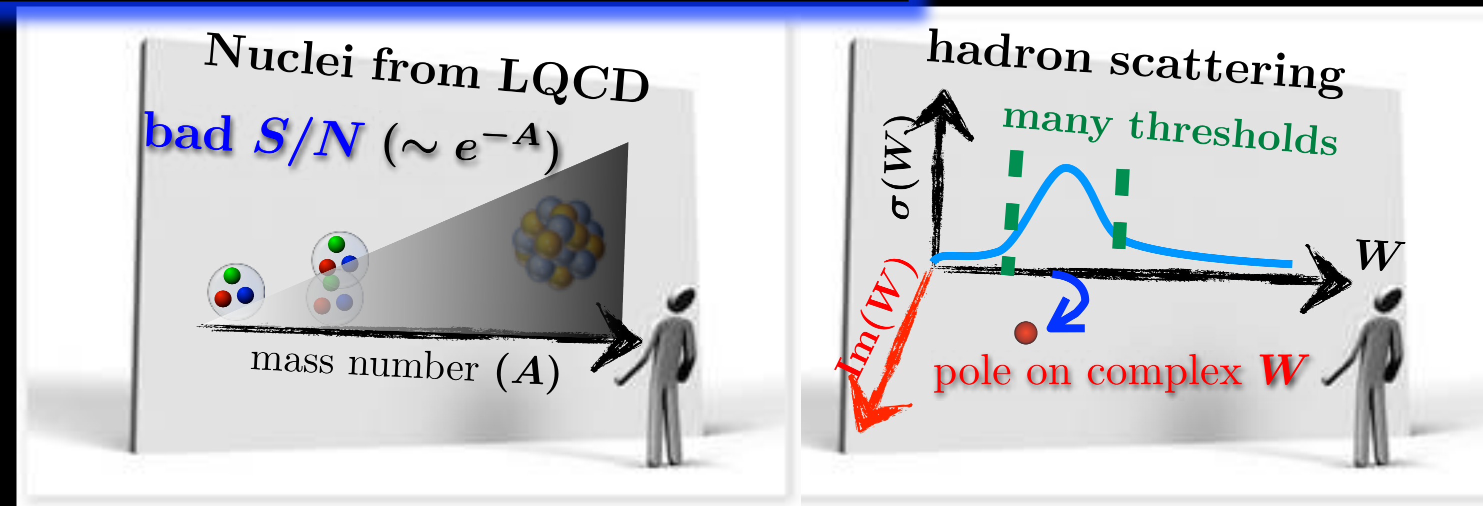
Y. Ikeda, N. Ishii, K. Murano, H. Nemura (RCNP, Osaka Univ.)

T. Inoue (Nihon Univ.)

F. Etminan (Univ. of Birjand)

Lattice2017
35th International Symposium on Lattice Field Theory
18-24 June 2017
Granada - Spain

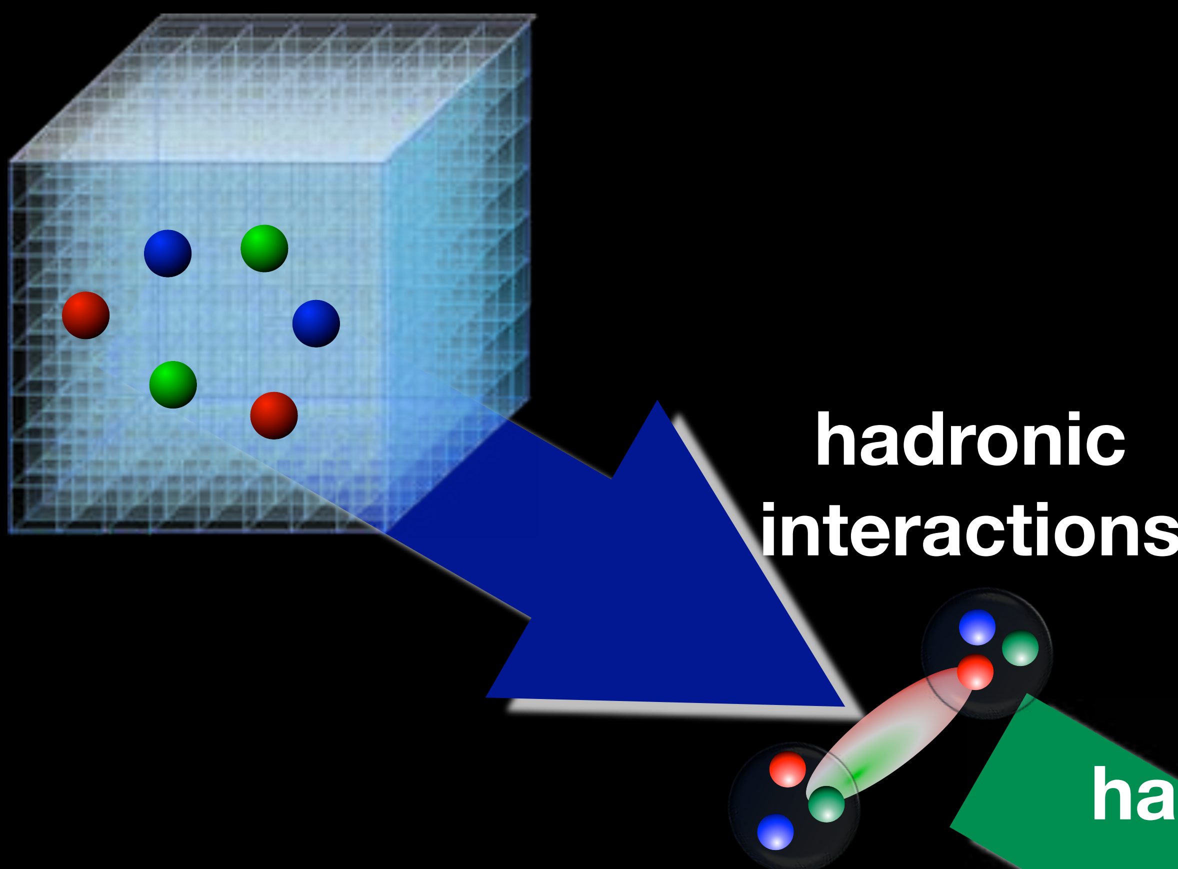
Strategy: from quarks to nuclei & neutron stars

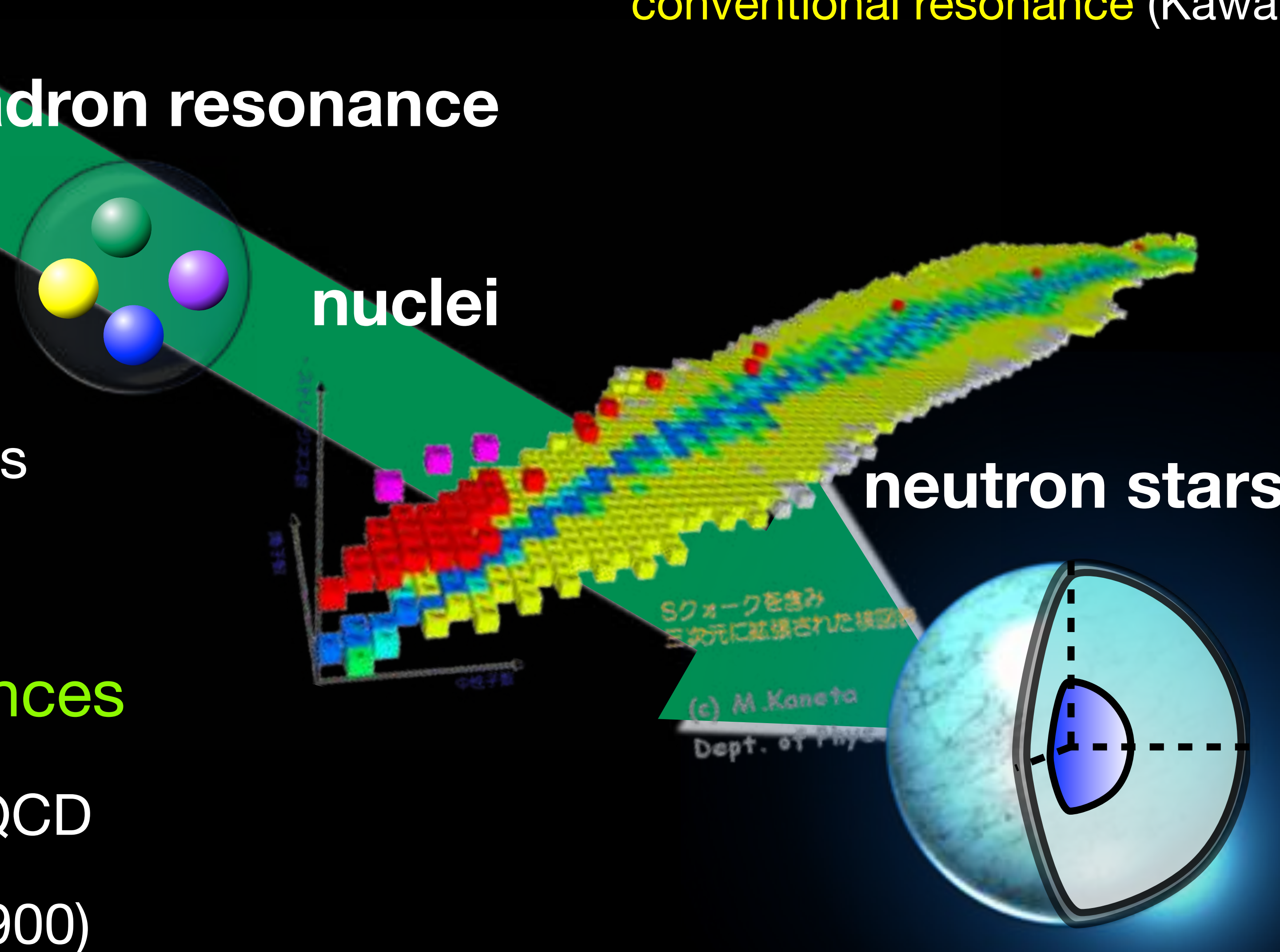


Physical point simulations (Doi, Ishii, Sasaki, Nemura)

Problems in plateau method (Aoki, Iritani)

conventional resonance (Kawai)





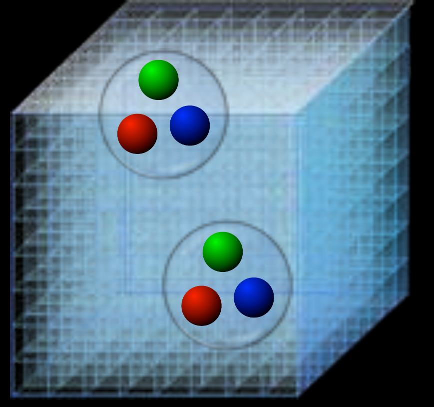
Part I: hadronic interactions

- difficulties in baryon-baryon systems
- solution = HAL QCD method

Part II: coupled-channel & resonances

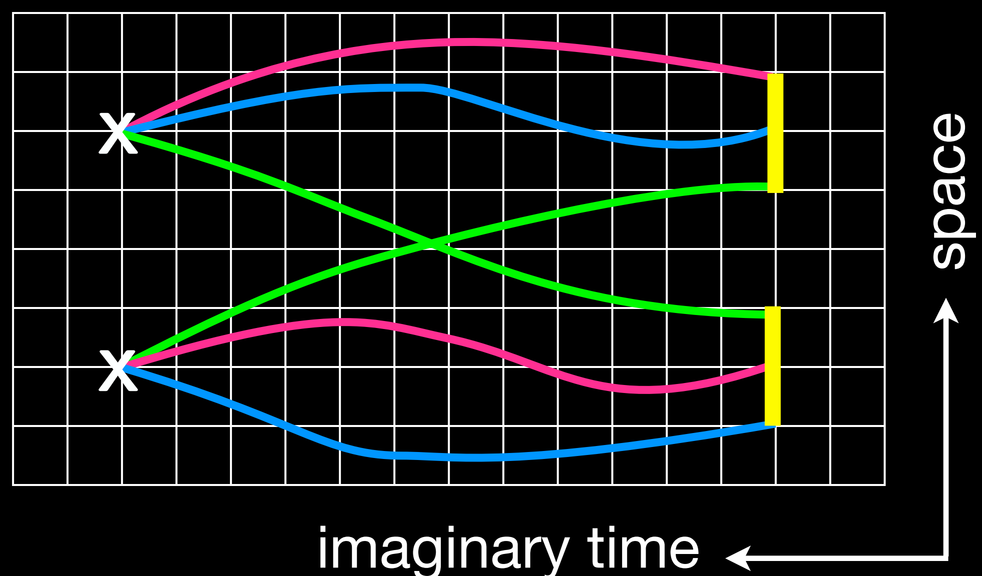
- coupled-channel scattering from LQCD
- nature of tetraquark candidate $Z_c(3900)$

Scattering observables



Hadronic interactions faithful to QCD S-matrix

$$C_{NN}(\vec{r},t) \equiv \langle 0|N_1(\vec{r},t)N_2(\vec{0},t)\mathcal{J}^\dagger(t=0)|0\rangle$$
$$= \sum_n A_n \psi_n(\vec{r}) e^{-W_n t}$$



Energy eigenvalues $W_n(L)$ & NBS (Nambu-Bethe-Salpeter) wave functions $\psi_n(r)$

- **Lüscher's Method**
  - $W_n(L)$ --> phase shift, binding energy

  Lüscher, Nucl. Phys. B354, 531 (1991).

➡ Serious difficulty to measure Wn(L) in BB systems

- **HAL QCD Method**
  - $\psi_n(r)$ --> **2PI kernel** ($\psi = \varphi + G_0 U \psi$)

    --> phase shift, binding energy

  Ishii, Aoki, Hatsuda, PRL 99, 022001 (2007).

  Ishii et al. [HAL QCD], PLB 712, 437 (2012).

Challenge in multi-baryon system (S/N issue)

$$C_N(t) = a_0 e^{-m_N t} + a_1 e^{-(m_N + m_\pi)t} + \cdots$$

$$\longrightarrow a_0 e^{-m_N t}$$

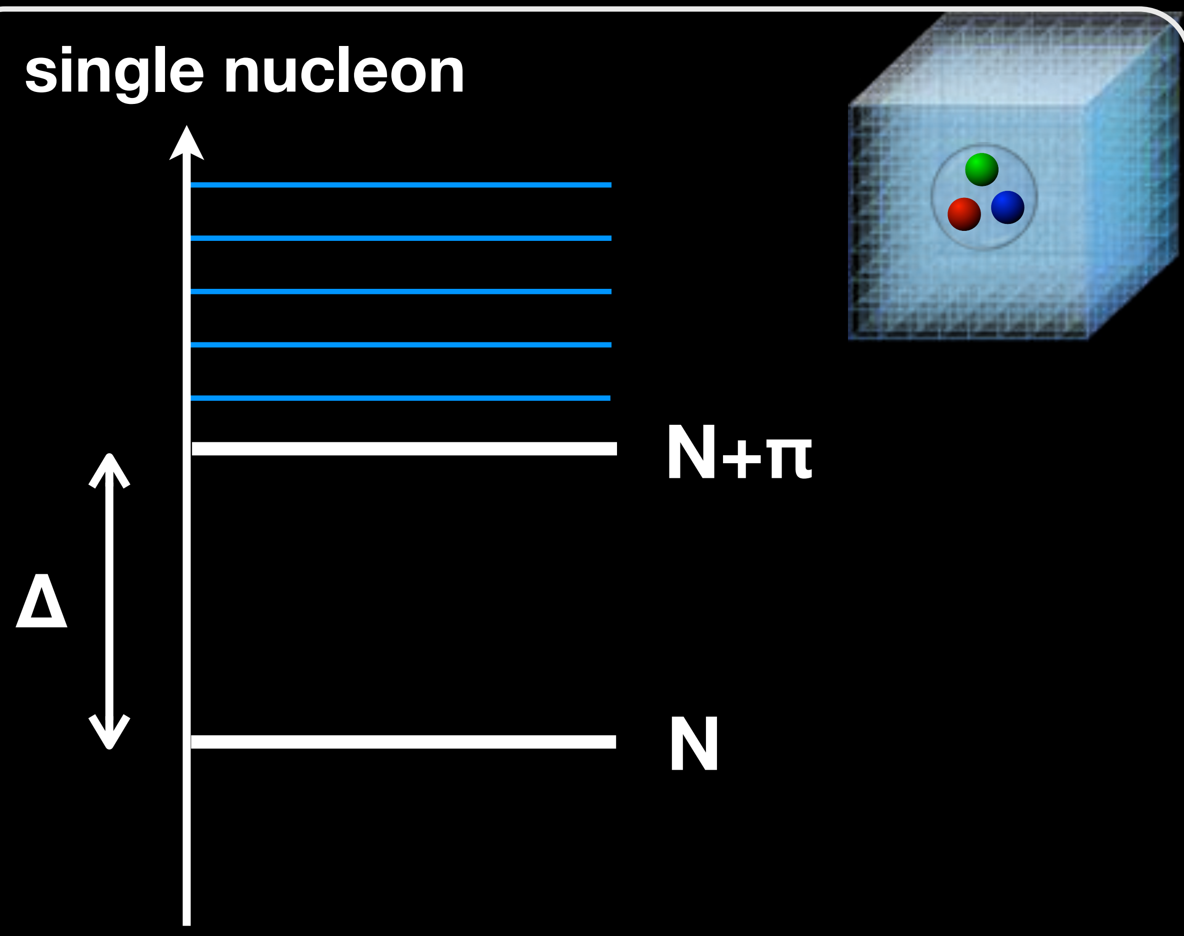
$$(t > t^*)$$

$$C_{NN}(t) = b_0 e^{-W_0 t} + b_1 e^{-W_1 t} + \cdots$$

$$\longrightarrow b_0 e^{-W_0 t}$$

$$(t > t^*)$$

single nucleon

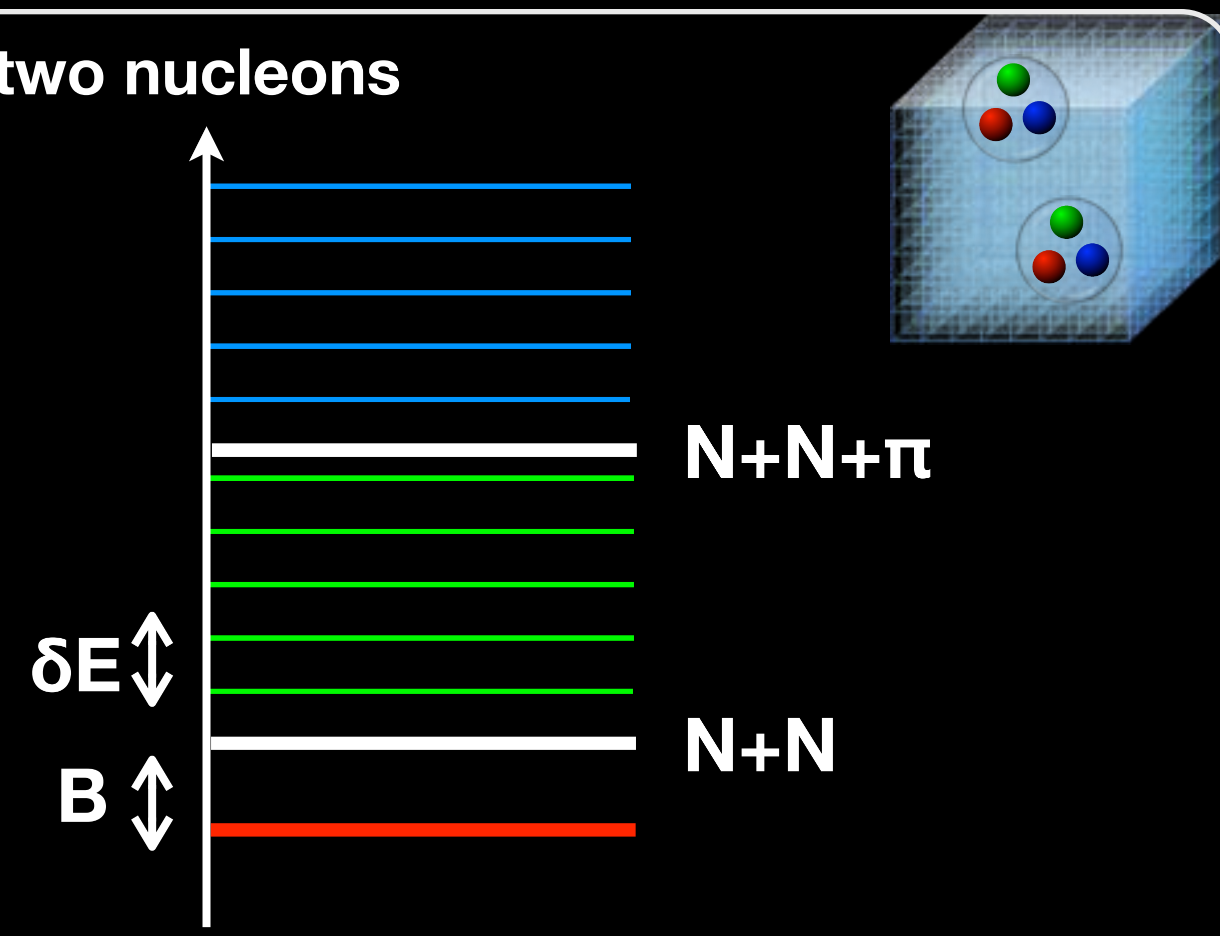


t* ~ Δ⁻¹ ~ 1 fm

$$\mathcal{S}/\mathcal{N} \sim \sqrt{N_{\rm conf.}} \times \exp\left[-(m_N - 3/2 m_\pi) t^*\right]$$

$$\sim \sqrt{N_{\rm conf.}} \times 10^{-2}$$

two nucleons



$t^* \sim \delta E^{-1} = m_N (L/2\pi)^2 \sim 10$ fm

$$\mathcal{S}/\mathcal{N} \sim \sqrt{N_{\rm conf.}} \times \exp\left[-2(m_N - 3/2 m_\pi) t^*\right]$$

$$\sim \sqrt{N_{\rm conf.}} \times 10^{-32}$$

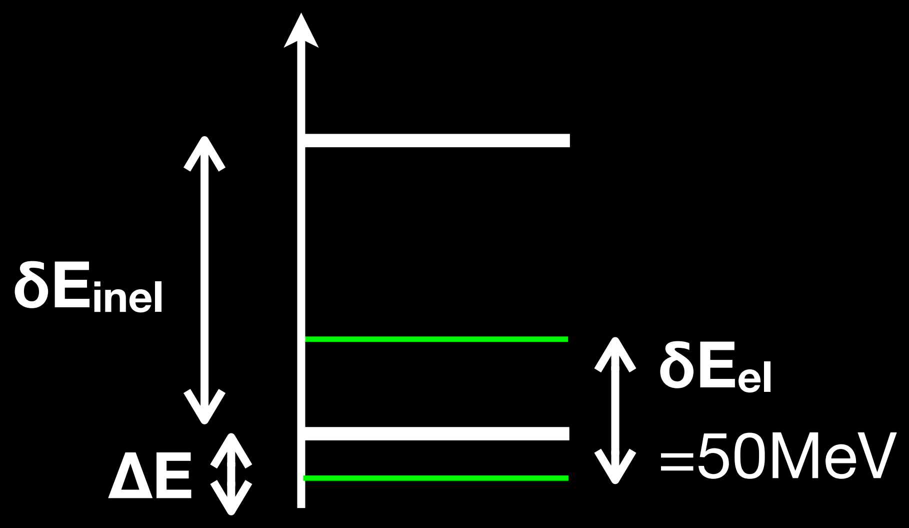
Demonstration of plateau method by mock-up data

"Mirage in temporal correlation functions for baryon-baryon interactions in lattice QCD"
Iritani, Doi et al. [HAL QCD], JHEP10 (2016) 101.

- Normalized correlation func. R(t) for two baryons

$$R(t) = \frac{C_{BB}(t)}{C_B(t)^2} = b_1 e^{-\Delta E t} + b_2 e^{-\delta E_{\rm el} t} + c_1 e^{-\delta E_{\rm inel} t}$$

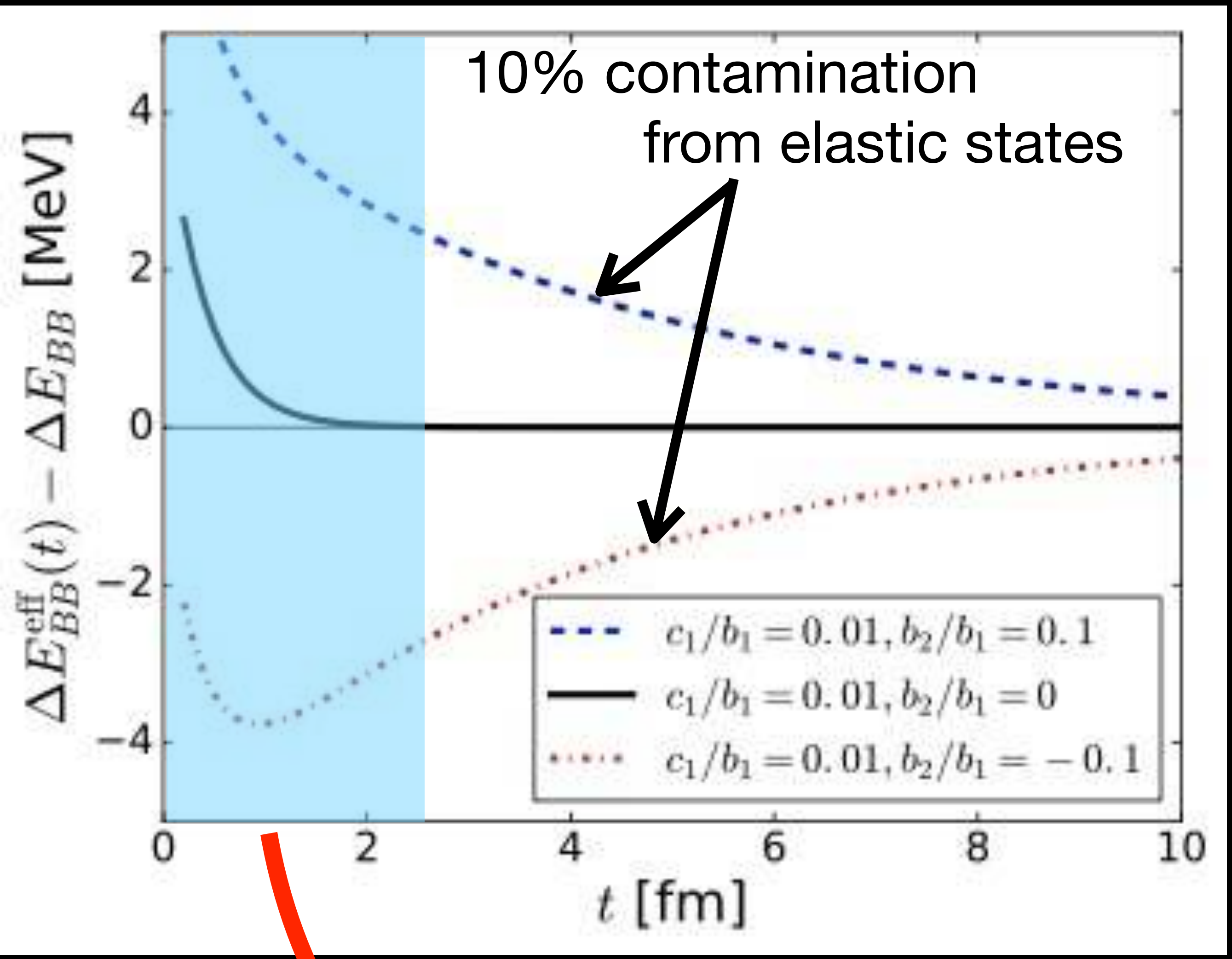
$$\Delta E^{\rm eff}(t) = \log\left[\frac{R(t)}{R(t+1)}\right] \xrightarrow[t > t^*]{} \Delta E$$



- Ground state energy $\Delta E = W_{BB} - 2m_B$
  ~1 MeV precision necessary (nuclear physics scale)
- Elastic scattering states δE_{el}
  δE_{el} = 50MeV, $b_2/b_1 = \pm 0.1, 0$ (10% contamination)
- Inelastic threshold δE_{inel}
  δE_{inel} = 500MeV, $c_1/b_1 = 0.01$ (1% contamination)

Demonstration of plateau method by mock-up data

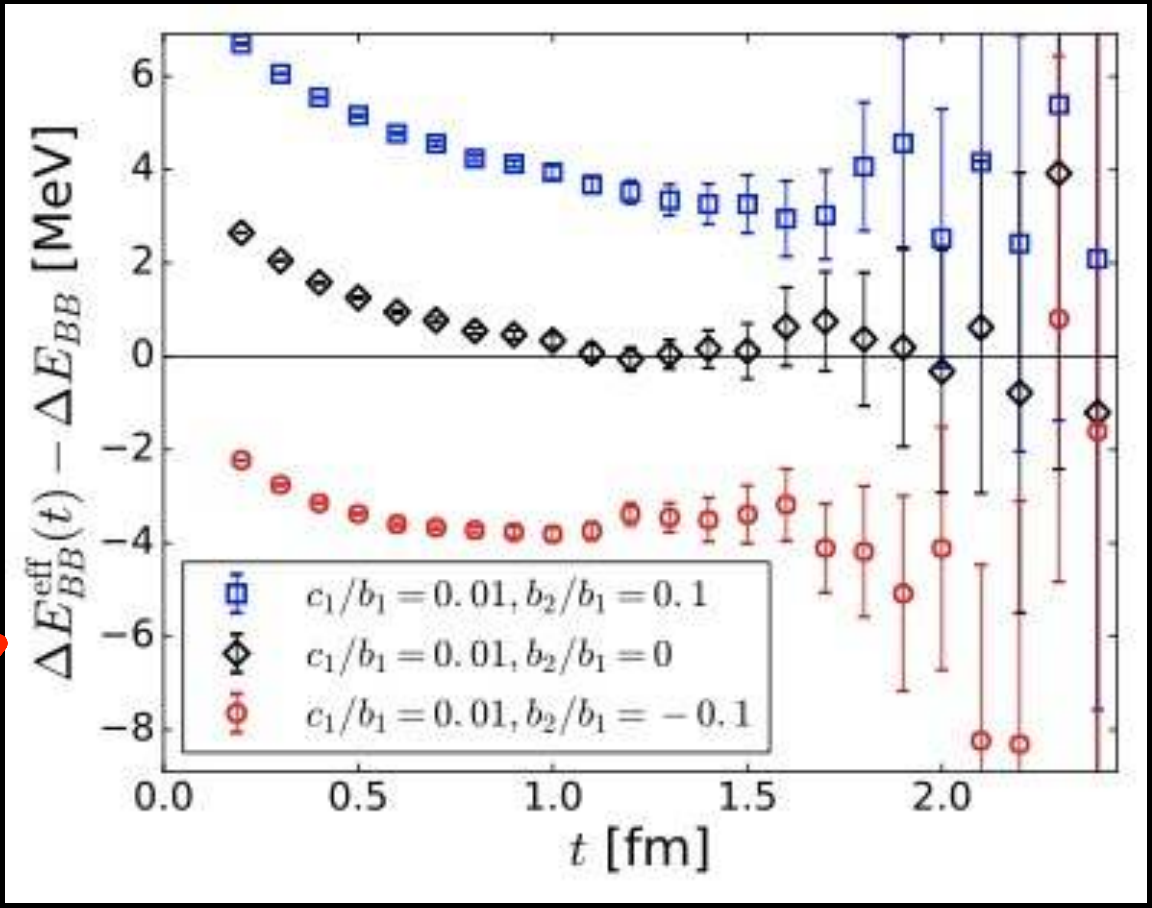
"Mirage in temporal correlation functions for baryon-baryon interactions in lattice QCD"
Iritani, Doi et al. [HAL QCD], JHEP10 (2016) 101.



➡ **True ground state for t > 8 fm with 10% contamination**

Fake plateaux or "Mirage" appear at t ~ 1 fm

Zoom + typical stat. error

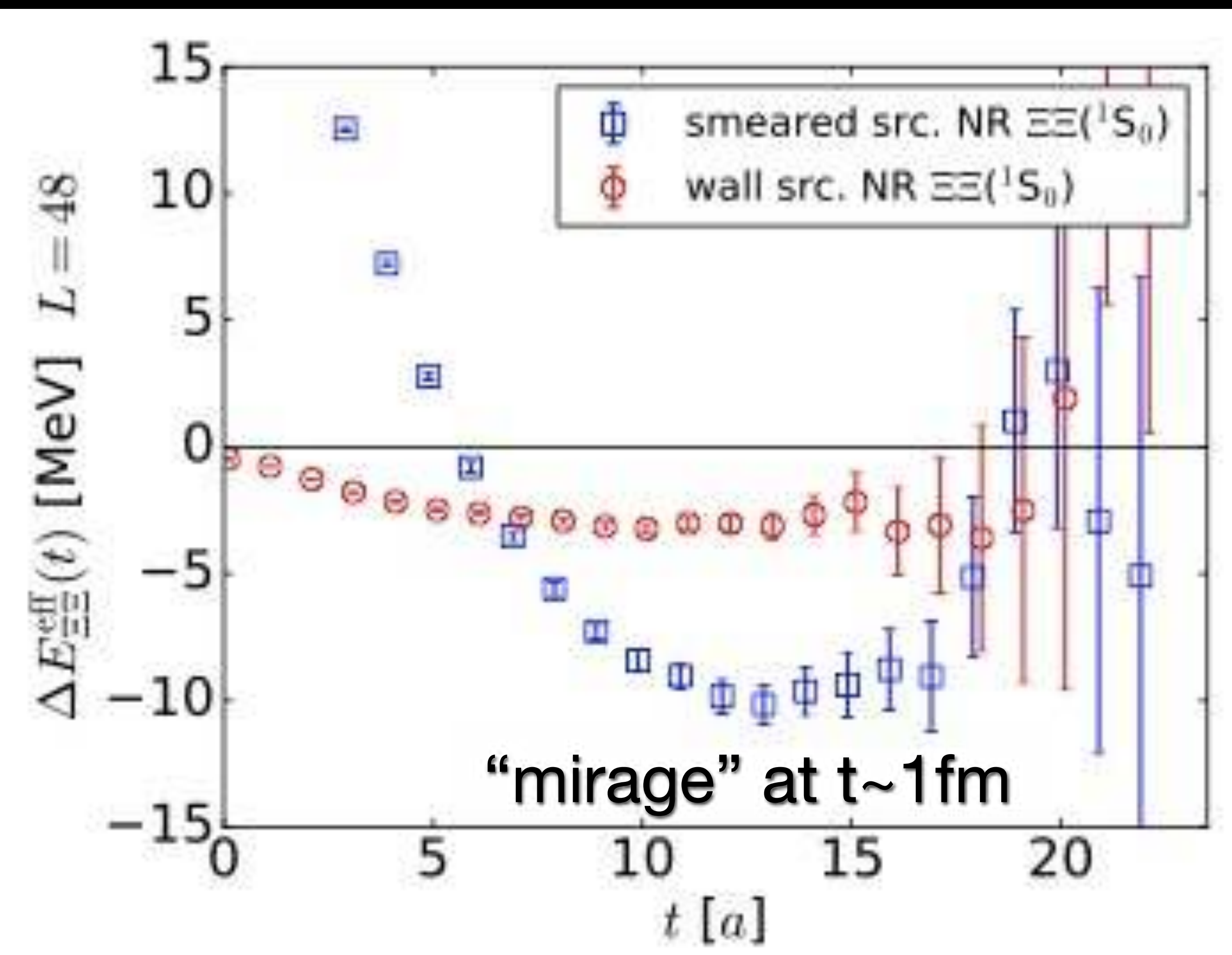


Actual data for ΞΞ ($^1S_0$) @m_π=0.51GeV, L=4.3fm, a=0.09fm

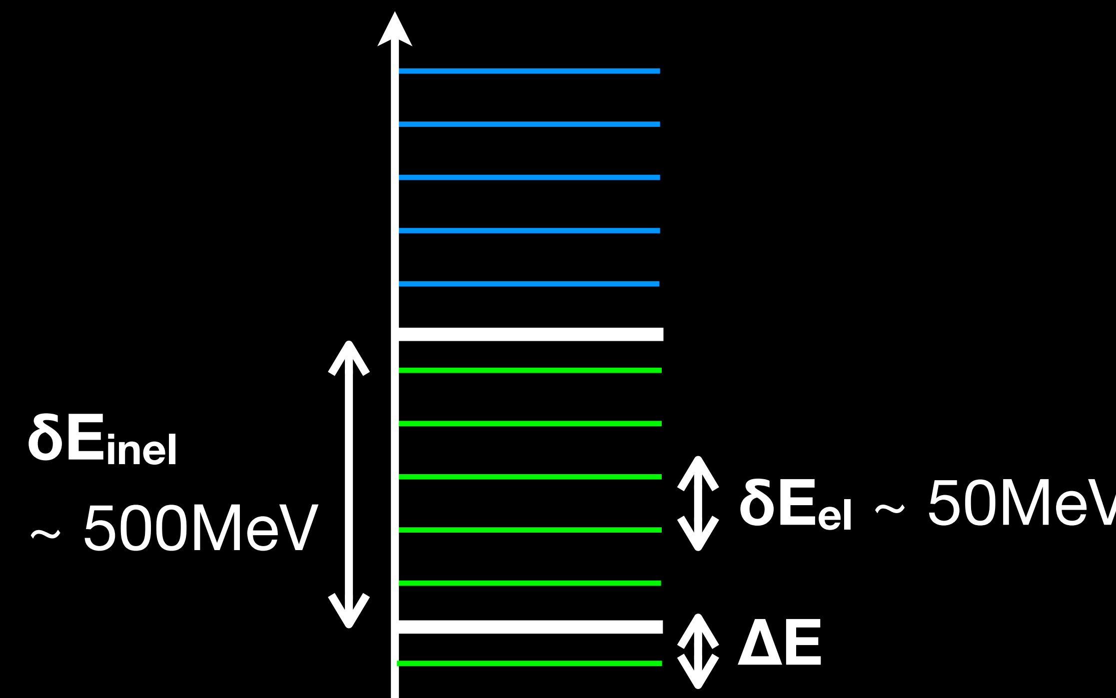
--> same setup as Yamazaki et al. ('12)

Source dependence in plateau method

$$R(t) = \sum_{\vec{x},\vec{y}} \langle 0 | B_1(\vec{x},t) B_2(\vec{y},t) \mathcal{J}^\dagger(t=0) | 0 \rangle / C_B(t)^2$$



$$\Delta E^{\rm eff}(t) = \log\left[\frac{R(t)}{R(t+1)}\right] \xrightarrow[t > t^*]{} \Delta E$$



➡ **True ground state is to appear at t > t* (t*~8 fm)**

- Data at t~1fm is too early to identify ground state energy
- Both "plateaux" can be fake!

New method to diagnose LQCD data in plateau method

-- “Sanity check” with Lüscher’s finite volume formula --

Our proposal

Iritani et al. [HAL QCD], arXiv:1703.07210.

see also talk by Aoki (Thur.)

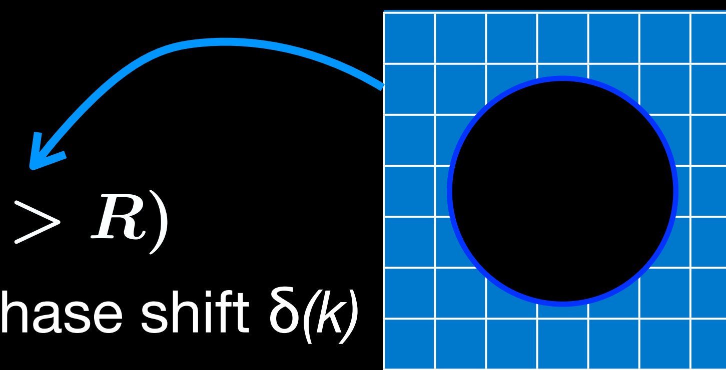
1. extract *k cot δ(k)* through Lüscher’s formula

- Lüscher’s finite V formula
  - energy ΔE --> momentum k --> k cot $\delta(k)$

$$k \cot \delta(k) = \frac{1}{\pi L} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{\vec{n}^2 - (kL/2\pi)^2}$$

$\psi_k(r > R)$

=> phase shift $\delta(k)$

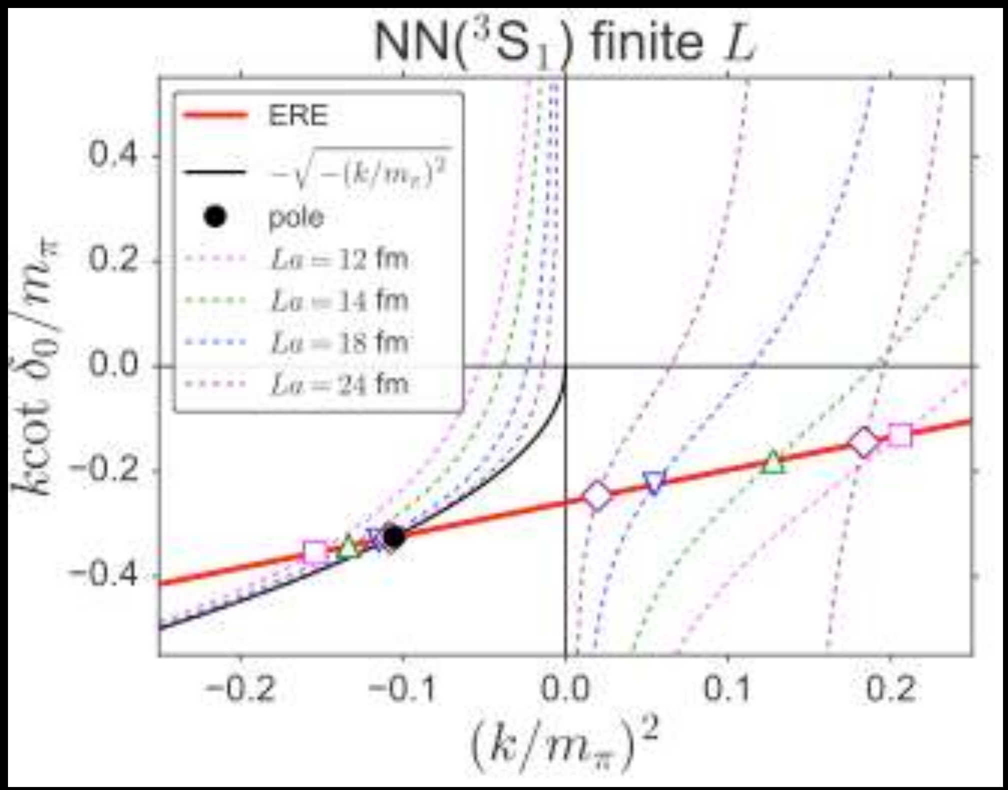


2. combine the results with effective range expansion (ERE)

- Effective range expansion (ERE)

$$k \cot \delta(k) = \frac{1}{a} + \frac{1}{2} r_e k^2 + \cdots$$

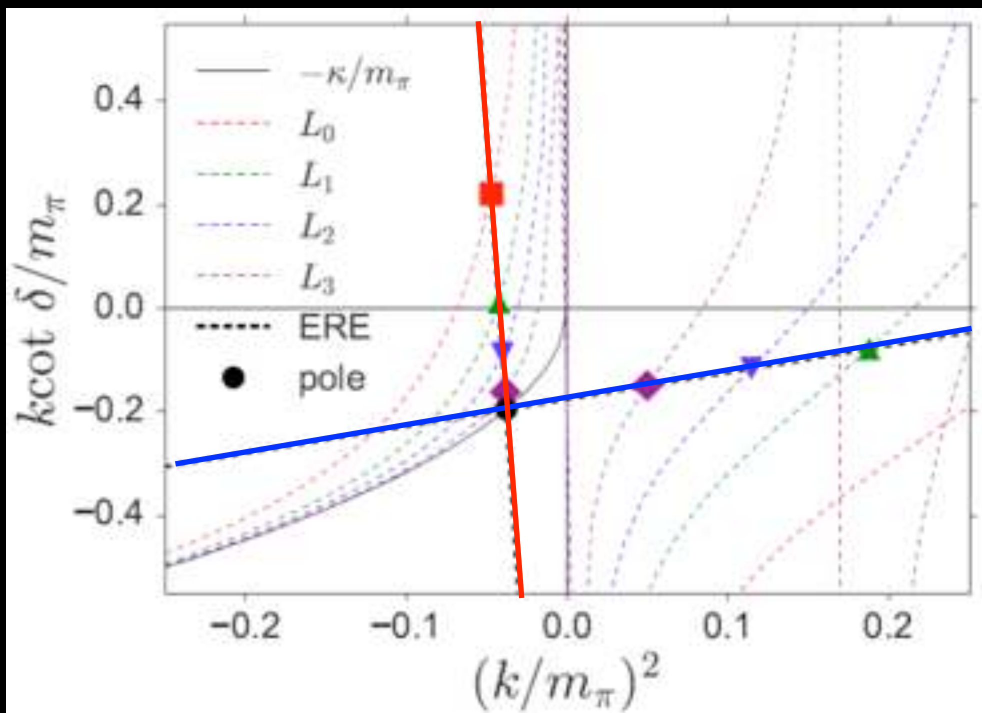
- An example of ERE w/ Lüscher’s formula (dotted lines) for bound state problem
- If the energy is extracted correctly, all data should be alined on ERE line at low-energy



"Sanity check" for all existing data

An exotic example

Iritani et al. [HAL QCD], arXiv:1703.07210.



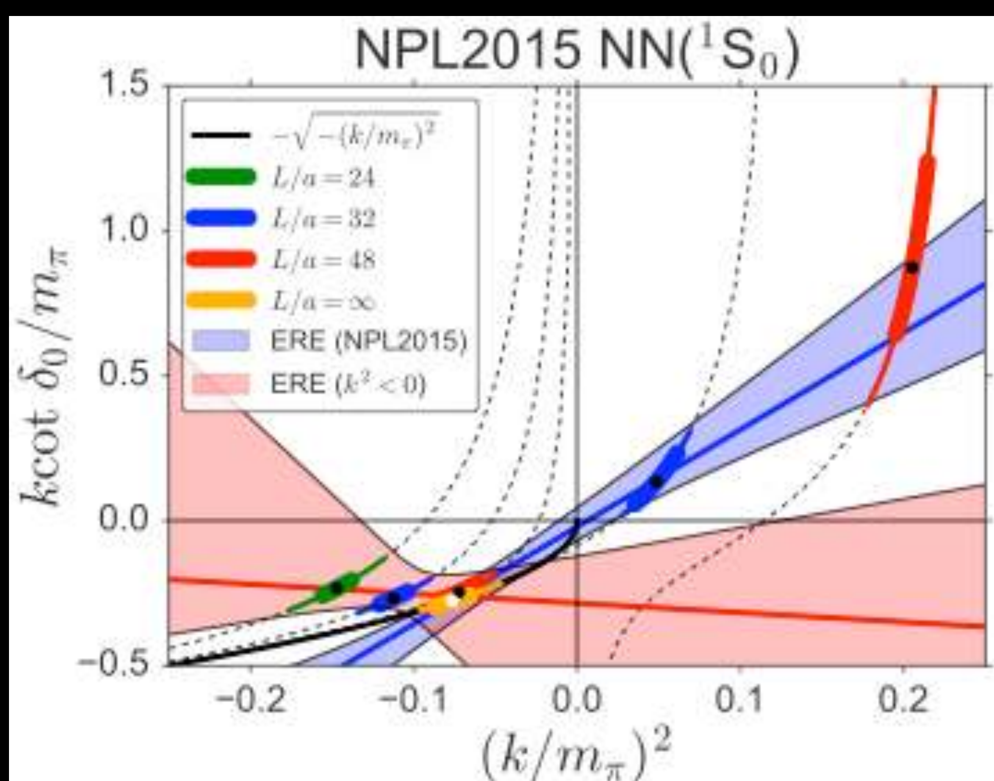
$$k\cot\delta(k) = \frac{1}{a} + \frac{1}{2} r_e k^2 + \cdots$$

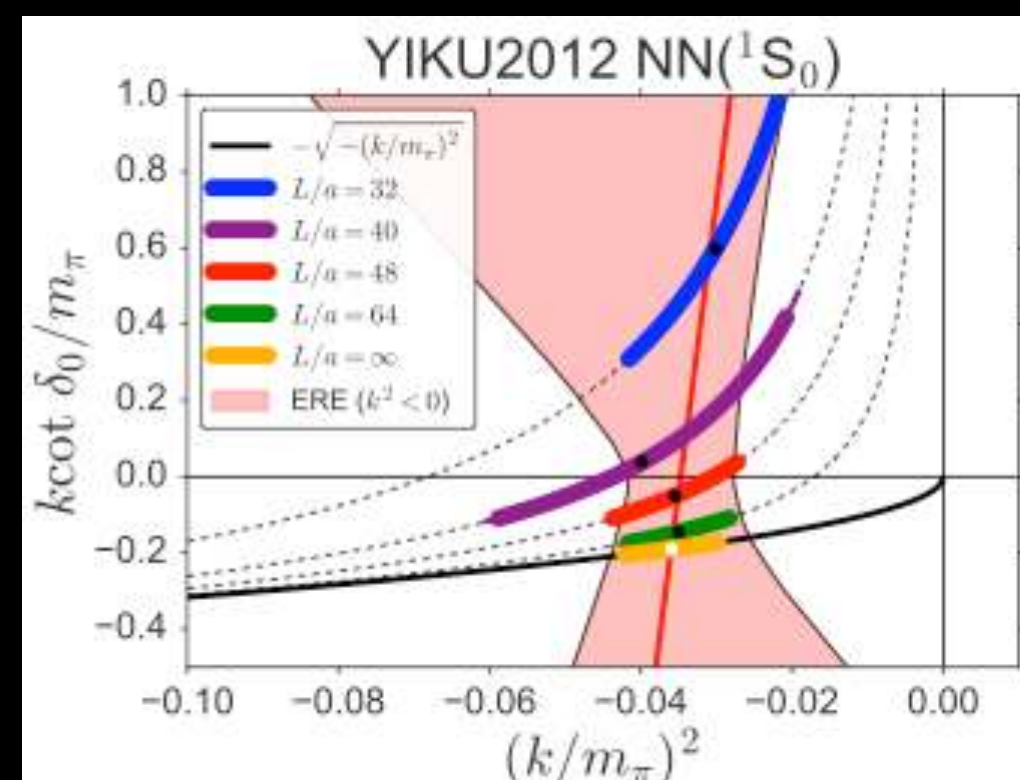
(i) inconsistent ERE for $k^2<0$ and for $k^2>0$

(ii) singular parameterizations $\longrightarrow r_e \simeq \pm\infty$

(iii) unphysical residue at pole position

➡ indicate the problem in LQCD data





(i) inconsistent ERE, (iii) unphysical residue

(ii) singular behavior

Summary table (fake plateaux & sanity checks)

Iritani et al. [HAL QCD], arXiv:1703.07210.

+ update [NPL2013 ($^3S_1$), No --> ?]

At least single “No” implies the result is “Mirage” in temporal correlator

(No data has passed these tests so far)

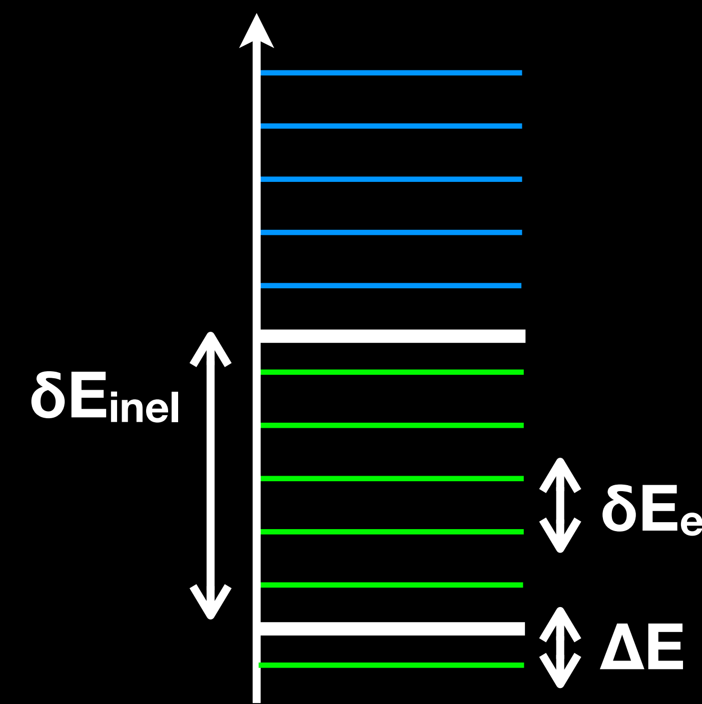
	$NN(^1S_0)$				$NN(^3S_1)$			
Data	Source independence	Sanity check (i)	Sanity check (ii)	Sanity check (iii)	Source independence	Sanity check (i)	Sanity check (ii)	Sanity check (iii)
YKU2011 [23]	†	No	No		†	No	No	
YIKU2012 [24]	No	†	No		No	†	No	
YIKU2015 [25]	†	†	No		†	†	No	No
NPL2012 [26]	†	†	No		†	†		
NPL2013 [27, 28]	No			No	No			?
NPL2015 [29]	†	No		No	†	No		No
CalLat2017 [30]	No	?		No	No	?		No

TABLE IV. A summary of sanity checks (i) consistency between $\mathrm{ERE}_{k^2>0,BE}$ and $\mathrm{ERE}_{k^2<0}$, (ii) non-singular ERE parameters and (iii) physical residue for the bound state pole, together with the source independence of ΔE. Here “No” means that the source independency/sanity check has failed, while the symbol † implies there is none or only insufficient study on the corresponding item. “Blank” implies that obvious contradiction is not found within the error bars, while it does not necessarily guarantee that the data are reliable. See appendix B for the meaning of the symbol ? on the Sanity check for NPL2013 and CalLat2017.

see talk by Aoki (Thur.)

A solution: HAL QCD method -- potential as a representation of S-matrix --

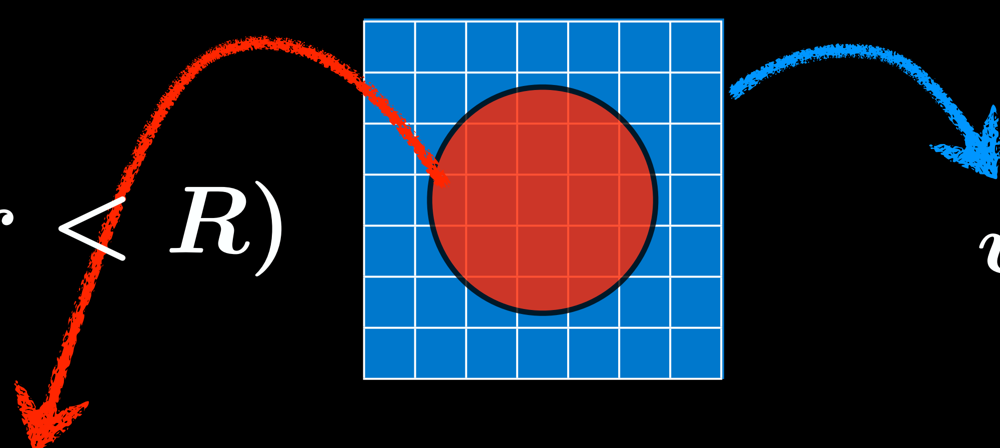
- **The scattering states do exist, and we cannot avoid contaminations**
- **We should tame the scattering states**



Spacial correlation (NBS wave function)

$$\Longrightarrow \psi_n(\vec{r}) \equiv \langle 0 | B_1(\vec{x}) B_2(\vec{y}) | W_n \rangle$$

$\psi_n(r < R)$



$\psi_n(r > R)$

=> Lüscher's method

- **HAL QCD Method**
  - **$\psi_n(r)$** --> **2PI kernel** **($\psi = \varphi + G_0 U \psi$)** --> S-matrix

    (phase shift, binding energy, ...)

Ishii, Aoki, Hatsuda, PRL 99, 022001 (2007).

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

Ishii et al. [HAL QCD], PLB 712, 437 (2012).

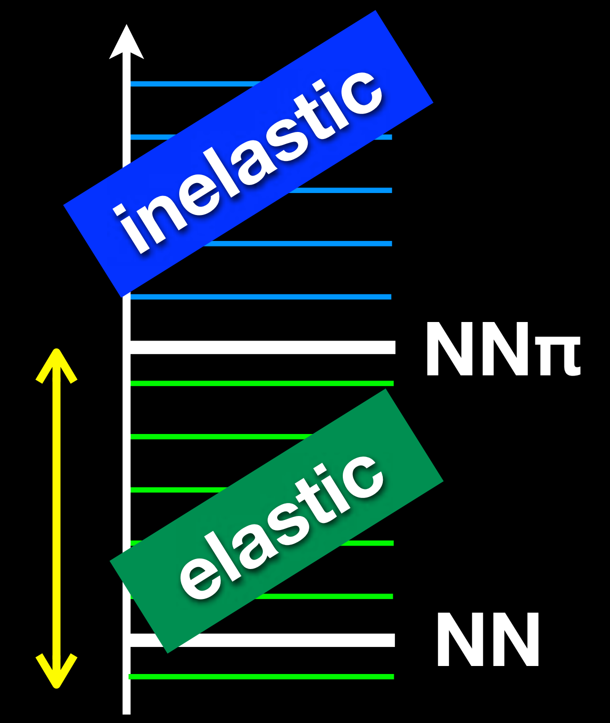
A solution: HAL QCD method -- potential as a representation of S-matrix --

✓ energy-independent potential U(r,r')

$$\int d\vec{r}' U(\vec{r},\vec{r}')\psi_n(\vec{r}') = (E_n - H_0)\,\psi_n(\vec{r})$$

➡ All elastic states share the same potential U(r,r')

$$\int d\vec{r}' U(\vec{r},\vec{r}')\psi_0(\vec{r}') = (E_0 - H_0)\,\psi_0(\vec{r}')$$
$$\int d\vec{r}' U(\vec{r},\vec{r}')\psi_1(\vec{r}') = (E_1 - H_0)\,\psi_1(\vec{r}')$$
$$\vdots$$



✓ All equations are combined into time-dependent Schrödinger-type eq.

$$\int d\vec{r}' U(\vec{r},\vec{r}')R(\vec{r}',t) = \left(-\frac{\partial}{\partial t} + \frac{1}{4m_B}\frac{\partial^2}{\partial t^2} - H_0\right) R(\vec{r},t)$$

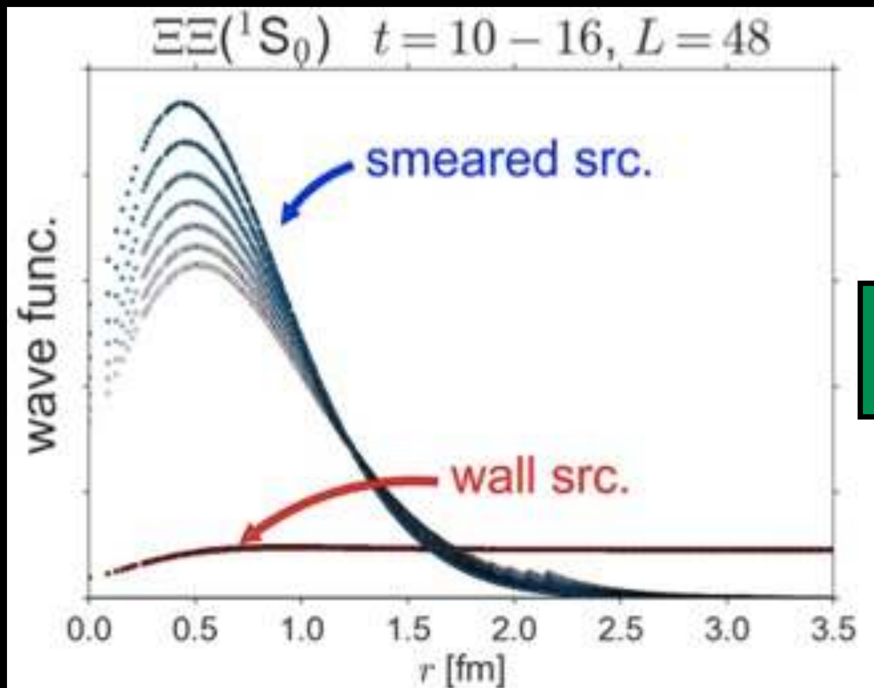
$$R(\vec{r},t) = A_0 e^{-(W_0-2m_B)t}\psi_0(\vec{r}) + A_1 e^{-(W_1-2m_B)t}\psi_1(\vec{r}) + \cdots$$

	Plateau method	HAL QCD method
Inelastic states	noise	noise
elastic states	noise	**signal**
necessary t (> t*)	**t*~10fm**	**t*~1fm**

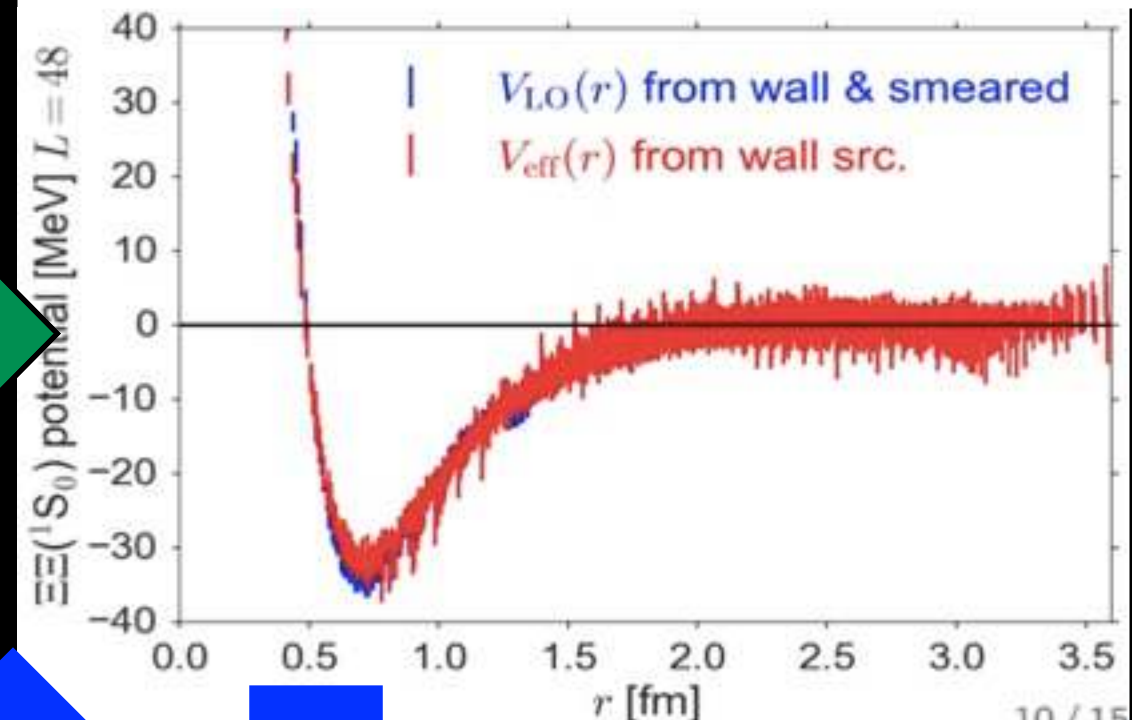
S/N improved exponentially

ΞΞ ($^1S_0$) in HAL QCD method @m_π=0.51GeV, L=4.3fm, a=0.09fm

source dependence of R(r,t)

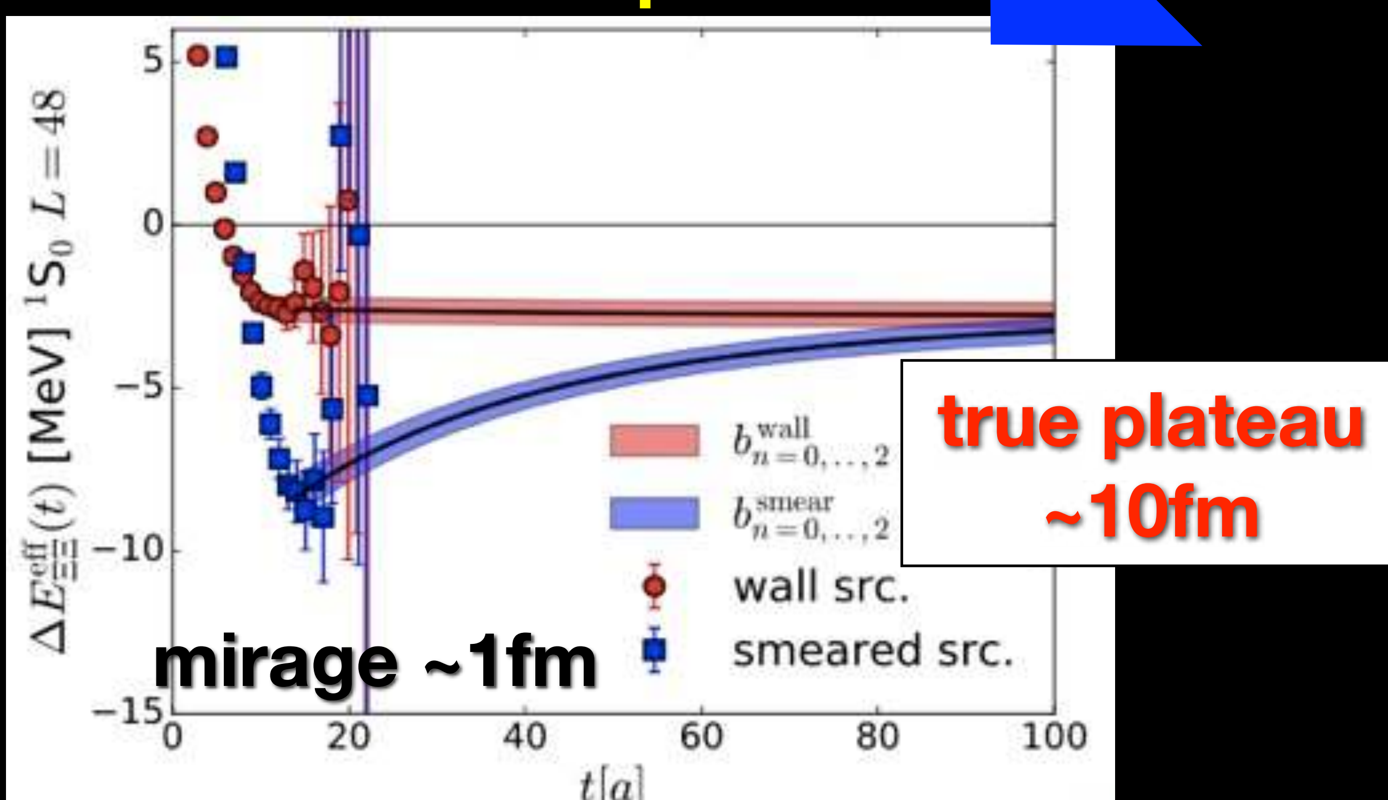


$$U(\vec{r}, \vec{r}') = \left[V_{\rm LO}(\vec{r}) + V_{\rm NLO}(\vec{r})\nabla^2 \cdots \right] \delta(\vec{r} - \vec{r}')$$

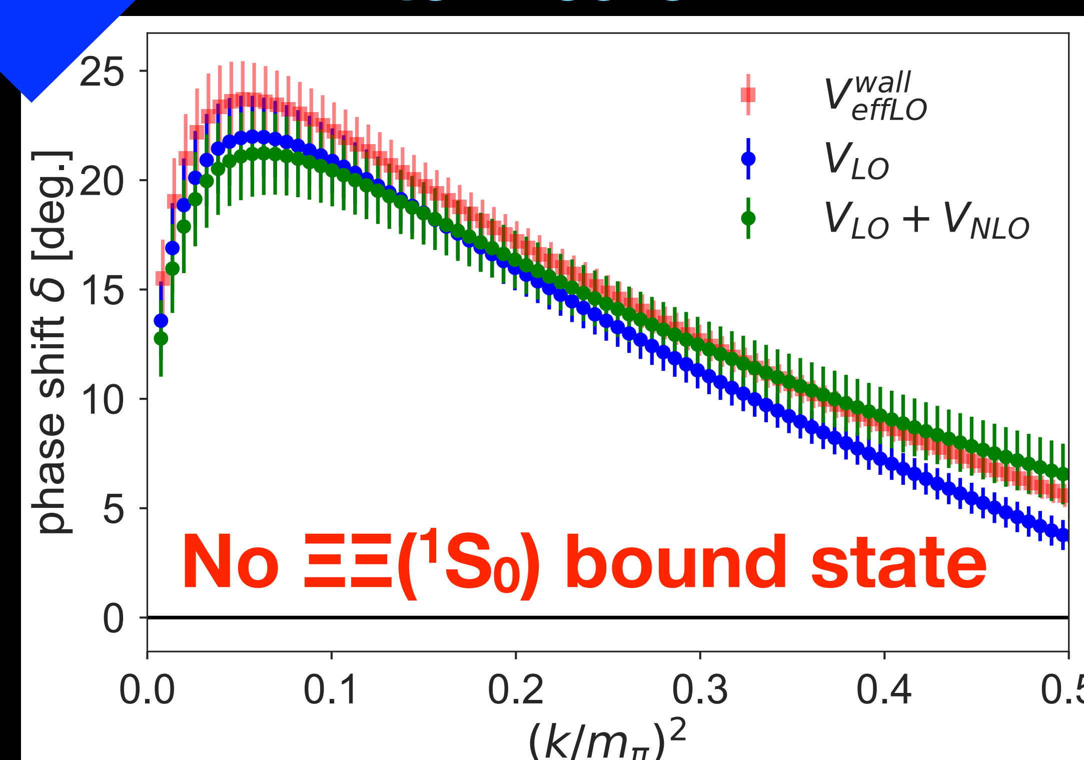


finite V calc.

➡ Fate of fake plateaux



infinite V calc.

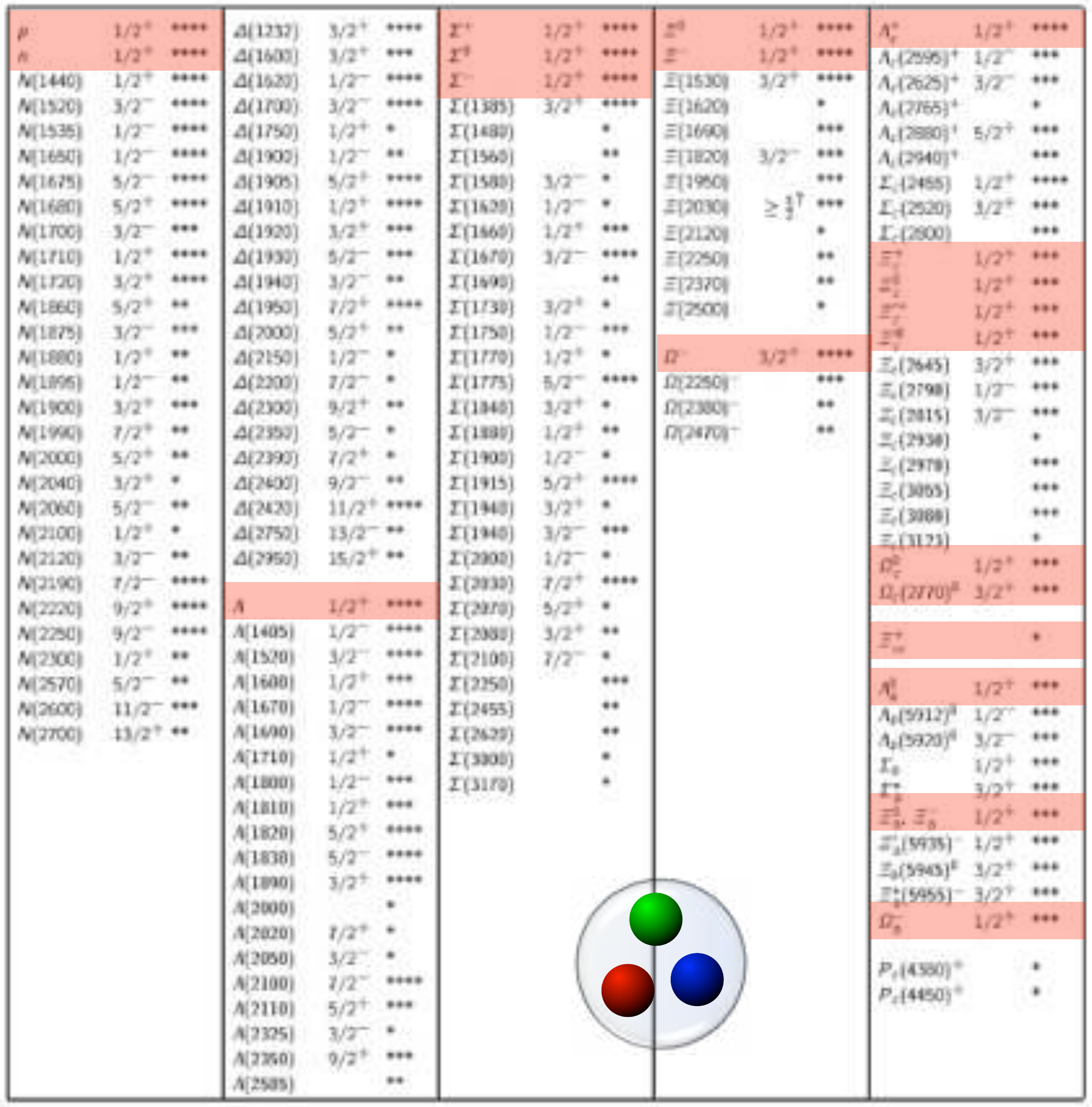


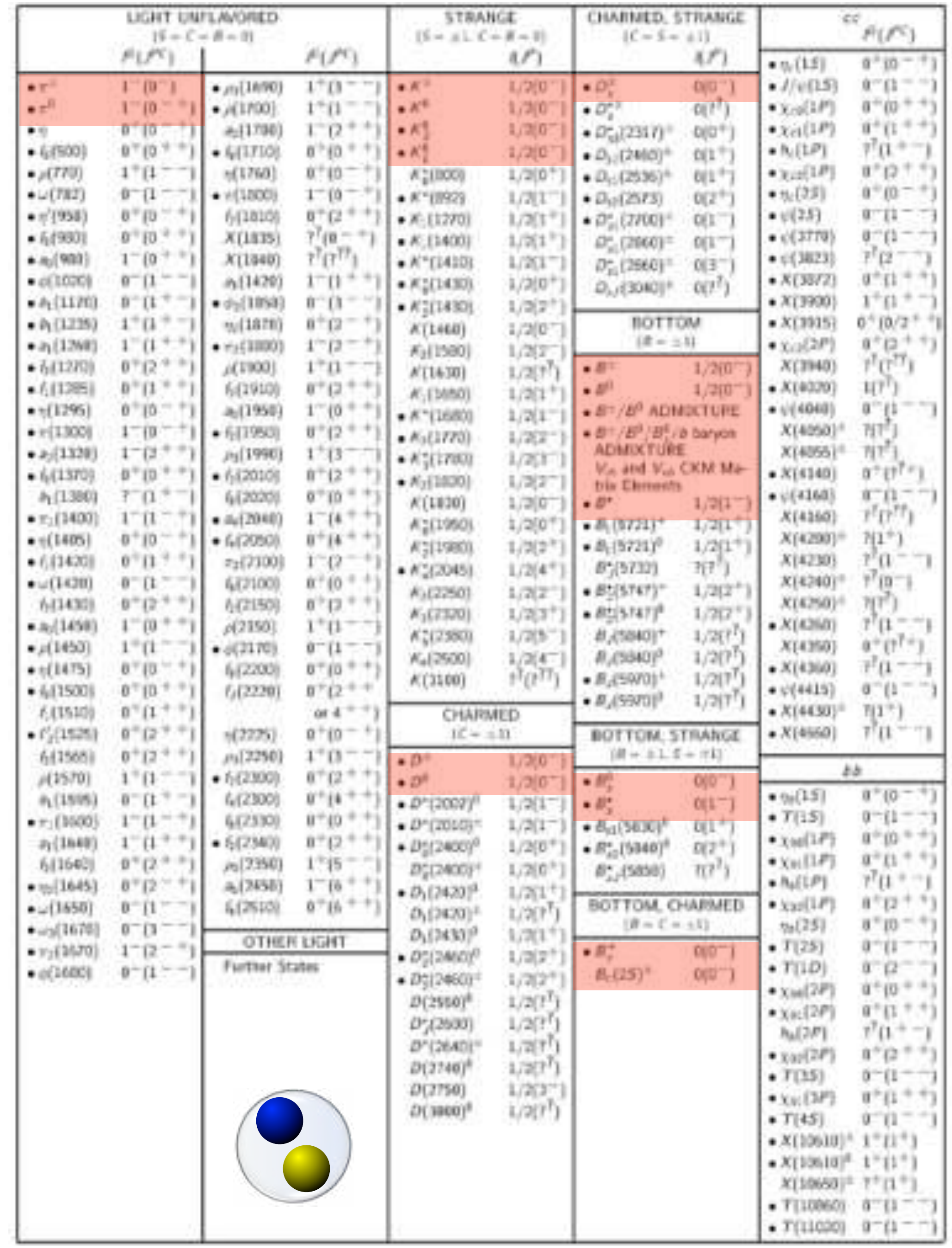
see talk by Iritani (Thur.)

Hadron resonances

- **Particle data group**

  http://www-pdg.lbl.gov/



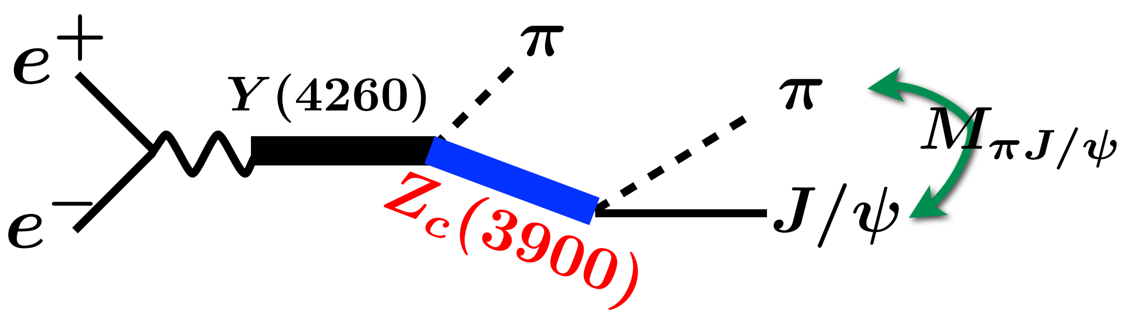


- Most hadrons are consistent with qqq / qq^{bar} quantum number (non-trivial)
- Only 10% is stable, others are unstable (resonances)
- To establish exotic resonances is important issue in hadron physics

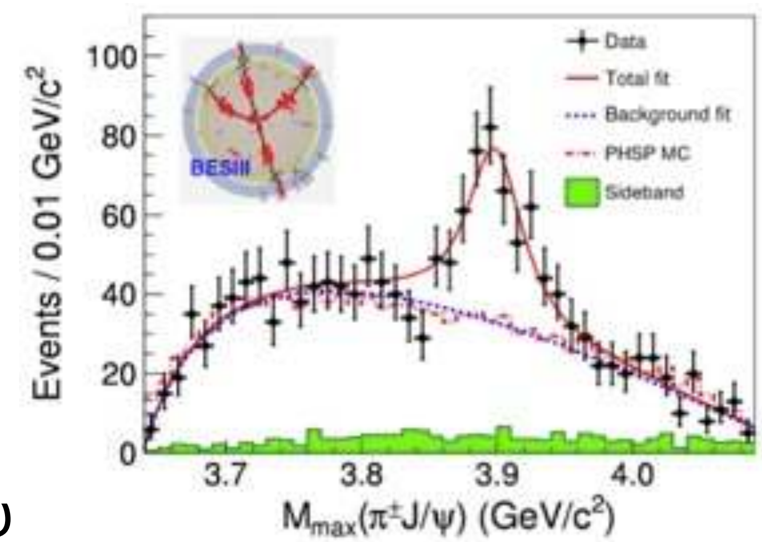
  (Challenge in LQCD simulations)

Tetraquark candidate $Z_c(3900)$

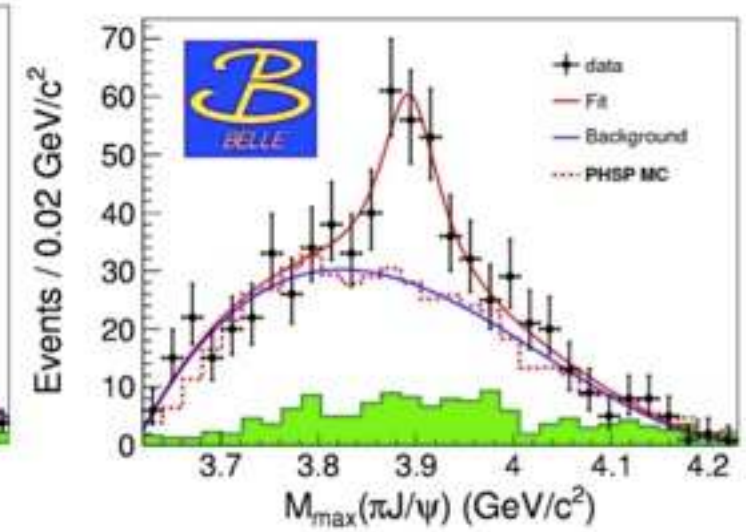
● **Expt. observations**



BESIII Coll., PRL110 (2013).



Belle Coll., PRL110 (2013).



- $e^+ + e^-$ --> Y(4260) --> π + $Z_c(3900)$

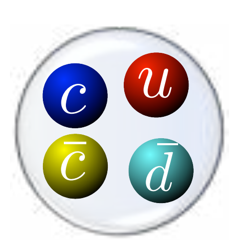
  ➡ $\pi^{+/-}$ + J/ψ

- peak in **$\pi^{+/-}$J/ψ invariant mass** (minimal quark content $c c^{bar} u d^{bar}$ <--> tetraquark candidate)
- M ~ 3900, Γ ~ 60 MeV (Breit-Wigner) --> just above $D^{bar}D^*$ threshold

  ($J^P=1^+$ <--> couple to **s-wave** meson-meson continuum)

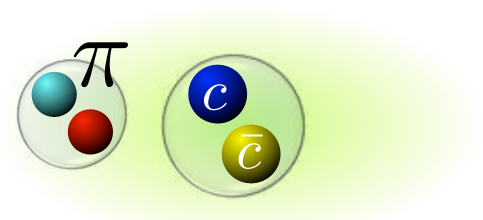
★ **structure of $Z_c(3900)$ studied by models**

tetraquark



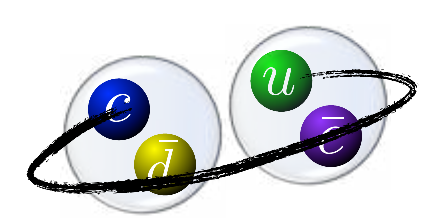
Maiani et al.('13)

J/ψ + π atom



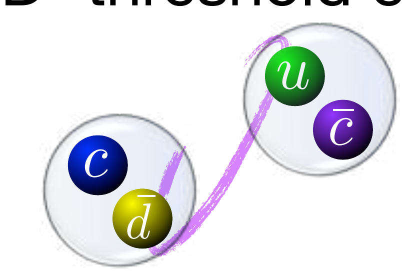
Voloshin('08)

$D^{bar}D^*$ molecule



Nieves et al.('11) + many others

$D^{bar}D^*$ threshold effect



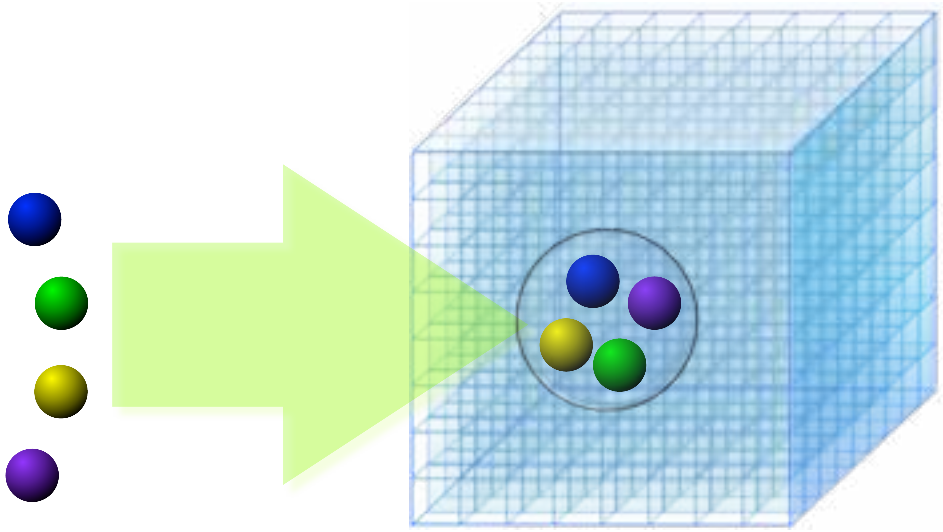
Chen et al.('14), Swanson('15)

genuine state expected **kinematical origin**

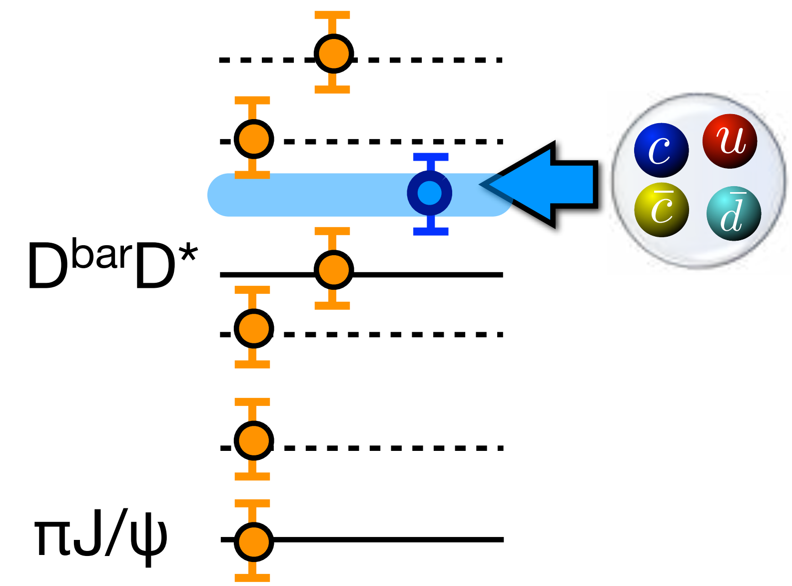
How to study $Z_c(3900)$ on the lattice?

✦ Conventional approach: temporal correlation

➡ identify all relevant $W_n(L)$ (n=0,1,2,3,...)



$$\langle 0|\Phi[c\bar{c}u\bar{d}](\tau)|W_n\rangle = e^{-W_n\tau}$$



✓ No positive evidence for $Z_c(3900)$ in $J^{PC}=1^{+-}$

variational method

S. Prelovsek et al., PLB 727 (2013), PRD91 (2015).

S.-H. Lee et al., PoS Lattice2014 (2014).

see also $D^{bar}D^*$ single channel calc. Y. Chen et al., PRD89 (2014).

★ Why is the peak observed in expt.?

- resonance? threshold effect?

➡ resonance search in manifest coupled-channel necessary

(To understand expt. signals for exotics from QCD is very challenging)

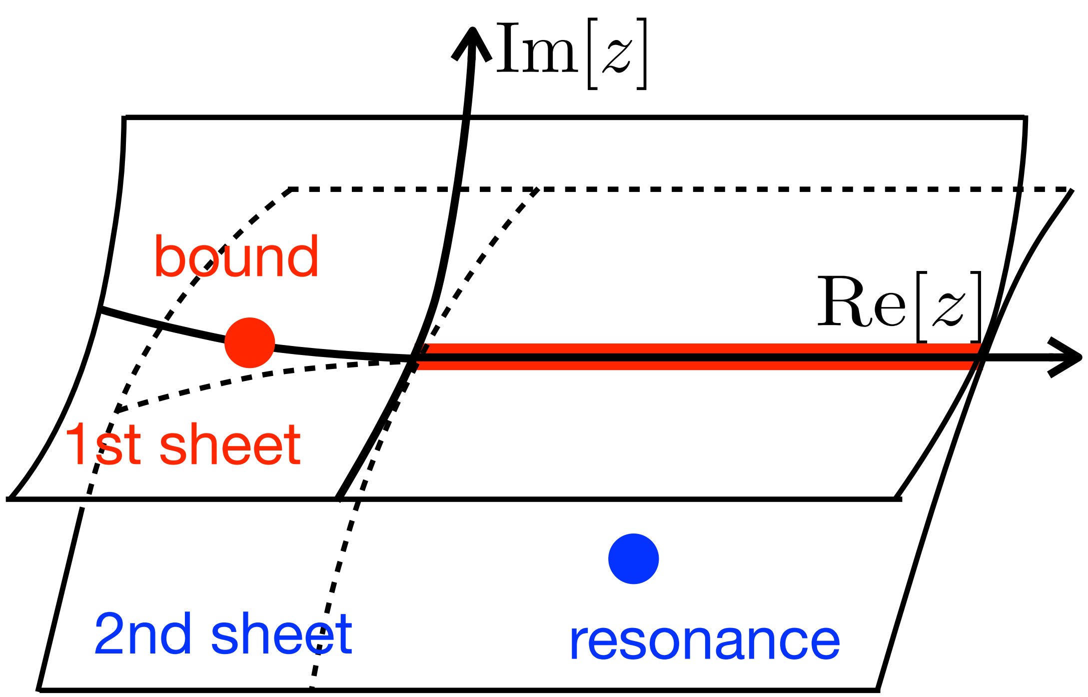
How to search for resonances on the lattice

★ **Resonance energy is NOT eigen-energy W_n** $\langle 0|\Phi[c\bar{c}u\bar{d}](\tau)|W_n\rangle = e^{-W_n\tau}$

$S(W)$ ⟵ **S-matrix from HAL QCD method**

- NBS wave func. ψ(r) --> “potential” --> infinite V calc. --> S(W)

Analyticity of S-matrix is **uniquely** determined **(S-matrix theory)**



bound state (1st sheet)

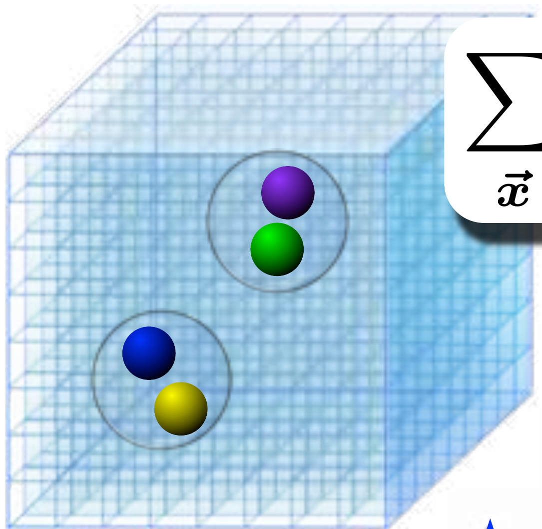
- pole position --> binding energy
- residue --> coupling to scattering state

resonance (2nd sheet)

- analytic continuation onto 2nd sheet
- pole position --> resonance energy
- residue --> coupling to scat. state, partial decay

Coupled-channel HAL QCD method

✦ **measure not only temporal but also spatial correlation**



$$\sum_{\vec{x}} \langle 0|\phi_1^a(\vec{x}+\vec{r},t)\phi_2^a(\vec{x},t)\mathcal{J}^\dagger(0)|0\rangle = \sqrt{Z_1^a Z_2^a}\sum_n A_n \psi_n^a(\vec{r}) e^{-W_n t}$$

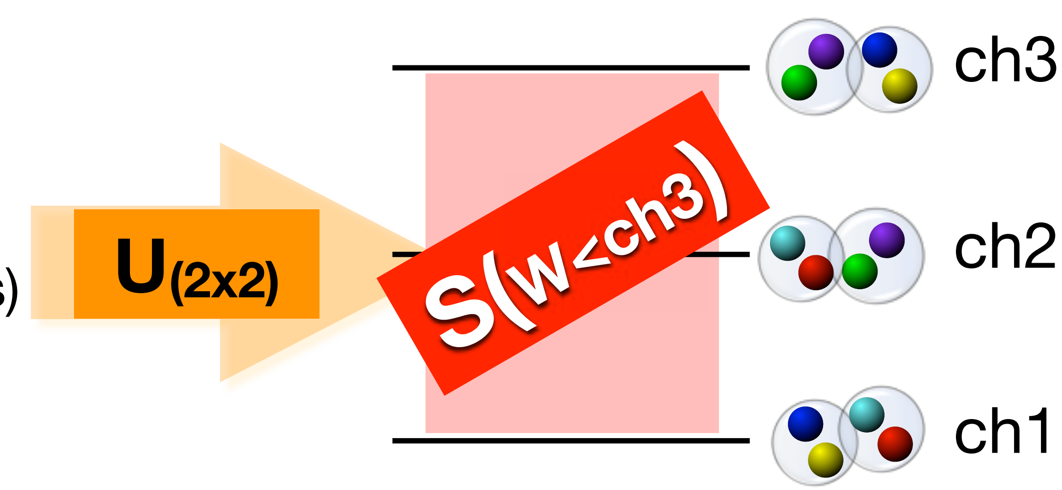
Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).
Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al. (HAL QCD), PLB712, 437(2012).

★ **spatial correlation --> identify channel wave function**

$$\left(\nabla^2 + (\vec{k}_n^a)^2\right)\psi_n^a(\vec{r}) = 2\mu^a \sum_b \int d\vec{r}'\, U^{ab}(\vec{r},\vec{r}')\psi_n^b(\vec{r}')$$

★ **coupled-channel potential U^{ab}(r,r'):**

- U^{ab}(r,r') is faithful to **coupled-channel S-matrix**
- U^{ab}(r,r') is **energy independent** (until new threshold opens)
- Non-relativistic approximation is not necessary
- U^{ab}(r,r') contains all 2PI contributions



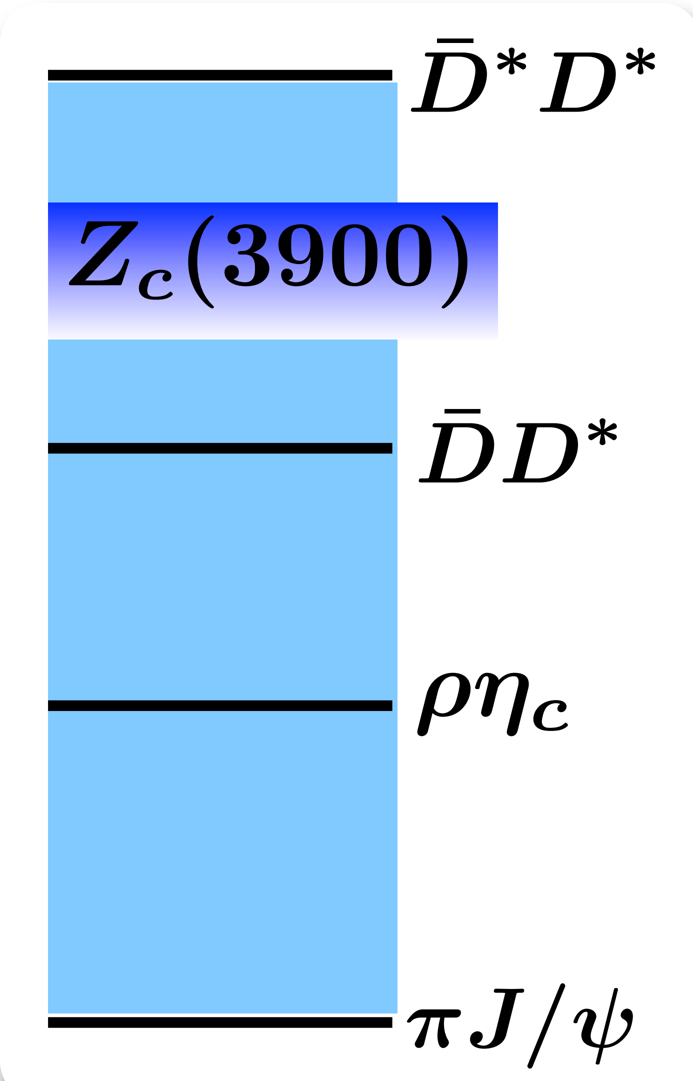
➡ **derive potential from time-dependent HAL QCD method**

Full details, Aoki et al. (HAL QCD), PRD87, 034512 (2013); Proc. Jpn. Acad., Ser. B, 87 (2011).

$Z_c(3900)$ in $I^G(J^{PC})=1^+(1^{+-})$

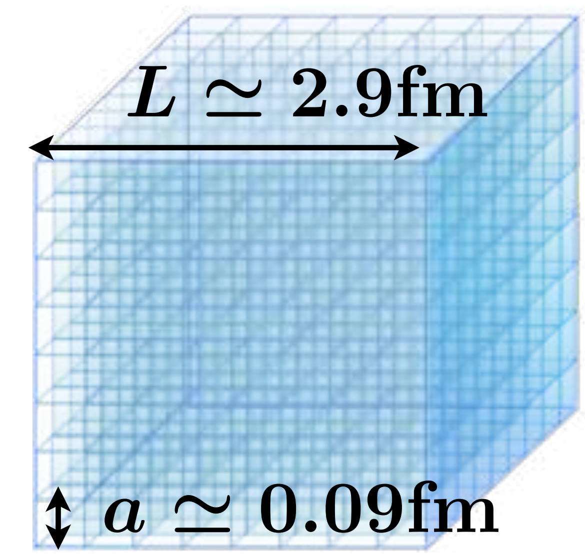
-- $\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$ coupled-channel --

Y. Ikeda et al., [HAL QCD], PRL117, 242001 (2016).



❖ $N_f=2+1$ full QCD

- Iwasaki gauge
- clover Wilson quark
- 32^3 x 64 lattice



❖ Tsukuba-type Relativistic Heavy Quark (charm)

- remove leading cutoff errors $O((m_c a)^n)$, $O(\Lambda_{QCD} a)$, ...

➡ We are left with $O((a\Lambda_{QCD})^2)$ syst. error (~ a few %)

light meson mass (MeV)

m_π= 411(1), 572(1), 701(1)

m_ρ= 896(8), 1000(5), 1097(4)

charm meson mass (MeV)

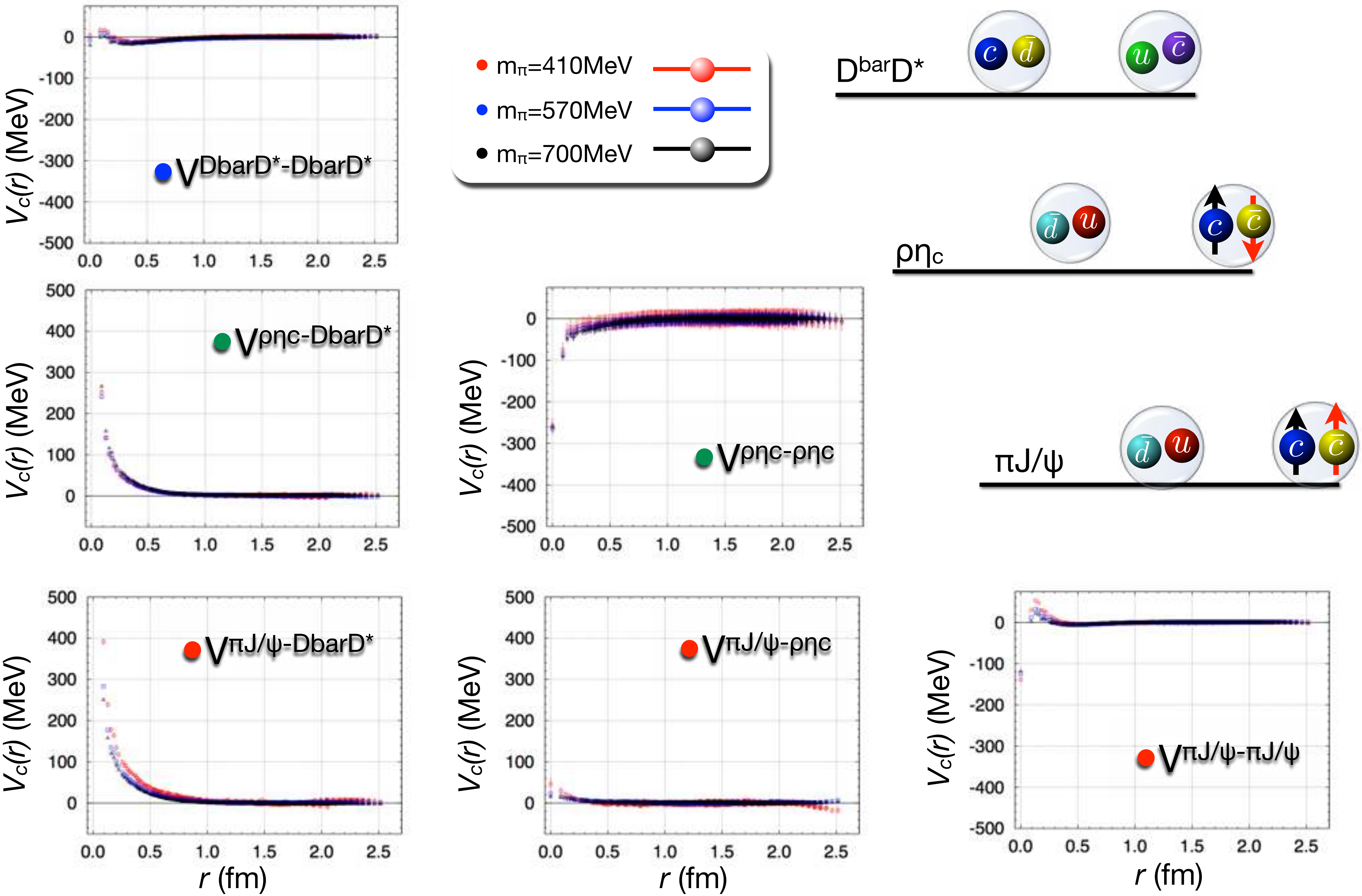
$m_{\eta c}$= 2988(1), 3005(1), 3024(1)

$m_{J/\psi}$= 3097(1), 3118(1), 3143(1)

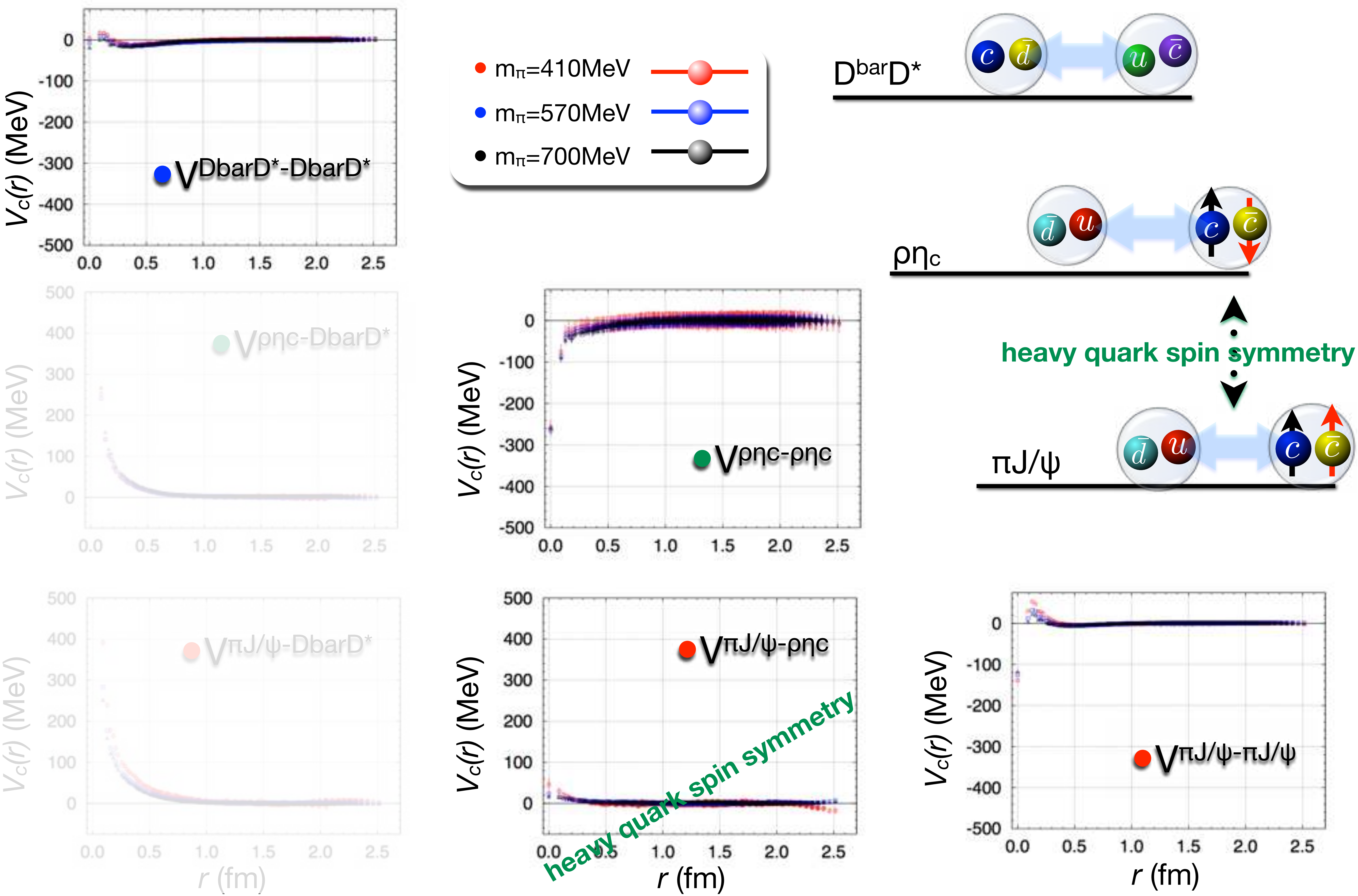
m_D= 1903(1), 1947(1), 2000(1)

m_{D^*}= 2056(3), 2101(2), 2159(2)

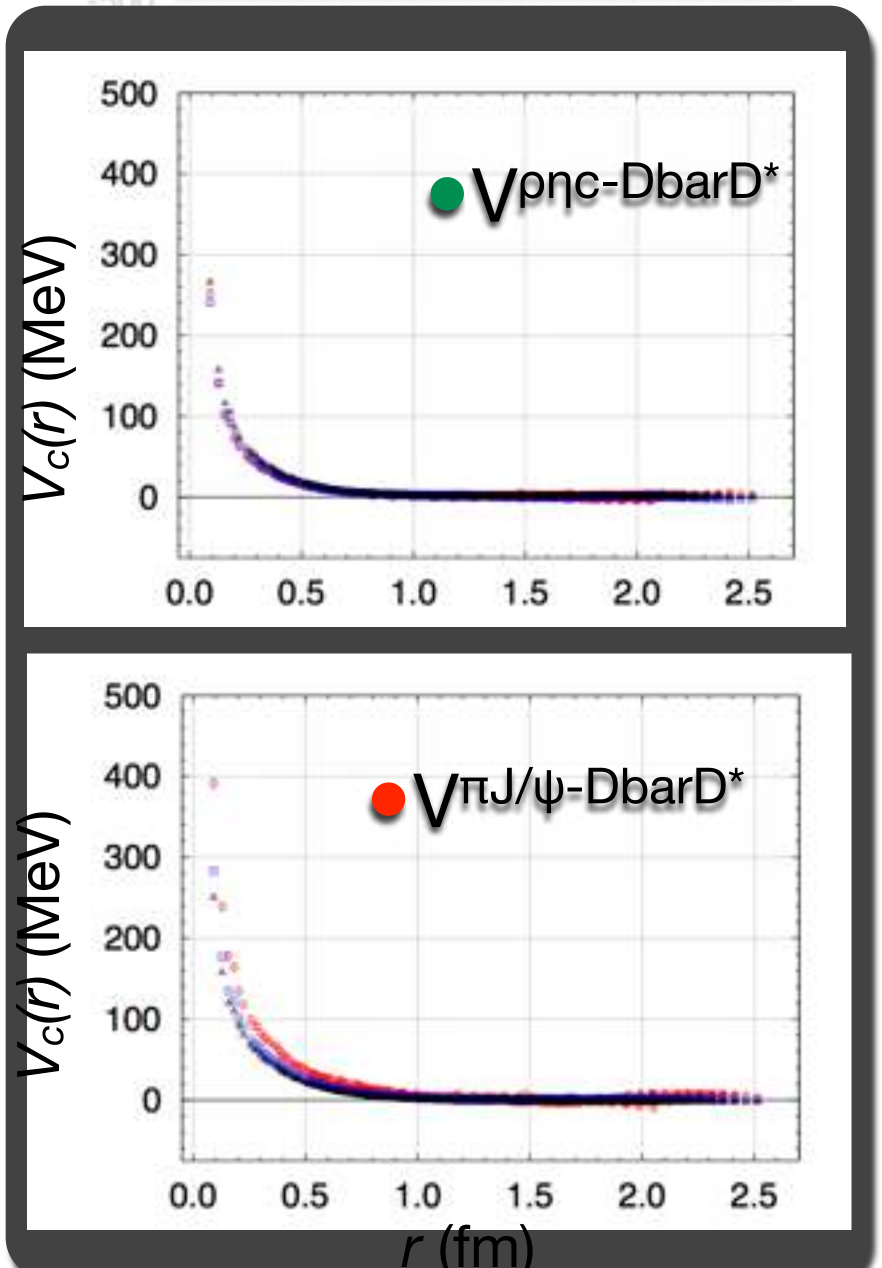
3x3 potential matrix ($\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$)



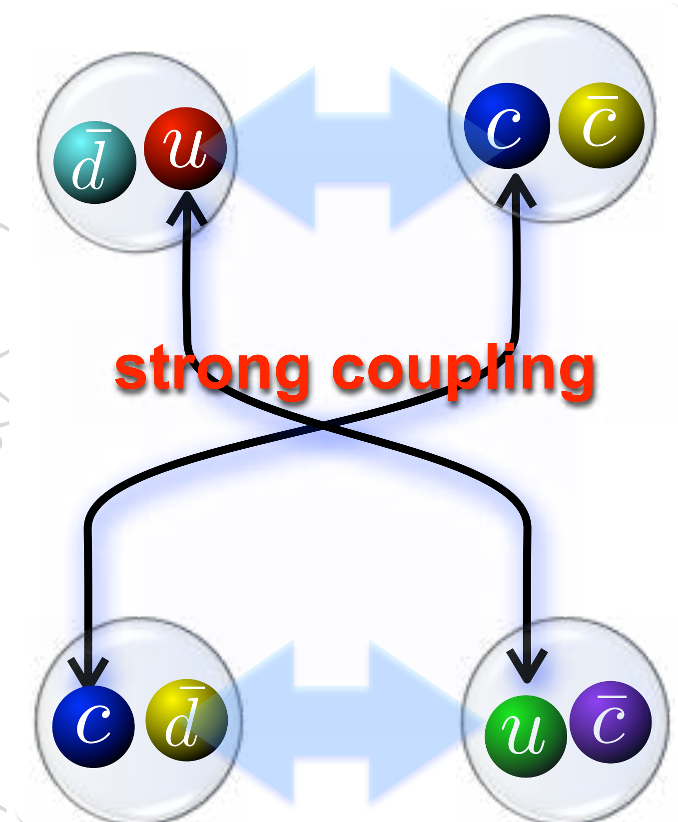
3x3 potential matrix (πJ/ψ - $\rho\eta_c$ - $D^{bar}D^*$)

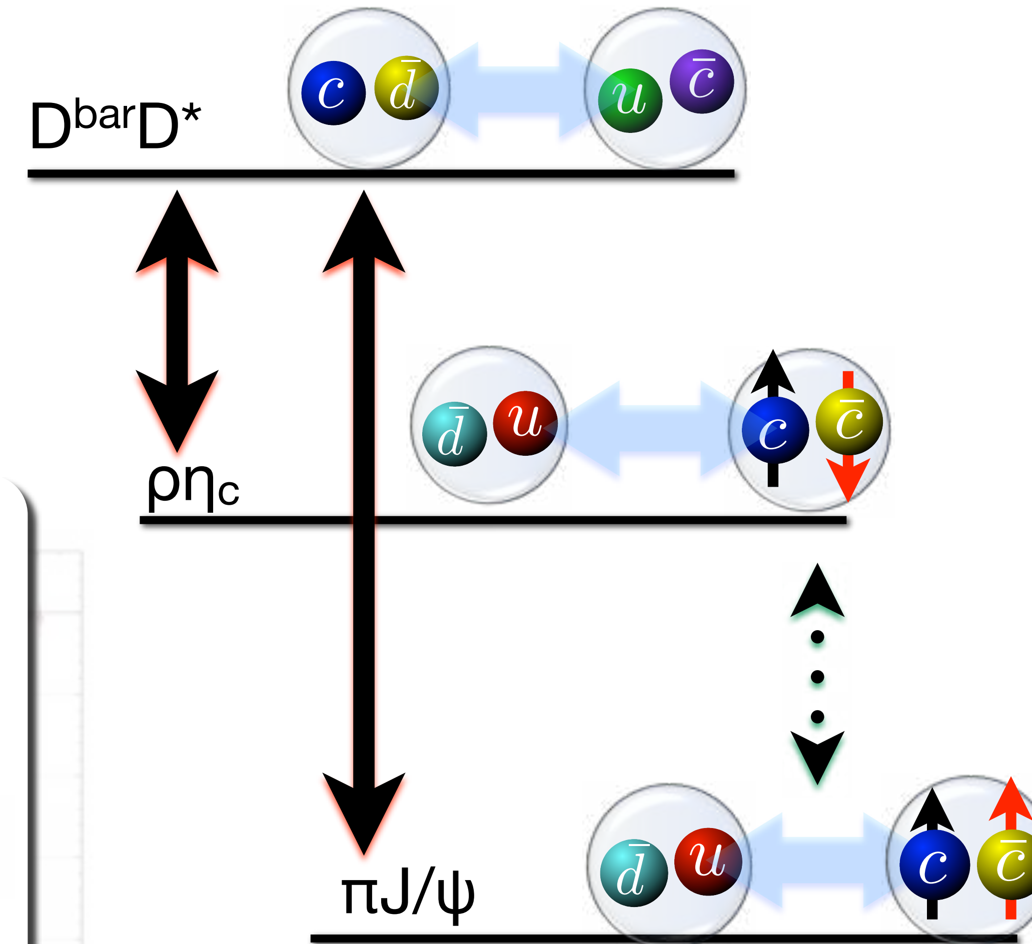


3x3 potential matrix (πJ/ψ - $\rho\eta_c$ - $D^{bar}D^*$)



- m_π=410MeV
- m_π=570MeV
- m_π=700MeV





- strong $V^{\pi J/\psi,DbarD^*}$ & $V^{\rho\eta c,DbarD^*}$

➡ charm quark exchange process

Structure of Z_c(3900)
studied by the most ideal scattering process

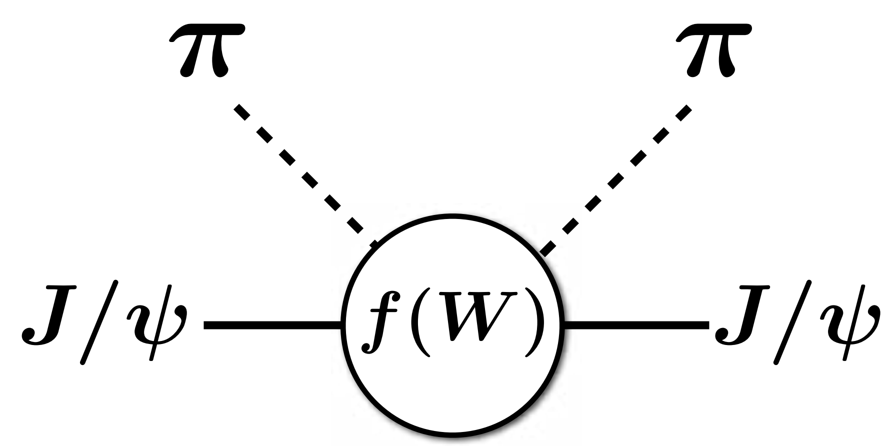
- **S-wave $\pi J/\psi$ - $\rho\eta_c$ - $D^{bar}D^*$ coupled-channel scattering**

➡ **Z_c(3900) is observed in $\pi J/\psi$ --> 2-body scattering is the most ideal reaction**

1. invariant mass distribution of 2-body scattering

\# of scat. particles proportional to imaginary part of amplitude

$$N_{\mathrm{sc}} \propto (\mathrm{flux}) \cdot \sigma(W) \propto \mathrm{Im}\, f(W)$$

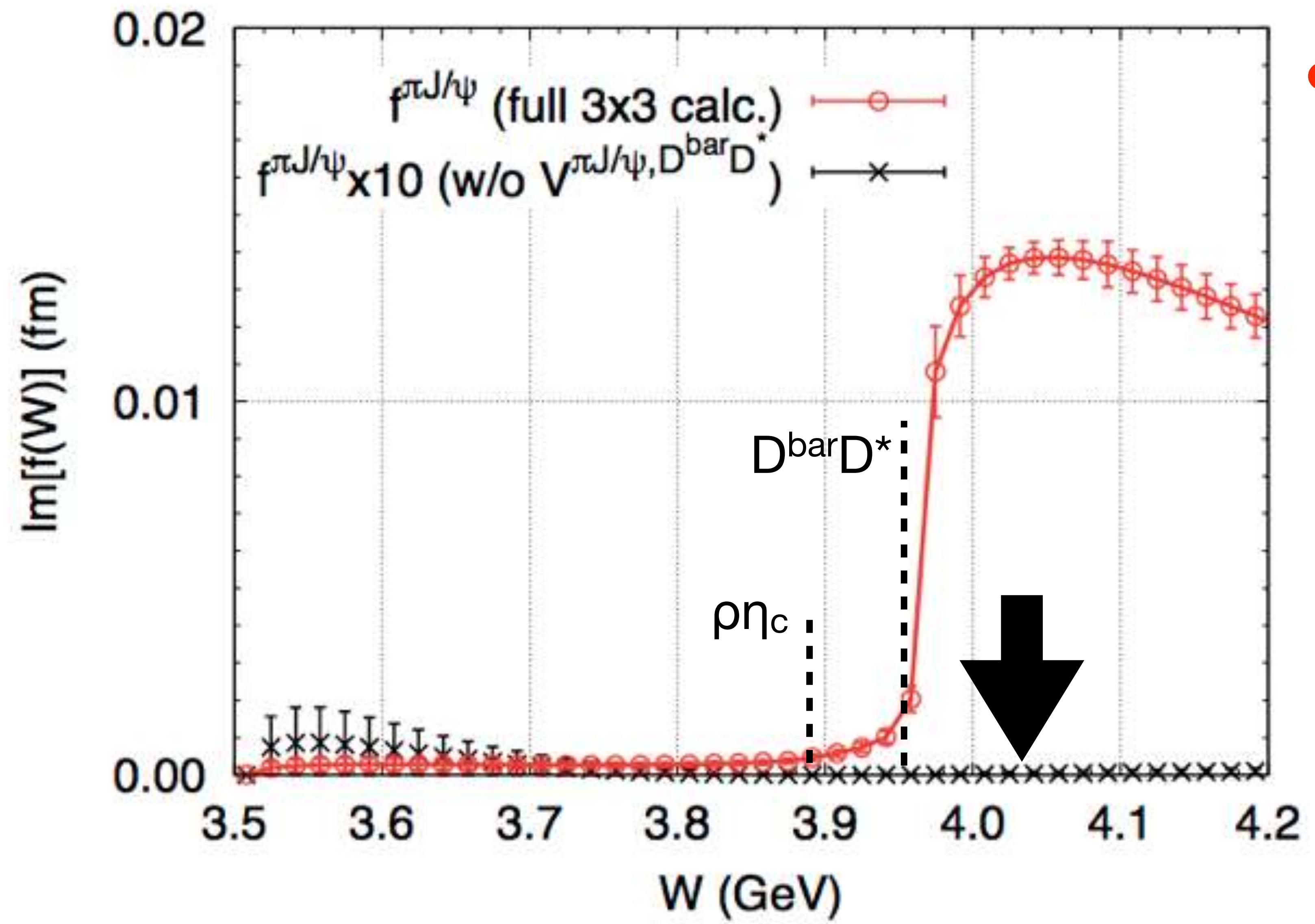


2. pole position of S-matrix

- analytic continuation of c.c. S-matrix onto complex energy plane
- understand nature of Z_c(3900)

- Results w/ m_π=410MeV are shown. (**weak quark mass dependence observed**)

Invariant mass of $\pi J/\psi$ 2-body scattering



- m_π=410MeV

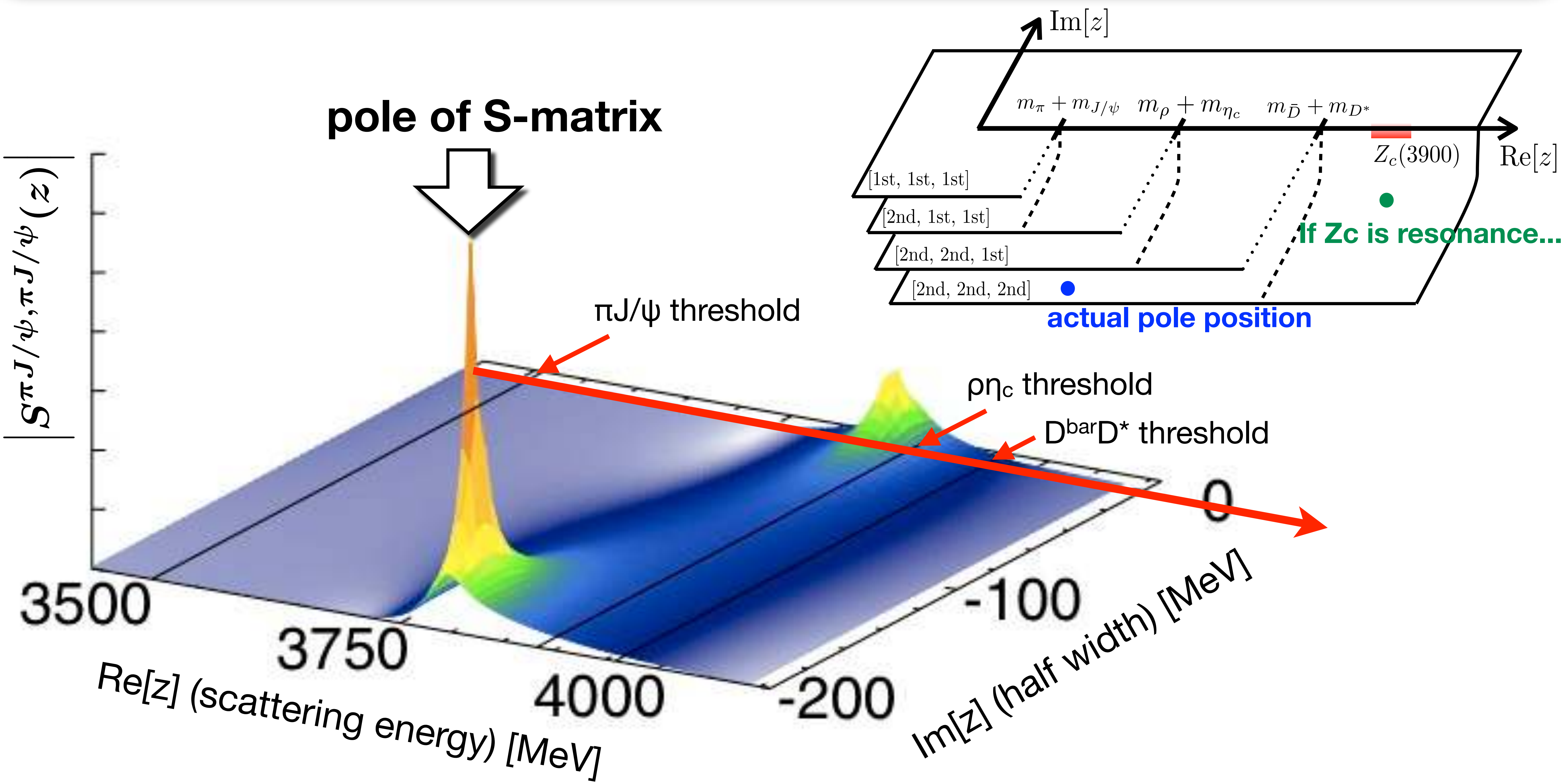
✓ **Enhancement just above $D^{bar}D^*$ threshold**

➡ **effect of strong $V^{\pi J/\psi,\ DbarD^*}$** (black --> $V^{\pi J/\psi,\ DbarD^*}=0$)

- line shape not Breit-Wigner

✓ **Is $Z_c(3900)$ a conventional resonance? --> pole of S-matrix**

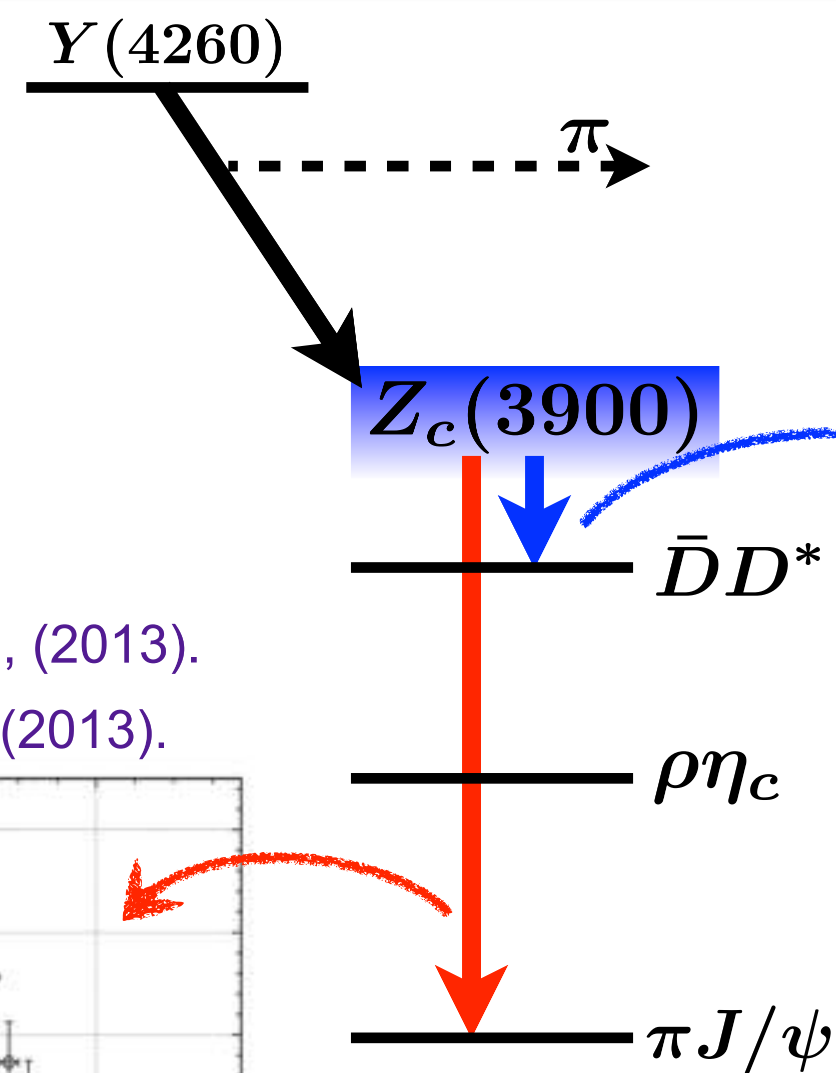
Pole of S-matrix ($\pi J/\psi$:2nd, $\rho\eta_c$:2nd, $D^{bar}D^*$:2nd)



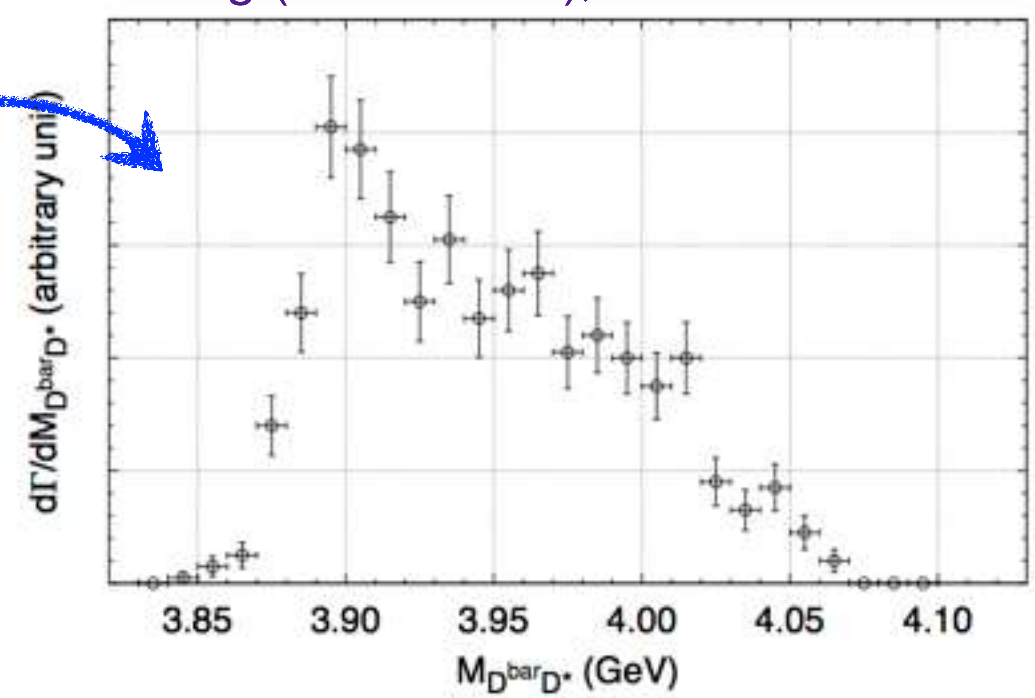
- Pole corresponding to "**virtual state**"
- Pole contribution to scat. observables is small (far from scat. axis)
- **Z_c(3900) is not a resonance but "threshold cusp" induced by strong $V^{\pi J/\psi, DbarD^*}$**

Comparison with expt. data:

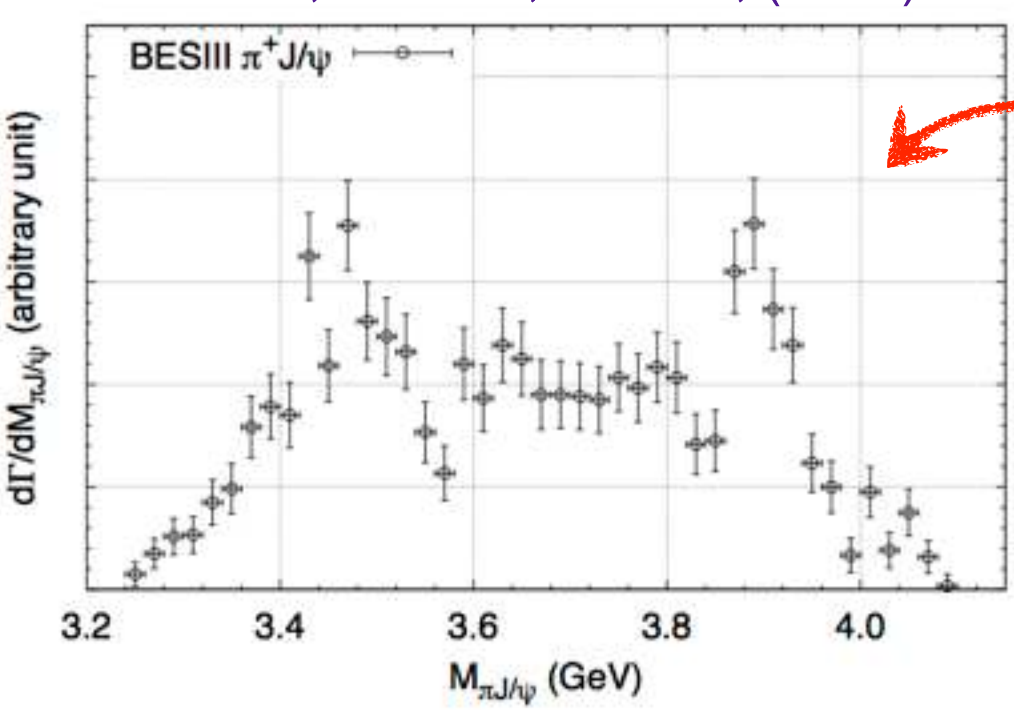
-- spectrum of Y(4260) 3-body decay --

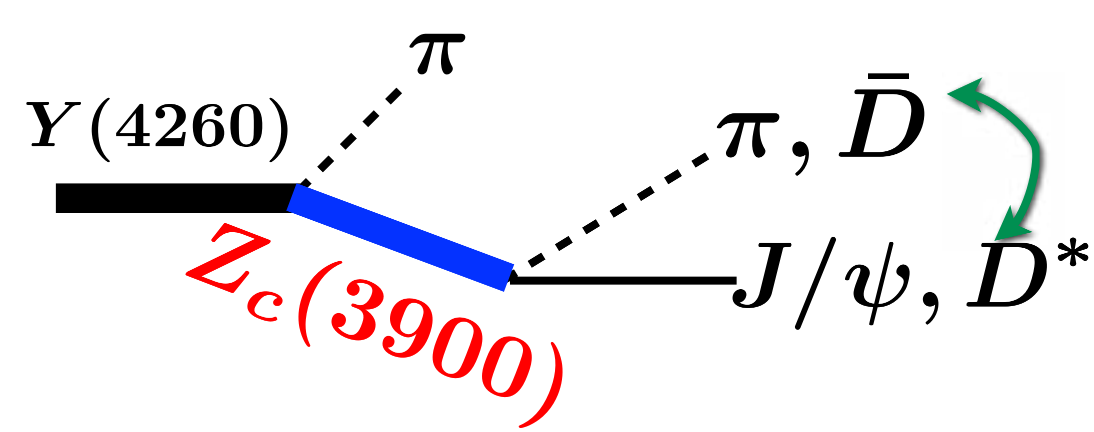


BESIII Coll., PRL112, 022001, (2014).
Wang (BESIII Coll.), MENU2016 talk



BESIII Coll., PRL110, 252001, (2013).
Belle Coll., PRL110, 252002, (2013).



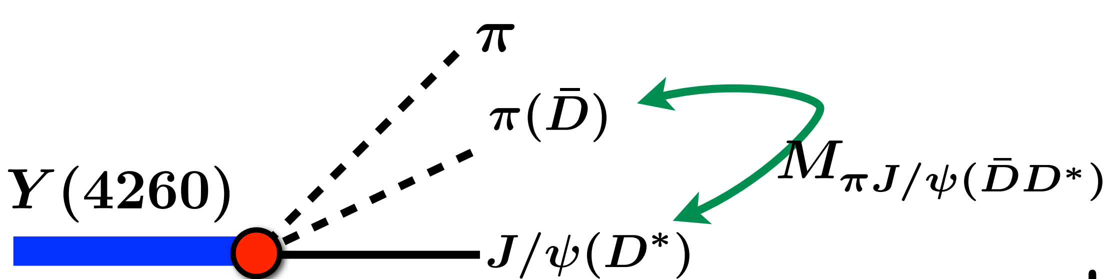


Y(4260) --> ππJ/ψ & πD^barD*

$$d\Gamma_{Y\to\pi+f} = (2\pi)^4\delta(W_3 - E_\pi(\vec{p}_\pi) - E_f(\vec{q}_f))d^3p_\pi d^3q_f|T_{Y\to\pi+f}(\vec{p}_\pi,\vec{q}_f;W_3)|^2$$

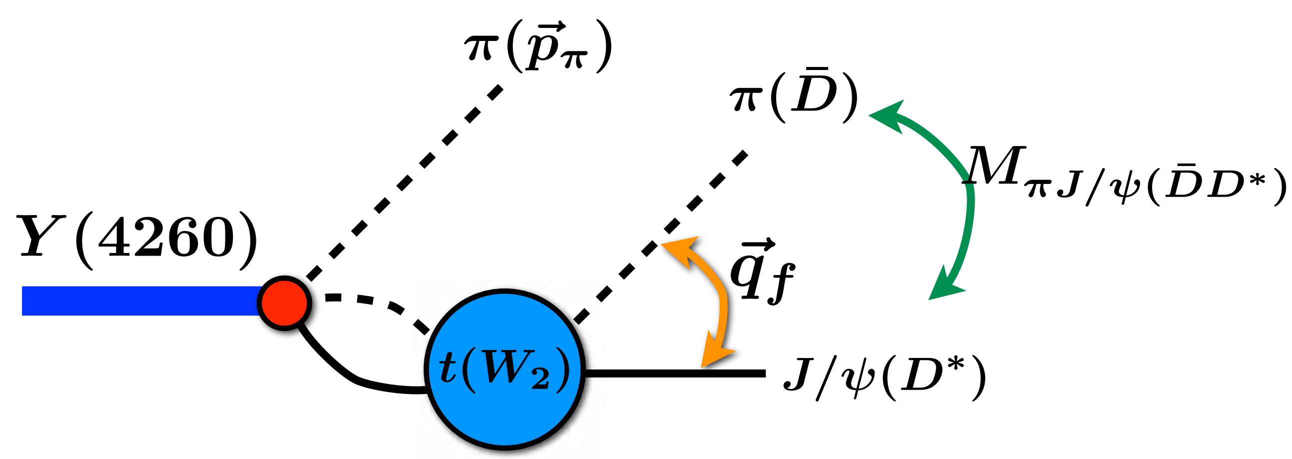
✔ **3-body T-matrix: $T_{Y\text{->}\pi+f}(W_3\text{=}4260\text{MeV})$**

$$T_{Y\to\pi+f}(\vec{p}_\pi,\vec{q}_f;W_3) = \sum_{n=\pi J/\psi,\bar{D}D^*} C^{Y\to\pi+n}\left[\delta_{nf} + \int d^3q' \frac{t_{nf}(\vec{q}',\vec{q}_f,\vec{p}_\pi;W_3)}{W_3 - E_\pi(\vec{p}_\pi) - E_n(\vec{q}',\vec{p}_\pi) + i\epsilon}\right]$$



➡ modeling primary vertex
(2 parameters)

+



➡t-matrix from $V^{LQCD}(r)$

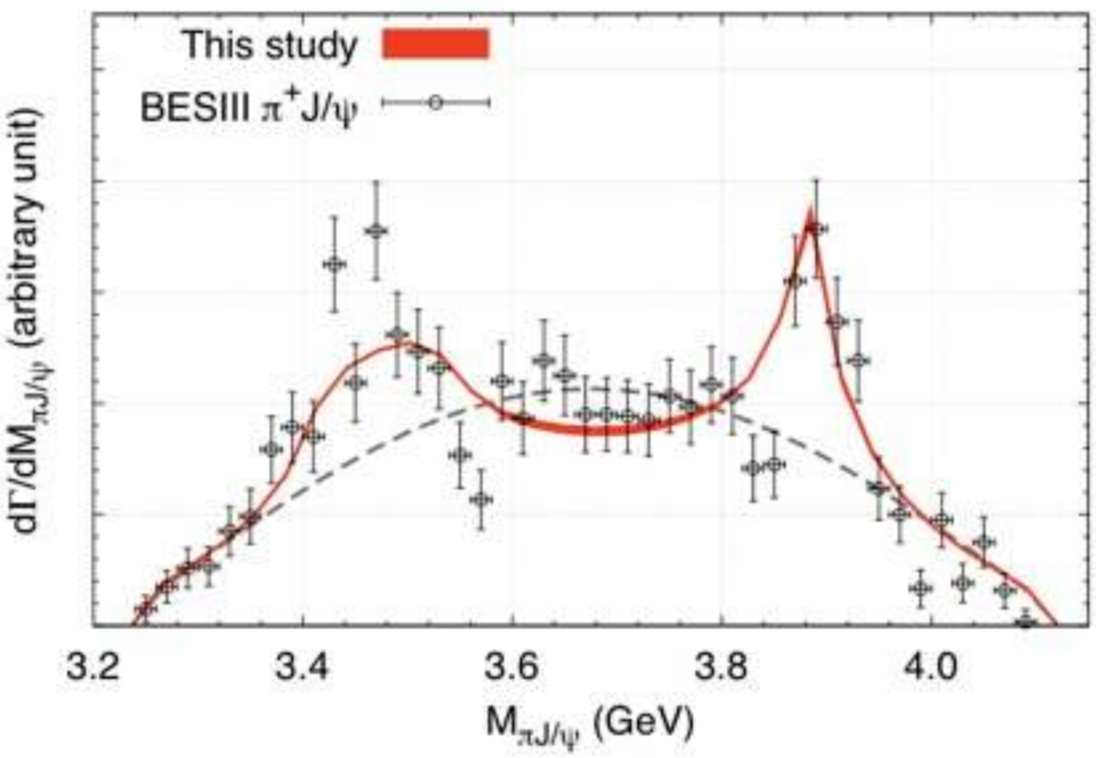
employ physical hadron masses to compare w/ expt. data

✔ $V^{LQCD}(r)$ is taken into account --> calculate t-matrix for subsystem

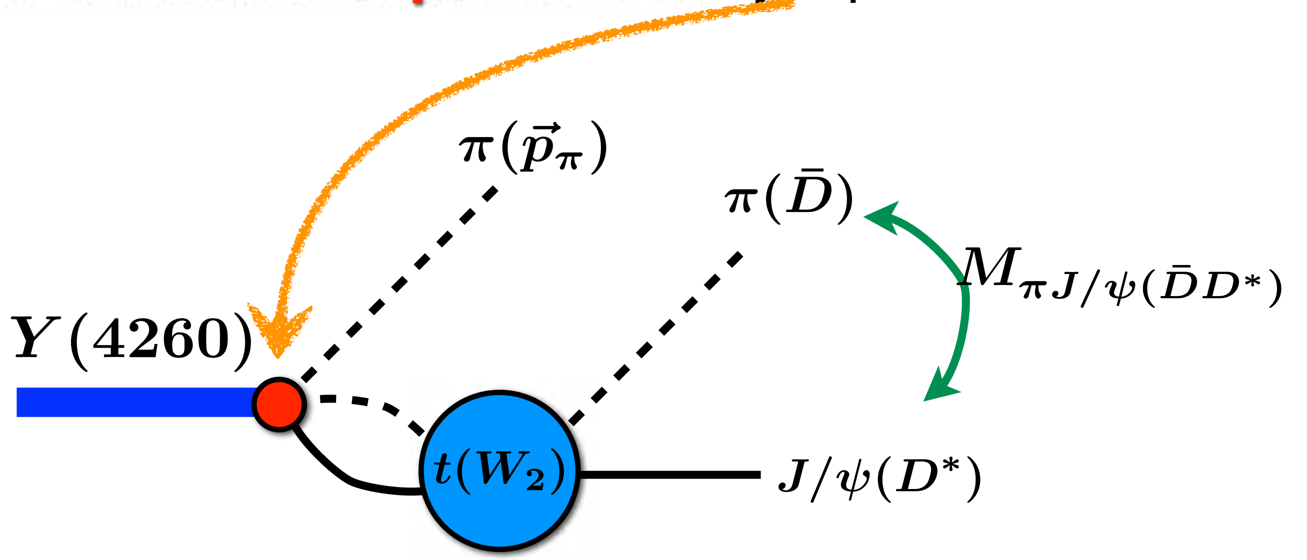
c.f., ~10 parameters needed in models

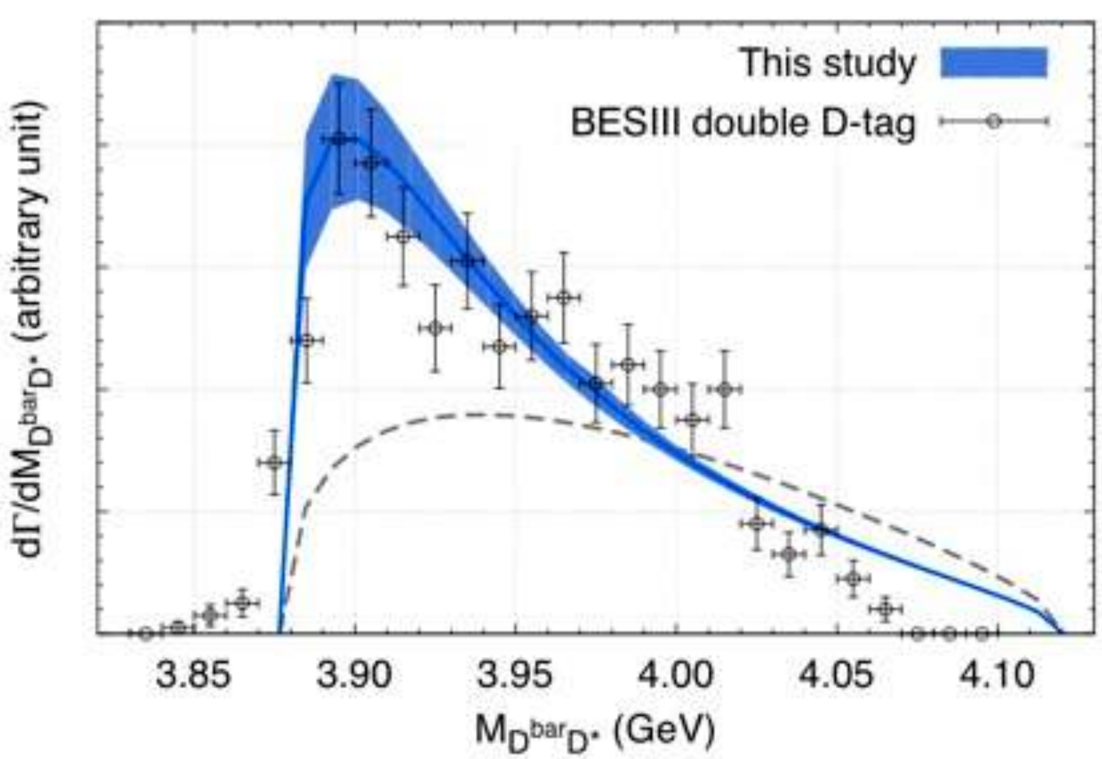
Invariant mass of 3-body decay

Y. Ikeda [HAL QCD] arXiv:1706.07300.



- **Expt. data well reproduced** by 2 parameters





- Without off-diagonal $V^{\pi J/\psi,\ DbarD^*}$ (dashed curves), peak structures are not reproduced.

conclusion: Z_c(3900) is threshold cusp caused by strong $V^{\pi J/\psi,\ DbarD^*}$

Summary

✤ "Mirages" in temporal correlation functions

- Fake plateaux can always appear due to contaminations from unavoidable scattering states
- One has to take data at t~10fm (at least the variational method mandatory)

Iritani, Doi et al. [HAL QCD], JHEP10 (2016) 101.
Iritani et al. [HAL QCD], arXiv:1703.07210.

✤ HAL QCD method

- All scattering states share the same 2PI kernel (= solution for "fake plateau" problem)
- Crucial for coupled-channel scatterings

✤ Tetraquark candidate $Z_c(3900)$

- $Z_c(3900)$ is threshold cusp induced by strong $V^{DbarD^*,\ \pi J/\psi}$
  - pole position very far from scat. axis
  - expt. data of Y(4260) decay well reproduced
  - no peak structure w/o $V^{DbarD^*,\ \pi J/\psi}$

Ikeda et al. [HAL QCD], PRL117, 242001 (2016).
Ikeda [HAL QCD], arXiv:170607300.

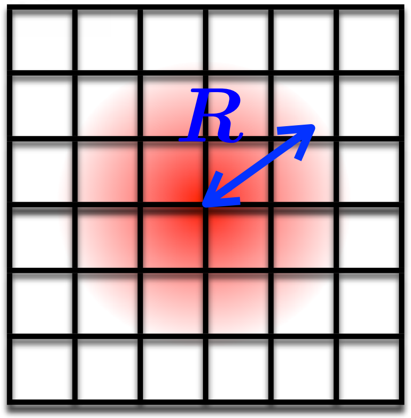
✤ Future: many hadron resonances & nuclear structures at physical point

Thank you for your attention!

"Potential" as representation of S-matrix

- NBS wave func. in interacting region --> half-off-shell T-matrix

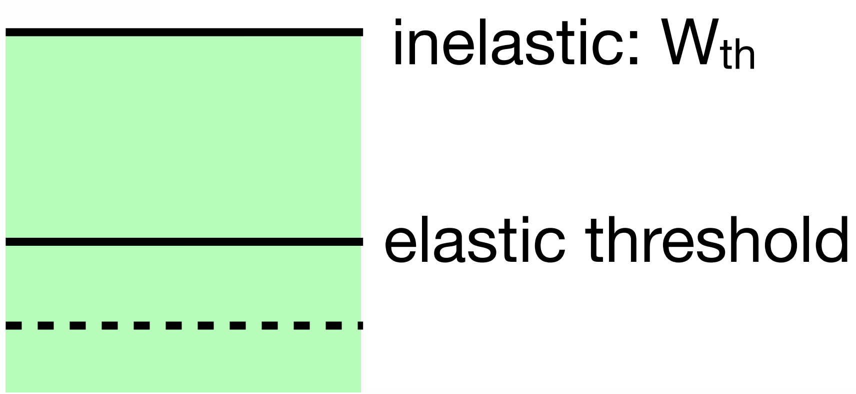
$$(\nabla^2 + \vec{k}_n^2)\psi_n(\vec{r}) = 2\mu\mathcal{K}_n(\vec{r}) \quad (|\vec{r}| < R)$$



- Energy-independent potential (faithful to phase shift)

$$U(\vec{r},\vec{r}') = \sum_{n \le W_{\rm th}} \mathcal{K}_n(\vec{r})\overline{\psi_n}(\vec{r}')$$

★ U(r,r') contains all 2PI contribution



above W_{th}: coupled-channel analysis

- "Potential" becomes a kernel of Schrödinger-type equation

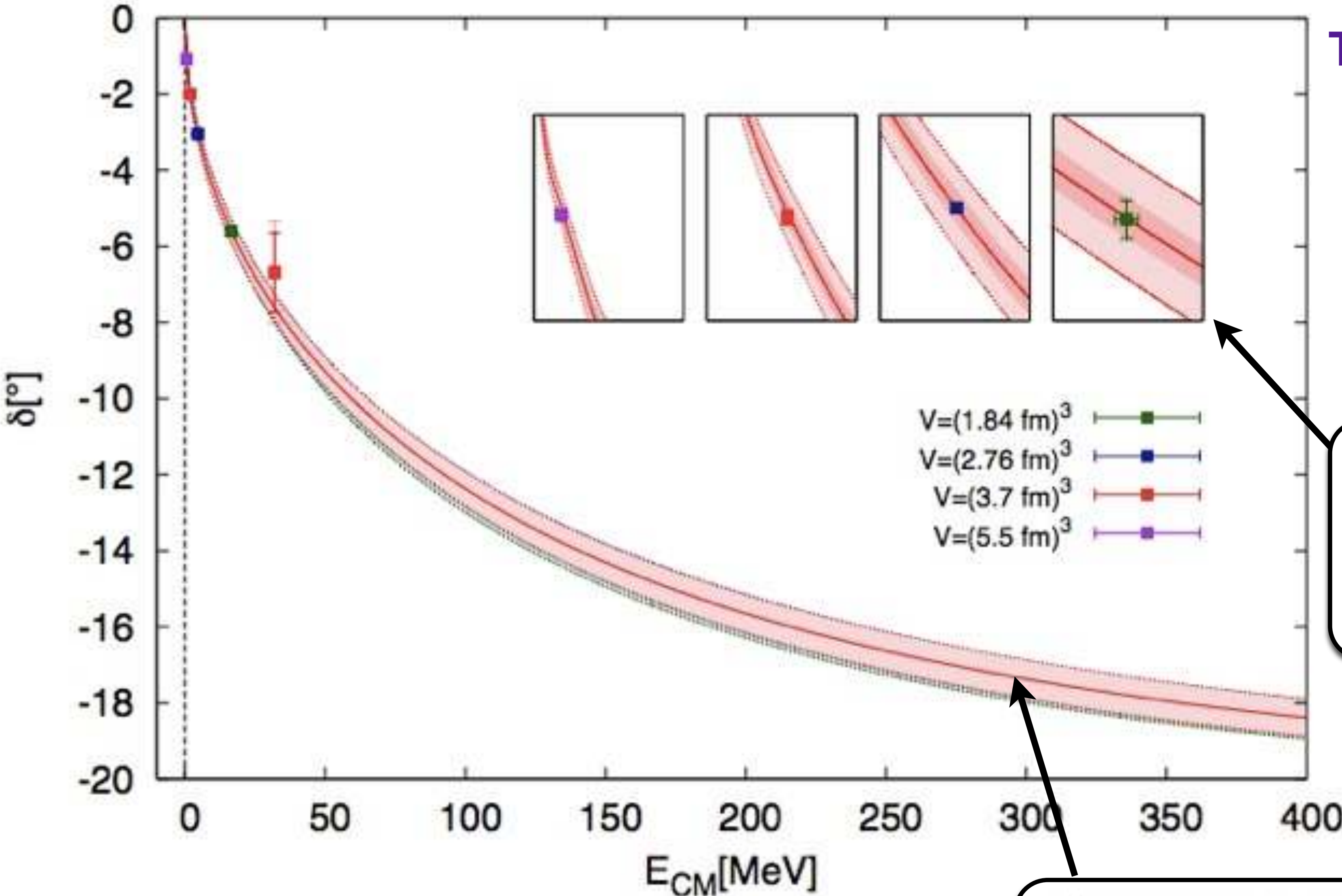
$$\left(\nabla^2 + \vec{k}_n^2\right)\psi_n(\vec{r}) = 2\mu \int d\vec{r}'\, U(\vec{r},\vec{r}')\psi_n(\vec{r}')$$

Ishii, Aoki, Hatsuda, PRL99, 02201 (2007).

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

HAL QCD & Lüscher's methods

I=2 ππ S-wave phase shift from HAL QCD & Lüscher's methods



T. Kurth et al., JHEP 1312 (2013) 015.

❖ Quench QCD

m_π ~ 940 MeV, a=0.115 fm

V x T = (16^3, 24^3, 32^3, 48^3) x 128

✦ **Lüscher's method (points)**

$$k\cot\delta(k) = \frac{1}{\pi L}\sum_{\vec{n}\in Z}\frac{1}{|\vec{n}|^2 - \tilde{k}^2}$$

✦ **HAL QCD (red band)**

$$\left[-\partial_\tau + \nabla^2/2\mu + \partial_\tau^2/8\mu\right] R(\vec{r},\tau) = V(\vec{r})R(\vec{r},\tau)$$

Both methods agree (theoretically identical)

Nonlocality of potentials

✓ **Velocity expansion:**

$$U(\vec{r},\vec{r}') = V(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$

(LO) (NLO)

$$\Rightarrow V(\vec{r},\nabla) = V_C(\vec{r}) + \vec{L}\cdot\vec{S}V_{LS}(\vec{r}) + \mathcal{O}(\nabla^2)$$

✓ **Derive (effective) leading order potential in velocity expansion:**

$$\left[\nabla^2/2\mu^a - \partial_\tau\right] R(\vec{r},\tau) = V_{\text{eff}}(\vec{r})R(\vec{r},\tau)$$

$$R(\vec{r},\tau) = A_1 e^{-\Delta W_1 \tau}\psi_1(\vec{r}) + A_2 e^{-\Delta W_2 \tau}\psi_2(\vec{r}) + \cdots$$

✓ **Check of nonlocality (energy-independent local potential is good approximation?)**

$$V_{\text{eff}}(\vec{r})\psi_1(\vec{r}) = \left(\frac{\nabla^2}{2\mu} + \frac{\vec{k}_1^2}{2\mu}\right)\psi_1(\vec{r})$$

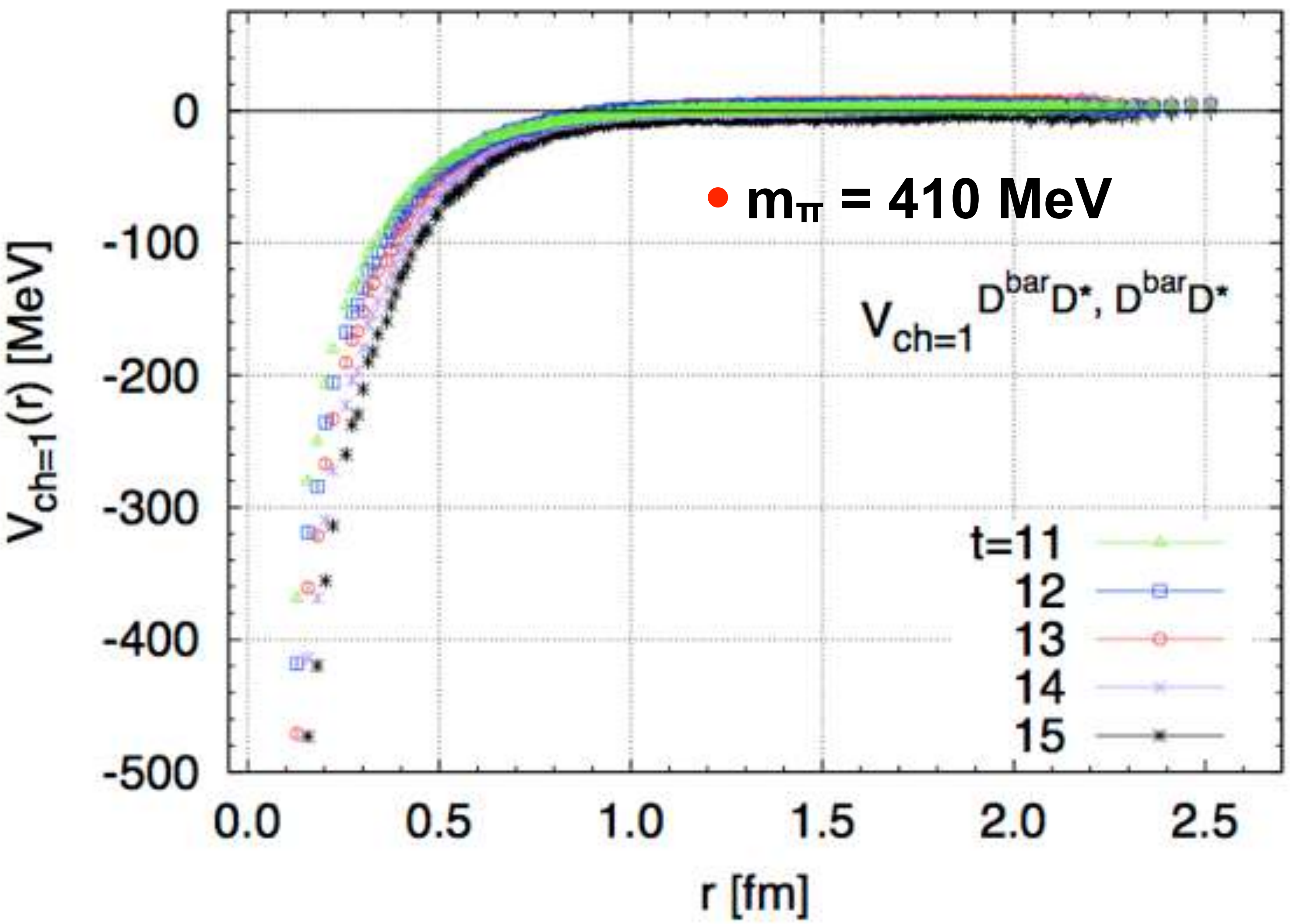
$$V_{\text{eff}}(\vec{r})\psi_2(\vec{r}) = \left(\frac{\nabla^2}{2\mu} + \frac{\vec{k}_2^2}{2\mu}\right)\psi_2(\vec{r})$$

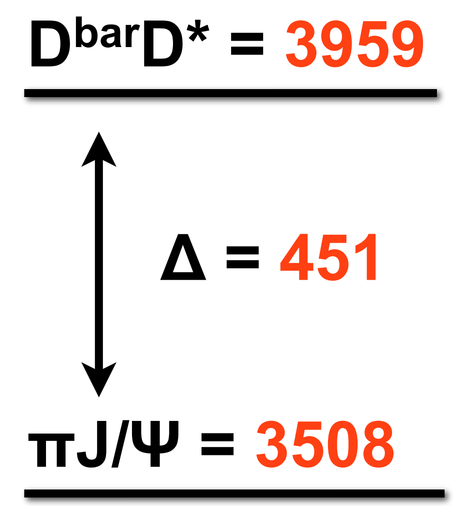
✓ **nonlocality --> energy-dependence of local potential**

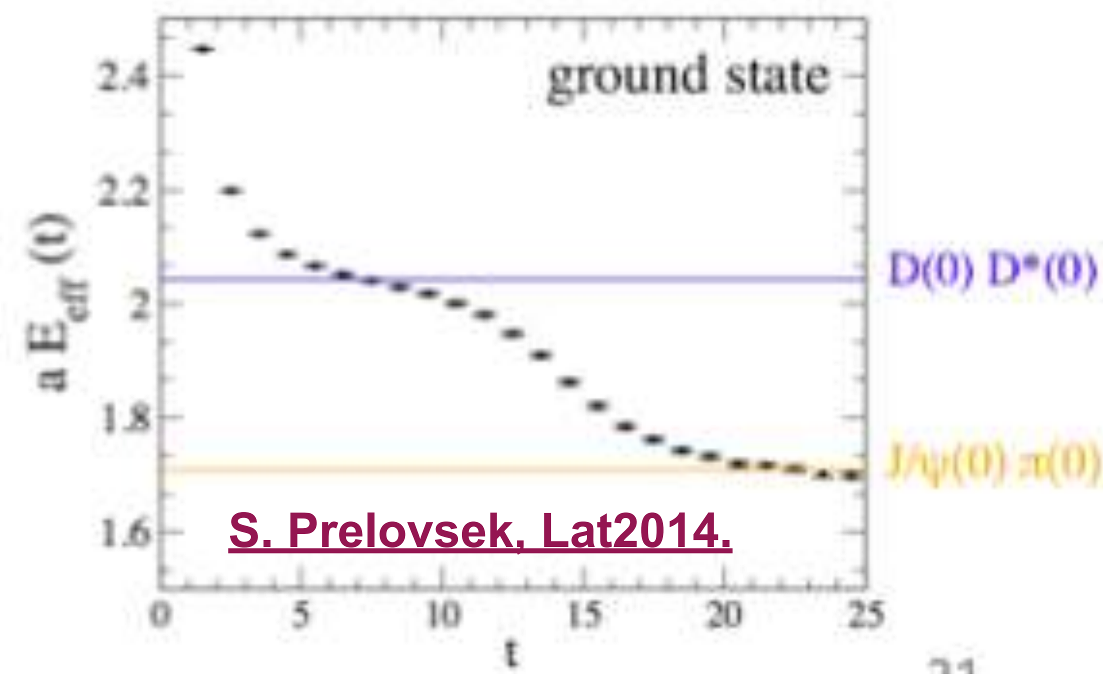
➡ **Local potential shows time-slice dependence**

$D^{bar}D^*$ potential (single-channel)

- **$D^{bar}D^*$: inelastic channel**







- **Time-slice dependence indicates**

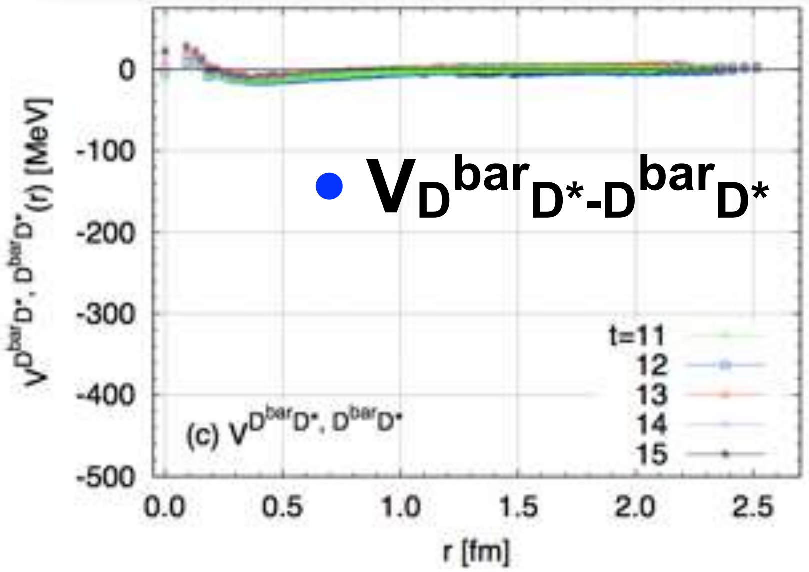
➡ coupling to lower channels (large contribution from πJ/ψ and/or $\rho\eta_c$)

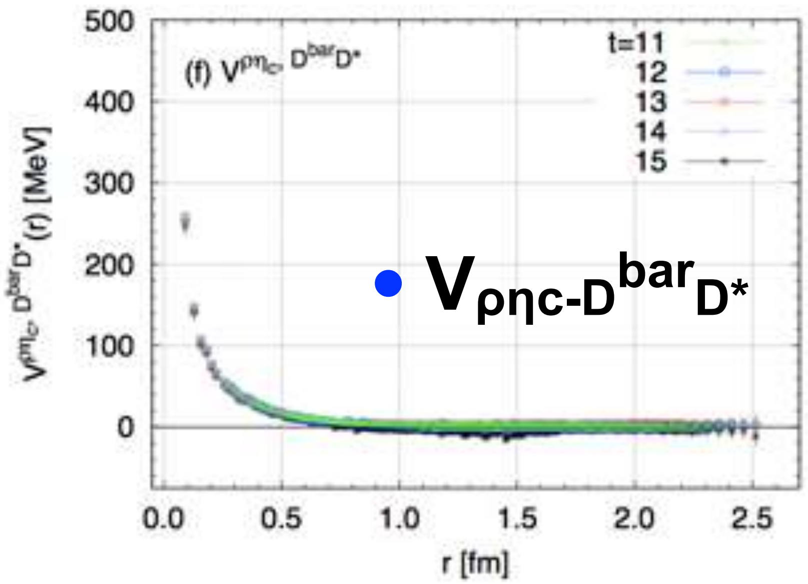
➡ single channel $D^{bar}D^*$ potential NOT well defined

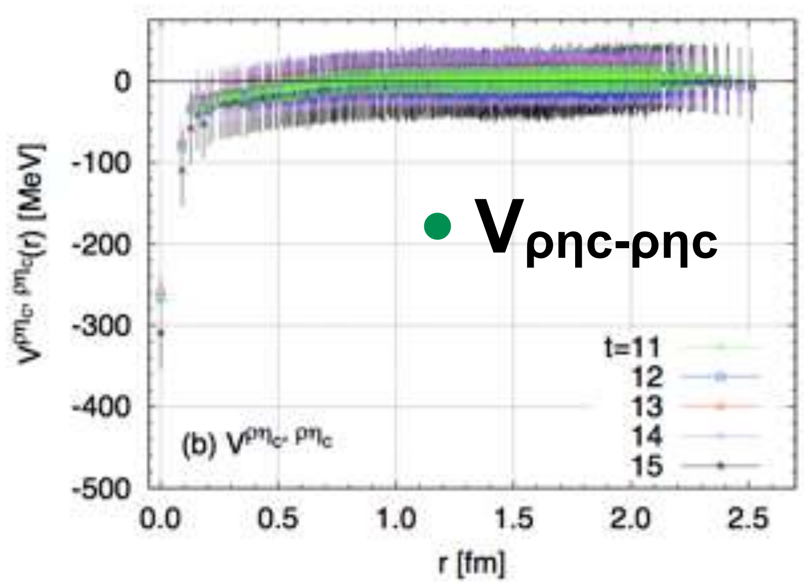
➡ coupled-channel analysis is necessary

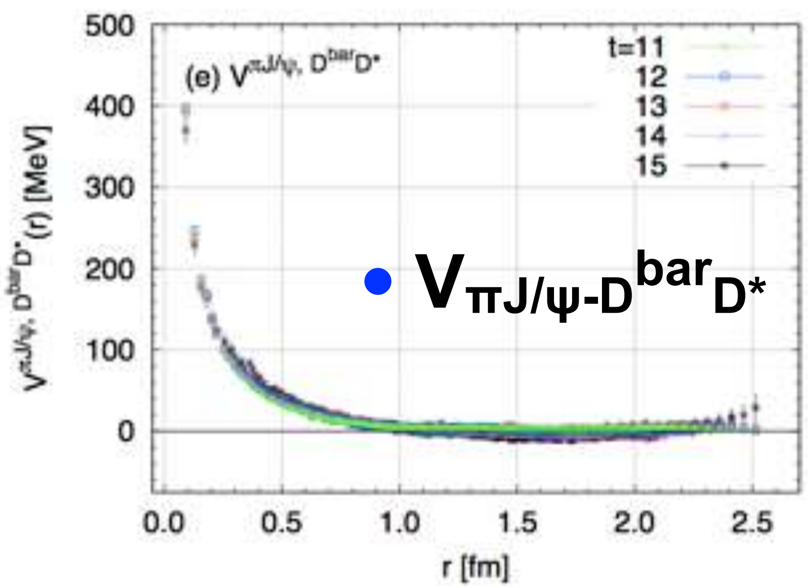
t-dependence on potential matrix

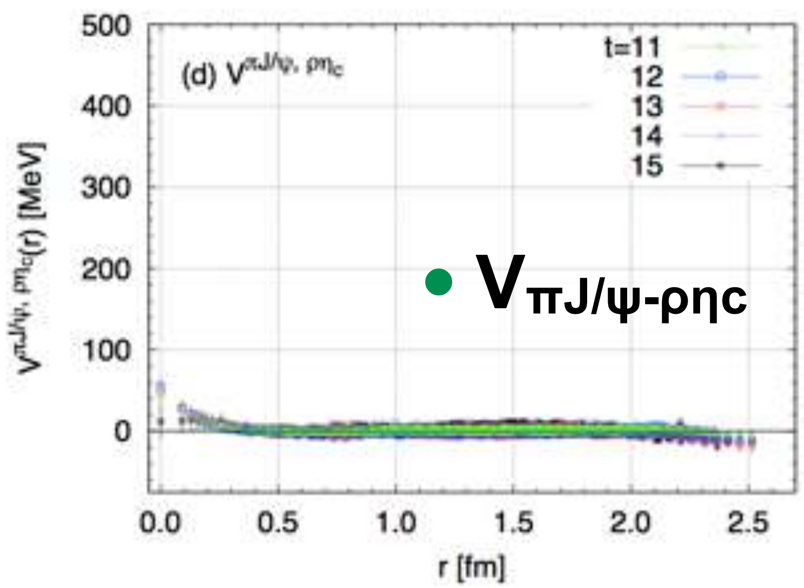
- check of non-locality of potential (m_π=410MeV)
  - all $V^{(t=11\text{--}15)}$ are consistent within stat. errors
  - ➡ non-locality of potential is weak

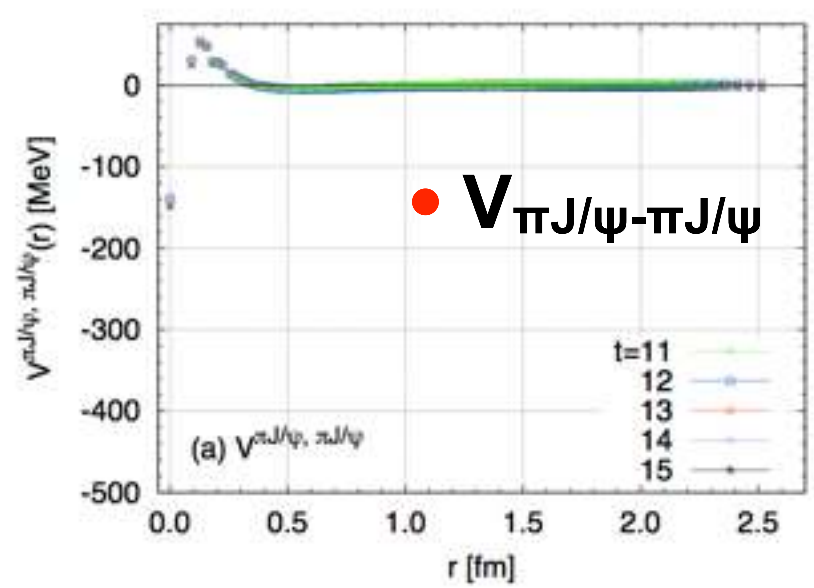






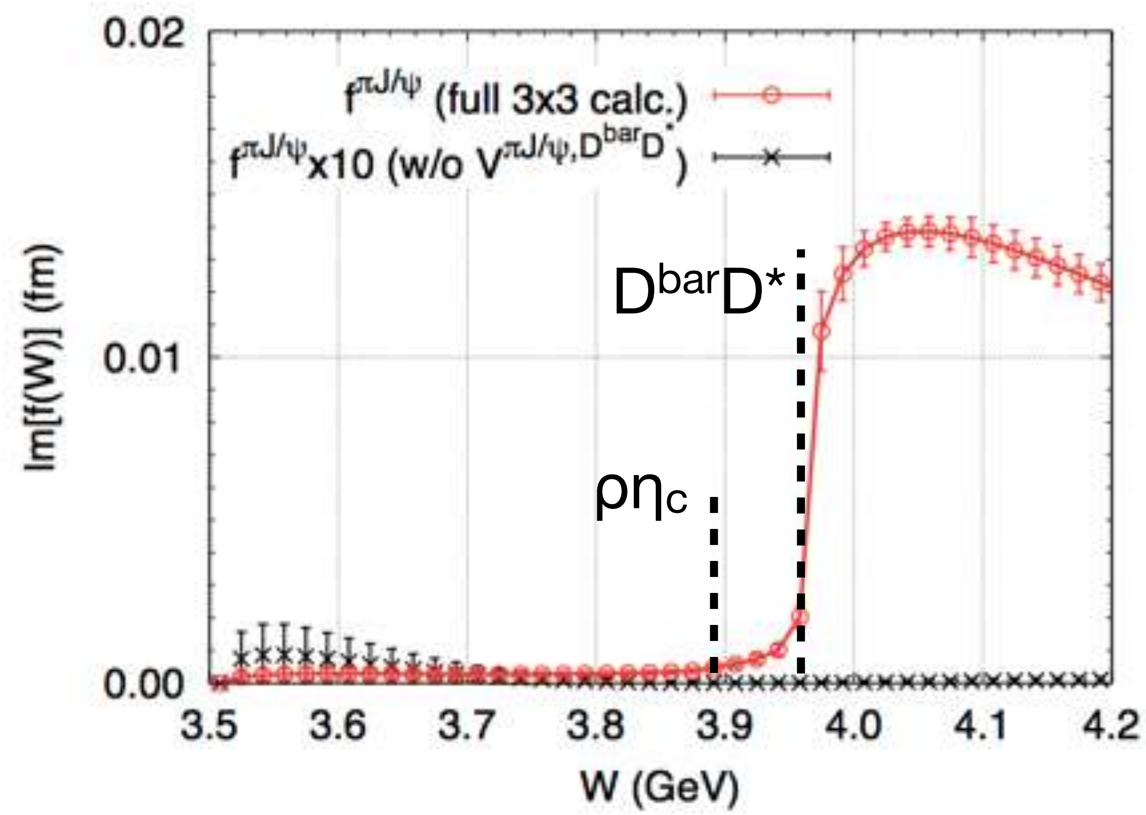




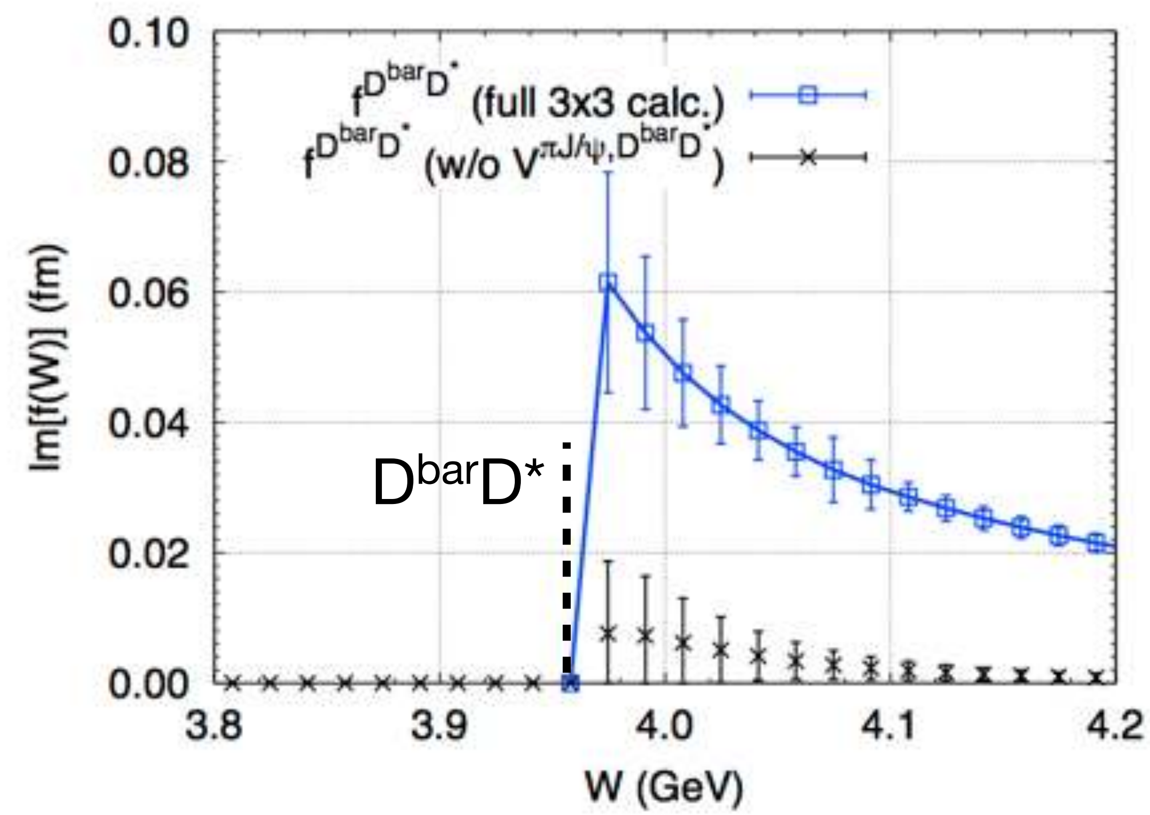


Amplitudes in πJ/ψ & $D^{bar}D^*$ (2-body scat.)

- πJ/ψ invariant mass



- $D^{bar}D^*$ invariant mass

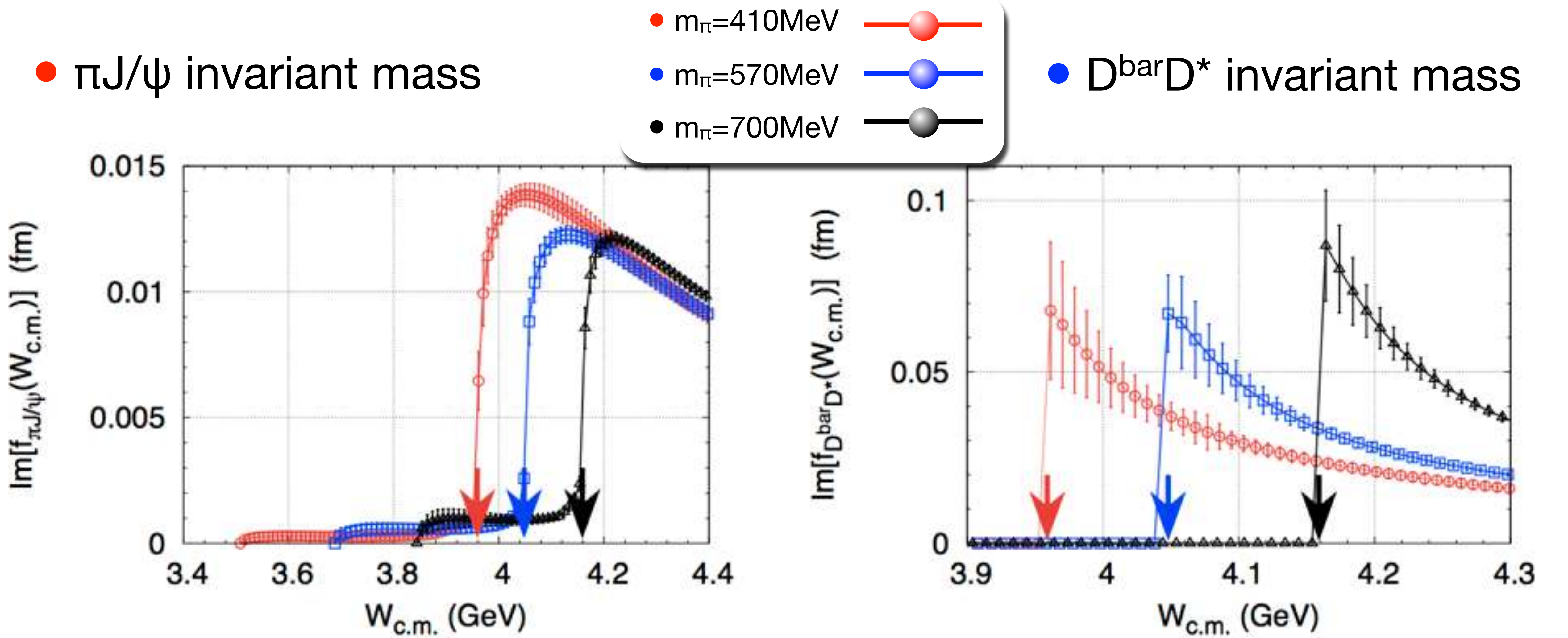


$$\frac{\Gamma(Zc(3900) \to \bar{D}D^*)}{\Gamma(Zc(3900) \to \pi J/\psi)} = 6.2(1.1)(2.7)$$

BESIII Coll., PRL112 (2014).

✓ **Obtained amplitudes consistent with decay ratio in expt.**

Invariant mass spectra of πJ/ψ & $D^{bar}D^*$



✓ Enhancement near $D^{bar}D^*$ threshold due to large πJ/ψ-$D^{bar}D^*$ coupling

- Peak in πJ/ψ invariant mass (Not Breit-Wigner line shape)
- Threshold enhancement in $D^{bar}D^*$ invariant mass (cusp behavior)

(No m_q dependence on qualitative behaviors of line shapes)

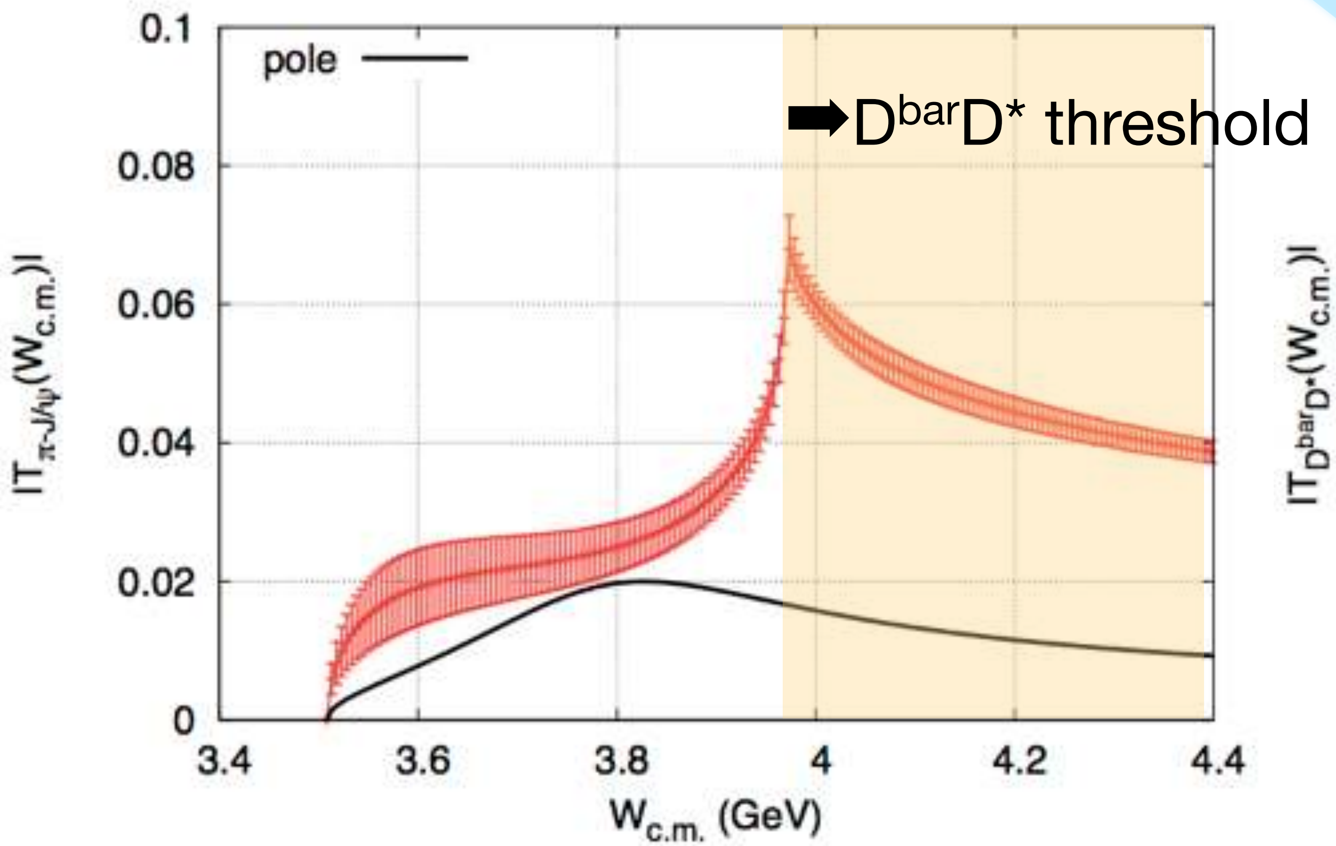
➡ Is Zc(3900) a conventional resonance?

T-matrix of πJ/ψ & $D^{bar}D^*$

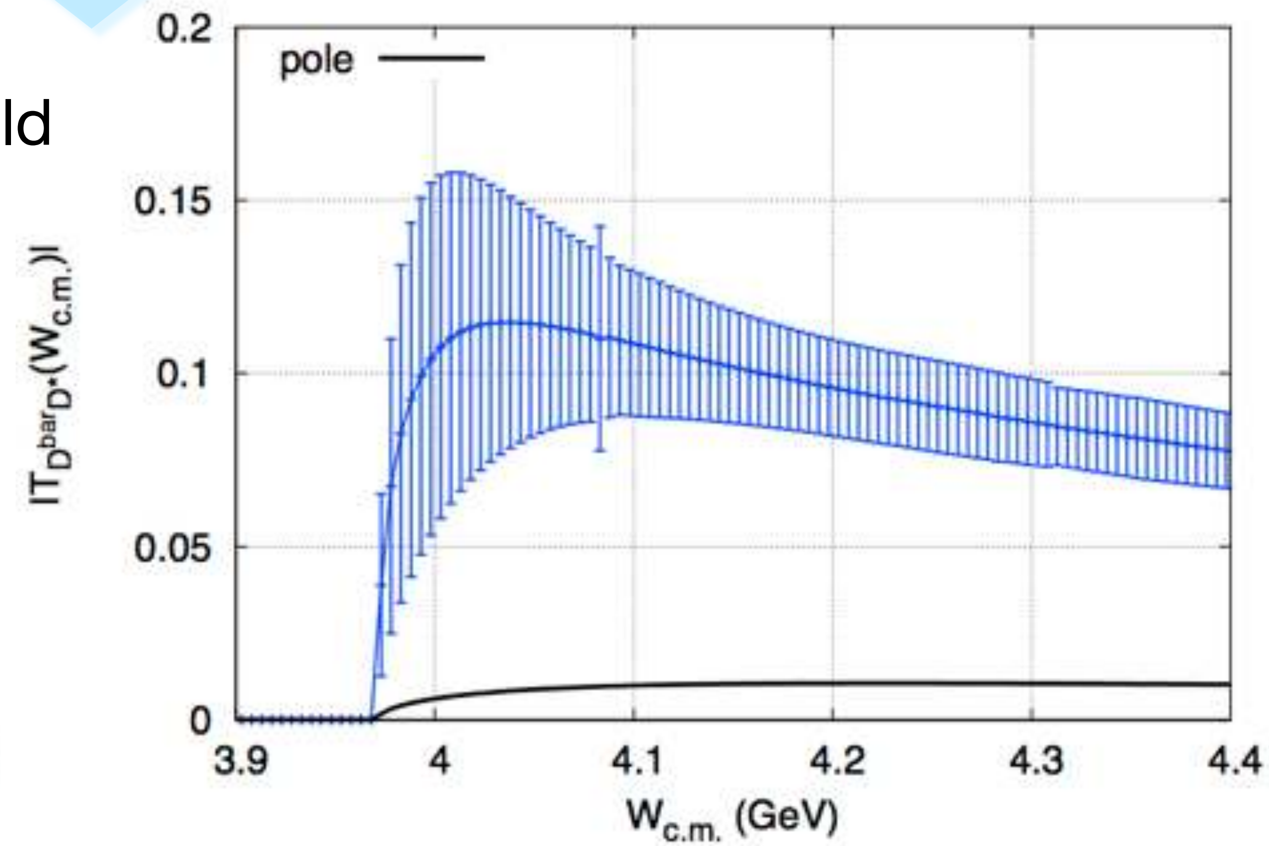
- calculate residues of T-matrices in πJ/ψ & $D^{bar}D^*$ channels

$$S(k) = 1 + 2iT(k)$$

- πJ/ψ-πJ/ψ T-matrix



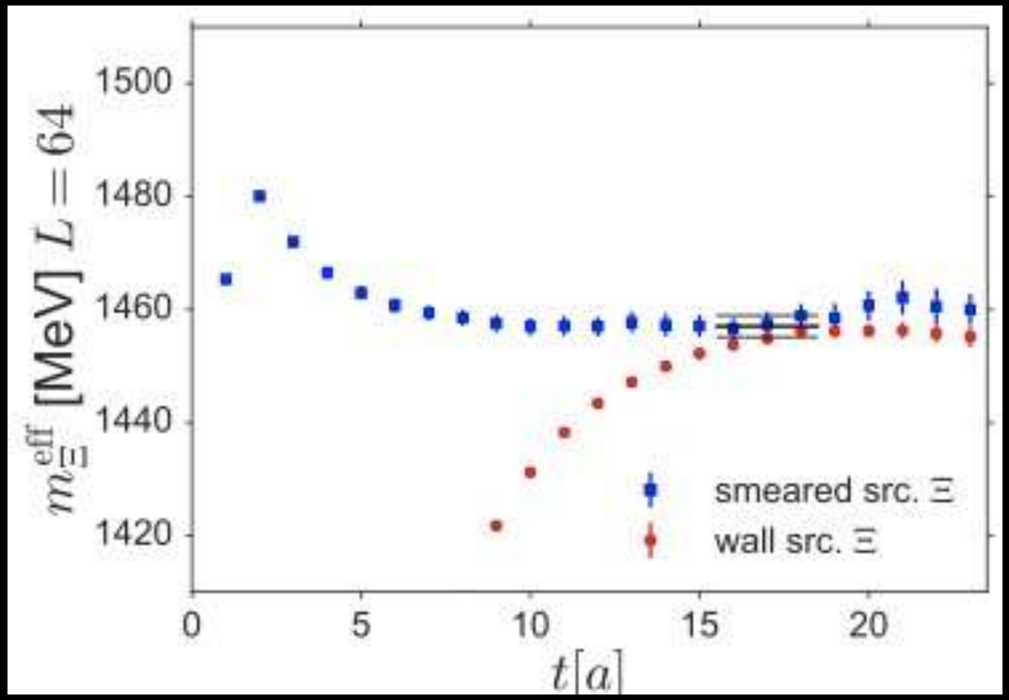
- $D^{bar}D^*$-$D^{bar}D^*$ T-matrix



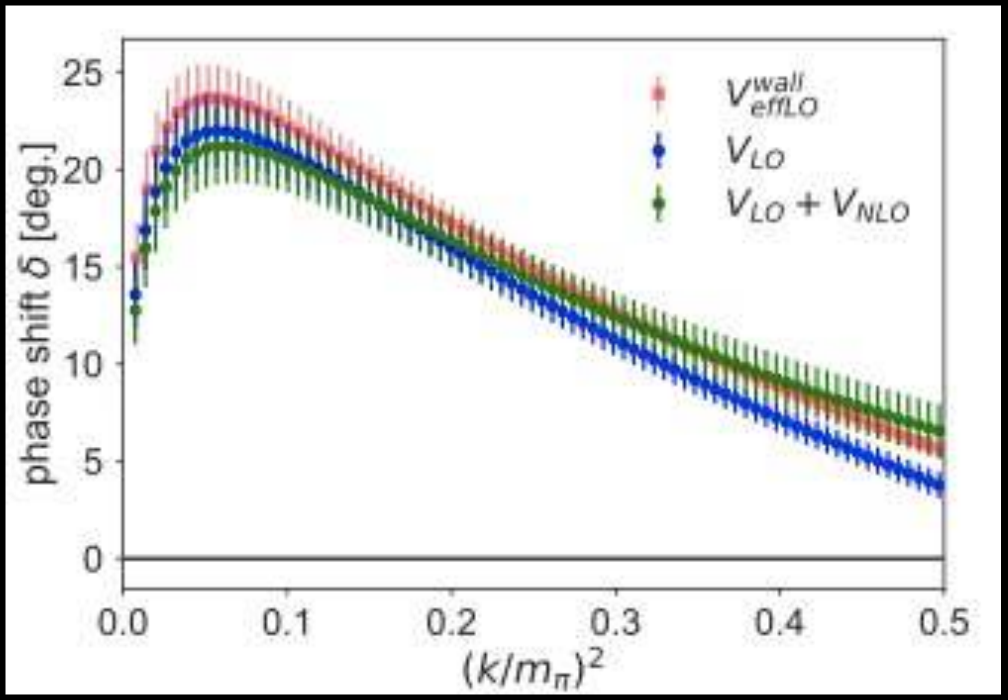
- contribution from virtual pole to T-matrix is small
- Z_c(3900) is cusp at $D^{bar}D^*$ threshold induced by off-diagonal $V^{\pi\psi,\ DbarD^*}$

HAL QCD data in ΞΞ ($^1S_0$) -- systematics --

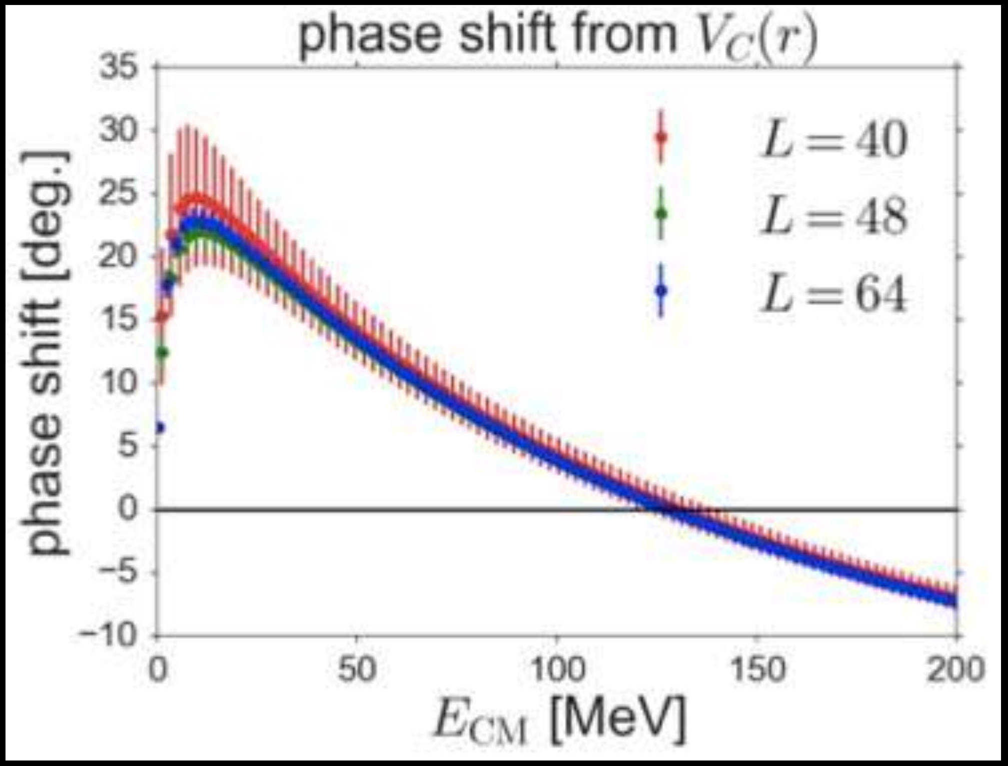
✓ single hadron effective mass



✓ convergence of derivative expansion



✓ Volume dependence of observable

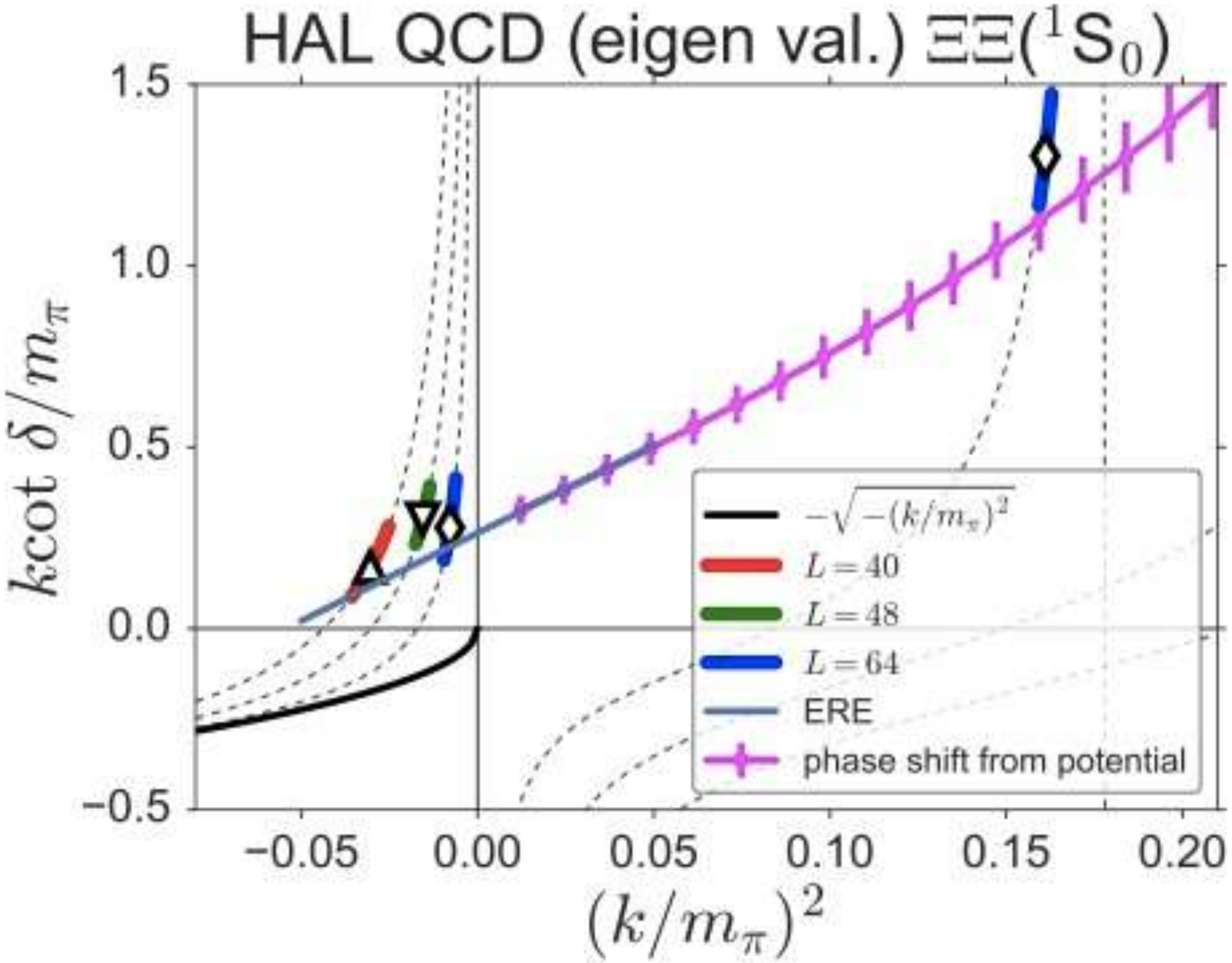


see talk by T. Iritani (Parallel, Hadron Spectroscopy and Interactions on Thur.)

Sanity check for HAL QCD data

✓ phase shift from Lüscher's formula shows **reasonable behavior**

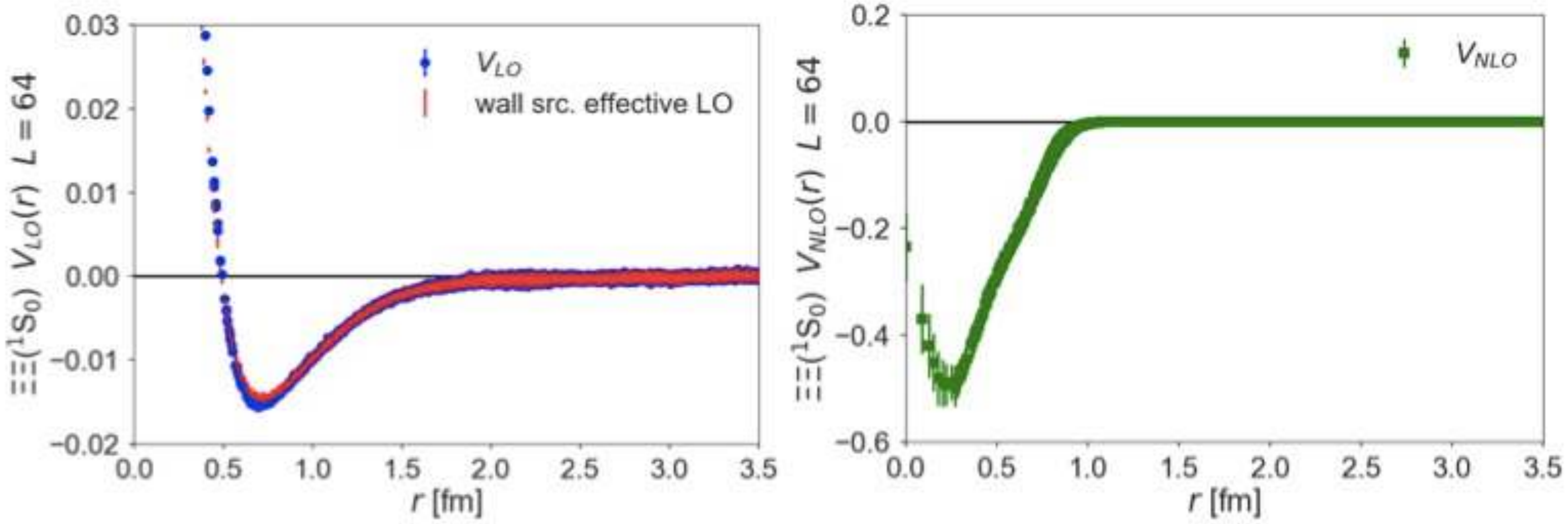
- finite volume energy shift
  $\Delta E_L \to k \cot \delta(k)$
  at $L = 40, 48, 64$
- potential $V(r)$
  $\longrightarrow$ phase shift δ
- Effective Range Expansion
  $k \cot \delta = \frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2$



NLO potential in ΞΞ ($^1S_0$) channel

$$U(\vec{r},\vec{r}') = [V_{\rm eff}(\vec{r})]\,\delta(\vec{r}-\vec{r}')$$
$$U(\vec{r},\vec{r}') = \left[V_{\rm LO}(\vec{r}) + V_{\rm NLO}(\vec{r})\nabla^2\right]\delta(\vec{r}-\vec{r}')$$

▶ $V_{\rm LO}(r)$ and $V_{\rm NLO}(r)$



Contamination from elastic states in ΞΞ ($^1S_0$) channel

HAL pot. ▶ eigenfunc/value Ψ_n, ΔE_n ▶ eigenmode decomposition

$$R^{\mathrm{wall/smear}}(\vec{r}, t) = \sum_n a_n^{\mathrm{wall/smear}} \Psi_n(\vec{r}) \exp(-\Delta E_n t)$$

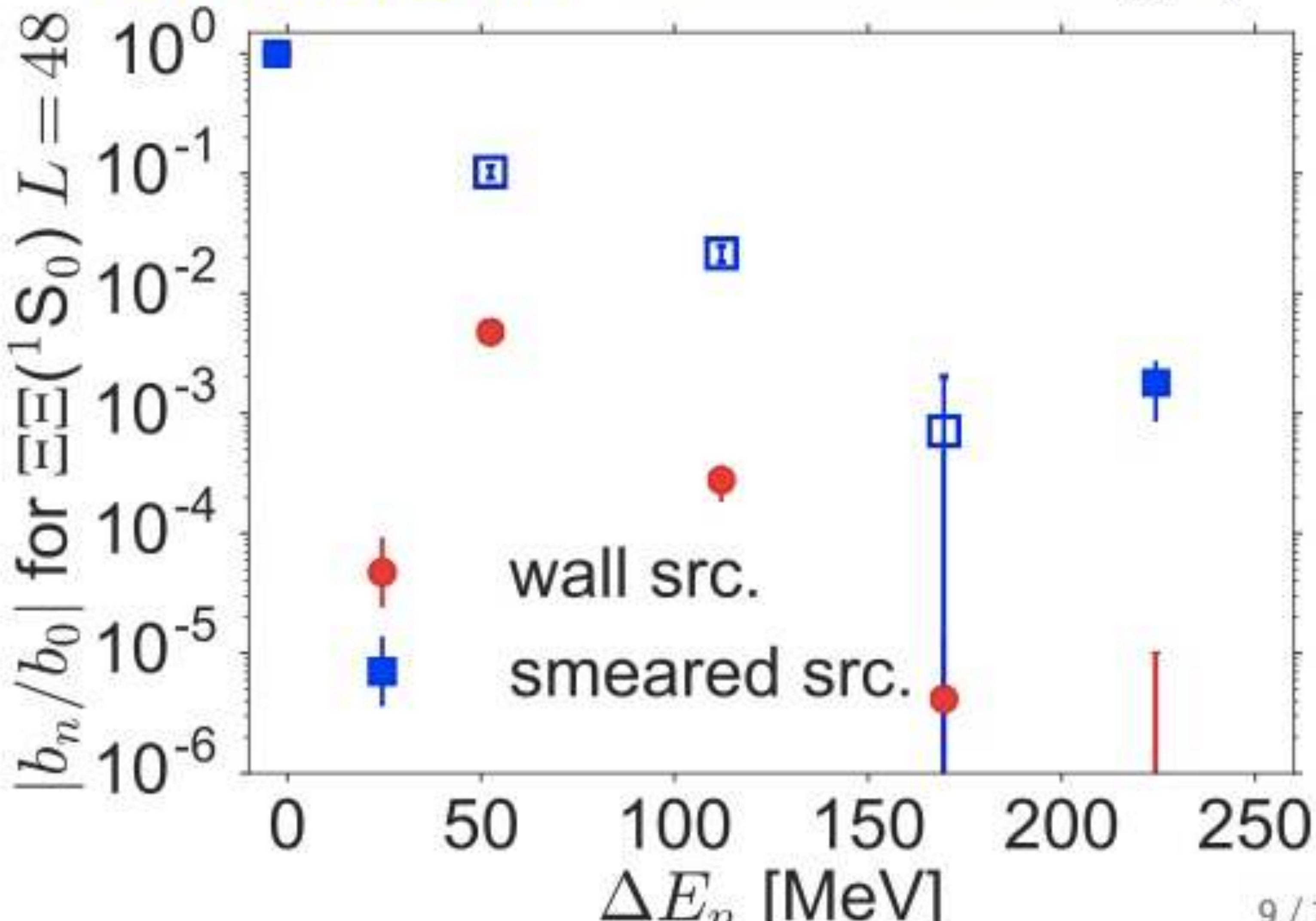
$$\therefore R(t) \equiv R(\vec{p} = 0, t) = \sum_r R(\vec{r}, t) = \sum_n b_n^{\mathrm{wall/smear}} e^{-\Delta E_n t}$$

□ ex. **1st excited state**

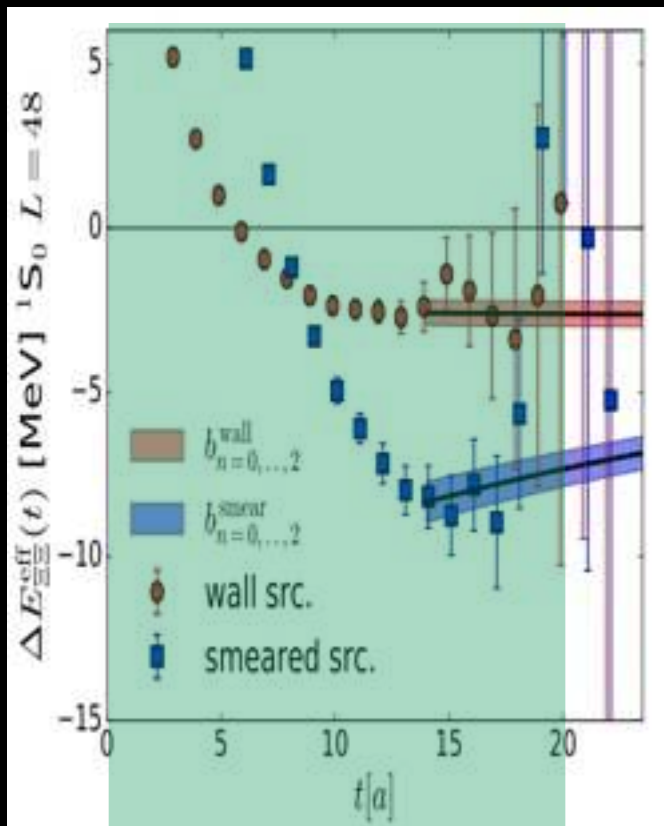
- **wall source**
  $b_1/b_0 \ll 1$ %
- **smeared source**†
  $b_1/b_0 \simeq -10$ %
- with energy gap
  $E_1 - E_0 \simeq 50$ MeV
  for $L^3 = 48^3$

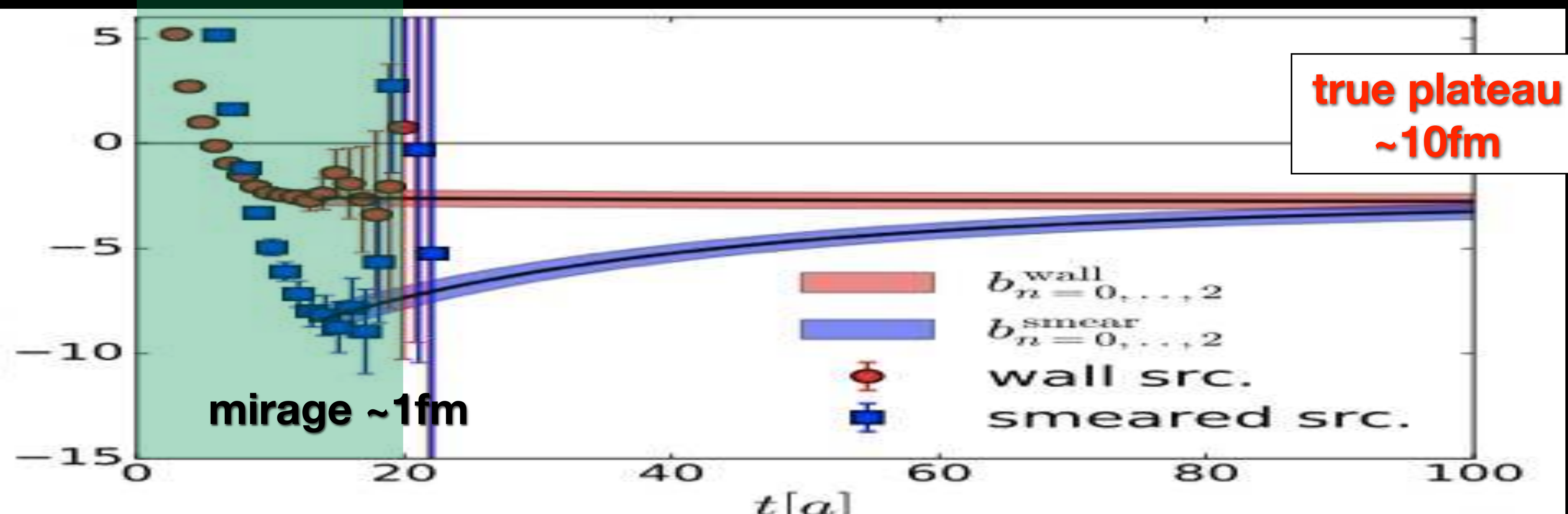
†unfilled symbols: $b_n/b_0 < 0$

"contamination" of excited states b_n/b_0



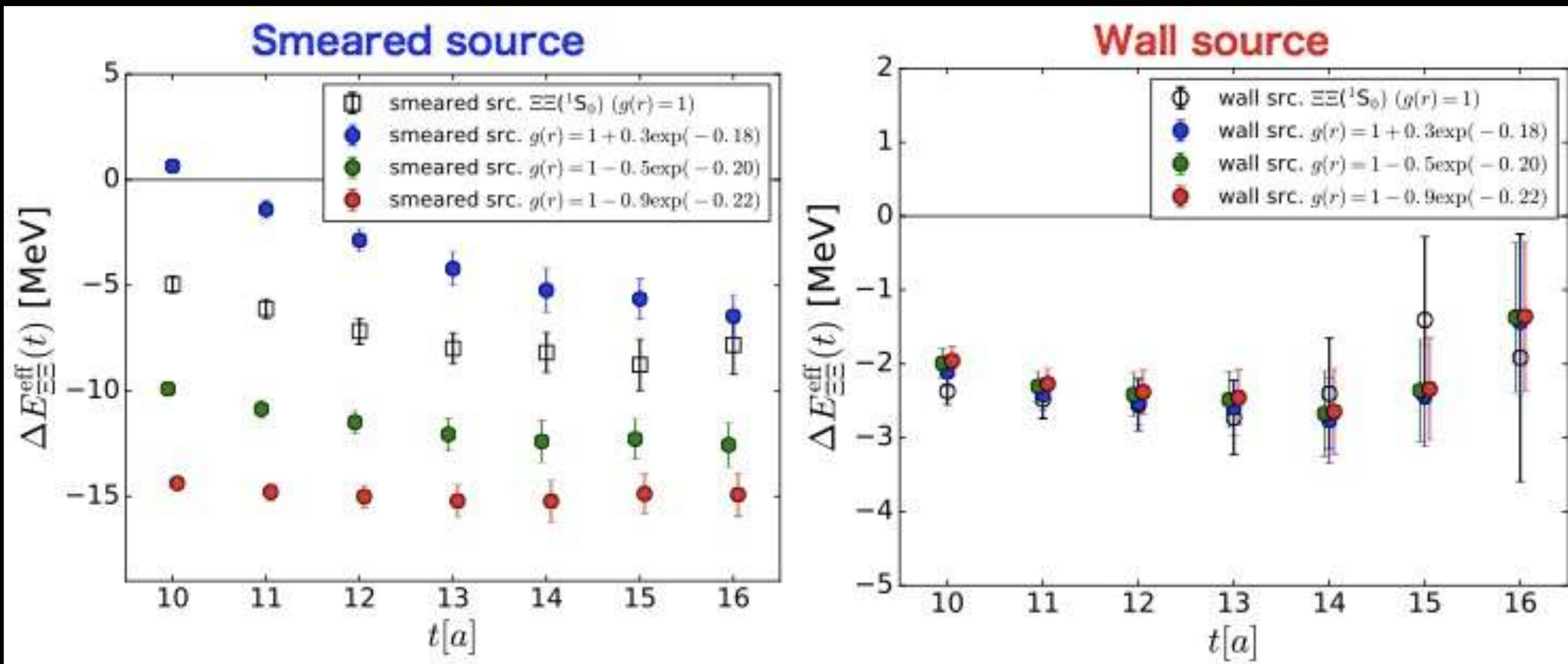
Fate of “fake plateau” in ΞΞ ($^1S_0$) channel





Sink operator dependence in ΞΞ ($^1S_0$) channel

$$R(t) = \sum_{\vec{r}} g(\vec{r}) \sum_{\vec{x}} \langle 0 | B_1(\vec{x}+\vec{r}, t) B_2(\vec{x}, t) \mathcal{J}^\dagger(t=0) | 0 \rangle / C_B(t)^2$$



Smeared source is very sensitive to the choice of sink operators.

On the other hand, wall source is insensitive.

On the comment on "sanity check" by NPLQCD

NPL2017C **Comment on "Are two nucleons bound in lattice QCD for heavy quark masses? - Sanity check with Lüscher's finite volume formula -"**

Data	$NN(^1S_0)$				$NN(^3S_1)$			
	Source independence	Sanity check (i)	Sanity check (ii)	Sanity check (iii)	Source independence	Sanity check (i)	Sanity check (ii)	Sanity check (iii)
NPL2013 [27,28]	No			No	No			No

NPL claimed that the above table in our paper should be replaced by

Data	$NN(^1S_0)$				$NN(^3S_1)$			
	Source independence	Sanity check (i)	Sanity check (ii)	Sanity check (iii)	Source independence	Sanity check (i)	Sanity check (ii)	Sanity check (iii)
NPL2013 [27,28]	Yes	Passed	Passed	Passed	Yes	Passed	Passed	Passed

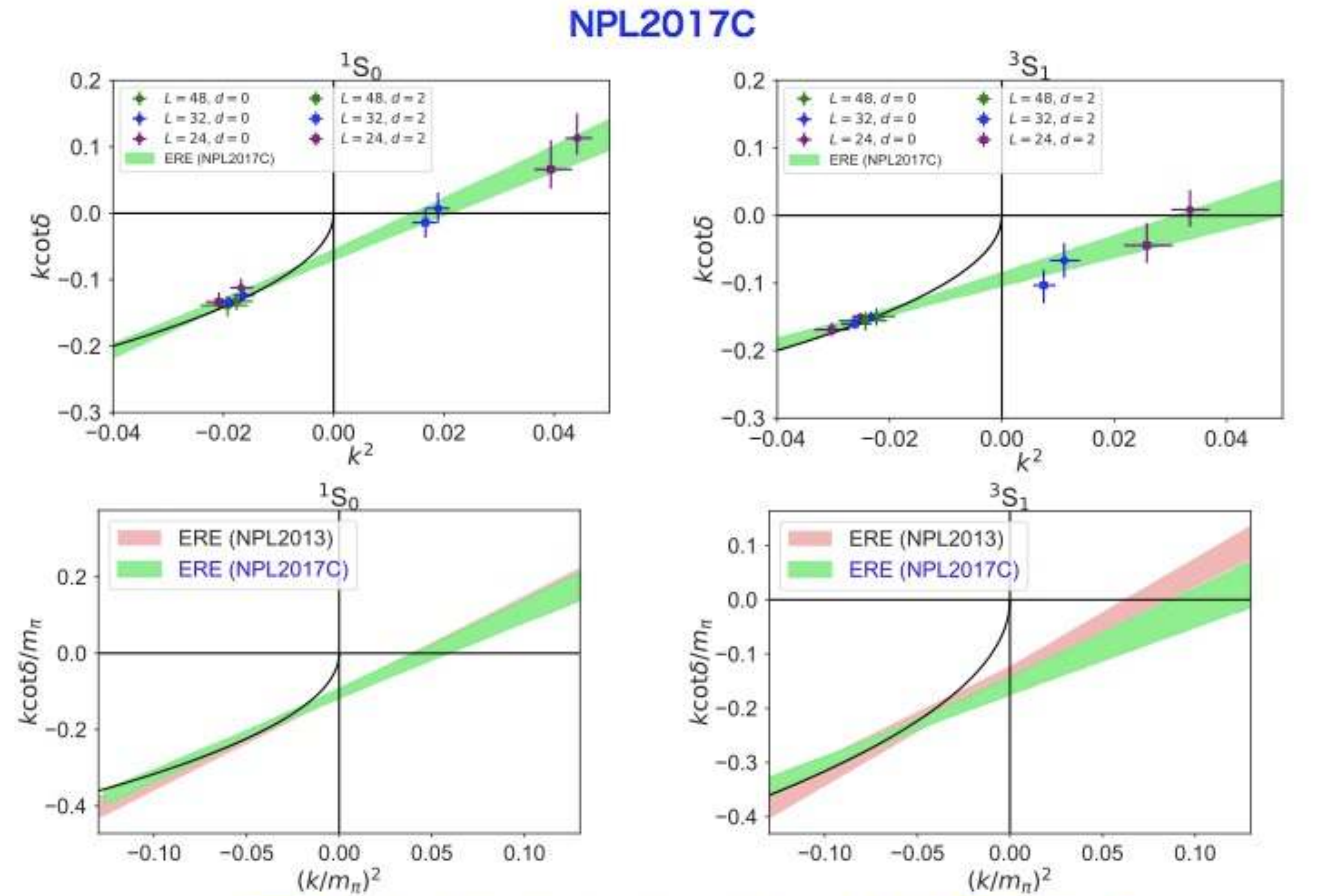
Source independence was already discussed:

χ^2/dof	3.70	4.22	2.37	2.75
significance	0.05	0.04	0.12	0.10

It is fair to say that source independence is NOT established.

see talk by S. Aoki (Parallel, Hadron Spectroscopy and Interactions on Thur.)

On the comment on “sanity check” by NPLQCD

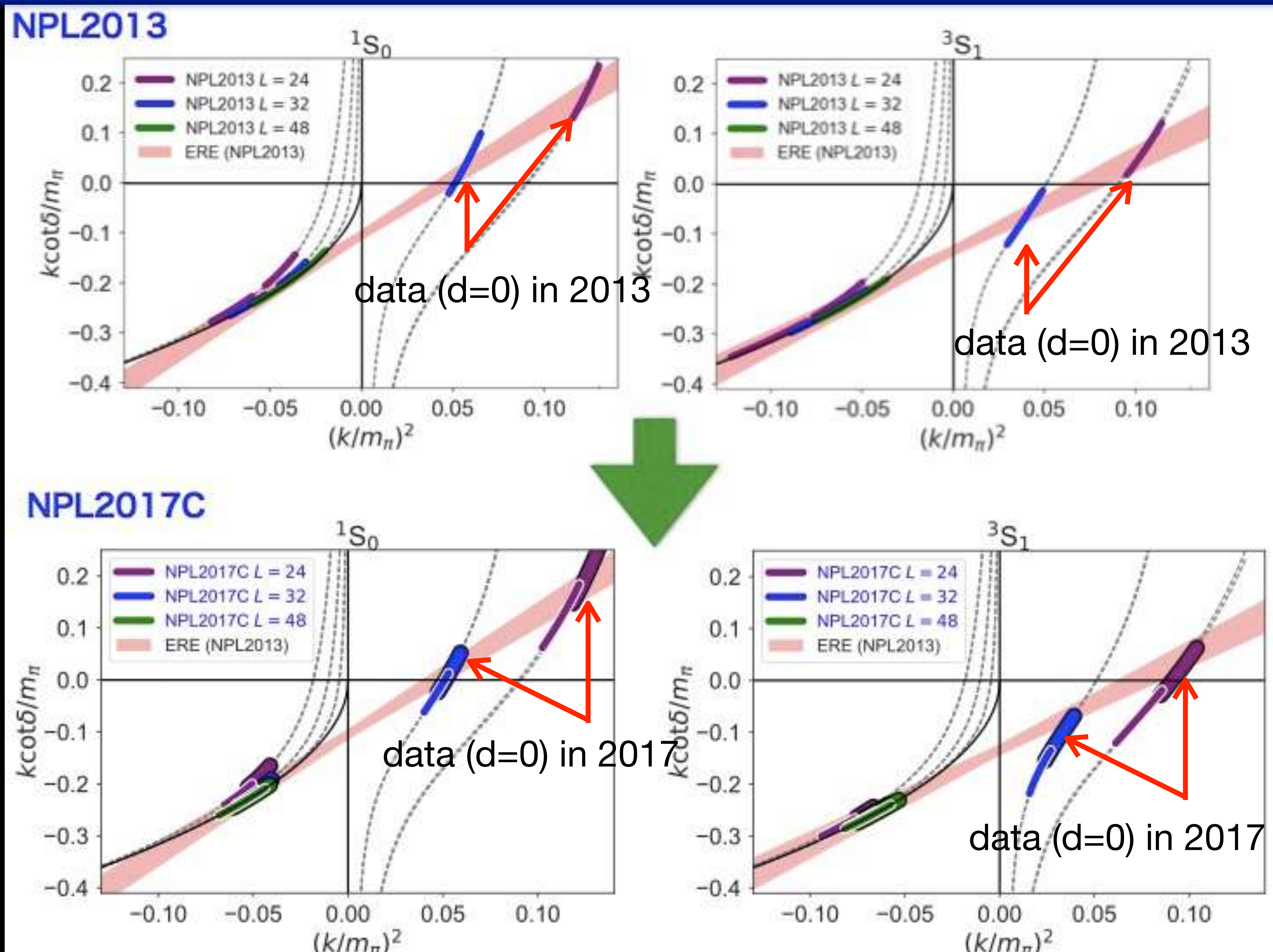


EREs look a litter better than their original EREs. Why ?

An inclusion of data in the moving frame (d=2) affect ?

see talk by S. Aoki (Parallel, Hadron Spectroscopy and Interactions on Thur.)

On the comment on “sanity check” by NPLQCD



The data have been modified without being mentioned...

➡ Our assessment to original NPL2013 data is valid

see talk by S. Aoki (Parallel, Hadron Spectroscopy and Interactions on Thur.)