Charmed and bottom pseudoscalar meson decay constants and quark masses from HISQ simulations

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Outline

1. HISQ ensembles with (2+1+1)-flavors of dynamical quarks
2. Decay constants of heavy-light mesons
3. Extraction of quark masses from heavy-light meson masses

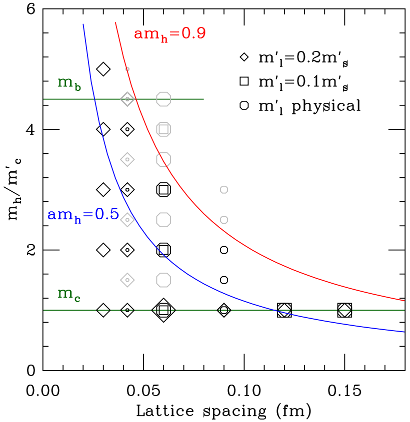
MILC ensembles with (2+1+1)-flavors of dynamical quarks

- Ensembles with physical mass for the strange quark:

$\approx a$ (fm)	m_l/m_s	size	L (fm)	$M_\pi L$	M_π (MeV)
0.15	1/5	$16^3 \times 48$	2.38	3.8	314
0.15	1/10	$24^3 \times 48$	3.67	4.0	214
0.15	1/27	$32^3 \times 48$	4.83	3.2	130
0.12	1/5	$24^3 \times 64$	3.00	4.5	299
0.12	1/10	$24^3 \times 64$	2.89	3.2	221
0.12	1/10	$32^3 \times 64$	3.93	4.3	216
0.12	1/10	$40^3 \times 64$	4.95	5.4	214
0.12	1/27	$48^3 \times 64$	5.82	3.9	133
0.09	1/5	$32^3 \times 96$	2.95	4.5	301
0.09	1/10	$48^3 \times 96$	4.33	4.7	215
0.09	1/27	$64^3 \times 96$	5.62	3.7	130
0.06	1/5	$48^3 \times 144$	2.94	4.5	304
0.06	1/10	$64^3 \times 144$	3.79	4.3	224
0.06	1/27	$96^3 \times 192$	5.44	3.7	135
0.042	1/5	$64^3 \times 192$	2.91	4.34	294
0.042	1/27	$144^3 \times 288$	6.12	4.17	134
0.03	1/5	$96^3 \times 288$	3.25	4.84	294

- The fermion action is “highly improved staggered quark” (HISQ) action
- Physical-mass ensembles at most lattice spacings

Heavy-Light Pseudoscalar Mesons



- We have 24 Ensembles:
  - 6 Lattice spacings
  - Several sea masses
- We calculate 2-point correlators for various light and heavy masses
  - light valence: $m_l \lesssim m_v \lesssim m_s$
  - heavy valence: $m_c \lesssim m_h \lesssim m_b$

  $\Rightarrow$ Decay constants & masses of HL mesons
- We use only $am_h < 0.9$ to avoid large discretization errors

Decay constants

We employ HMrPQASχPT to perform a combined correlated, multidimensional fit to ~ 500 data
$\Rightarrow$ reduces statistical errors
$\Rightarrow$ controls systematic errors of extrapolations

Meson masses

We use a method based on HQET to extract quark masses from heavy-light meson masses

Scale setting and calculating tuned quark masses

- Scale setting for chiral analysis is done using F_{p4s}
  (the decay constant of a fiducial pseudoscalar meson with both valence masses equal to $m_{p4s} \equiv 0.4 m_s$)
- This method yields a precise determination of both the lattice spacing a and the quark mass am_{p4s} (and in turn $m_s = 2.5 m_{p4s}$)
- The values of F_{p4s} and quark mass ratio m_s/m_l are determined by analyzing light-light data from the same ensembles
  $\Rightarrow$ Various systematic errors (such as FV, EM, continuum extrapolation *etc.*) in estimate of F_{p4s} and tuned quark masses must be incorporated to our estimate of uncertainties

Constructing fit function for decay constants

- We use a cascade of EFTs to construct our fit functions
- We start from the following schematic form for decay constants of H_{v} mesons

$$\Phi_{H_{\mathsf{v}}} = C\ (1+\mathsf{SET})\ (1+\mathsf{HQET})\ (1+\mathsf{HMrAS\chi PT}) \left(\frac{m_c'}{m_c}\right)^{3/27} \tilde{\Phi}_0$$

- These terms correspond to different effective field theories

Symanzik Effective Theory (SET)

$$c_1\alpha_s\,(a\Lambda)^2 + \cdots + c_3\alpha_s\,(am_h)^2 + \cdots$$

Wilson coefficient C

$$\Big[\alpha_s(M_{H_s})\Big]^{-6/25}\Big(1+\mathcal{O}(\alpha_s)\Big)$$

HQET (and integrating out sea-charm)

$$k_1\frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}} + \cdots + k_1'\frac{m_c}{m_c'}$$

Integrating out charm quark

$$\frac{\Lambda^{(3)}_{\mathrm{QCD}}(m_c')}{\Lambda^{(3)}_{\mathrm{QCD}}(m_c)} \approx \left(\frac{m_c'}{m_c}\right)^{2/27}$$

HMrPQASχPT at NLO

chiral non-analytic terms

$+L_{\mathsf{v}}m_{\mathsf{v}} + L_{\mathsf{s}}(2m_l+m_s) + L_a a^2$

Chiral terms contain effects of

- Taste splittings in “pion” masses & new logs.
- Hyperfine and flavor splittings
- Finite lattice volume

Constructing fit function for decay constants

- We improve the fit function by adding more (analytic) terms
- Building blocks for constructing "natural" expansions:
  1) Dimensionless light quark masses and a measure of the taste splitting

$$x_{u,d,s,\mathrm{v}} \equiv \frac{1}{8\pi^2 f_\pi^2} 2Bm_{u,d,s,\mathrm{v}} \ , \qquad x_{\bar{\Delta}} \equiv \frac{1}{8\pi^2 f_\pi^2} a^2 \bar{\Delta}$$

     where B is the LEC that gives the Goldstone pion mass $M_\pi^2 = B(m_u + m_d)$
  2) For parameterizing heavy-quark discretization error, we use "natural" variable

$$x_h = \left(\frac{am_h}{\frac{\pi}{2}} \right)^2$$

     where the factor $\frac{\pi}{2}$ is obtained from the radius of convergence of the tree-level mass-dependent correction to the Naik term in HISQ action
  3) For parameterizing higher order effects in HQET, we use

$$\frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}} \qquad \text{with } \Lambda_{\mathrm{HQET}} = 800\,\mathrm{MeV}$$

  4) For parameterizing generic lattice artifacts for staggered quarks, we use

$$(a\Lambda)^2 \qquad \text{with } \Lambda = 600\,\mathrm{MeV}$$

The fit function and data

- With the building blocks we construct a fit function with 60 free parameters
  - Building blocks are such that the free parameters are expected to be of order 1
  - We use priors of 0 ± 1 for higher order parameters
- We use 532 lattice data for decay constants of heavy-light mesons
  - 6 Lattice spacings (ranging from 0.03 fm to 0.15 fm)
  - several sea quark masses
    (with physical-mass ensembles at five lattice spacings [0.042 fm to 0.15 fm])
  - several valence quark masses
    m_h/m'_c: 0.9, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5 and 5 (if $am_h < 0.9$)
    m_v/m'_s: 1, 0.8, 0.6, 0.4, 0.2, [0.1, 0.07 and 0.04]
  - The data at 0.03 fm and 0.042 fm ensembles are adjusted for non-equilibration of topological charge for both heavy-light and light-light sectors
    ([C. Bernard and D. Toussaint (in preparation)], and
    [C. Bernard and D. Toussaint PoS(LATTICE2016)189])

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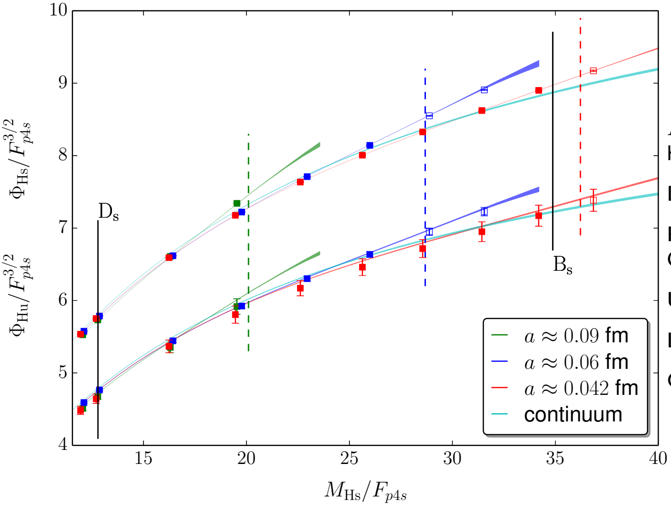
- Simulation of the B systems at 0.03 fm
  - We have a non-physical mass ensemble with $m'_l/m'_s = 0.2$
  - No extrapolation from lighter heavy-quark masses is needed ($am_b \approx 0.6$)
  - A chiral extrapolation to physical point for light-quark masses is needed (provided by HMrPQASχPT)

A snapshot of the fit and data



M_{Hs} as proxy of heavy-quark mass m_h

Dashed lines: $am_h = 0.9$

Filled Symbols: included
Open Symbols: excluded

Upper curves: $m_{\mathrm{v}} = m_s$

Lower curves: $m_{\mathrm{v}} = m_u$

Cyan bands: extrap. to cont.

χ^2/d.o.f $= 500/472$, P-value=0.2

Preliminary results for decay constants

Preliminary results

$$f_{D^+} = 212.7 \pm 0.3_{\rm stat} \pm 0.3_{\rm sys} \pm 0.2_{f_\pi,\rm PDG}\ \rm MeV$$
$$f_{D_s} = 250.0 \pm 0.3_{\rm stat} \pm 0.2_{\rm sys} \pm 0.2_{f_\pi,\rm PDG}\ \rm MeV$$
$$f_{B^+} = 189.8 \pm 0.8_{\rm stat} \pm 0.3_{\rm sys} \pm 0.2_{f_\pi,\rm PDG}\ \rm MeV$$
$$f_{B_s} = 231.2 \pm 0.7_{\rm stat} \pm 0.3_{\rm sys} \pm 0.2_{f_\pi,\rm PDG}\ \rm MeV$$

The systematic error includes

- Systematic errors in calculation of scale setting quantities and tuned quark masses:
  - Continuum extrapolation
  - Finite volume
  - EM contribution to meson masses that are used to fix the quark masses (Decay constants are pure-QCD quantities; EM contributions to the relation between decay constants and physical decay rates are not included here by definition but would be relevant for phenomenology)
- Uncertainty in adjustment for non-equilibration of topological charge

Extraction of quark masses from heavy-light meson masses

- We develop a method based on HQET to extract $\overline{\text{MS}}$-renormalized masses of quarks from masses of heavy-light mesons
- HQET description of a HL meson mass in terms of its heavy quark mass

$$M_H = m_Q + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2(m_Q)}{2m_Q} + \mathcal{O}(1/m_Q^2)$$

  - $\bar{\Lambda}$: energy of quark and gluons inside the system
  - $\mu_\pi^2/2m_Q$: kinetic energy of the heavy quark inside the system
  - $\mu_G^2(m_Q)/2m_Q$: hyperfine energy due to heavy quark's spin (can be estimated from B^*-B splitting $\Rightarrow \mu_G^2(m_b) \approx 0.35\,\text{GeV}^2$)
  - m_Q is the pole mass of the heavy quark

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  - m_Q is the pole mass of the heavy quark (not a practical interpretation because of renormalon problem in pole mass)
  - As a practical choice we use the renormalon-subtracted (RS) mass (defined by subtracting the leading infrared renormalon from the pole mass [Pineda hep-ph/0105008])
  - We modify the RS mass to avoid introducing any renormalon-subtraction scale

Mapping bare quark masses to the $\overline{\mathsf{MS}}$ and RS masses

1) Introduce a "reference mass" and for a generic heavy quark h construct the ratio

$$\frac{m_h^{\overline{\mathsf{MS}}}(\mu)}{m_r^{\overline{\mathsf{MS}}}(\mu)} = \frac{am_h}{am_r}\Big(1 + K_1\big((am_h)^2 - (am_r)^2\big) + \cdots\Big)$$

(Parameterize higher order lattice artifacts by adding more terms)

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2) Set the reference mass to a tuned quark mass. We set $am_r = am_{p4s}$ ($m_{p4s} = 0.4 m_s$ is a convenient choice when "p4s" scale setting is used)

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4) Map the bare mass of a generic quark am_h to its $\overline{\text{MS}}$-renormalized mass by

$$m_h^{\overline{\text{MS}}}(2\text{GeV}) = m_{p4s}^{\overline{\text{MS}}}(2\text{GeV}) \times \frac{am_h}{am_{p4s}}\Big(1 + K_1\big((am_h)^2 - (am_{p4s})^2\big) + \cdots\Big)$$

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5) Use continuum-limit relations to map $m_h^{\overline{\text{MS}}}(2\text{GeV})$ to the RS mass m_h^{RS}

Mapping bare quark masses to the $\overline{\text{MS}}$ and RS masses

1) Introduce a "reference mass" and for a generic heavy quark h construct the ratio

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5) Use continuum-limit relations to map $m_h^{\overline{\text{MS}}}(2\text{GeV})$ to the RS mass m_h^{RS}

6) Finally, plug m_h^{RS} in HQET description of masses of heavy-light mesons

Constructing fit function for meson masses

- Dependence on light-quark masses
  - We include leading non-analytic dependence on light quark masses

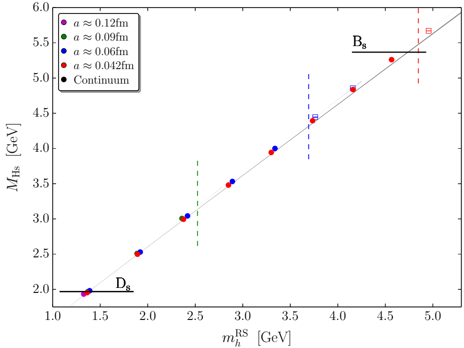
    One-loop chiral corrections to HL meson masses are calculated in HMrPQASχPT

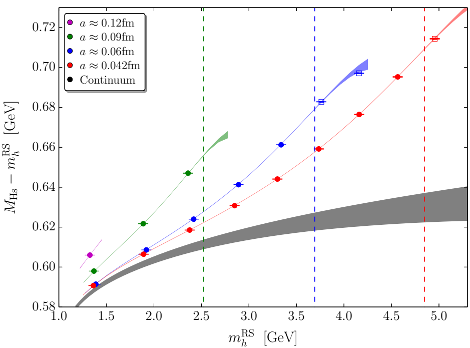
  - In addition to chiral non-analytic terms, we include linear, quadratic and cubic mass-dependent analytic terms
  - We also construct an alternative fit function only with mass-dependent analytic terms
- Using our "building blocks" we construct a fit function with 53 fit parameters (50 parameters for the alternative fit function)

The fit function and meson masses

- We use 274 lattice data for decay constants of heavy-light mesons
  - 5 Lattice spacings (ranging from 0.03 fm to 0.12 fm)
  - several sea quark masses
    (with physical-mass ensembles at four lattice spacings [0.042 fm to 0.12 fm])
  - several valence quark masses
    m_h/m'_c: 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5 and 5 (if $am_h < 0.9$)
    m_v/m'_s: 1, 0.8, 0.6, 0.4, 0.2, [0.1, 0.07 and 0.04]
- The fit gives χ^2/d.o.f $= 162/221$
  (and χ^2/d.o.f $= 195/224$ for the alternative fit without chiral terms)

A snapshot of the fit and data





Dashed lines: $am_h \approx 0.9$; open symbols: data points omitted from fit
Vertical axis: heavy-strange meson masses
Horizontal axis: the fit values for the RS mass projected to continuum (no lattice artifacts)

- After extrapolating to continuum, M_{D_s} and M_{B_s} are used to determine the charm- and bottom-quark masses

Preliminary results

Preliminary results for strange-, charm- and bottom-quark masses and their ratios in a theory with 4 dynamical quarks

$$
\begin{aligned}
m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) &= 92.4 \pm 0.4_{\mathrm{stat}} \pm 0.5_{\mathrm{sys}}\,\mathrm{MeV} \\
\overline{m}_c &= 1270 \pm 4_{\mathrm{stat}} \pm 10_{\mathrm{sys}}\,\mathrm{MeV} \\
\overline{m}_b &= 4198 \pm 11_{\mathrm{stat}} \pm 9_{\mathrm{sys}}\,\mathrm{MeV} \\
m_c/m_s &= 11.755 \pm 0.013_{\mathrm{stat}} \pm 0.034_{\mathrm{sys}} \\
m_b/m_s &= 53.91 \pm 0.08_{\mathrm{stat}} \pm 0.11_{\mathrm{sys}} \\
m_b/m_c &= 4.586 \pm 0.005_{\mathrm{stat}} \pm 0.010_{\mathrm{sys}}
\end{aligned}
$$

where $\overline{m}_h = m_h^{\overline{\mathrm{MS}}}(\mu = \overline{m}_h)$
The systematic error includes

- Systematic errors in determination of scale setting quantities and tuned quark masses: continuum extrapolation, finite volume and EM contribution
- Uncertainty in the strong coupling constant $\alpha_s^{\overline{MS}}(5\,\mathrm{GeV}; n_f = 4) = 0.2128(25)$ [HPQCD, arXiv:1408.4169]
- EM contribution to meson masses that are used to fix the quark masses

Preliminary results

Preliminary results for HQET LECs

$$\begin{aligned} \overline{\Lambda}^{\mathrm{RS}}(1\mathrm{GeV}) &= 637 \pm 21_{\mathrm{stat}} \pm 23_{\mathrm{sys}}\,\mathrm{MeV} \\ \mu_G^2(m_b) &= 0.35 \pm 0.02_{\mathrm{stat}} \pm 0.02_{\mathrm{sys}}\,\mathrm{GeV}^2 \\ \mu_\pi^2 &= 0.02 \pm 0.09_{\mathrm{stat}} \pm 0.07_{\mathrm{sys}}\,\mathrm{GeV}^2 \end{aligned}$$

Note that

- The values of HQET LECs depend on our choice for subtracting the infrared renormalons of the pole mass
- The prior value of $\mu_G^2(m_b)$ is set to $0.35 \pm 0.07\,\mathrm{GeV}^2$ [P. Gambino and C. Schwanda, arXiv:1307.4551]

Preliminary results

- To calculate the light quark masses we combine our determination of $m_s^{\overline{\mathrm{MS}}}(2\mathrm{GeV})$ and mass ratios m_s/m_l and m_d/m_u

  Preliminary results for light quark masses

$$
\begin{aligned}
m_d^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) &= 4.656 \pm 0.020_{\mathrm{stat}} \pm 0.049_{\mathrm{sys}}\,\mathrm{MeV} \\
m_l^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) &= 3.396 \pm 0.014_{\mathrm{stat}} \pm 0.021_{\mathrm{sys}}\,\mathrm{MeV} \\
m_u^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) &= 2.136 \pm 0.016_{\mathrm{stat}} \pm 0.038_{\mathrm{sys}}\,\mathrm{MeV}
\end{aligned}
$$

- m_u and m_d values depend on separate calculation of EM effects on light-light mesons; that calculation is being finalized

Conclusion

- Decay constants
  - We employed HMrPQASχPT to perform a combined correlated, multidimensional fit to 532 data at multiple lattice spacings
    $\Rightarrow$ reduces statistical errors
    $\Rightarrow$ controls systematic errors of extrapolations
  - We presented preliminary results for decay constants f_{D^+}, f_{D_s}, f_{B^+} and f_{B_s}
- Quark masses
  - We developed a method based on HQET to extract quark masses from heavy-light meson masses
  - We employed HMrPQASχPT to describe the dependence of heavy-light mesons on masses of light valence and sea quarks, and we performed a combined correlated, multidimensional fit to 274 data at mult. lattice spacings
  - We presented preliminary results for strange-, charm- and bottom-quark masses and their ratios
  - We presented preliminary results for HQET LECs: $\overline{\Lambda}$, μ_π^2 and $\mu_G^2(m_b)$
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Thanks for your attention!

back-up slides

Minimal renormalon subtracted (MRS) scheme

- The RS mass is defined by subtracting the leading renormalon of the pole mass
- In the RS mass a finite part, in addition to the leading renormalon, is subtracted from the pole mass
- The finite part depends on the subtraction scale ν_f (factorization scale)
- In the minimal renormalon subtracted (MRS) scheme we subtract only the renormalon part of pole mass (*i.e.*, the ambiguous part of the pole mass in the Borel plane)
- Define

$$\mathcal{J}^{\mathrm{MRS}}(\mu) \equiv \frac{N_m}{2\beta_0}\,\mu\, e^{-1/[2\beta_0\alpha(\mu)]} \sum_{n=0}^{\infty} \frac{1}{n!(n-b)} \left[\frac{1}{2\beta_0\alpha(\mu)}\right]^n$$

where $b = \frac{\beta_1}{2\beta_0^2}$ and α is defined in a scheme with $\beta(\alpha) = -\beta_0\alpha^2/(1 - \frac{\beta_1}{\beta_0}\alpha)$

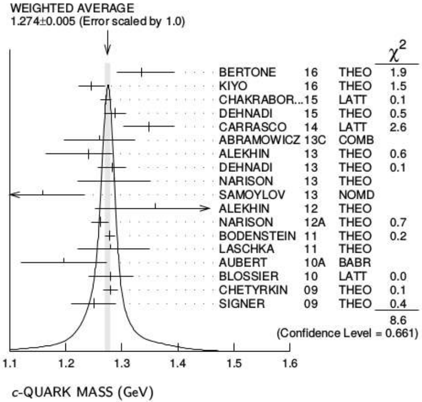
- To convert the quark mass and $\overline{\Lambda}$ form the MRS scheme to the RS scheme :

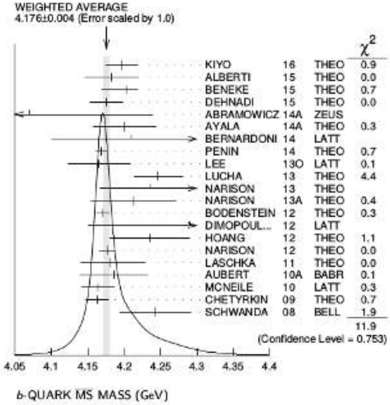
$$m^{\mathrm{RS}}(\nu_f) = m^{\mathrm{MRS}} - \mathcal{J}^{\mathrm{MRS}}(\nu_f)$$
$$\overline{\Lambda}^{\mathrm{RS}}(\nu_f) = \overline{\Lambda}^{\mathrm{MRS}} + \mathcal{J}^{\mathrm{MRS}}(\nu_f)$$

for instance we have

$$\overline{\Lambda}^{\mathrm{MRS}} = 553 \pm 21_{\mathrm{stat}} \pm 20_{\mathrm{sys}}\,\mathrm{MeV}$$
$$\overline{\Lambda}^{\mathrm{RS}}(1\mathrm{GeV}) = 637 \pm 21_{\mathrm{stat}} \pm 23_{\mathrm{sys}}\,\mathrm{MeV}$$

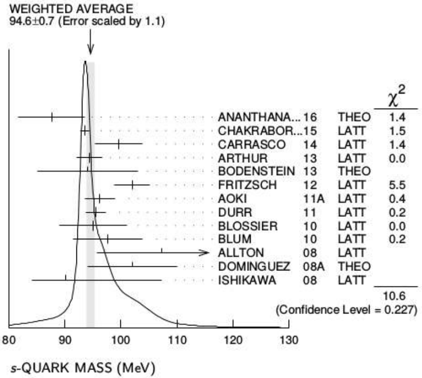
PDG summary plot for charm and bottom quark masses





$$\overline{m}_c = 1270 \pm 4_{\rm stat} \pm 10_{\rm sys}\ {\rm MeV}$$
$$\overline{m}_b = 4198 \pm 11_{\rm stat} \pm 9_{\rm sys}\ {\rm MeV}$$

PDG summary plot for strange quark masses



$$m_s^{\overline{\rm MS}}(2\,{\rm GeV}) \quad = \quad 92.4 \pm 0.4_{\rm stat} \pm 0.5_{\rm sys}\,{\rm MeV}$$

Stability plot for decay constants