# Electric Dipole Moment Results from Lattice QCD

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# Calculation Parameters

- Publicly available PACS-CS gauge fields from [www.jldg.org].
- $N_f = 2 + 1$ , $a = 0.090$ fm , $32^3 \times 64$ volume.
- Gauge-invariant Gaussian smearing $\alpha = 0.71$
- 64 smearing sweeps at source, combination of 16, 32, 64 at sink.
- No disconnected quark loop contributions to current observables calculated.
- Vector current renormalisation of 0.7354, from work done in [**Aoki:2010wm**]

| $m_\pi \approx$ | 411 MeV | | 701 MeV | |
|---|---|---|---|---|
| Gauge Fields | 200 | | 399 | |
| Observable | $\alpha$ | $G_3$ | $\alpha$ | $G_3$ |
| Sources Per GF (Avg) | 77.12 | 47.8 | 67.24 | 53.96 |
| Total # of Measurements | 15,423 | 9,560 | 26,828 | 21,530 |

# CP Violating Operator Definitions

- $\theta$-term using Topological charge density:

$$-i\theta q(x) \equiv -i\theta \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}\left[G^{\mu\nu}(x)G^{\rho\sigma}(x)\right]$$

- Weinberg operator term:

$$-i\frac{\alpha_{\widetilde{G}}}{\Lambda^2}W(x) \equiv -i\frac{\alpha_{\widetilde{G}}}{\Lambda^2}\, f^{ABC}\widetilde{G}^{A\nu}_{\mu}(x)G^{B\rho}_{\nu}(x)G^{C\mu}_{\rho}(x)$$

- Since $W(x)$ is dimension 6, we have coefficient being $\frac{\alpha_{\widetilde{G}}}{\Lambda^2}$!
- This implies that knowing $\alpha_{\widetilde{G}}$ could give information about scale of BSM physics?

We will look at the blue operators in our Lattice QCD calculations!

# CP Violating Observables

- $\langle Q_t \rangle = 0$ and $\langle W \rangle = 0$ as $\mathcal{L}_{QCD}$ has no $\not{CP}$

- Topological Susceptibility:

$$\chi_{top} = \frac{1}{V} \int d^4x d^4y \, \langle q(x) q(y) \rangle = \frac{1}{V} < Q_t^2 > \quad , \quad \left[ Q_t = \int d^4x q(x) \right]$$

- Analogous Weinberg operator observable:

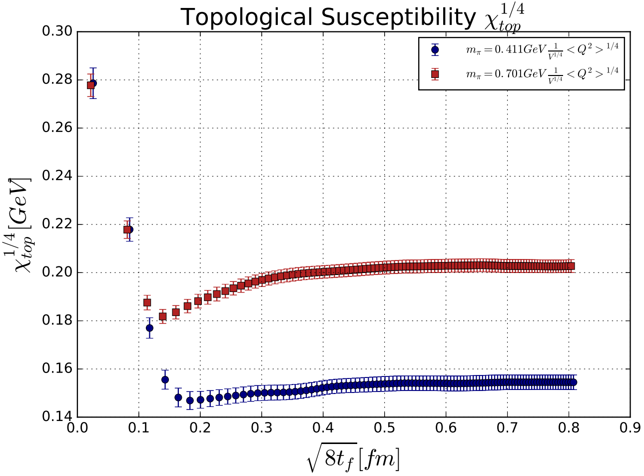
$$\chi_W = \frac{1}{V} \int d^4x d^4y \, \langle W(x) W(y) \rangle = \frac{1}{V} < W^2 > \quad , \quad \left[ W = \int d^4x W(x) \right]$$

- BUT! These quantities pose troubles when trying to renormalise and bring to the continuum limit.

- We employ the Gradient Flow technique to overcome this problem. [Lüscher, 2010-2013]

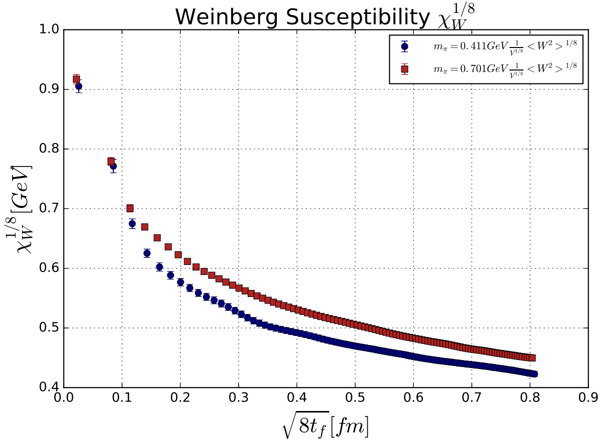
# Topological Susceptibility with Gradient Flow

$$\chi_{top}(t_f) = \frac{1}{V}\int d^4x d^4y \langle q(x,t_f) q(y,t_f)\rangle .$$



# Analogous Weinberg Susceptibility with Gradient Flow

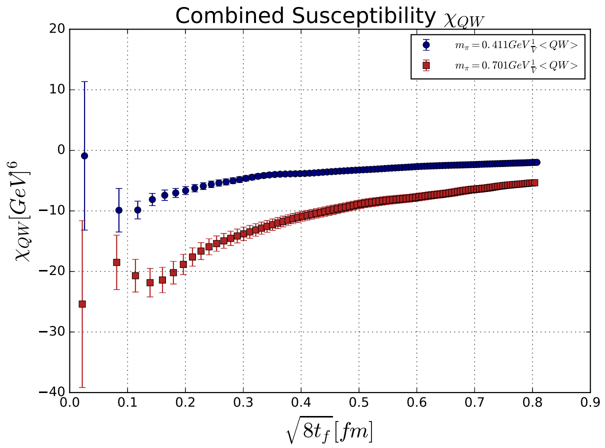
$$\chi_W(t_f) = \frac{1}{V}\int d^4x d^4y \, \langle W(x,t_f)W(y,t_f)\rangle \, .$$




flow time dependence at larger $t_f$ needs to be analysed.

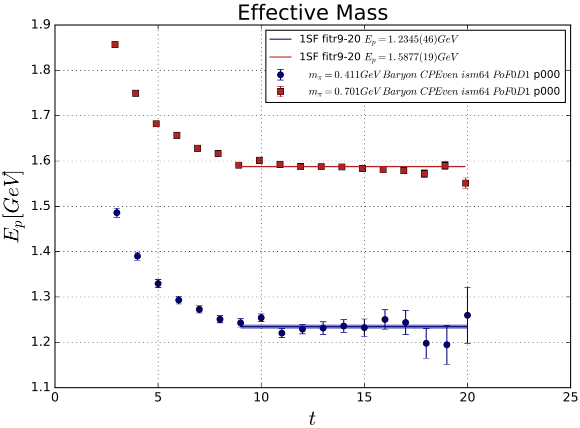
# Combined Susceptibility with Gradient Flow??

$$\chi_{QW}(t_f) = \frac{1}{V}\int d^4x d^4y \, \langle q(x,t_f) W(y,t_f) \rangle \, .$$




The combination might “soften” the flow time dependence?...

# Fermionic Observables: Baryon Mass Examples



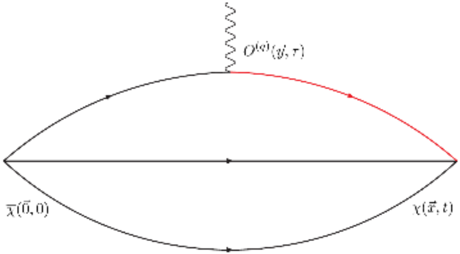

$$G_2(\Gamma; \vec{p}, t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \mathrm{Tr}\left\{\Gamma \langle \chi(\vec{x}, t)\overline{\chi}(0)\rangle\right\}$$

$$\text{Effective Mass}(t) \equiv \log\left(\frac{G_2(\Gamma_4; \vec{0}, t)}{G_2(\Gamma_4; \vec{0}, t+1)}\right) \xrightarrow{t \gg 0} m$$

# Probing the Baryon

- Three-point correlator defined as:

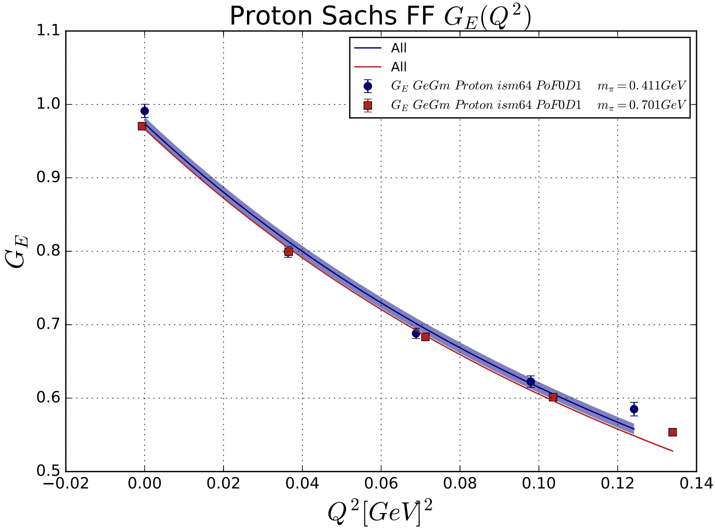
$$G_3(\Gamma;\vec{p}',t;\vec{q},\tau;O^{(q)}) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}'\cdot\vec{x}} e^{i\vec{q}\cdot\vec{y}} \mathrm{Tr}\left\{\Gamma\langle\chi(\vec{x},t)O^{(q)}(\vec{y},\tau)\overline{\chi}(0)\rangle\right\}$$




- Selecting $O^{(q)} = \mathcal{J}_\mu^{(q)}$ with appropriate $\Gamma$ gives access to the vector form-factors $F_1(Q^2)$ & $F_2(Q^2)$.
- Define $Q^2 \equiv -q^2$, i.e. the transfer momentum.

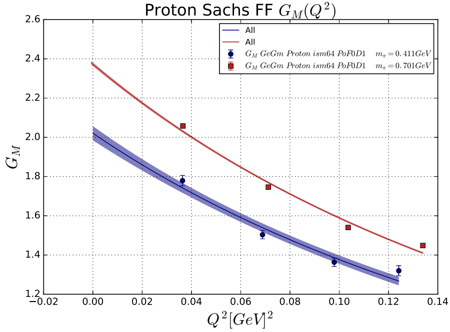
# Fermionic Observables: Proton Form Factor

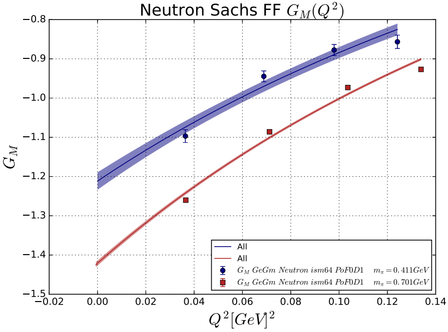
$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$



# Fermionic Observables: Proton/Neutron Form Factors

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$



# QCD Lagrangian with $\not{CP}$

- Standard Model QCD Lagrangian has the form:

$$\mathcal{L}_{QCD} = \frac{1}{4} G^{(a)}_{\mu\nu} G^{(a)\mu\nu} + \sum_q \overline{\psi}_q \left( \gamma^\mu D_\mu - m_q \right) \psi_q .$$

- Induce CP violations in $\mathcal{L}_{QCD}$ by adding terms:

$$\mathcal{L}_{\not{CP}} = -i\theta q(x) - i \frac{\alpha_{\widetilde{G}}}{\Lambda^2} W(x) - \ldots$$

- The terms $\theta$ & $\frac{\alpha_{\widetilde{G}}}{\Lambda^2}$ scales the $\not{CP}$ effect due to $q$ & $W$ respectively.
- Many more terms can create $\not{CP}$ , including others that involve electro-weak interactions or fermionic operators.

# Action Modification and the Small $\theta$ Expansion.

[Shindler:2014oha]

- Including a $\not{CP}$ term in the Lagrangian, the action becomes:

$$S_\theta = S_{QCD} - i\theta Q_t \quad , \quad S_{QCD} = \int d^4x \; \mathcal{L}_{QCD}(x)$$

- Since we have small $\theta$, any expectation value can be expanded:

$$\begin{aligned}\langle O \rangle_\theta &= \frac{1}{Z}\int DU_\mu det[M] O e^{-(S_{QCD} - i\theta Q_t)} \\ &= \frac{1}{Z}\int DU_\mu det[M] O e^{-S_{QCD}}[1 + i\theta Q_t + \mathcal{O}(\theta^2)] \\ &= \langle O \rangle_{\theta=0} + i\theta \langle O Q_t \rangle_{\theta=0} + \mathcal{O}(\theta^2)\end{aligned}$$

- The $\theta$ term can be substituted for any other $\not{CP}$ term.

# Attempting to Extract the EDM from Lattice QCD

- The EDM of P/N is related to the CP-odd Form Factor:

$$\frac{F_3^{P/N}(Q^2)}{2M_N} \xrightarrow{small\ Q^2} d_{P/N} + S_{P/N} Q^2 + \mathcal{O}(Q^4) \quad *$$

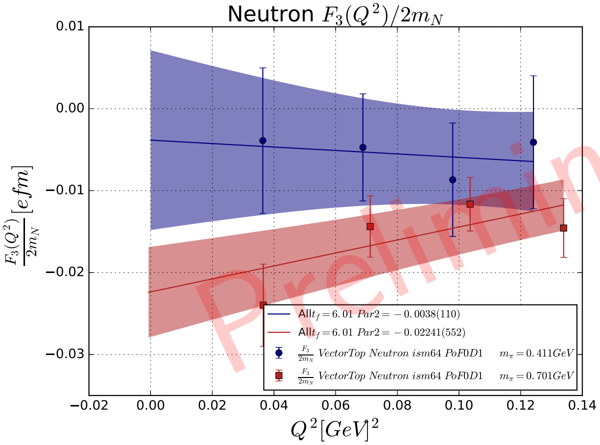
- $F_3(Q^2)$ is contained in the combination of $G_3$ and $Q_t$.

$$G_3^{Q_t}(\Gamma; \vec{p}', t; \vec{q}, \tau; \mathcal{J}_\mu) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}'\cdot\vec{x}} e^{i\vec{q}\cdot\vec{y}} \mathrm{Tr}\left\{\Gamma \left\langle \chi(\vec{x}, t) \mathcal{J}_\mu(\vec{y}, \tau) \overline{\chi}(0) Q_t(t_f) \right\rangle\right\}$$

- But $G_3^{Q_t}(\Gamma; \vec{p}', t; \vec{q}, \tau; \mathcal{J}_\mu) = 0$ for all cases when $Q^2 = 0$
- To fix this, we fit the resulting $F_3$ at $Q^2 > 0$ using the form above *.

[**Berruto:2004cr**; **Shintani:2005xg**; **Shindler:2015aqa**]

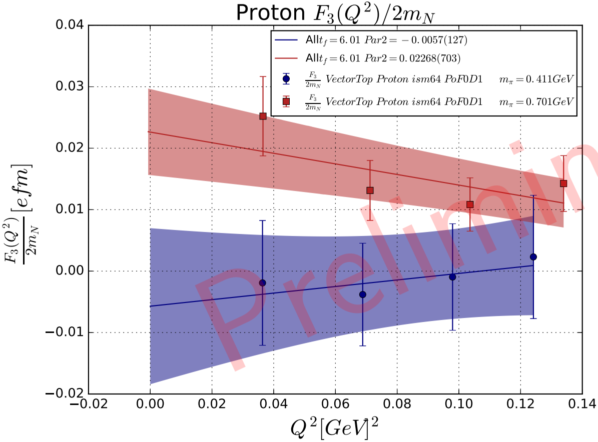
# CP-odd Neutron Form Factor $F_3(Q^2)$ for $\theta$ term




- At $m_\pi = 411$ MeV
- $\left|d_N^{(\theta)}\right| < 0.015\ \theta\ fm$

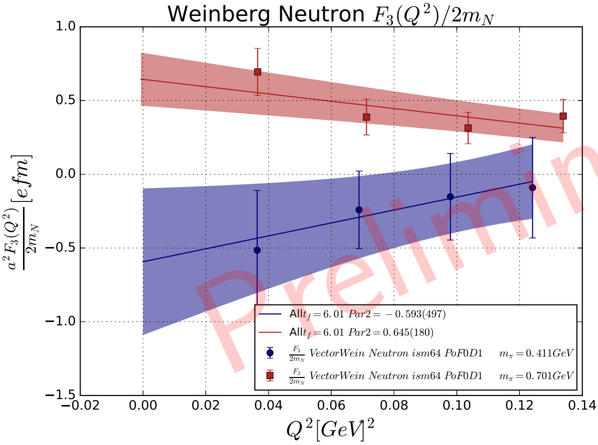
- [Ottnad:2009jw]
- $\chi PT$ + experiment
- $\theta < 2.5 \times 10^{-10}$.
- $|d_N| < 0.0012\ \theta\ fm$

# CP-odd Proton Form Factor $F_3(Q^2)$ for $\theta$ term



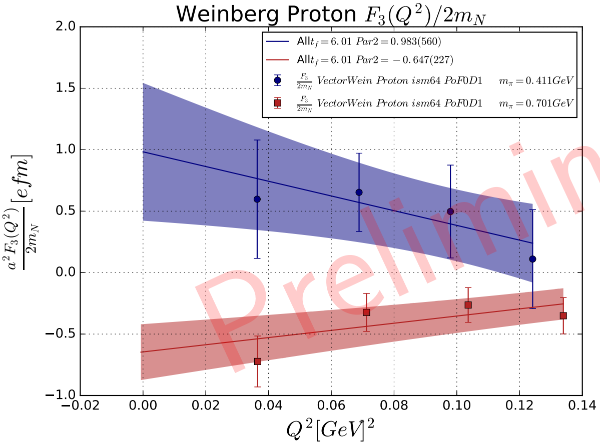

- At $m_\pi = 411$ MeV
- $\left|d_P^{(\theta)}\right| < 0.02\ \theta\ fm$

# CP-odd Neutron Form Factor $F_3(Q^2)$ for Weinberg term



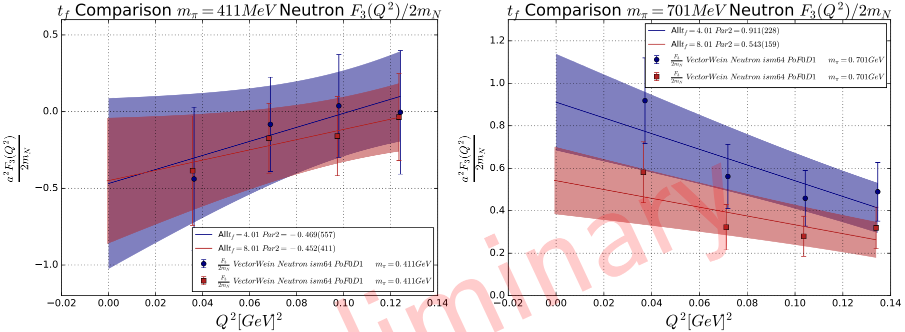

- At $m_\pi = 411$ MeV.
- $\left|d_N^{(W)}\right| < 1.25\ \frac{\alpha_{\widetilde{G}}}{\Lambda^2}\ fm$
- Experimentally: [Olive:2016xmw]
- $|d_n| < 3 \times 10^{-13} efm.$
- Assume $|d_n| \approx \left|d_N^{(W)}\right|$
- $\Lambda \gtrsim 403\ \sqrt{\alpha_{\widetilde{G}}}$ TeV

# CP-odd Proton Form Factor $F_3(Q^2)$ for Weinberg term




- At $m_\pi = 411$ MeV.
- $\left| d_N^{(W)} \right| < 1.6 \frac{\alpha_{\tilde{G}}}{\Lambda^2} \; fm$

# Flow time dependence of Weinberg nEDM



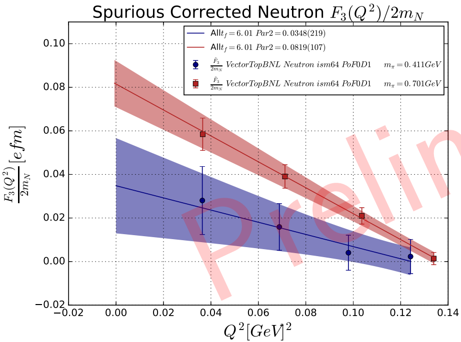

- $t_f \in [4, 8]$ corresponds to $\sqrt{8t_f} \in [0.4, 0.8] fm$
- In this range, we found no discritization effects.
- $m_\pi = 701 MeV$ results need to consider this flow time depenance.

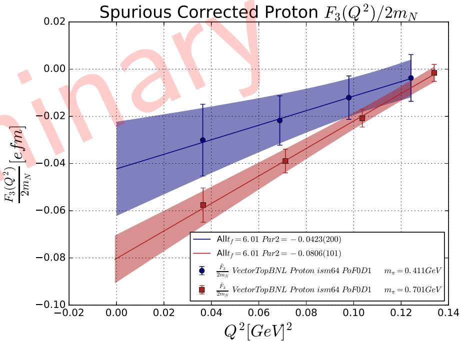
# Spurious Contribution Corrections for $\theta$ Term

**[Abramczyk:2017oxr]**

- We employ the correction suggested by the previous talk.
- Their notation denotes the lattice results $F_{2,3} \to \tilde{F}_{2,3}$
- The corrected result is now:

$$F_3 = sin(2\alpha)\tilde{F}_2 + cos(2\alpha)\tilde{F}_3 \qquad F_2 = cos(2\alpha)\tilde{F}_2 - sin(2\alpha)\tilde{F}_3$$

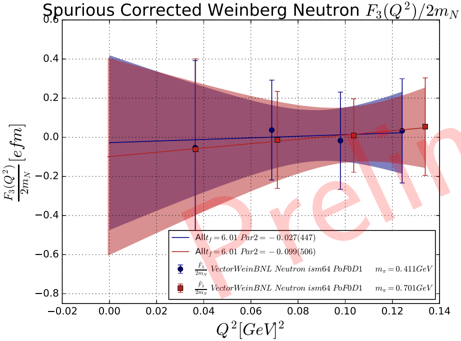


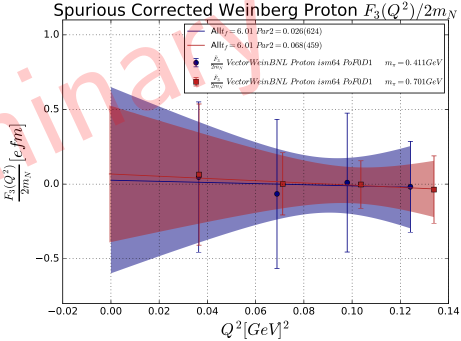



# Spurious Contribution Corrections for Weinberg Term

**[Abramczyk:2017oxr]**

- The same correction would be needed for the Weinberg contribution as well.



# The "*To Do*" List

- Short term:
  - Boost statistics to see if $F_3$ is non-zero.
  - Calculate another pion mass to constrain chiral extrapolation.
  - More consideration of the systematics associated with the analysis.
- Future work:
  - Calculate different lattice spacings to understand $a \to 0$.
  - Calculate different box sizes to understand $L \to \infty$.
  - Incorporate disconnected quark loop contributions to $\langle N \mathcal{J}_\mu N Q_t \rangle$.

# Summary

- Preliminary extraction of the $\theta$ contribution to nEDM shows

$$\left| d_N^{(\theta)} \right| < 0.015 \ \theta \ fm.$$

- with spurious contribution corrections:

$$\left| d_N^{(\theta)} \right| < 0.06 \ \theta \ fm.$$

- Still statistically consistant with $d_N = 0$.

- Preliminary extraction of the Weinberg contribution to nEDM shows:

$$\left| d_N^{(W)} \right| < 1.25 \ \frac{\alpha_{\widetilde{G}}}{\Lambda^2} \ fm.$$

- with spurious contribution corrections:

$$\left| d_N^{(W)} \right| < 0.5 \ \frac{\alpha_{\widetilde{G}}}{\Lambda^2} \ fm.$$

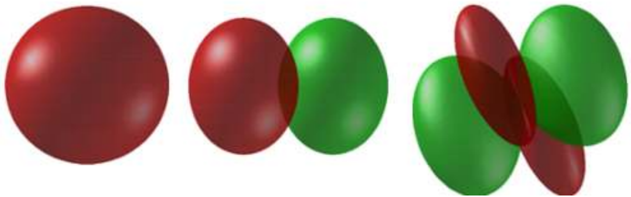
- This might give us insight into the scale of new physics $\Lambda$.

# Acknowledgements


- This work was supported in part by Michigan State University through computational resources, greatly aided by the new Laconia cluster, provided by the Institute for Cyber-Enabled Research.
- Jülich Supercomputing Centre. (2015). JUQUEEN: IBM Blue Gene/Q Supercomputer System at the Jlich Supercomputing Centre. Journal of large-scale research facilities, 1, A1. http://dx.doi.org/10.17815/jlsrf-1-18

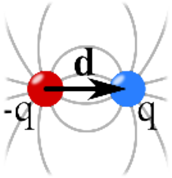
# The Electric Dipole Moment

- Can be visualized as the second order term of a multi-pole expansion.



[N. Butakov 2013]

- Question: does the Neutron (or Proton) have a dipole moment?
- Experimentally, the nEDM has a bound : $|d_n| < 3.0 \times 10^{-13}$ e fm **[Olive:2016xmw]**.



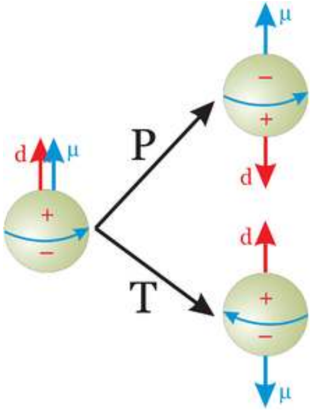

- If the Proton charge radius was 1 AU,
- $|d_n| < 5.1$ cm.

[Photonics101]

# EDM in Relation to CP-Violation

- C,P,T refer to charge, parity and time reversal operators respectively.

- CPT is conserved in nature.
- What about CP and T individually?
- An EDM is an example of a system that violates CP symmetry.
- Or: EDMs provides evidence for CP violation.

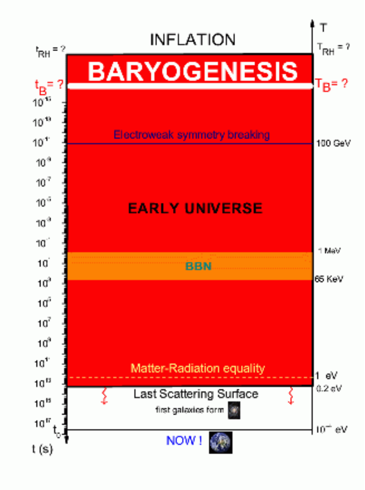



[A. Knecht]

# Why nEDM and CP-Violation?: Baryogenesis

- Sakharov Criteria is required to explain the matter/anti-matter asymmetry:
  - Baryon number violation
  - CP & C violation
  - Departure from Thermal Equilibrium

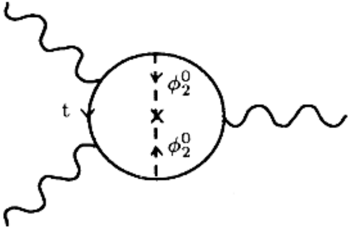
[www.memim.com]




- CP violation ensures that the rate of $B$-production $>$ rate of $\bar{B}$-production

# Why nEDM and CP-Violation?: Beyond SM

- SM terms that break CP are:
  - The dimension 4 gluonic operator referred to as *the $\theta$ term*.
  - Electroweak CP violation which indicates CKM $\neq$ CKM*
- nEDM is dominated by the strong force **[Dar:2000tn]**.
- BSM has modifications which involve dimension 5 and 6 operators that break CP.




**[PhysRevLett.63.2333]**

For example, one of the Higgs doublet $\phi_2^0$ could couple to the top quark.

- Integrated out, this gives a 3-gluon "*Weinberg vertex*"

# Attempting to Extract the EDM from Lattice QCD

- The EDM of P/N is related to the CP-odd Form Factor:

$$\frac{F_3^{P/N}(Q^2)}{2M_N} \xrightarrow{small\ Q^2} d_{P/N} + S_{P/N}Q^2 + \mathcal{O}(Q^4)$$

- The Schiff moment $S_{P/N}$ and the EDM $d_{P/N}$ are analogous to the heavy nuclei $d_A$ & $S_A$:

$$d_A = \mathcal{A}_A S_A \quad , \quad \text{For some heavy nucleus A}$$

- Where the Schiff moment can be related back to the P/N EDM through:

$$S_A = (a_0\bar{g}_0 + a_1\bar{g}_1) + (\alpha_N d_N + \alpha_P d_P)$$

[Chien:2015xha]

- $F_3(Q^2)$ is contained in the combination of $G_3$ and $Q_t$.

$$G_3^{Q_t}(\Gamma;\vec{p}\,',t;\vec{q},\tau;\mathcal{J}_\mu) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}\,'\cdot\vec{x}} e^{i\vec{q}\cdot\vec{y}} \mathrm{Tr}\left\{\Gamma\langle\chi(\vec{x},t)\mathcal{J}_\mu(\vec{y},\tau)\overline{\chi}(0)Q_t(t_f)\rangle\right\}$$

[Berruto:2004cr; Shintani:2005xg; Shindler:2015aqa]

# The Lattice QCD Approach

1. Lattice QCD looks at the Feynman path integral.

2. We attempt to discretized space-time and derive a discrete action $S$:

$$S \equiv \int d^4x \ \mathcal{L}(x) \quad , \quad S = S_G + S_F = S_G + \sum_q \overline{\psi}_q M_q \psi_q$$

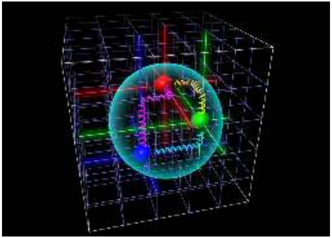
3. As in statistical mechanics, the expectation value of some observable $\langle O \rangle$ can be written down as:

$$\begin{aligned}\langle O \rangle &= \frac{1}{Z} \int DU_\mu D\psi_q D\overline{\psi}_q O e^{-(S_G + S_F)} \\ &= \frac{1}{Z} \int DU_\mu \prod_q det[M_q] O e^{-S_G}.\end{aligned}$$

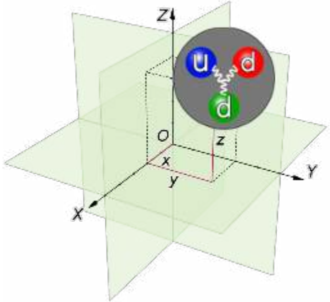
4. The integration over discretized gluonic fields $U_\mu$ can be estimated via Monte Carlo techniques.

# Key Systematics and Difficulties in Lattice QCD

| | |
|---|---|
| Continuum limit | $a \to 0$ |
| Infinite volume limit | $L_{x,y,z,t} \to \infty$ |
| Chiral limit | $(m_\pi)_{Lat} \to (m_\pi)_{Phys}$ |
| State isolation | $\sum E_{\vec{p}} \to E^{(n)}_{\vec{p}}$ (for state $n$) |
| Signal to noise | $\Delta O(t) \xrightarrow{t\ large} \infty$ |



Continuum Limit




[AND You INC.]

[Wiki]

# Extracting the EDM via the CP-odd Form Factor $F_3$

- To extract the $\theta$ contribution to the Neutron (and Proton) EMD from Lattice QCD, we require:
  - The Topological Charge $Q_t$, which we use to construct:
    - Mixing angle $\alpha_N$,
    - Modified three-point correlator $G_3^Q$,
  - CP even form factors $F_1(Q^2)$ & $F_2(Q^2)$,
  - Nucleon lattice mass $m_N$.

- For the Weinberg operator contribution to the EDM, we repeat the process, replacing $\theta$ with the Weinberg operator.

# Gradient Flow

[Lüscher, 2010-2013]

- Create new variable *flow time* $t_f$ (of dimension 2) for the gauge fields to be extended too:

$$A_\mu(x) \to B_\mu(x, t_f) \qquad \text{with} \qquad B_\mu(x, 0) = A_\mu(x)$$

- Solving the differential equation gives the *flowed* gauge field $B_{\mu t_f}(x)$

$$\partial_{t_f} B_\mu = D_{\nu,t_f} G_{\mu\nu,t_f},$$

- with the definitions

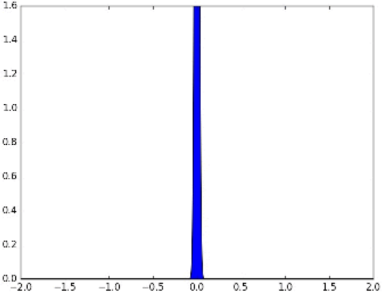
$$D_{\nu,t_f} \equiv \partial_\nu + [B_\nu, \ \cdot \ ] \quad , \quad G_{\mu\nu,t_f} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu + \left[B_\mu, B_\nu\right].$$

- Utilising the expansion of $B_\mu$ in powers of the bare coupling, an integral solution can be found and computed.

# Gradient Flow

[Lüscher, 2010-2013]

- Flowing over $t_f$ has an effect of "*smoothing*" over 4-D space-time of a radius $\sqrt{8t_f}$.




- Lattice QCD calculations are simplified greatly, as gauge fields for $t_f > 0$ requires no renormalization. [Lüscher and Weisz, 2011]

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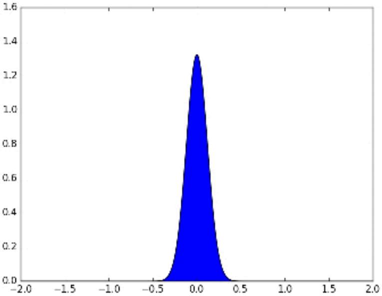


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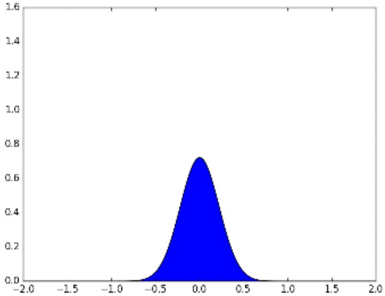


- Lattice QCD calculations are simplified greatly, as gauge fields for $t_f > 0$ requires no renormalization. [Lüscher and Weisz, 2011]

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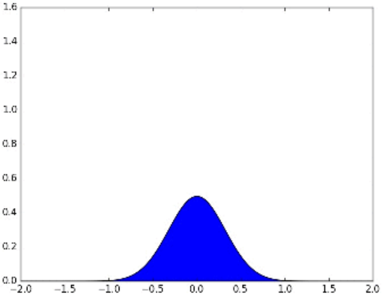


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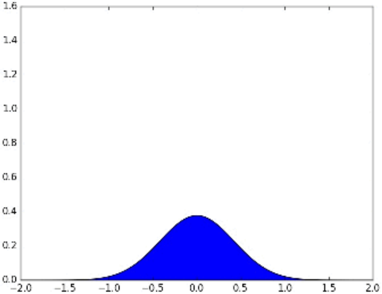


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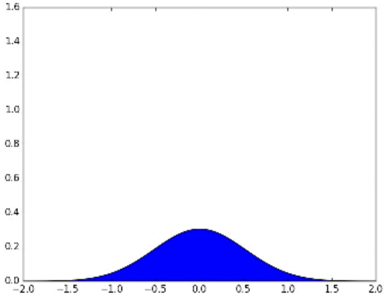


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# Nucleon Spinor Mixing Angle from Two-Point Correlators

[Berruto:2004cr; Shintani:2005xg]

- Mixing angle $\alpha_N$ relates the nucleon spinors in a $\theta$ vacuum to the regular one:

$$u_N^{\theta}(\vec{p}, s) = e^{i\alpha_N(\theta)\gamma_5} u_N(\vec{p}, s) \quad , \quad \alpha_N(\theta) = \alpha_N^{(1)}\theta + \mathcal{O}(\theta^3)$$

- Combining $G_2$ and $Q_t$ together, $\alpha_N^{(1)}$ can be extracted:

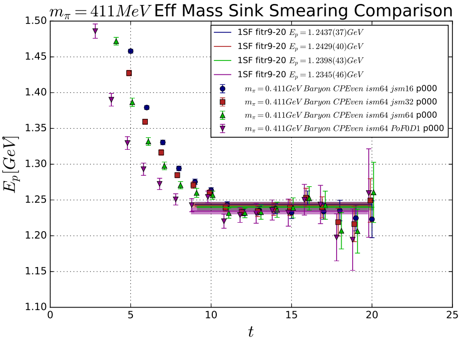
$$G_2^{Q_t}(\Gamma; \vec{p}, t, t_f) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \mathrm{Tr}\left\{\Gamma \left\langle \chi(\vec{x}, t)\overline{\chi}(0) Q_t(t_f) \right\rangle\right\}$$

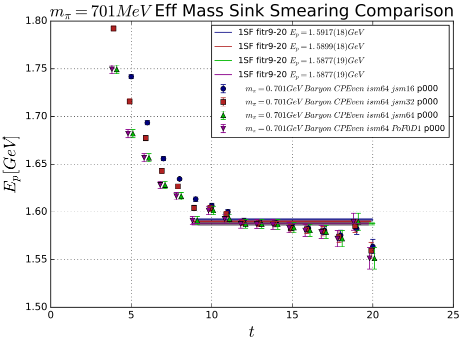
$$\frac{G_2^{Q_t}(\gamma_5\Gamma_4; \vec{0}, t, t_f)}{G_2(\Gamma_4; \vec{0}, t)} \xrightarrow{t \gg 0} \alpha_N^{(1)}$$

- Again, making sure no flow time dependence $t_f$.
- Large $t$ ensures both correlators are *in the ground state*.

[Shindler:2015aqa]

# Effective Mass Sink Smearing Dependence

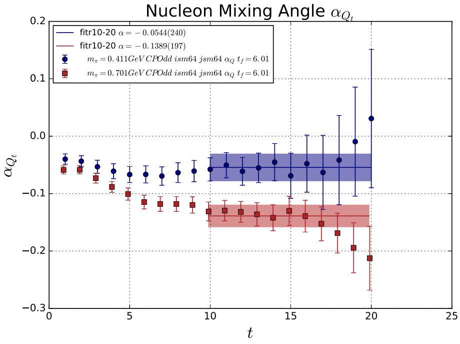


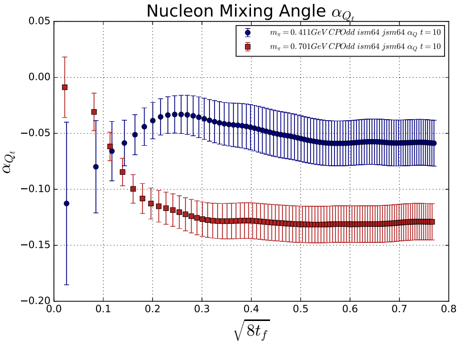




- Minimal variation between largest smearing and variational method.
- Decided to pick 64 sweeps of smearing for the sink for the heaviest pion mass instead of undergoing a variational aproach.

# Mixing Angle for $\theta$ Term

$$\frac{G_2^{Q_t}(\gamma_5\Gamma_4;\vec{0},t,t_f)}{G_2(\Gamma_4;\vec{0},t)} \xrightarrow{t\gg 0} \alpha_N^{(1)}$$

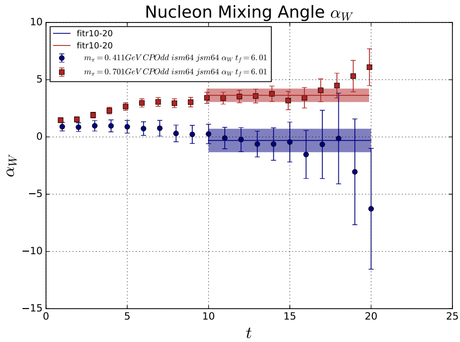


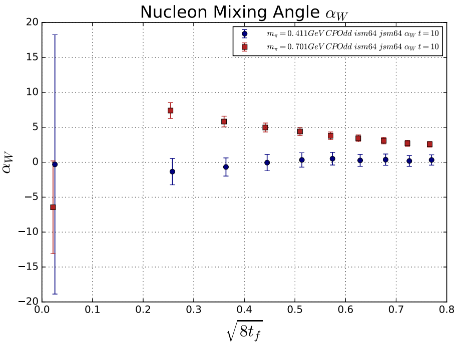




- Note: in the chiral limit, $\alpha_N \to 0$

# Mixing Angle for Weinberg Operator Term

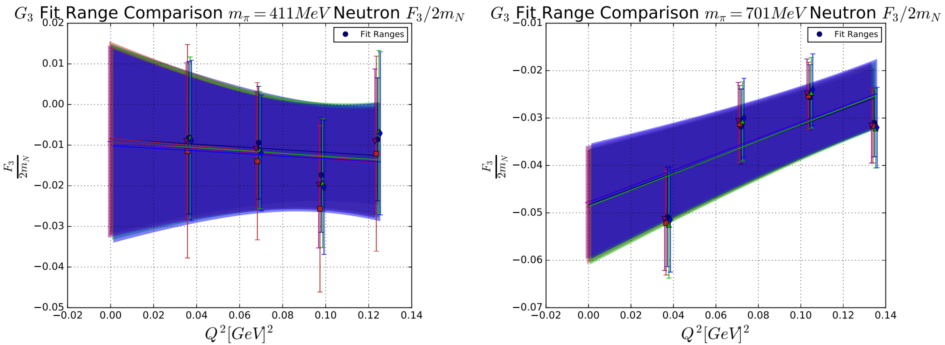
$$\frac{G_2^W(\gamma_5\Gamma_4;\vec{0},t,t_f)}{G_2(\Gamma_4;\vec{0},t)} \xrightarrow{t\gg 0} \alpha_N^{(1)}$$




- The flow time dependence from $\langle W^2 \rangle$ is not present in $\alpha_N^{(1)}$

# Fit Range Comparison for $\theta$ Term




- Fit ranges for the Ratio function range from $3 \to 8$ up to $5 \to 9$.