Improved convergence of Complex Langevin simulations

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In collaboration with Felipe Attanasio and Gert Aarts

Complex Langevin

[Parisi, Klauder, 1983]

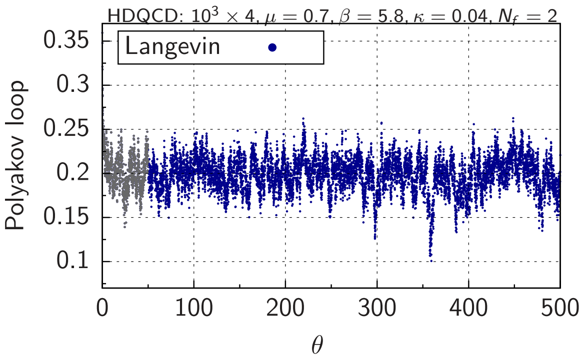
[Aarts, Stamatescu, 2008]

- Complexify degrees of freedom $SU(3) \to SL(3,\mathbb{C})$

$$U_{x,\mu} = \exp\left[\mathrm{i}\, a\, \lambda^c \left(A^c_{x,\mu} + \mathrm{i}\, B^c_{x,\mu}\right)\right]$$

- Evolve links according (1st order) Langevin equation

$$U_{x,\mu}(\theta+\varepsilon) = \exp\left[\mathrm{i}\, \lambda^a \left(-\varepsilon\, D^a_{x,\mu}\, S + \sqrt{\varepsilon}\, \eta^a_{x,\mu}\right)\right] U_{x,\mu}(\theta)$$



Complex Langevin

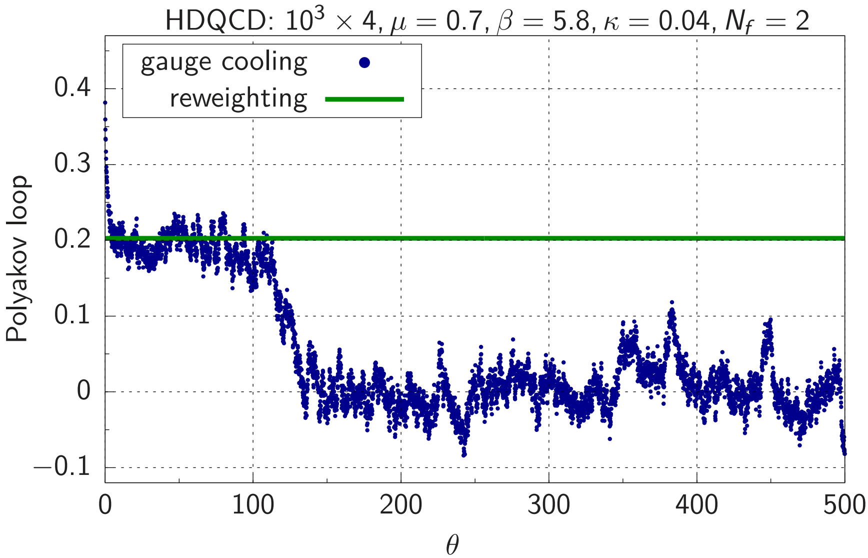
- However, $SL(3,\mathbb{C})$ is not a compact group...
- **Convergence** $\Leftrightarrow$
  - Action S is holomorphic
  - "Imaginary" direction of $SL(3,\mathbb{C})$ falls off quickly enough
- Measure distance to SU(3) manifold

$$d = \mathrm{Tr}\left(U_{x,\mu}\, U^{\dagger}_{x,\mu} - 1\right)^2$$

- Gauge cooling is essential, but sometimes not sufficient...

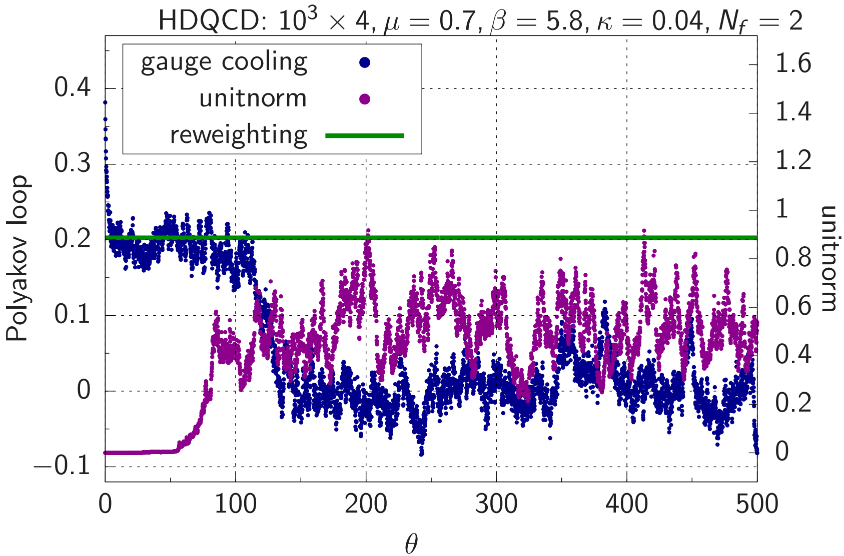
$$U_{x,\mu} \rightarrow \Omega_x\, U_{x,\mu}\, \Omega^{-1}_{x+\mu}$$

Instabilities



- Tunneling to wrong results.

Instabilities



- Tunneling to wrong results, when unitnorm grows too large.

Dynamic stabilization

- Adding a trivial force to the Langevin dynamics

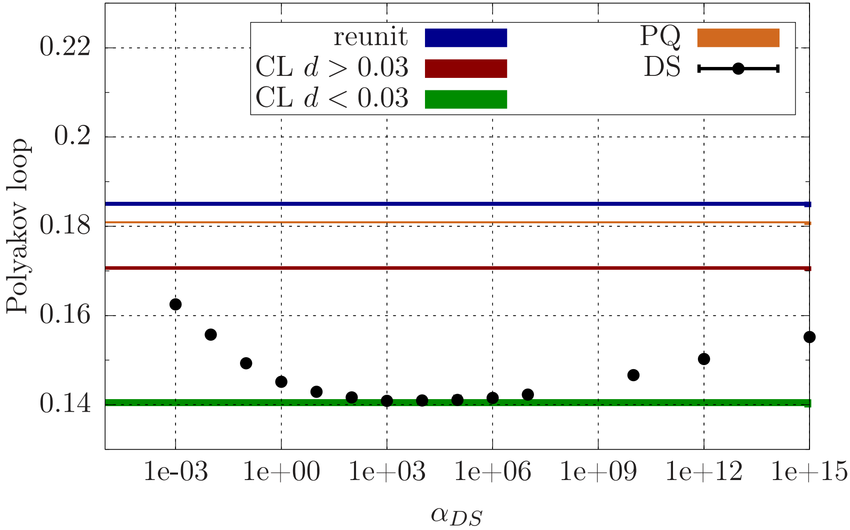
$$U_{x,\nu}(\theta+\varepsilon) = \exp\left[\mathrm{i}\lambda^a \left(\varepsilon K^a_{x,\nu} + \mathrm{i}\,\varepsilon\,\alpha_{DS}\,M^a_x + \sqrt{\varepsilon}\,\eta^a_{x,\nu}\right)\right] U_{x,\nu}(\theta)$$

  where

$$M^a_x = \mathrm{i} b^a_x \left(\sum_c b^c_x\, b^c_x\right)^3 \text{ and } b^a_x = \mathrm{Tr}\left[\lambda^a \sum_\nu U_{x,\nu} U^\dagger_{x,\nu}\right].$$

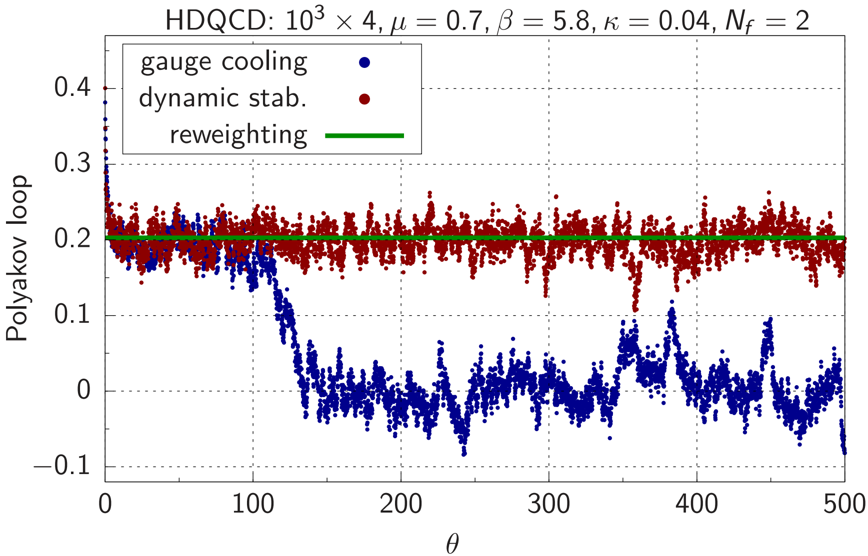
- Design objectives
  - Do not change SU(3) forces
  - Remove larger excursions into non-SU(3) direction
  - Irrelavant in the continuum limit $a \to 0$

Dynamic stabilization



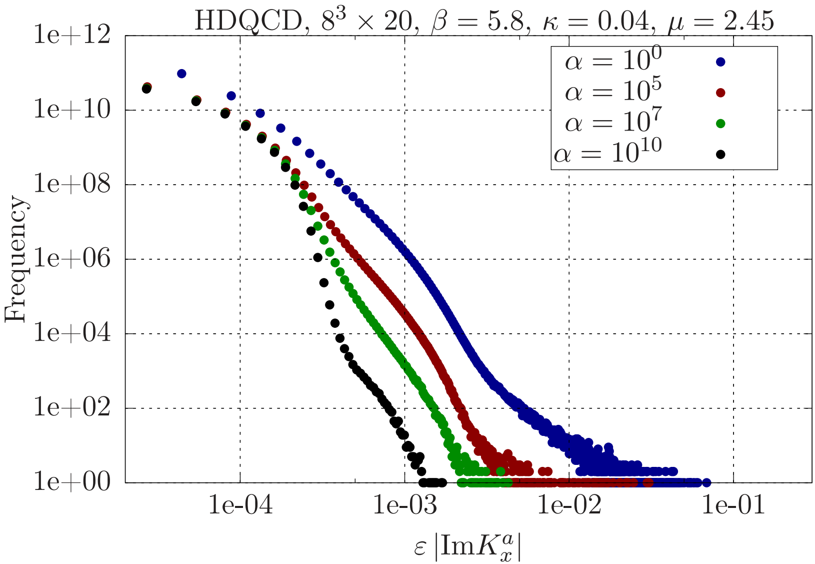
- A plateau shows agreement with results from small norm

Dynamic stabilization



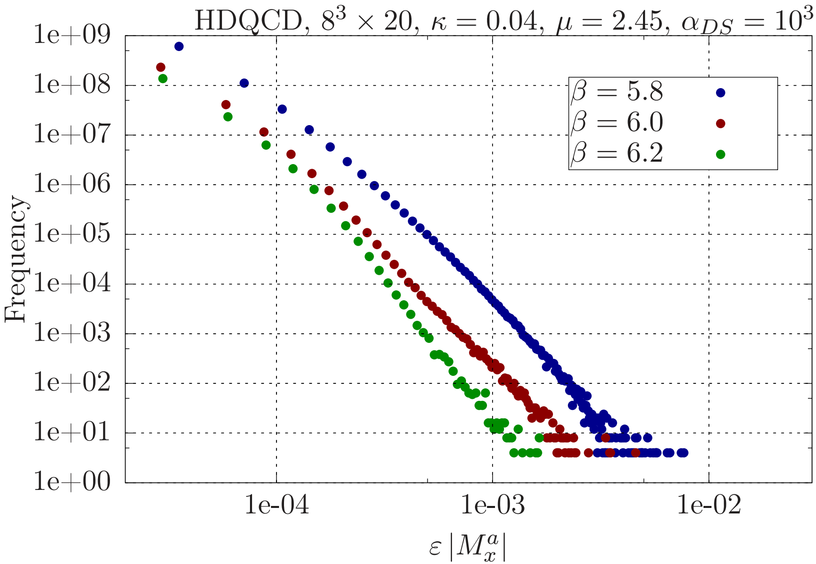
- Improved stability using dynamic stabilization

Dynamic stabilization



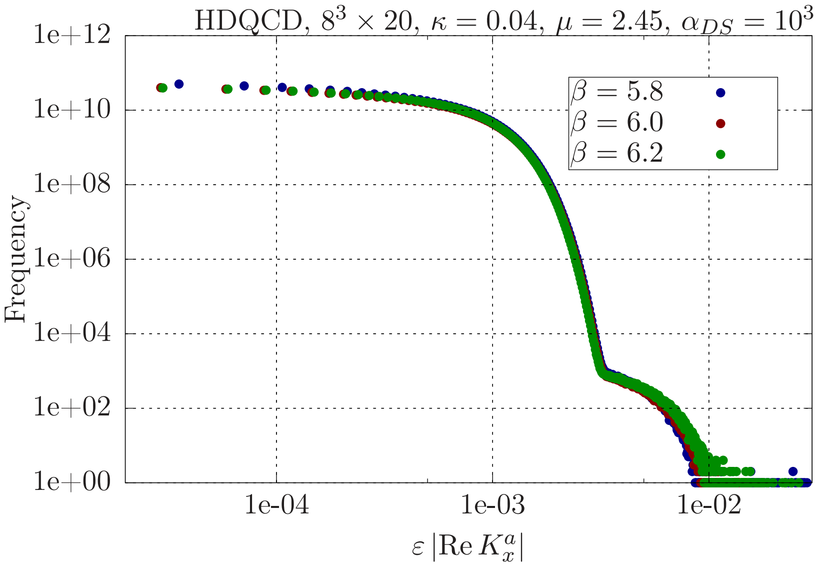
- The forces in the non SU(3) directions become more compact

Dynamic stabilization



- The dynamic stabilization forces shrinks with larger β

Dynamic stabilization



- The original SU(3) forces remains unchanged

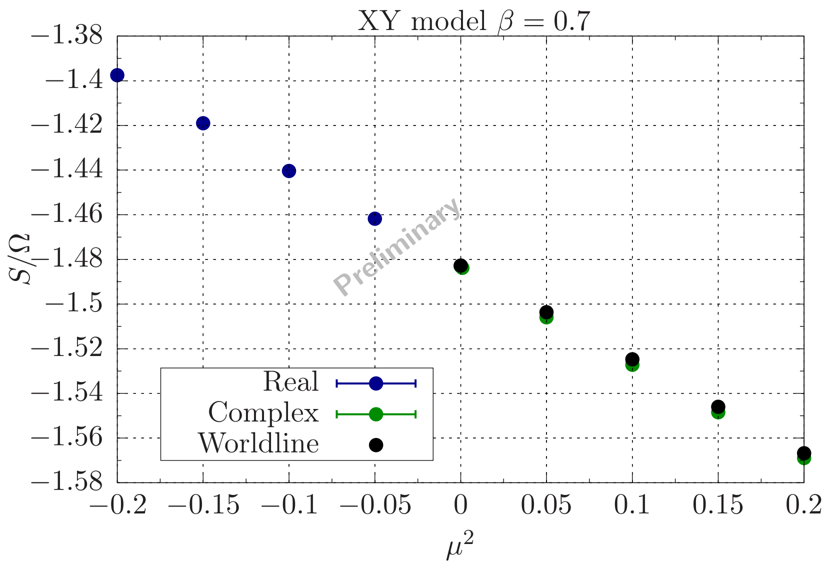
XY model with finite μ

[Aarts, James, 2005]

- Toy model to test Complex Langevin
- Has a phase transition at $\beta_c \sim 0.45$

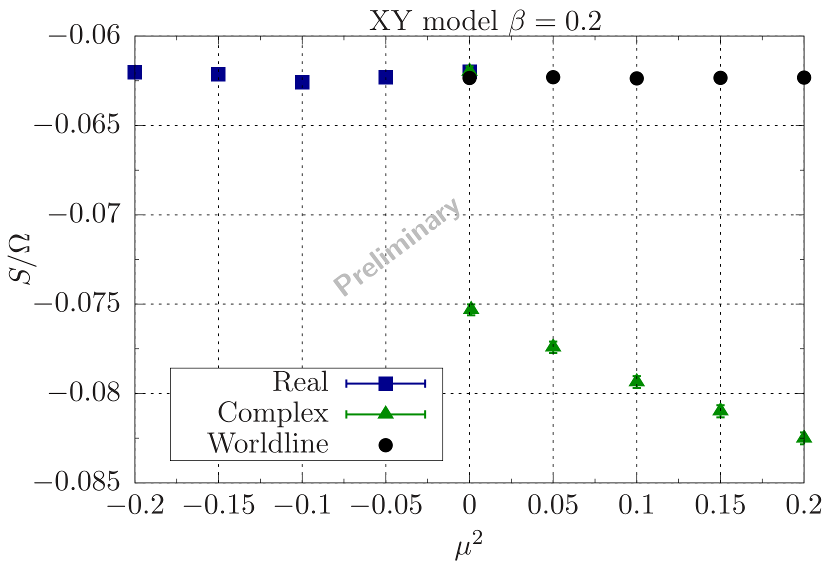
$$S = -\beta \sum_x \sum_{\nu=0}^{2} \cos(\phi_x - \phi_{x+\hat{\nu}} - i\mu\delta_{\nu,0})$$

XY model with finite μ



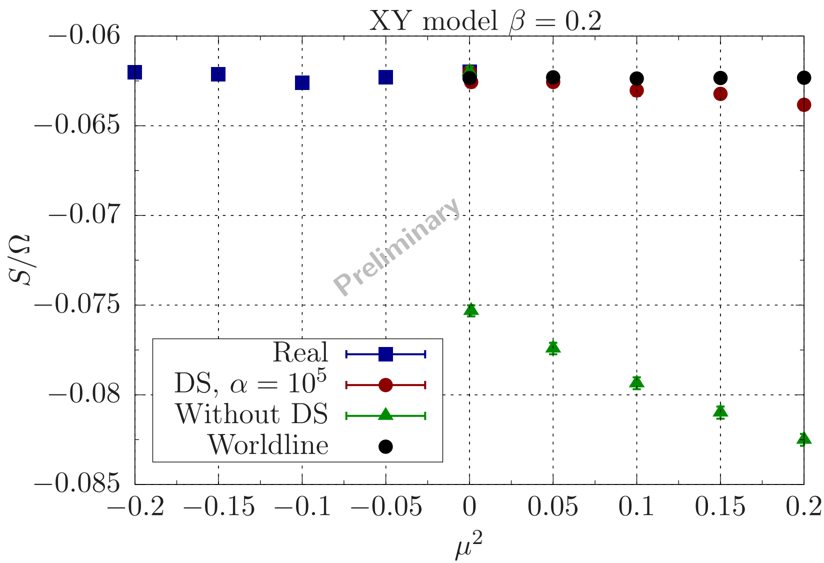
- In the ordered phase Complex Langevin agrees with (dual) Worldline formulation

XY model with finite μ



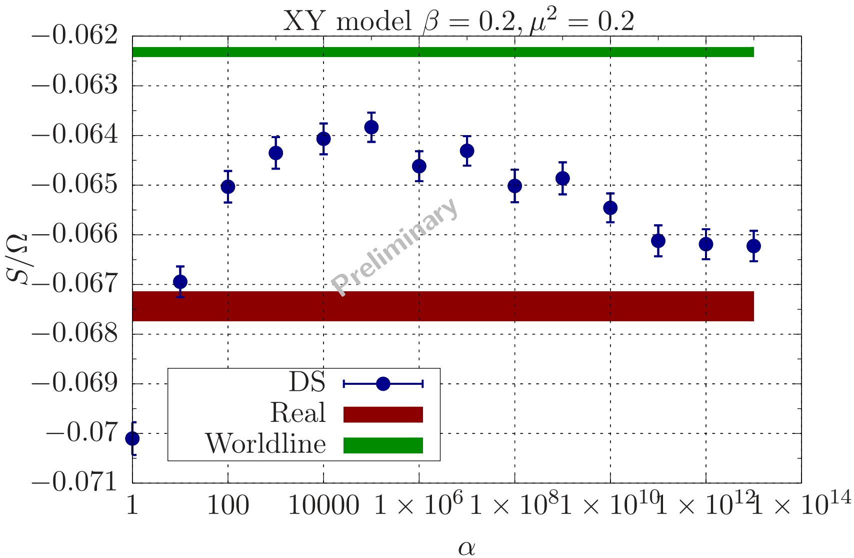
- In the disordered phase Complex Langevin fails spectacularly

XY model with finite μ



- Adding dynamic stabilization

XY model with finite μ



- For large μ^2 a discrepancy remains

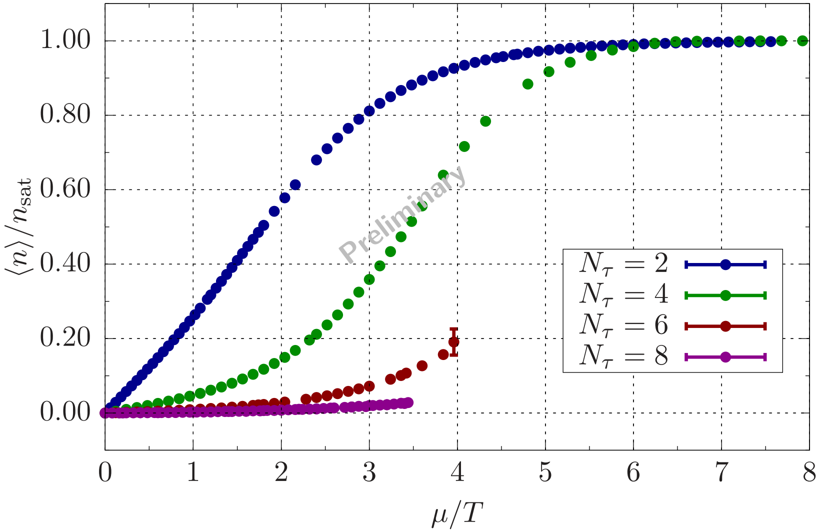
Extension to full QCD

- **Full QCD**
  - Absorb the Fermion determinant in the action and compute force

$$D^a_{x,\mu} S = D^a_{x,\mu} S_{YM} - \frac{N_f}{4} \mathrm{Tr}\left[M^{-1} D^a_{x,\mu} M\right]$$

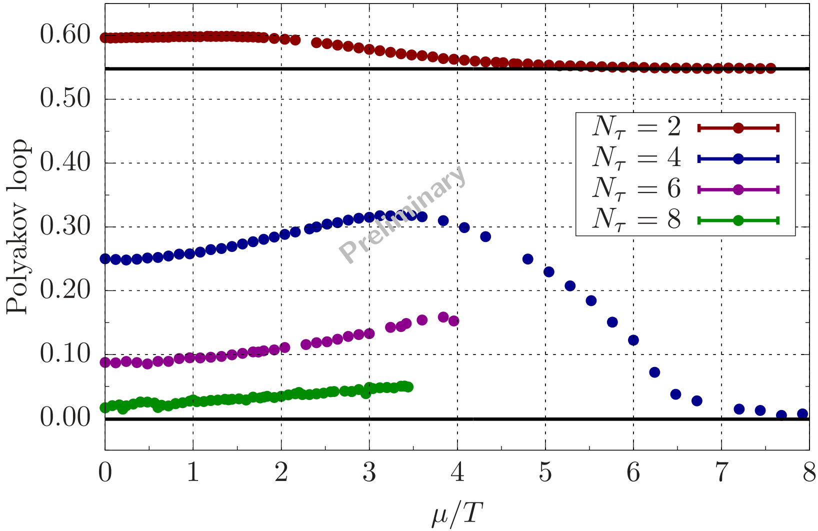
  - Every Langevin update needs inversion of the Dirac operator
    $\Rightarrow$ Expensive!

Simulation of dynamical QCD @ high T



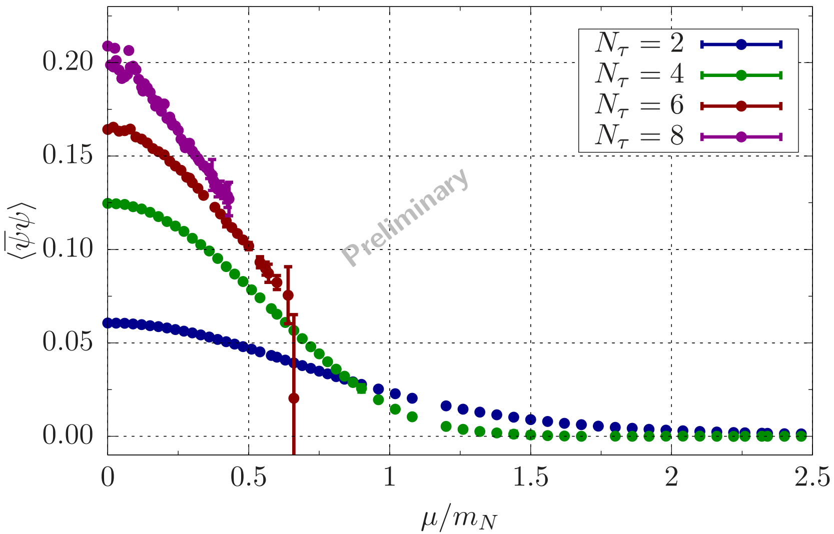
- High temperature region under control, more effort at low T

Simulation of dynamical QCD @ high T



- At large μ the fermion determinant is constant (Lattice artefact)

Simulation of dynamical QCD @ high T

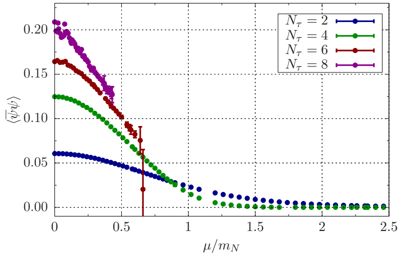


- Chiral condensate @ high T and μ

Future work

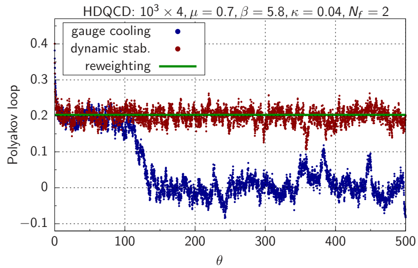
Conclusion

- Complex Langevin simulation can be used to study the QCD phase diagram
- Dynamical stabilisation improves convergence
- Work on the convergence, especially around μ_c.



Future work

- Start proper Full QCD simulations to identify phase structure of QCD.
- A lot of work to be done!



Thank you for your attention!