

QCD Phase Boundary in the Strong Coupling Regime

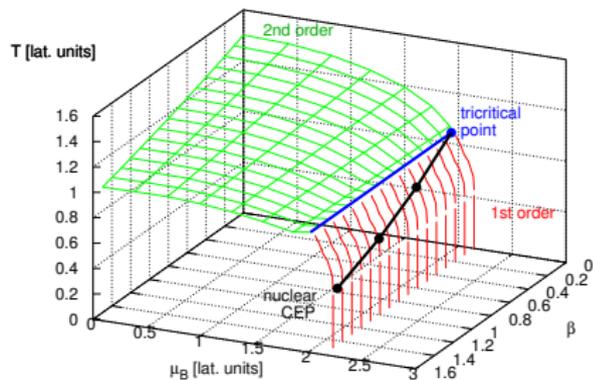
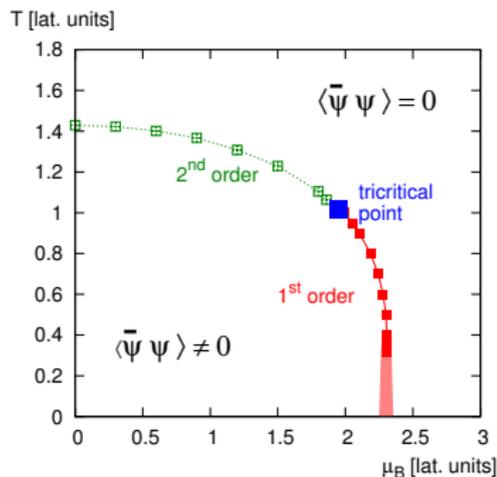
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Lattice 2017
Granada

Introduction

- ▶ The phase boundary of lattice QCD for staggered fermions in the $\mu_B - T$ has been established via a dual representation in the strong coupling limit.
- ▶ Extending this phase boundary towards finite inverse gauge coupling is challenging.



(a) Phase diagram in the strong coupling limit (b) [Langelage, de Forcrand, Philipsen & U., PRL 113(2014)]

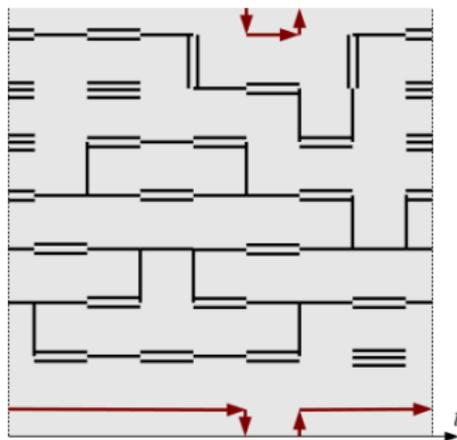
- ▶ We present numerical results from the direct simulations away from the strong coupling limit, taking into account the $O(\beta^2)$ corrections via plaquette occupation numbers using Metropolis update in addition to the worm algorithm.
- ▶ This allows to study the relation between the nuclear and chiral transition as a function of β .

At the Strong Coupling Limit ($\beta = 0$)

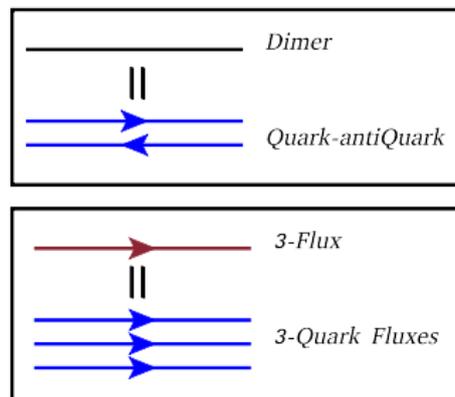
- ▶ The partition function after Grassmann integration,

$$Z_F(m_q, \mu, \gamma) = \sum_{k, n, \ell} \underbrace{\prod_{b=(x, \hat{\mu})} \frac{(N_c - k_b)!}{N_c! k_b!} \gamma^{2k_b \delta_{\hat{0}, \hat{\mu}}}}_{\text{meson hoppings}} \underbrace{\prod_x \frac{N_c!}{n_x!} (2am_q)^{n_x}}_{\text{chiral condensate}} \underbrace{\prod_{\ell} w(\ell, \mu)}_{\text{baryon hoppings}}$$

- ▶ k_b : bond occupation at bond b
- ▶ n_x : monomer numbers at site x
- ▶ ℓ : baryon loop



(c) MDP ensemble in the chiral limit



(d) singlet and triplet fluxes

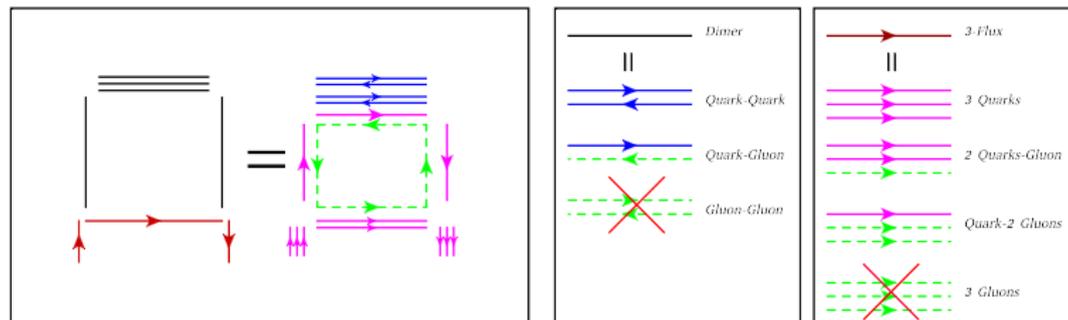
- ▶ If we turn on the β , the gauge flux have to be taken into account.
- ▶ At finite β , weights are modified.

$$\begin{aligned}
 Z(m_q, \mu, \gamma) = & \sum_{\{k, n, \ell, n_p\}} \underbrace{\prod_{b=(x, \mu)} \frac{(N_c - k_b)!}{N_c! (k_b - |f_b|)!} \gamma^{2k_b \delta_{\hat{0}, \hat{\mu}}}}_{\text{singlet hoppings } M_x M_y} \underbrace{\prod_x \frac{N_c!}{n_x!} (2am_q)^{n_x}}_{\text{chiral condensate } \bar{\psi}\psi} \\
 & \times \underbrace{\prod_{\ell_3} w(\ell_3, \mu)}_{\text{triplet hoppings } \bar{B}_x B_y} \underbrace{\prod_{\ell_f} \tilde{w}(\ell_f, \mu)}_{\text{weight modification}} \underbrace{\prod_P \frac{\left(\frac{\beta}{2N_c}\right)^{n_P + \bar{n}_P}}{n_P! \bar{n}_P!}}_{\text{gluon propagation}}
 \end{aligned}$$

- ▶ f_b : the number of gauge fluxes at bond b
- ▶ n_p : plaquette(counterclockwise) occupation number
- ▶ \bar{n}_p : plaquette(clockwise) occupation number
- ▶ $\tilde{w}(\ell_f)$: weight modification at $O(\beta^2)$ → see W.Unger's talk

- ▶ Grassmann constraint is modified

$$n_x + \sum_{\hat{\mu}=\pm\hat{0},\dots,\pm\hat{d}} \left(k_{\hat{\mu}}(x) + \frac{N_c}{2} |\ell_{\hat{\mu}}(x)| \right) = N_c + \sum_{\hat{\nu}=\pm\hat{0},\dots,\pm\hat{d}} \frac{1}{2} |f_{\hat{\nu}}(x)|$$



(e) At the corner of a plaquette, the color constraints is $N_c + 1$

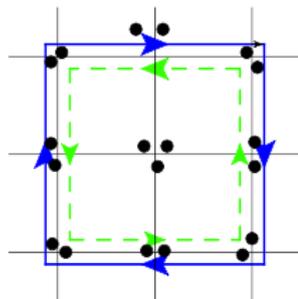
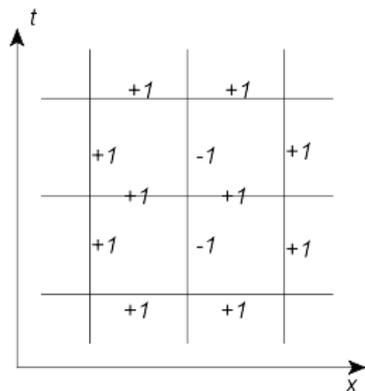
(f) Dimer and 3-Flux can be composed of quark and gluon fluxes

Sign problem of $U(N)$ ensembles at finite β

- ▶ $U(N)$ ensembles in the strong coupling limit do not have sign problem.
- ▶ The sign does not come from the gauge flux but comes only from the quark loop (ℓ_q).

$$\sigma(\ell_q) = (-1)^{w(\ell_q) + N_-(\ell_q) + 1} \prod_{b=(x, \hat{\mu}) \in \ell_q} \eta_{\hat{\mu}}(x)$$

- ▶ At finite β , $U(N)$ ensembles have sign problem.



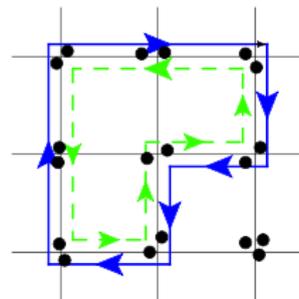
winding=0

negative quark flux=4

of quark loops=1

$\eta(x)=1$

sign=-1



winding=0

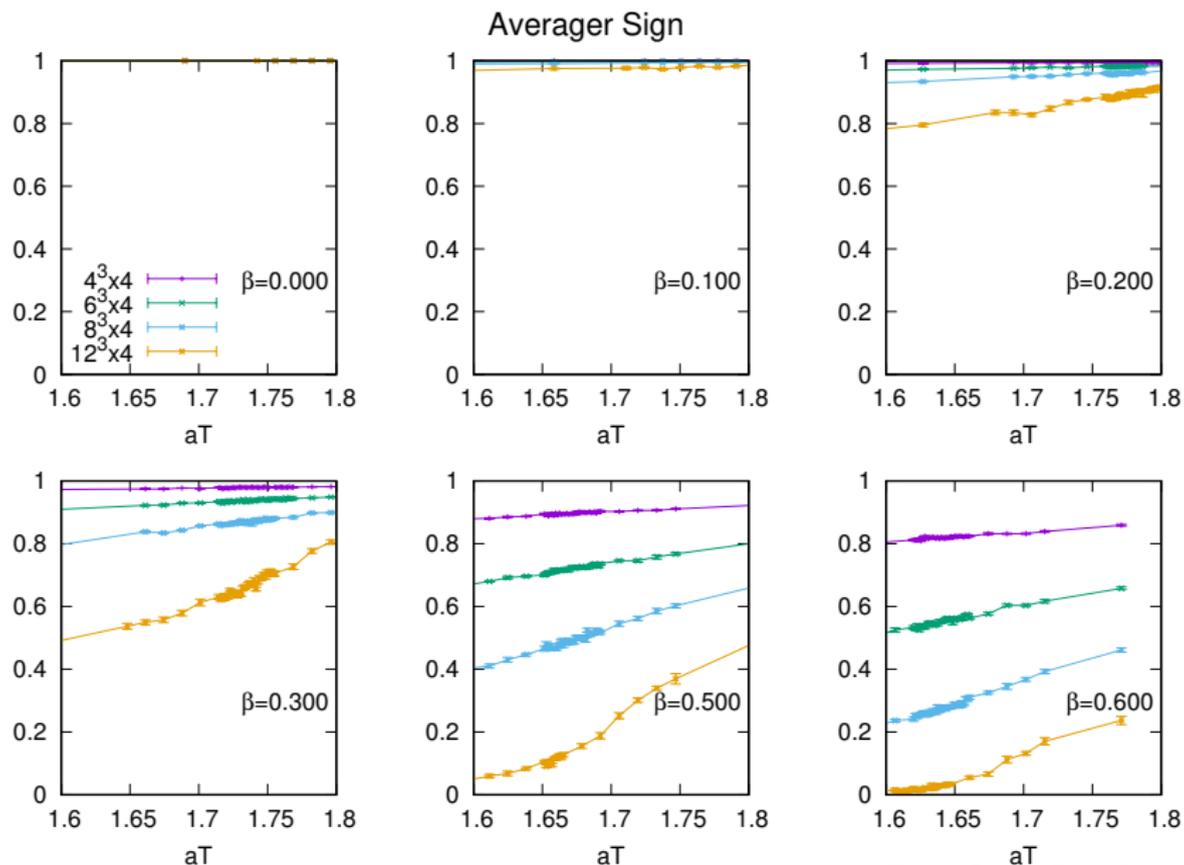
negative quark flux=4

of quark loops=1

$\eta(x)=-1$

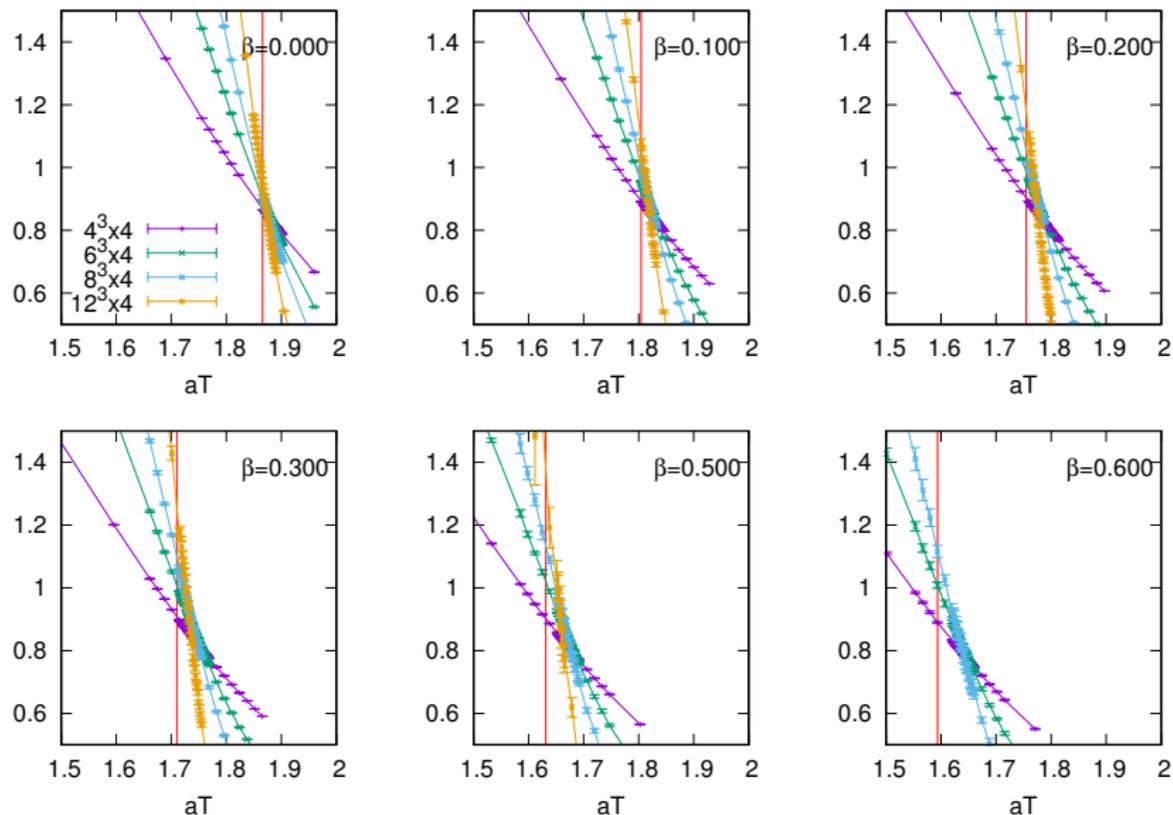
sign=1

Induced sign problem for $U(3)$

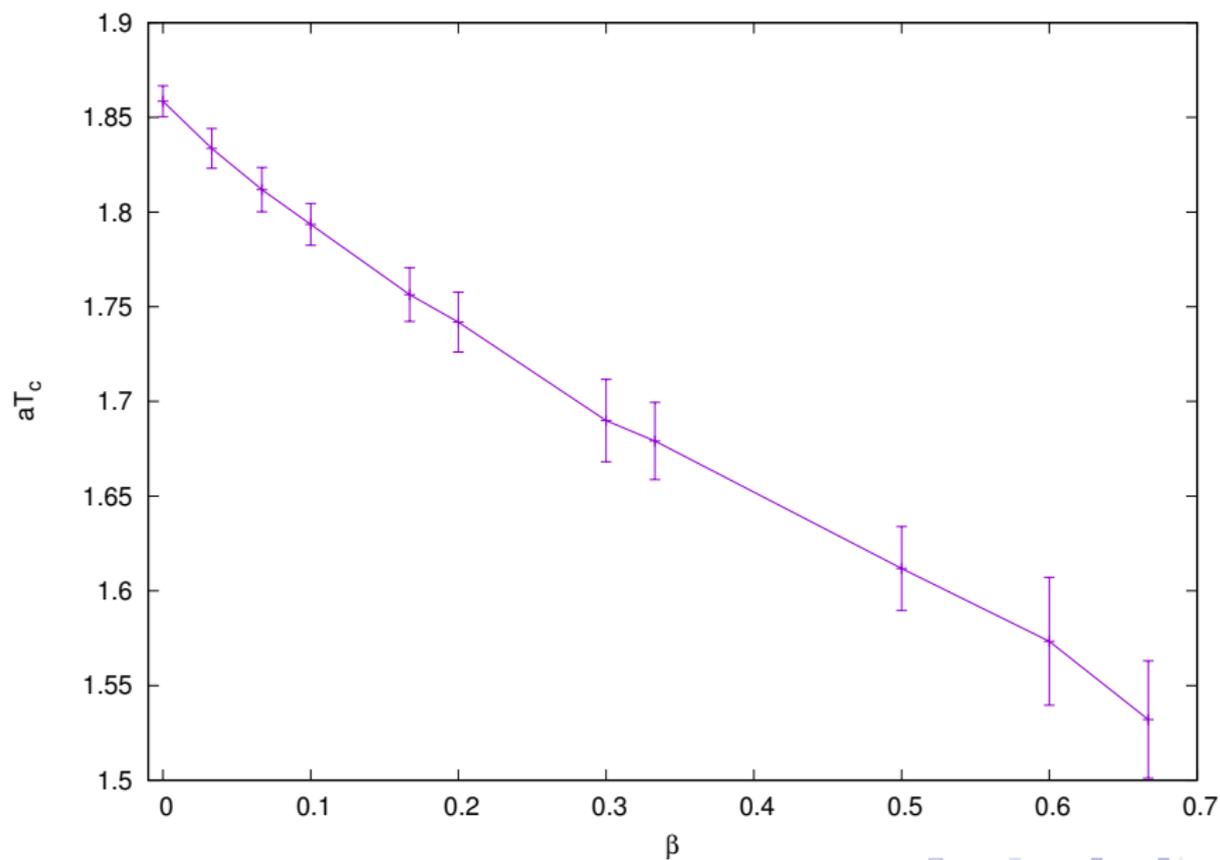


Chiral susceptibility for $U(3)$

Rescaled Chiral Susceptibility

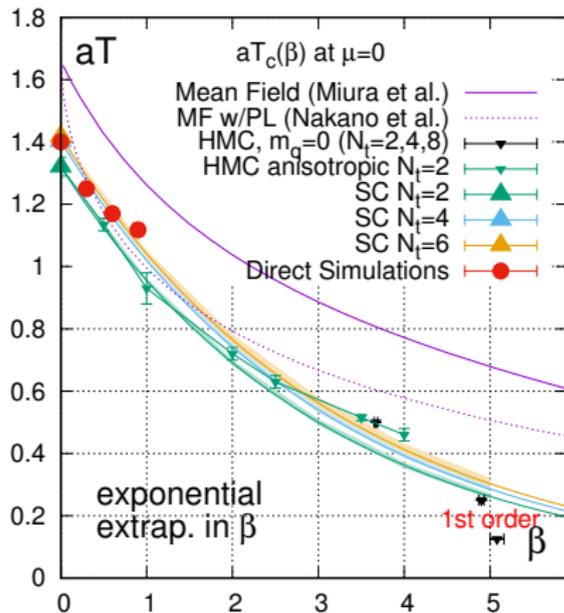
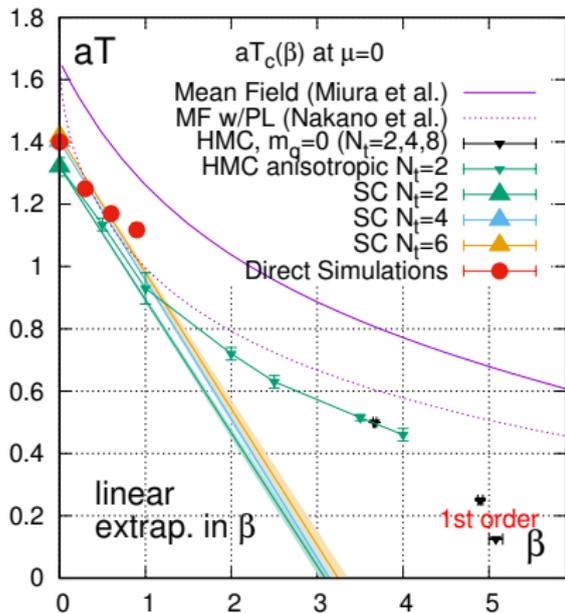


Critical temperature aT_c in β for $U(3)$



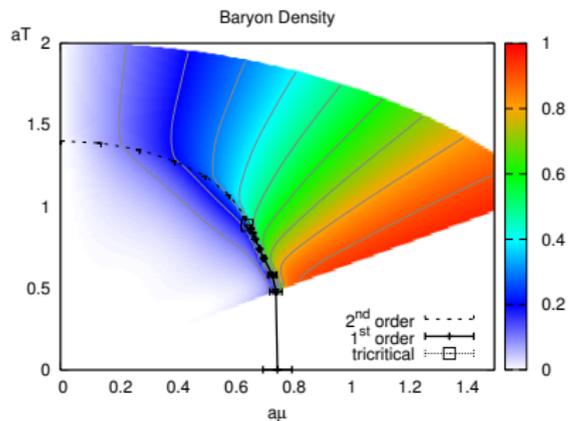
Chiral Transition at finite β and $\mu = 0$

Via **direct sampling** of plaquettes: $\mathcal{O}(\beta^2)$ corrections for SU(3)

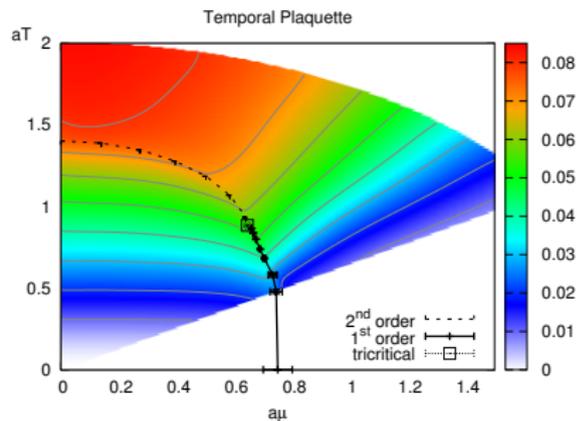


- ▶ Comparison to mean field and reweighting [Nakano, Miura, Ohnishi, PRD 83,2011]
- ▶ Exponential extrapolation slightly favored

Baryon Density and Plaquette at strong coupling

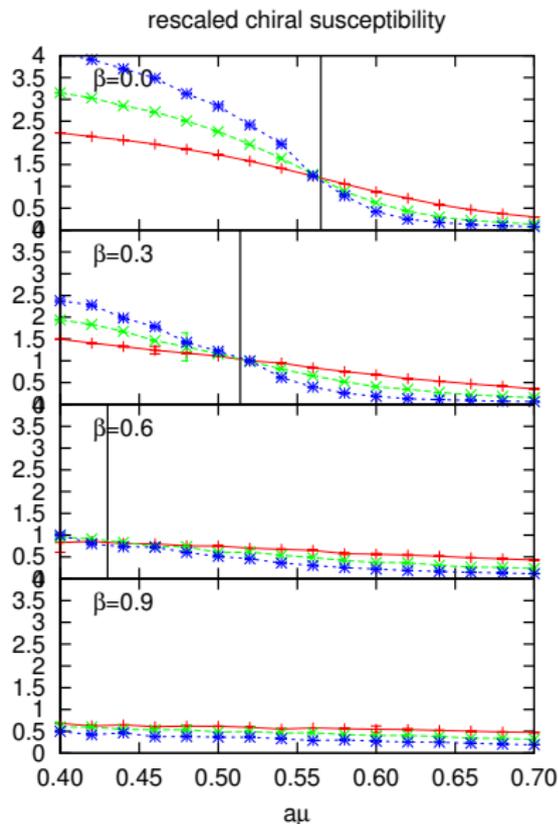
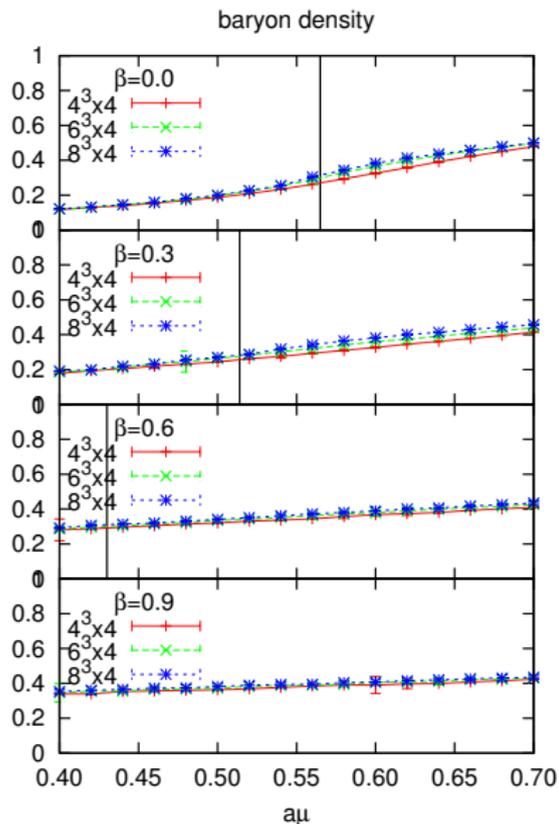


(g) Nuclear transition

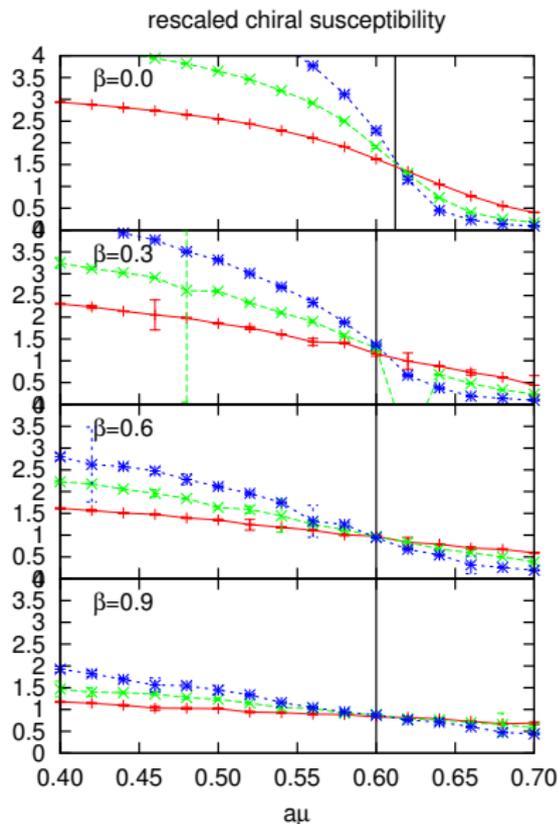
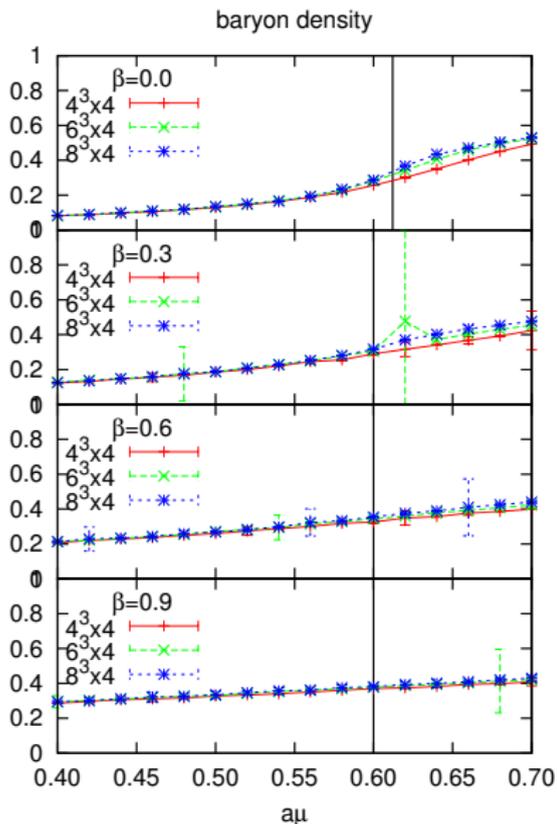


(h)

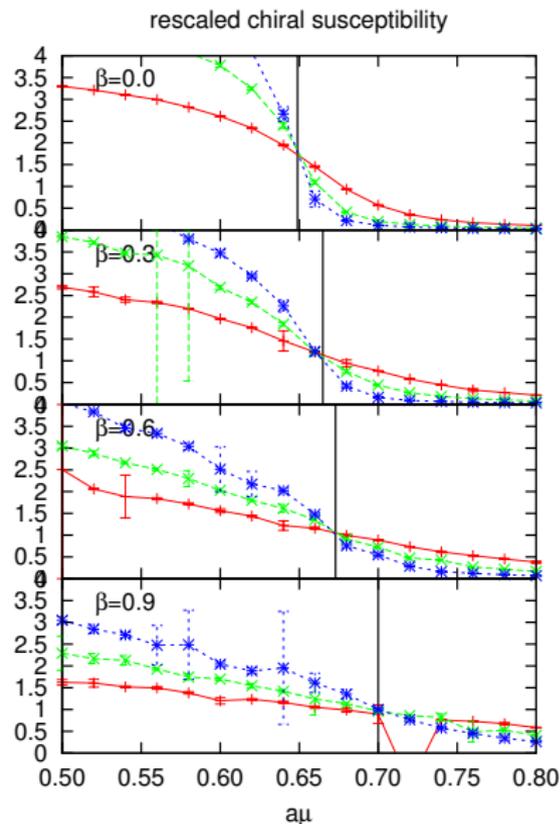
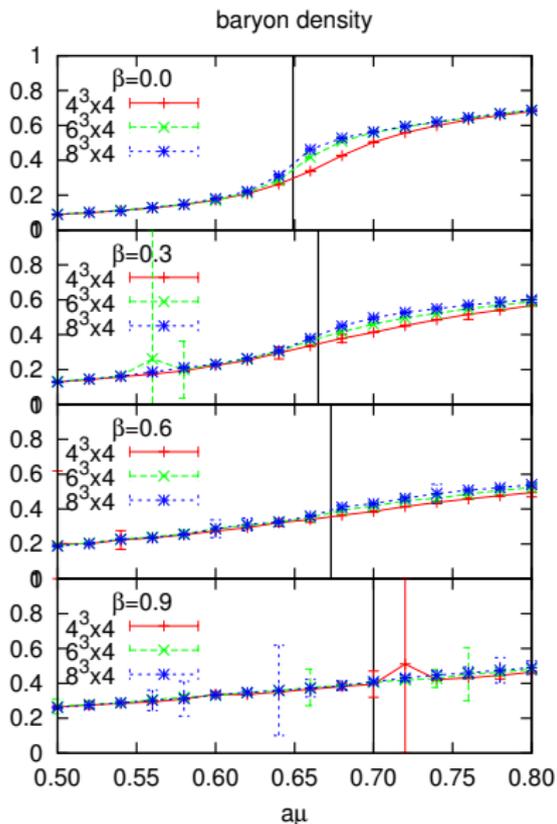
Phase boundary for $SU(3)$ at finite β , finite μ and $aT = 1.1 > aT_{\text{tric}}$



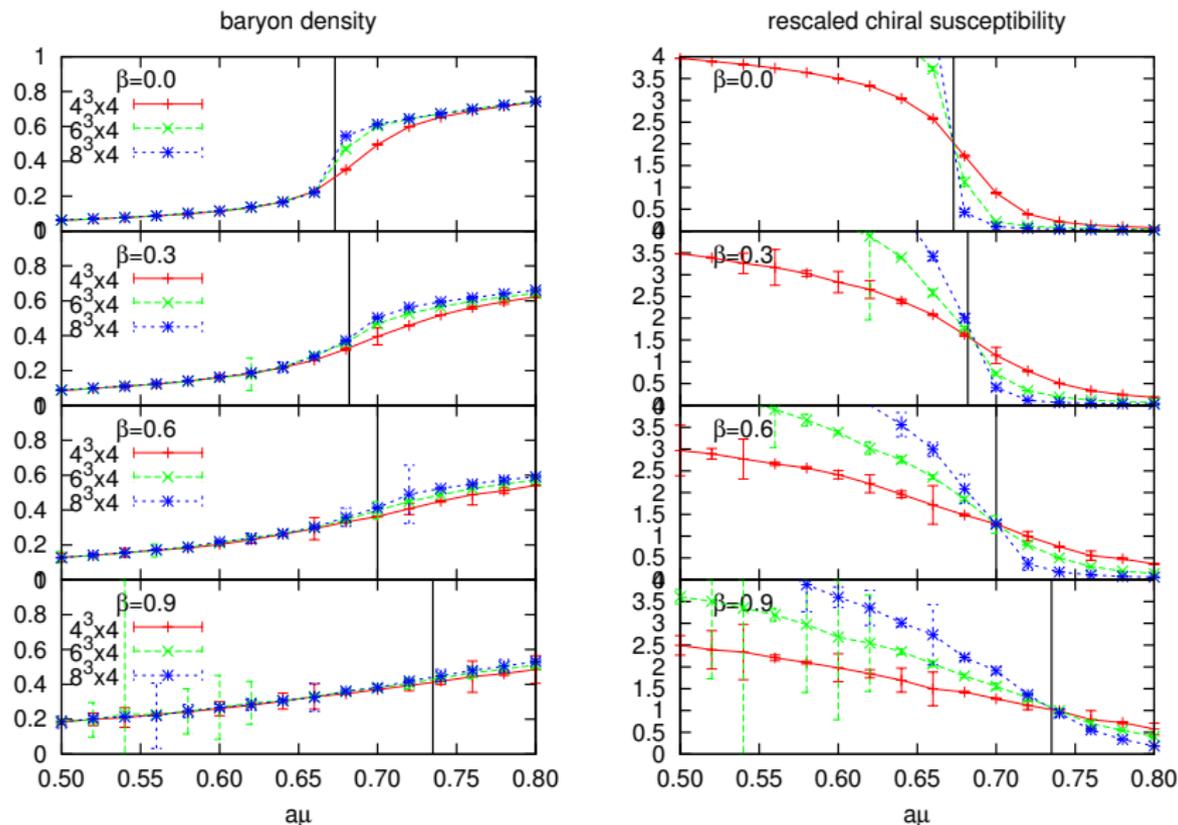
Phase boundary for $SU(3)$ at finite β , finite μ and $aT = 1.0 \sim aT_{\text{tric}}$



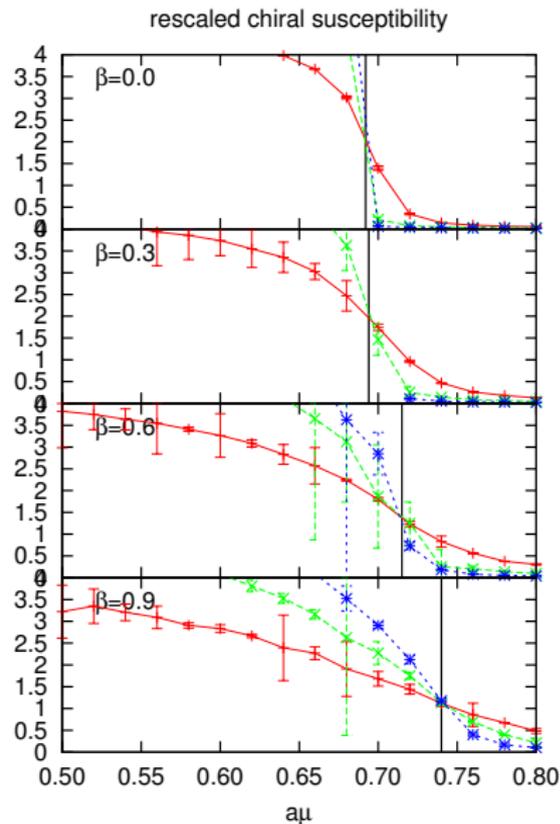
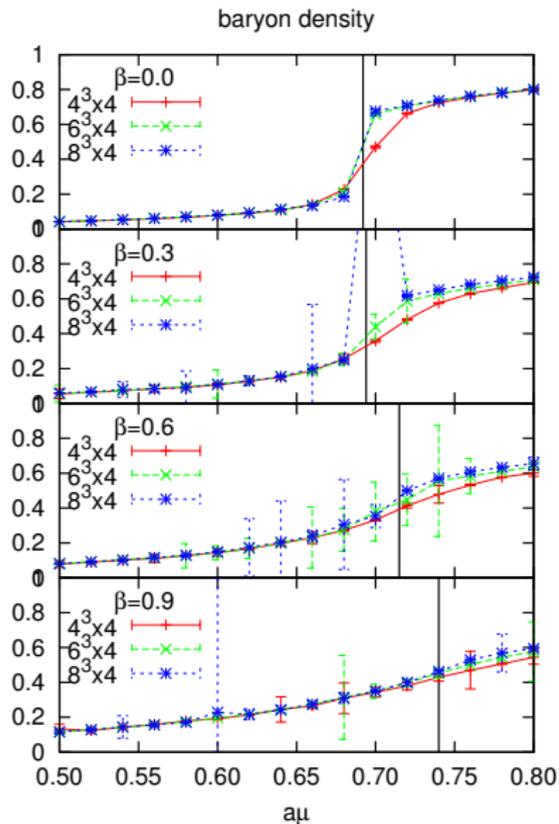
Phase boundary for $SU(3)$ at finite β , finite μ and $aT = 0.9 < aT_{\text{tric}}$



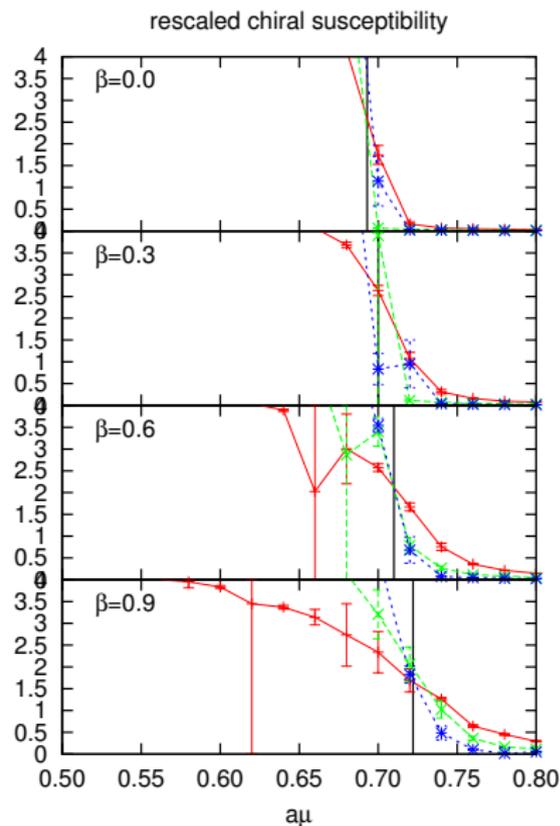
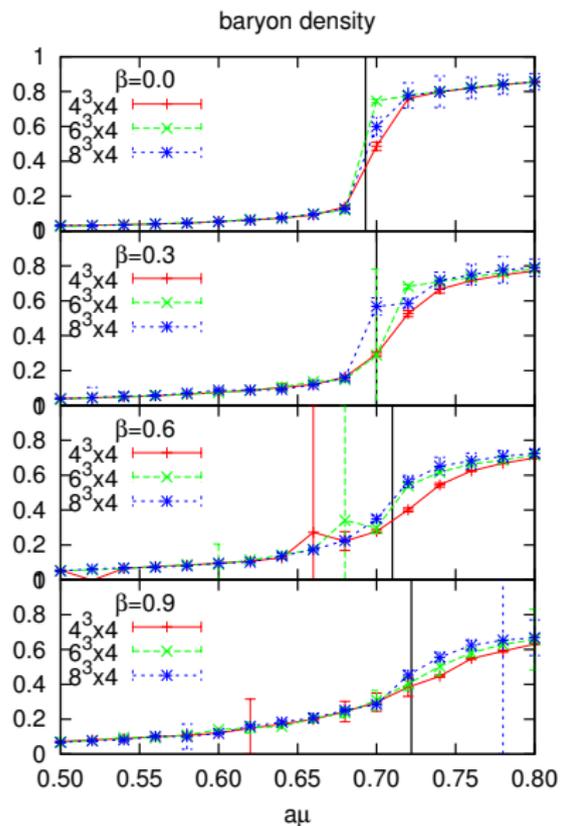
Phase boundary for $SU(3)$ at finite β , finite μ and $aT = 0.81 < aT_{\text{tric}}$



Phase boundary for $SU(3)$ at finite β , finite μ and $aT = 0.72 < aT_{\text{tric}}$

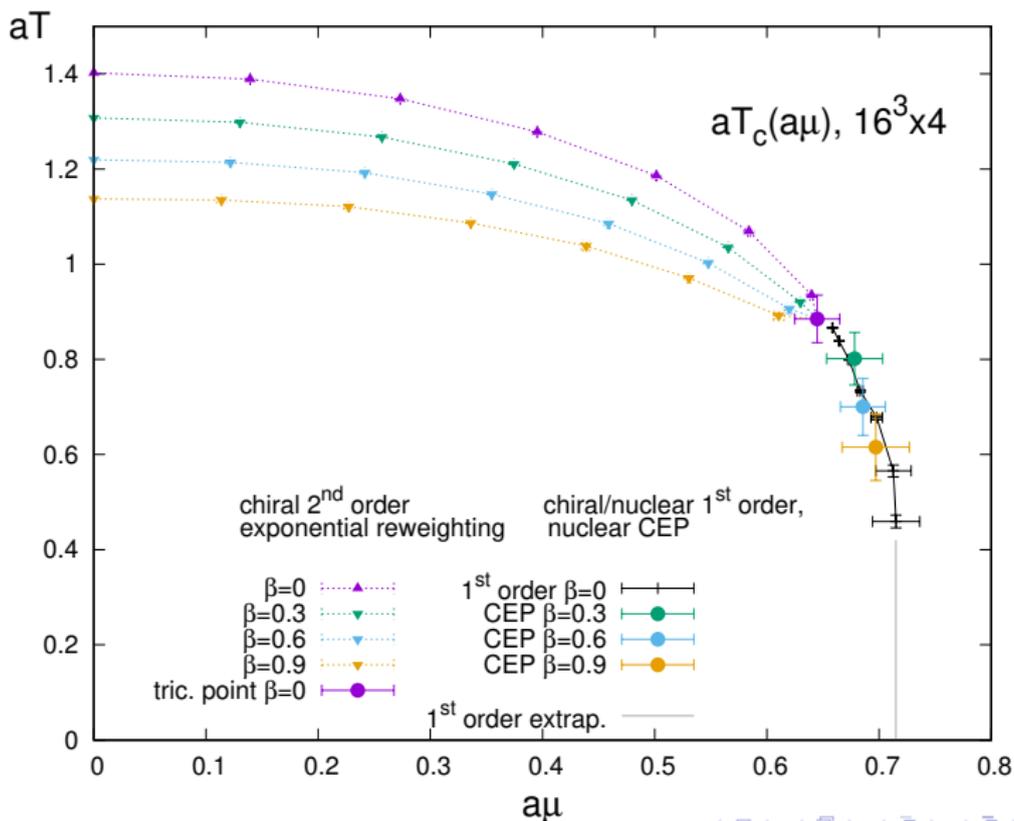


Phase boundary for $SU(3)$ at finite β , finite μ and $aT = 0.64 < aT_{\text{tric}}$

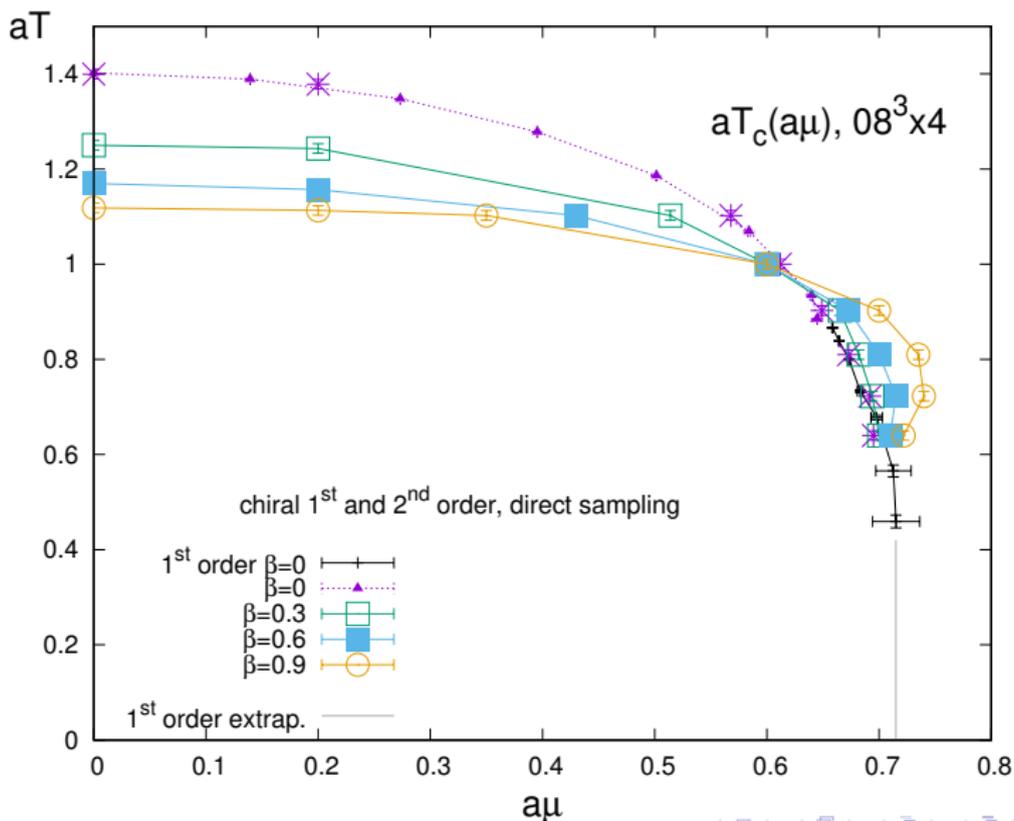


Phase Diagram in the Strong Coupling Regime

[Langelage, de Forcrand, Philipsen & U., PRL 113(2014)]



Phase Diagram in the Strong Coupling Regime, New Preliminary data



Conclusions

Results

- ▶ We obtain the phase boundary in the strong coupling regime via direct simulations.
- ▶ $\mathcal{O}(\beta)^2$ corrections are included
- ▶ Below aT_{tric} , $a\mu_c^{\text{chiral}}(aT)$ increases with β , in contrast to reweighting.
- ▶ sign problem severe for $\beta > 1$.

Questions to be addressed next

- ▶ indication that the tri-critical point moves to larger μ ?
- ▶ β dependence of nuclear transition $a\mu_c^{\text{chiral}}(aT)$ are not settled yet.

Backup

Backup: Connection Between Strong Coupling and Continuum Limit?

One of several **possible scenarios** for the extension to the continuum:

