35th International Symposium on Lattice Field Theory

# Relaxation time of the fermions in the magnetic field

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1. Chiral Magnetic Effect

2. New relaxation mechanism

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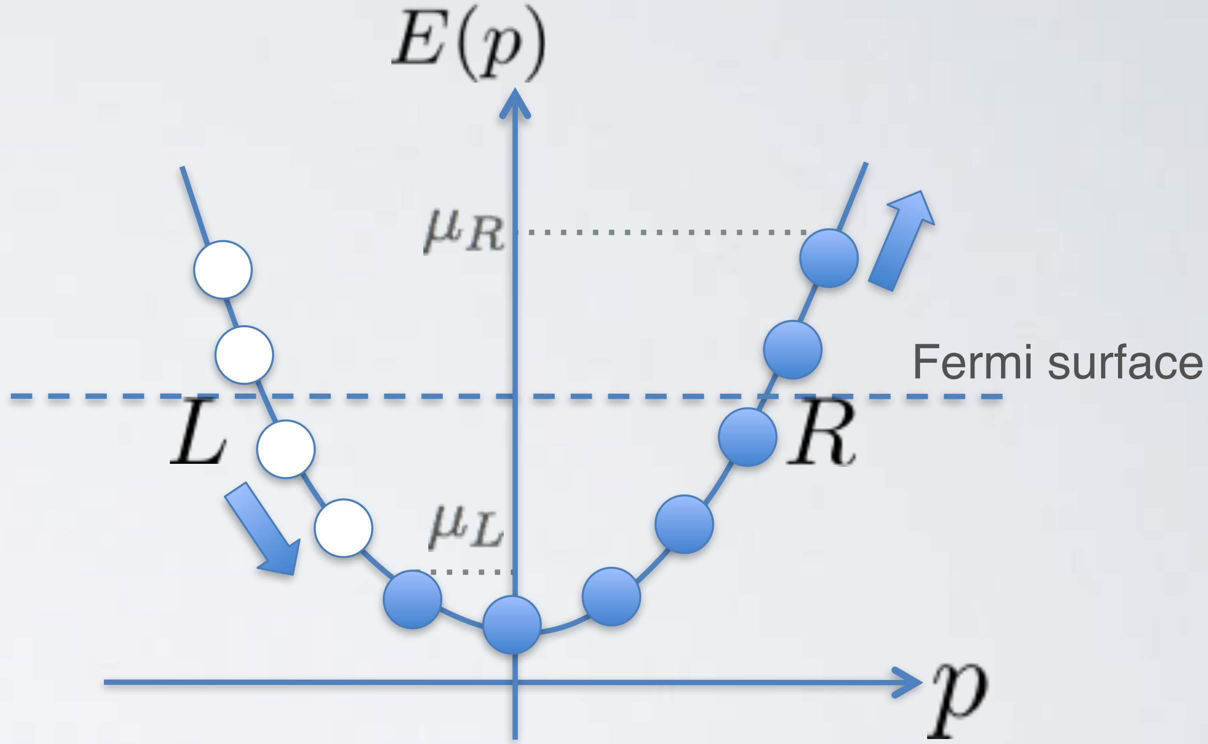
# Chiral Magnetic Effect

[Nielsen-Ninomiya 1983]

With $\mathbf{B}$ field, electron forms Landau levels.

With $\mathbf{E}$ field, electron moves from LH to RH corn.




$$\dot{N}_R - \dot{N}_L = \frac{e^2}{2\pi}\mathbf{E}\cdot\mathbf{B}$$

(Chiral Anomaly)

Supply the energy from $\mathbf{J_A}$

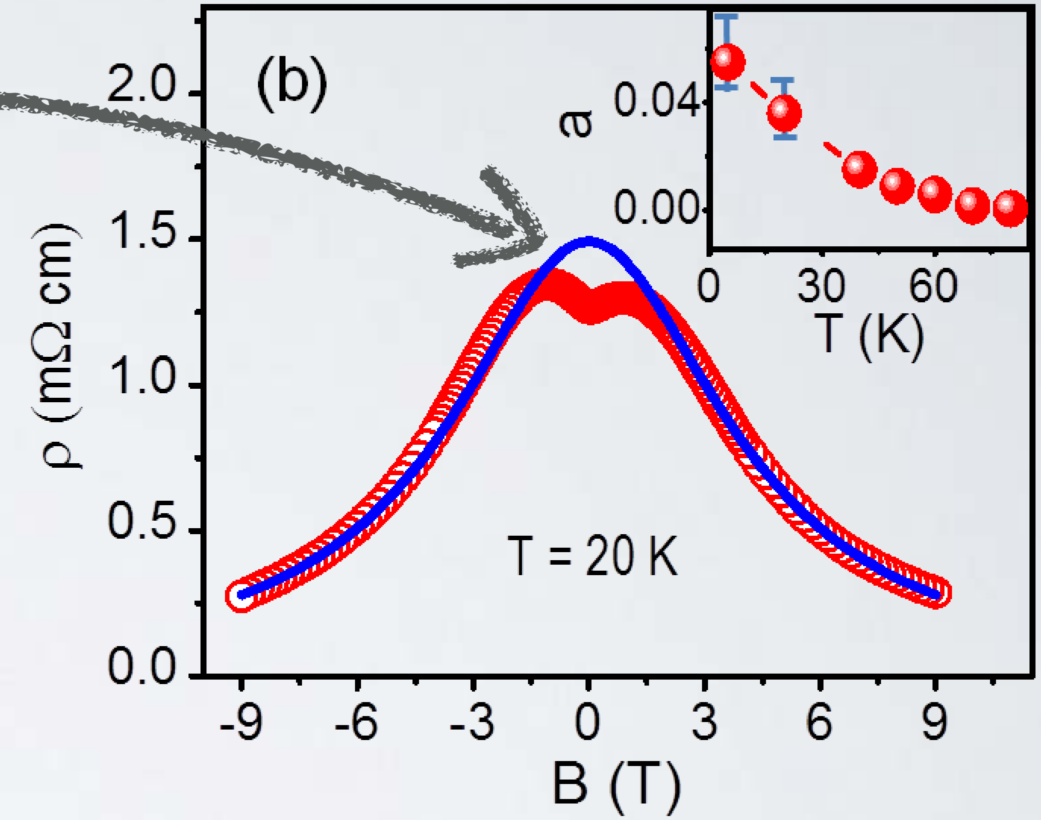
$$\mathbf{J}_A = \frac{e^2}{4\pi^2}\mathbf{B}(\mu_R - \mu_L)$$

# Experiment in Weyl semimetal

fitted by

$$\rho = \frac{1}{\sigma_0 + a(T)B^2}$$

Q. Li, et al. [arXiv:1412.6543]




Relaxation time approximation: $\dot{N}_R|_{\text{coll}} = -\frac{1}{\tau_I}\delta N_R$

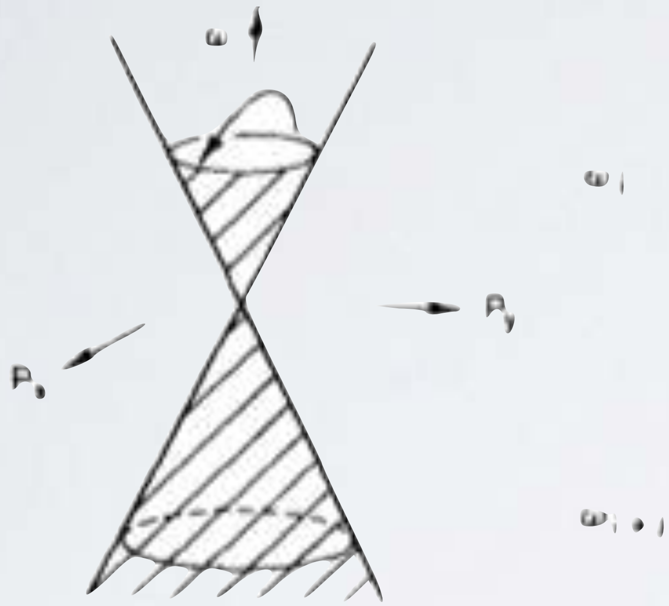
$$\mu_R - \mu_L = 2ev_F E \tau_I$$

depend on $\mathbf{B}$ ?

# Scattering by impurity

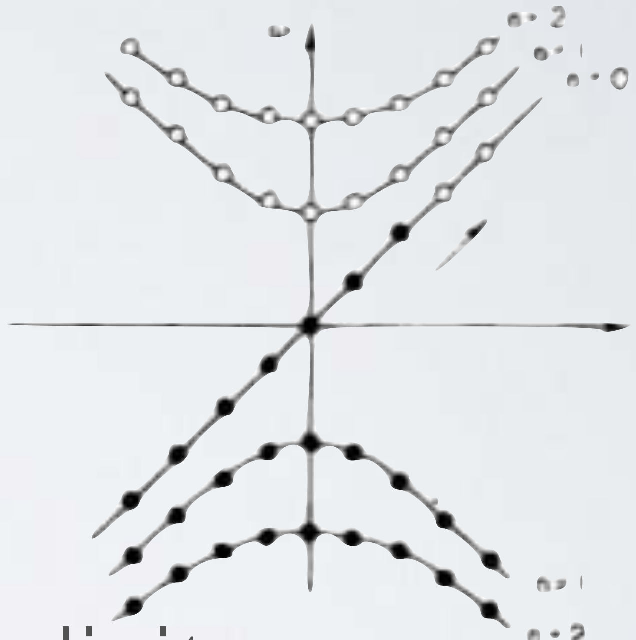
Nielsen-Ninomiya [1983]

$\mathbf{B} = 0$



Electrons can transfer to the other momentum states in the same corn **(easy to be scattered)**

$\mathbf{B} \neq 0$



Due to helicity conservation, electron cannot transfer into the same corn in low energy **(very hard to be scattered)**

Relaxation time should depend on $\mathbf{B}$ !

# Goal

Using the continuum theory (effective theory) for Dirac fermion with small mass,

we estimate the transfer probability within the corn by impurity scattering, and reveal

➢ **behavior of CME when the excited states in Landau levels contribute it**

with $T = 0$.

Review

# Transport theory

[Nielsen-Ninomiya 1983]
[Argyres-Adams 1956]

Boltzmann eq. :

$$\frac{\partial}{\partial t} f(n, P_y, P_z.t) - eE\frac{\partial}{\partial P_z} f(n, P_y, P_z, t) = \left(\frac{\partial}{\partial t} f(n, P_y, P_z)\right)_{\text{collision}}$$

$f(n, P_y, P_z, t)$ : probability distribution function

tiny deviation from the equilibrium: $f = f_0 + \delta f$

Definition of the collision term:

$$\left(\frac{\partial}{\partial t} f(n, P_y, P_z)\right)_{\text{collision}} = -\sum_{n'} \int d^2P' W(n, \mathbf{P} \to n', \mathbf{P}')(\delta f(n, \mathbf{P}, t) - \delta f(n', \mathbf{P}', t))$$

$W$ :transfer probability in unit time

Review

# Relaxation time approximation

$$\left(\frac{\partial}{\partial t} f(n, P_y, P_z)\right)_{\text{collision}} \approx -\frac{\delta f(n, \mathbf{P}, t)}{\tau(n, \mathbf{P})}$$

➡ Static solution of the Boltzmann equation

$$\delta f(n, \mathbf{P}) = \tau(n, \mathbf{P}) e E \frac{\partial}{\partial P_z} f_0(n, \mathbf{P})$$

Relaxation time $\tau$ : determined by definition of collision term

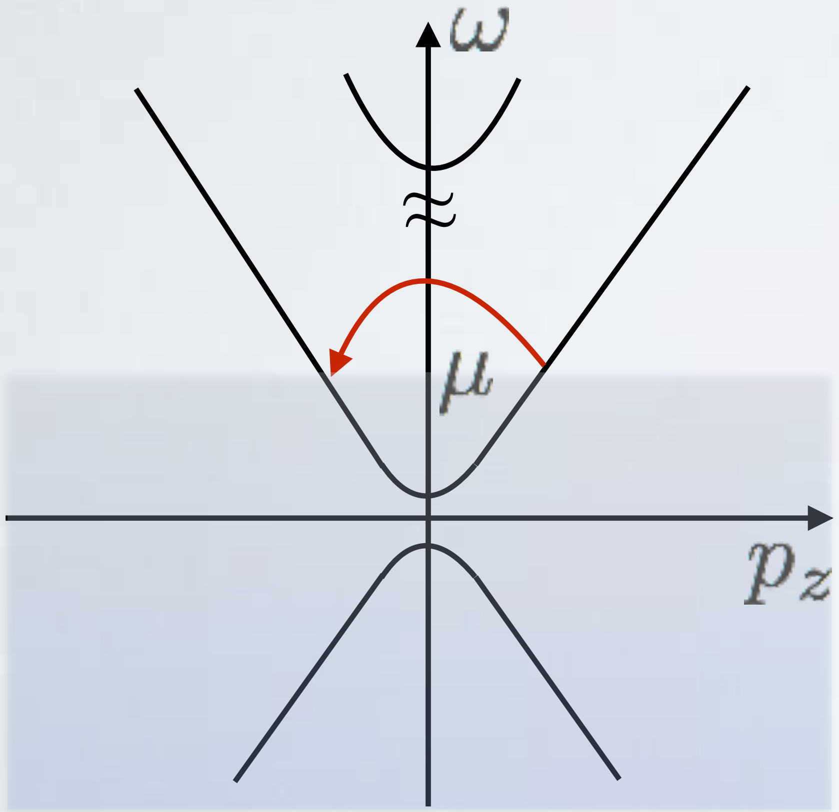
Substituting these into the definition of the collision term.

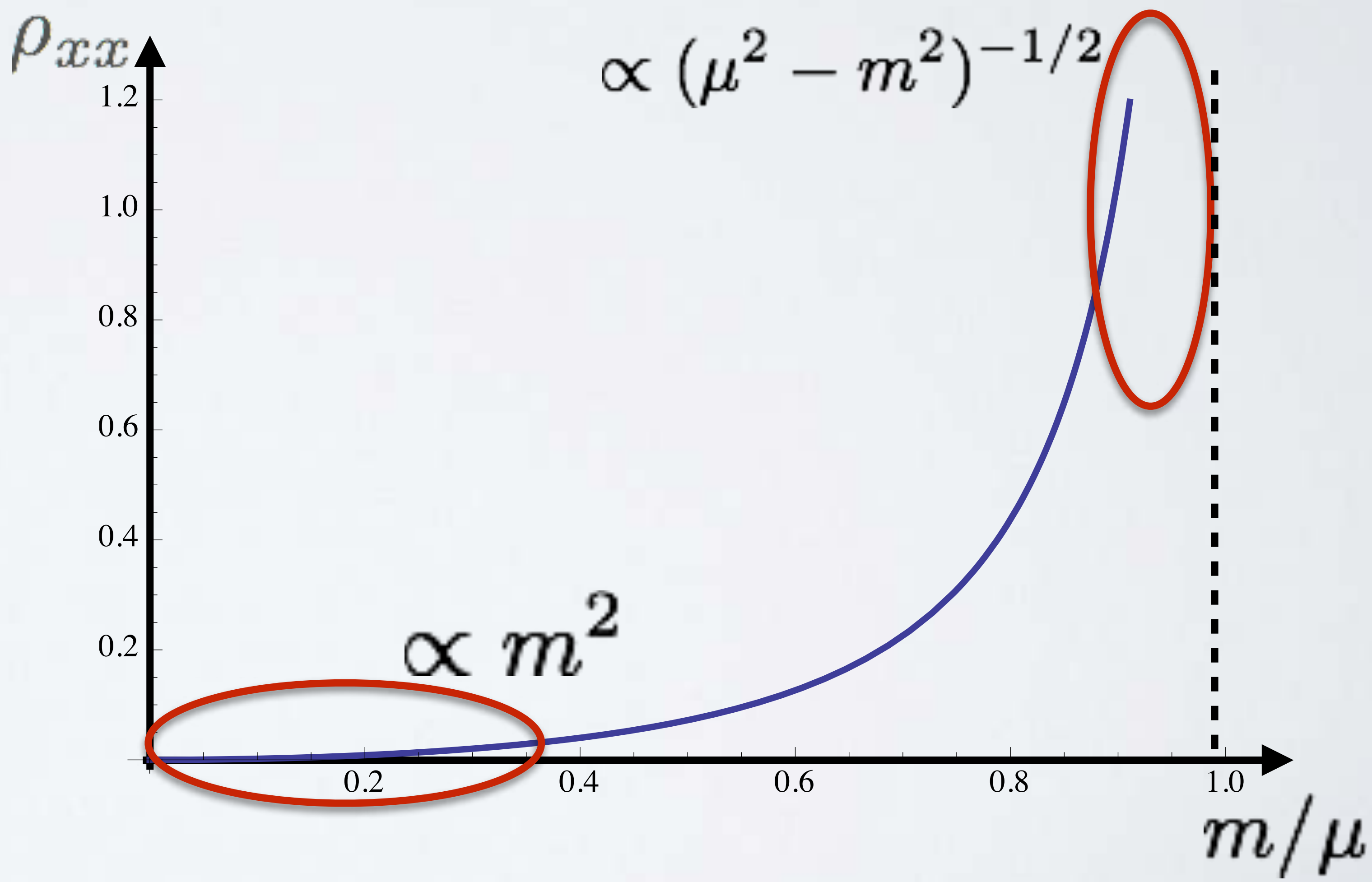
Equation for the relaxation time

$$P_z = \sum_{n'} \int d^2\mathbf{P}' W(n, \mathbf{P} \to n', \mathbf{P}')(\tau(n', \mathbf{P}') P'_z - \tau(n, \mathbf{P}) P_z)$$

# Strong B

$$\rho_{xx} \propto \frac{m^2}{\mu\sqrt{\mu^2 - m^2}} \frac{1}{4(\mu^2 - m^2) + 1/r_s^2}$$

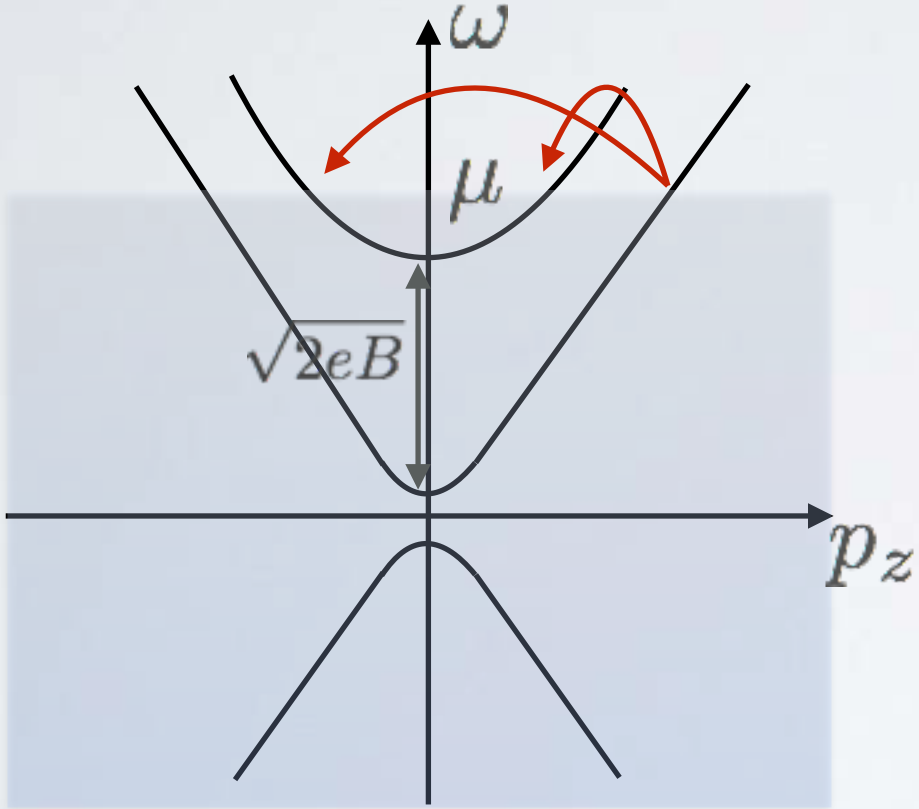



**massless fermion cannot be scattered in strong B**

# How about in weak B

Scattering within the corn does not occur in strong B.



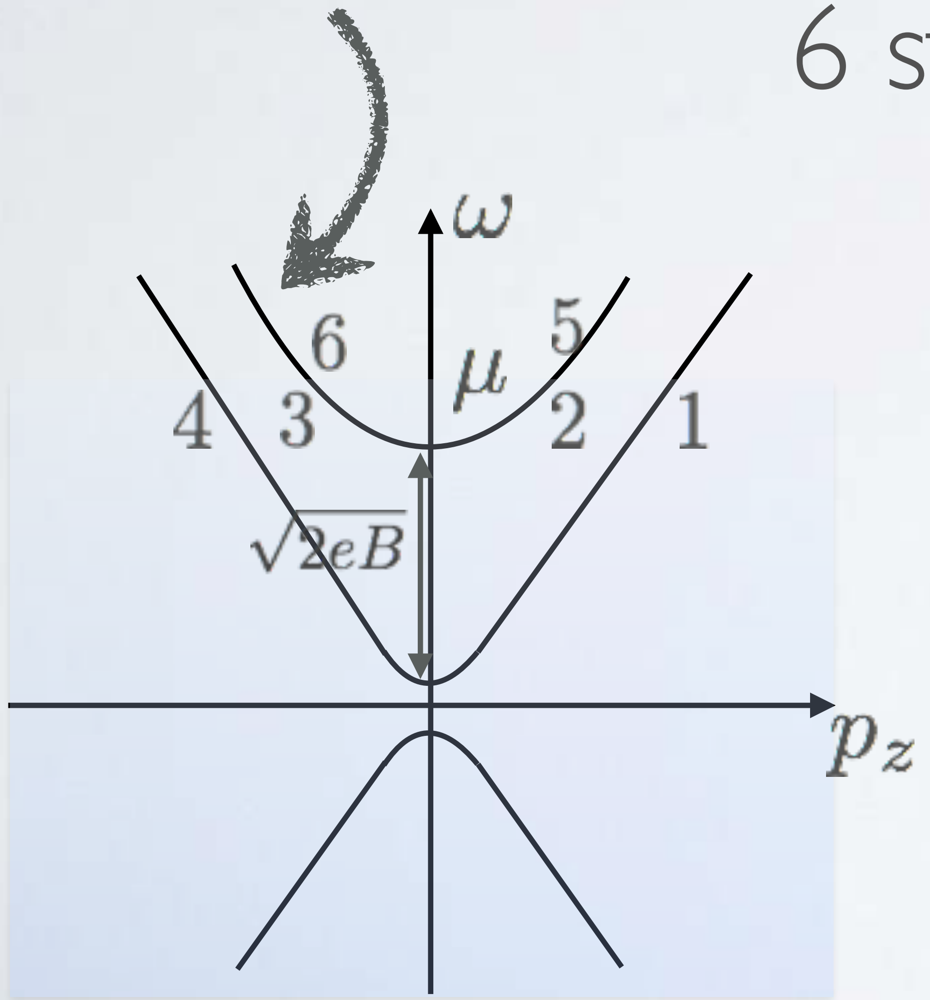

Does the scattering through these states occur in massless limit?

**The answer is Yes.**

# 1st excited states

$$\omega = \pm\sqrt{2eB\left(n+\frac{1}{2}\right) + p_3^2 + m^2 - eB\sigma^3}$$

doubly degenerated




6 states on Fermi energy

**6 relaxation time for each states**

$$\tau_I \ (I = 1, \cdots, 6)$$

We can calculate the each transfer probability $w_{IJ}$, and obtain the simultaneous equations for each $\tau_I$.

# Simultaneous equations

$$P_1 = \frac{\mu}{v}\left[\tau_1\left\{(w_{12}+w_{13}+w_{15}+w_{16})\frac{P_1}{P_2}+w_{14}\right\} - \tau_2 w_{12} + \tau_3 w_{13} + \tau_4 w_{14} - \tau_5 w_{15} + \tau_6 w_{16}\right]$$

$$P_2 = \frac{\mu}{v}\left[-\tau_1 w_{12} + \tau_2\left\{(w_{12}+w_{13})\frac{P_2}{P_1}+w_{23}+w_{25}+w_{26}\right\} + \tau_3 w_{23} + \tau_4 w_{13} - \tau_5 w_{25} + \tau_6 w_{26}\right]$$

$$-P_2 = \frac{\mu}{v}\left[-\tau_1 w_{13} - \tau_2 w_{23} - \tau_3\left\{(w_{13}+w_{12})\frac{P_2}{P_1}+w_{23}+w_{25}+w_{26}\right\} + \tau_4 w_{12} - \tau_5 w_{26} + \tau_6 w_{25}\right]$$

$$P_1 = \frac{\mu}{v}\left[-\tau_1 w_{14} - \tau_2 w_{13} + \tau_3 w_{12} - \tau_4\left\{(w_{13}+w_{12}+w_{16}+w_{15})\frac{P_1}{P_2}+w_{14}\right\} - \tau_5 w_{16} + \tau_6 w_{15}\right]$$

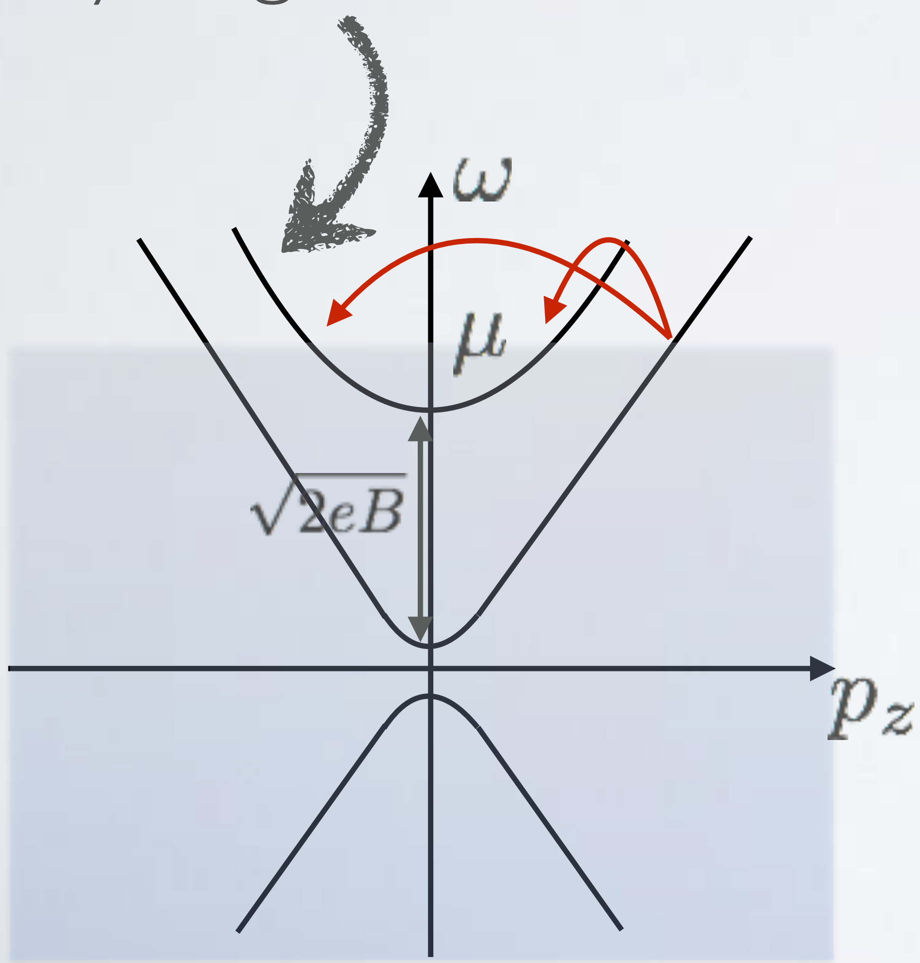
$$P_2 = \frac{\mu}{v}\left[-\tau_1 w_{15} - \tau_2 w_{25} + \tau_3 w_{26} + \tau_4 w_{16} + \tau_5\left\{(w_{15}+w_{16})\frac{P_2}{P_1}+w_{25}+w_{26}+w_{56}\right\} + \tau_6 w_{56}\right]$$

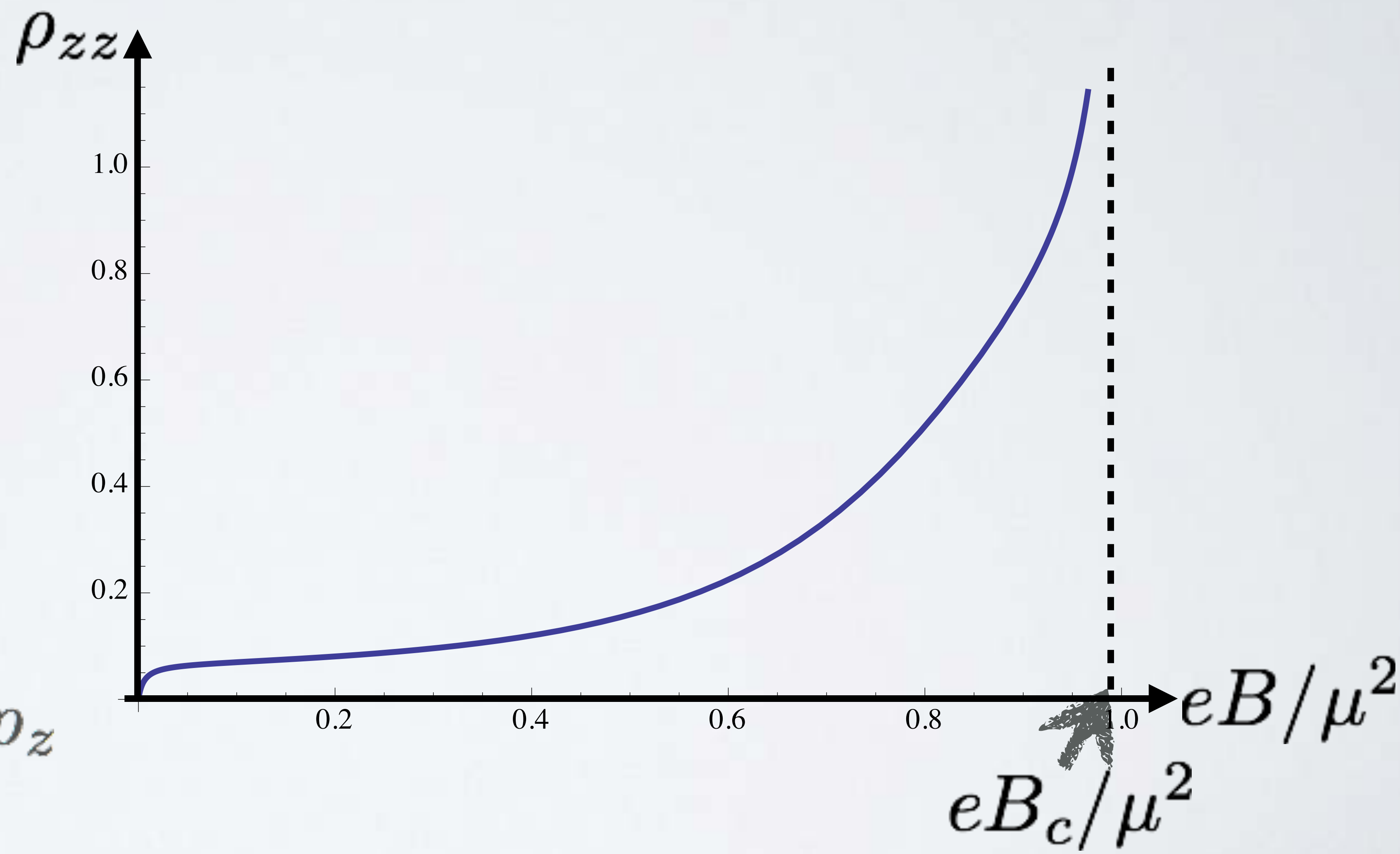
$$-P_2 = \frac{\mu}{v}\left[-\tau_1 w_{16} - \tau_2 w_{26} + \tau_3 w_{25} + \tau_4 w_{15} - \tau_5 w_{56} - \tau_6\left\{(w_{16}+w_{15})\frac{P_2}{P_1}+w_{26}+w_{25}+w_{56}\right\}\right]$$

# Weak B

$$J \propto eB\left[P_1(\tau_1+\tau_4)+P_2(\tau_2+\tau_3+\tau_5+\tau_6)\right]$$

doubly degenerated




Fermi energy touches the 1st excited states

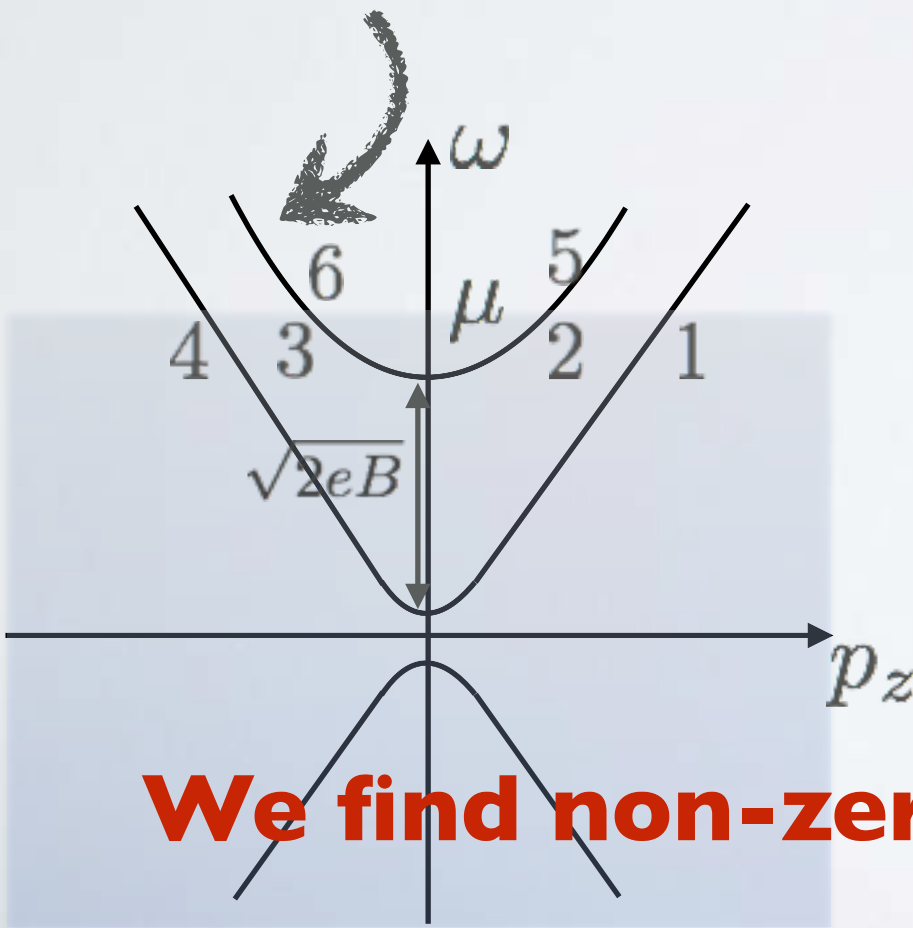
# Even in massless limit

For example we find,

$$w_{12} = \left(\frac{4\pi e^2}{\kappa}\right)^2 N_I \frac{[(\mu+m)^2 + P_1 P_2]^2}{4\mu^2(\mu+m)^2} \frac{1}{4\pi} \frac{1}{2eB} \frac{1}{\gamma(P_1,P_2)} [1 - (1+\gamma(P_1,P_2)) I(\gamma(P_1,P_2))]$$

**Non-zero in massless limit!**

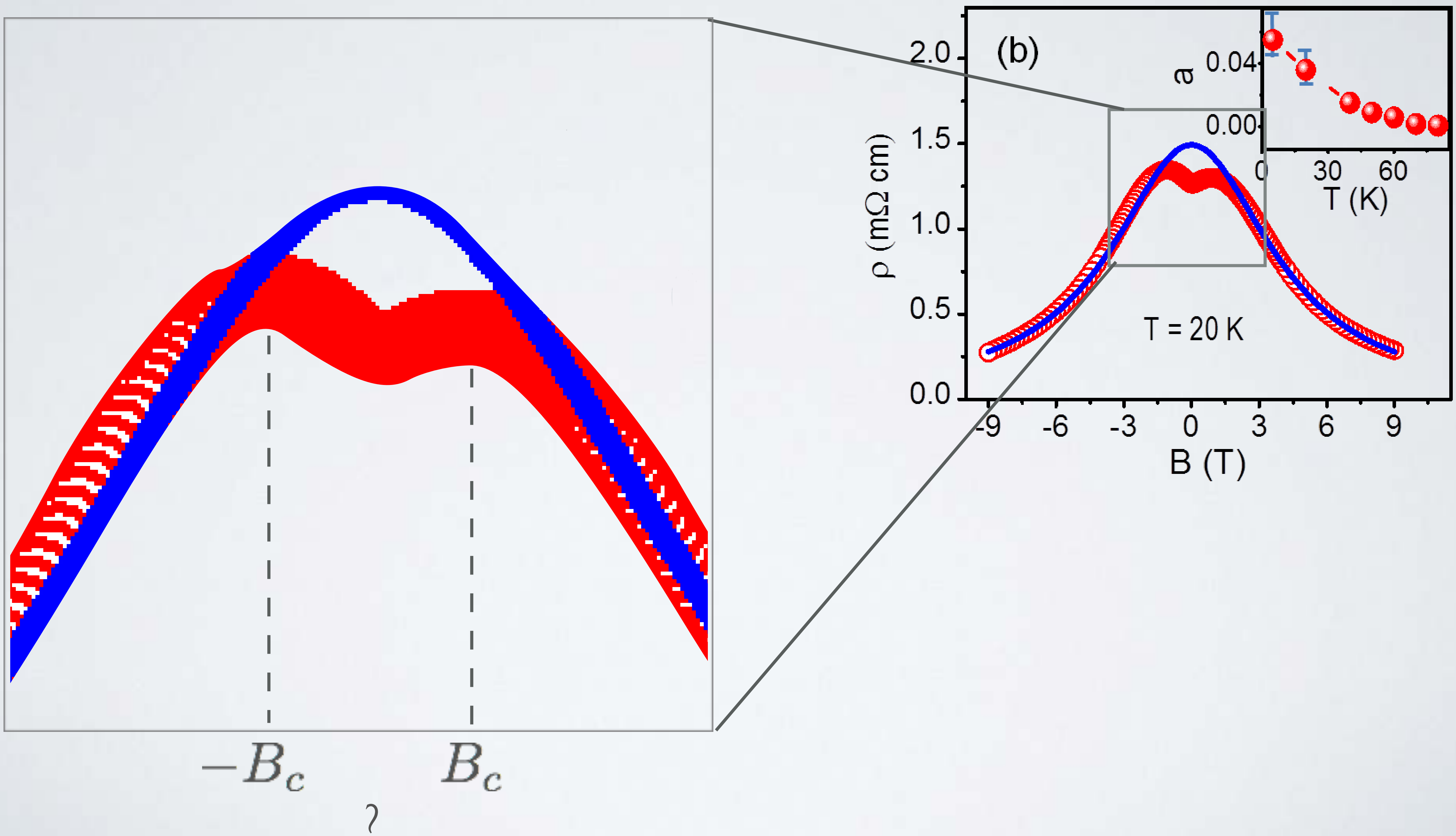
doubly degenerated




Relaxation through the 1st excited states remains even in massless limit, which didn't occur for only lowest states.

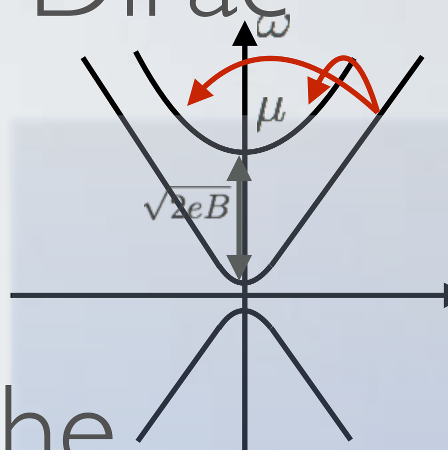
**We find non-zero resistivity in massless case.**

# Experiment in Weyl semimetal



# Summary

① We study the 1st excited states contribution to relaxation, using the continuum theory for Dirac fermion with small mass.




② Starting from Boltzmann eq. and assuming relaxation time approximation, we found the equation for relaxation times.

$$P_z = \sum_{n'} \int d^2\mathbf{P}' W(n, \mathbf{P} \to n', \mathbf{P}')(\tau(n', \mathbf{P}')P'_z - \tau(n, \mathbf{P})P_z)$$

③ Solving these equations, we found the relaxation mechanism for massless fermion in weak magnetic field.