Dirac spectral density and mass anomalous dimension in 2+1 flavor QCD

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Photo: KEK, Japan, 2016

◉ This work

◇ We test perturbative QCD by Dirac eigenvalue density

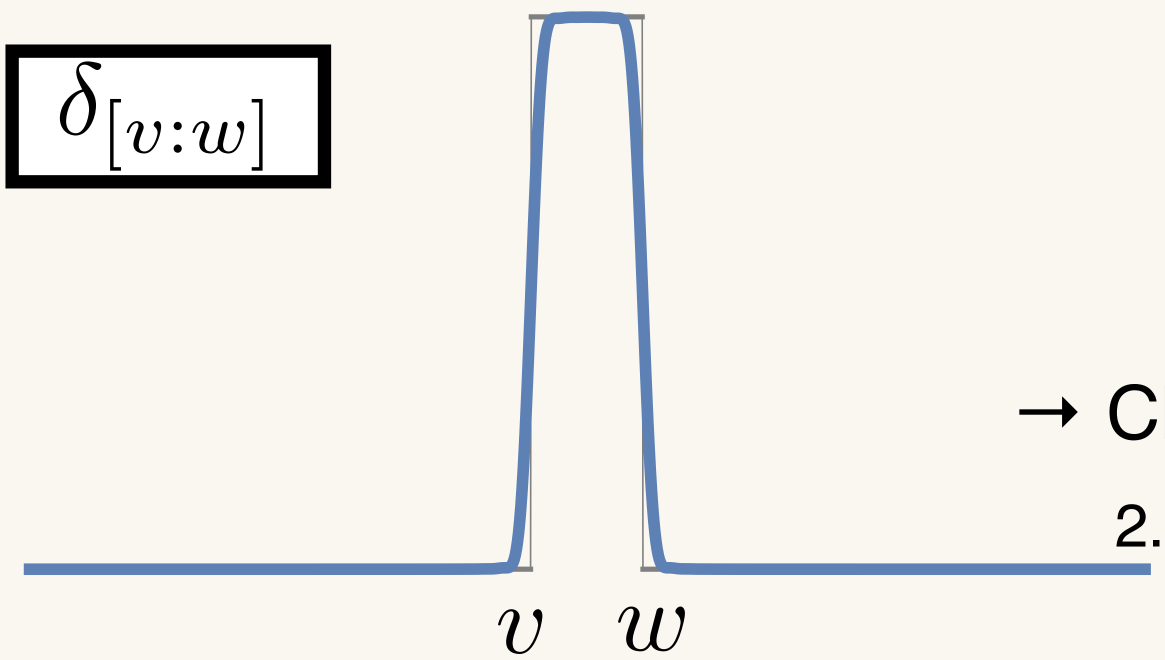
(1): Calculate eigenvalue density on the lattice
using a stochastic method

(2): Reduce discretization error using
modified Mobius domain-wall action

(3): Compare with $O(\alpha_s^4)$ perturbative expansion

◉ Stochastic estimation [E.Di.Napoli et.al.(2013)]

◇ Stochastic calculation of Dirac spectral density $\rho(\lambda)$ with eigenvalue filtering



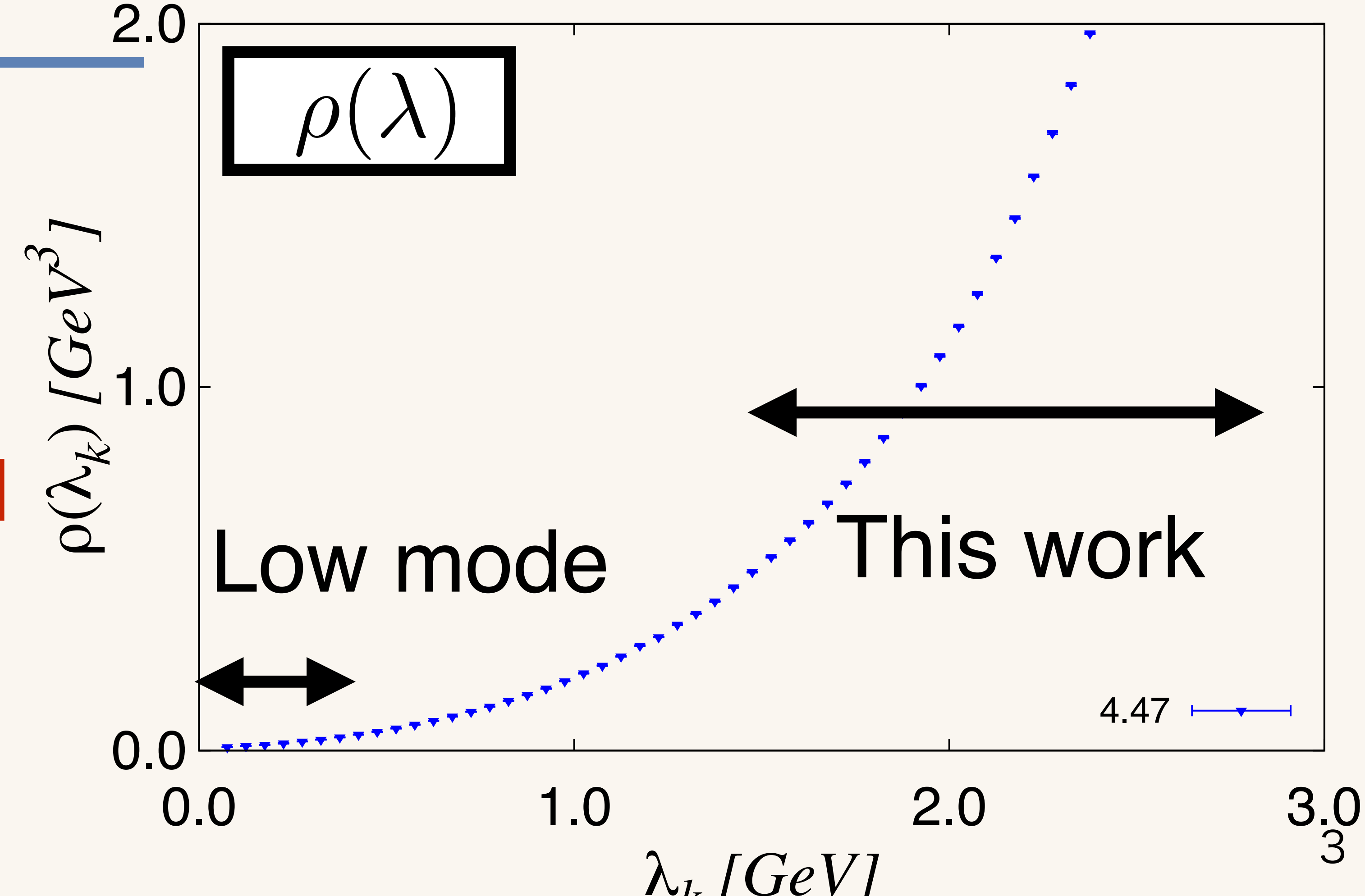
$$\delta_{[v:w]} \simeq \sum_j \gamma_{j\,[v:w]} T_j(D^\dagger D)$$

→ Chebyshev polynomial approximation

◇ Low mode:
→Chiral condensate

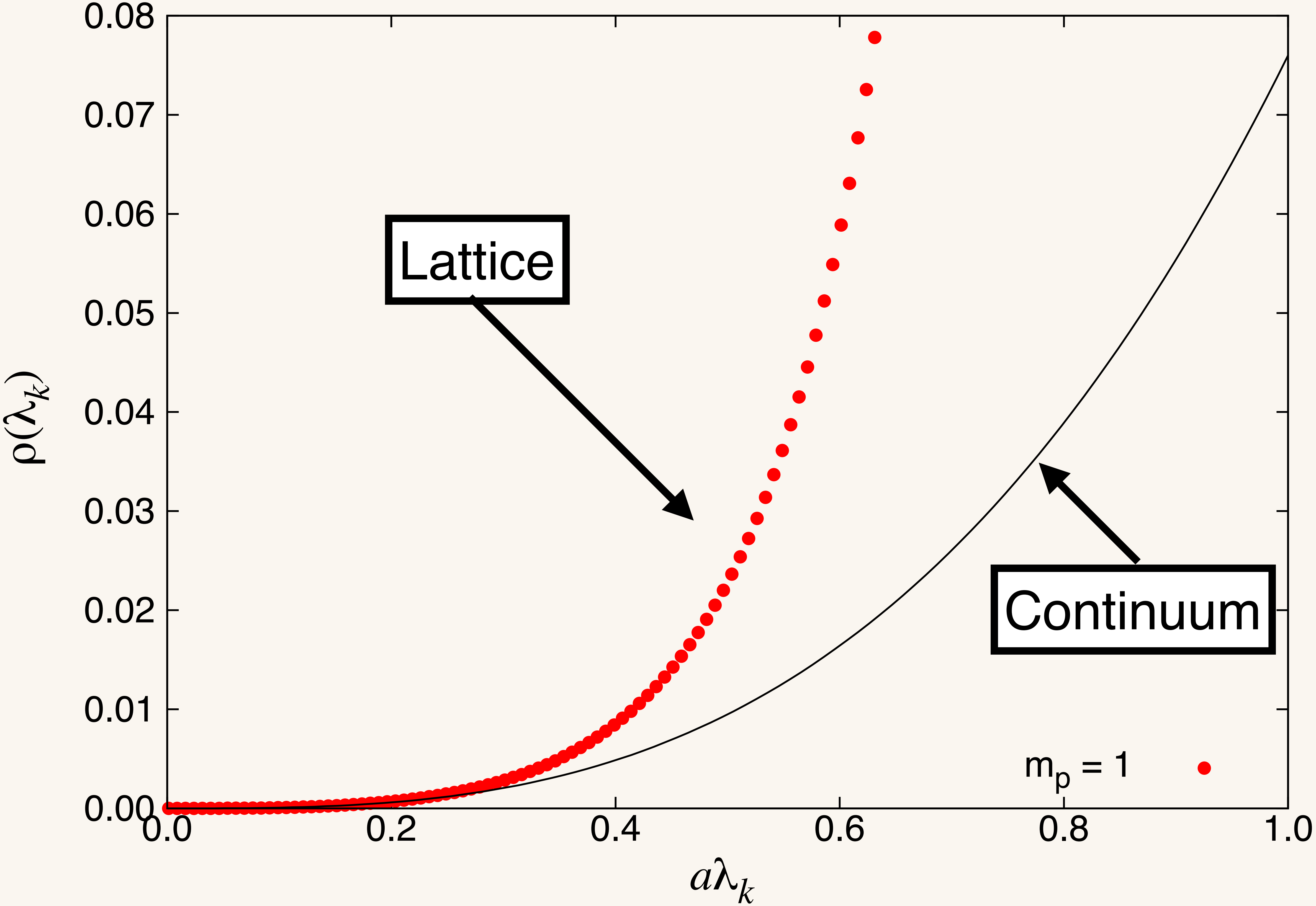
[G.Cossu et al. arXiv:1607.01099]

◇ High modes:
→This work



Improved (Mobius) domain-wall fermion

◉ Discretization error for the eigenvalue spectrum (Free theory)



◉ Mobius domain-wall fermion

◇ Wilson kernel

$$aD_{\mathrm{w}} \equiv i\sum \gamma_\mu \mathrm{sin} ap_\mu + r\sum(1-\mathrm{cos} ap_\mu) - M_0$$

◇ Mobius kernel [R.C.Brower et.al. arXiv:1206.5214]

$$aD_{\mathrm{Mobius}} \equiv \frac{(b+c)aD_{\mathrm{w}}}{2+(b-c)aD_{\mathrm{w}}}$$

◇ In this talk, we focus on our choice,

$$(r, M_0, b+c, b-c) = (1, 1, 2, 1)$$

◉ Pauli–Villars mass generalization

◇ Overlap operator construction

$$aD_{\mathrm{OV}}(m_f=0) \equiv [\mathcal{P} D_{\mathrm{DW}}^{-1}(1) D_{\mathrm{DW}}(m_f=0)\mathcal{P}]$$
$$= \frac{1+\gamma_5 \mathrm{sign}_{L_5}[\gamma_5 aD_{\mathrm{Mobius}}]}{2}$$

↓

General Pauli–Villars mass

$$aD_{\mathrm{OV}}(m_f=0) \equiv [\mathcal{P} D_{\mathrm{DW}}^{-1}(m_p) D_{\mathrm{DW}}(m_f=0)\mathcal{P}]$$
$$= m_p \frac{1+\gamma_5 \mathrm{sign}_{L_5}[\gamma_5 aD_{\mathrm{Mobius}}]}{(1+m_p)+(1-m_p)\gamma_5 \mathrm{sign}_{L_5}[\gamma_5 aD_{\mathrm{Mobius}}]}$$

◇ Eigenvalue

$$a^2\lambda^2(m_p) = \frac{a^2\lambda^2(m_p=1)}{1-(1-1/m_p^2)a^2\lambda^2(m_p=1)}$$

◇ Chirality (Ginsparg-Wilson)

$$\{D_{\mathrm{OV}}^{-1}, \gamma_5\} = 2a\gamma_5$$

[P.H.Ginsparg, K.G.Wilson (1982)]

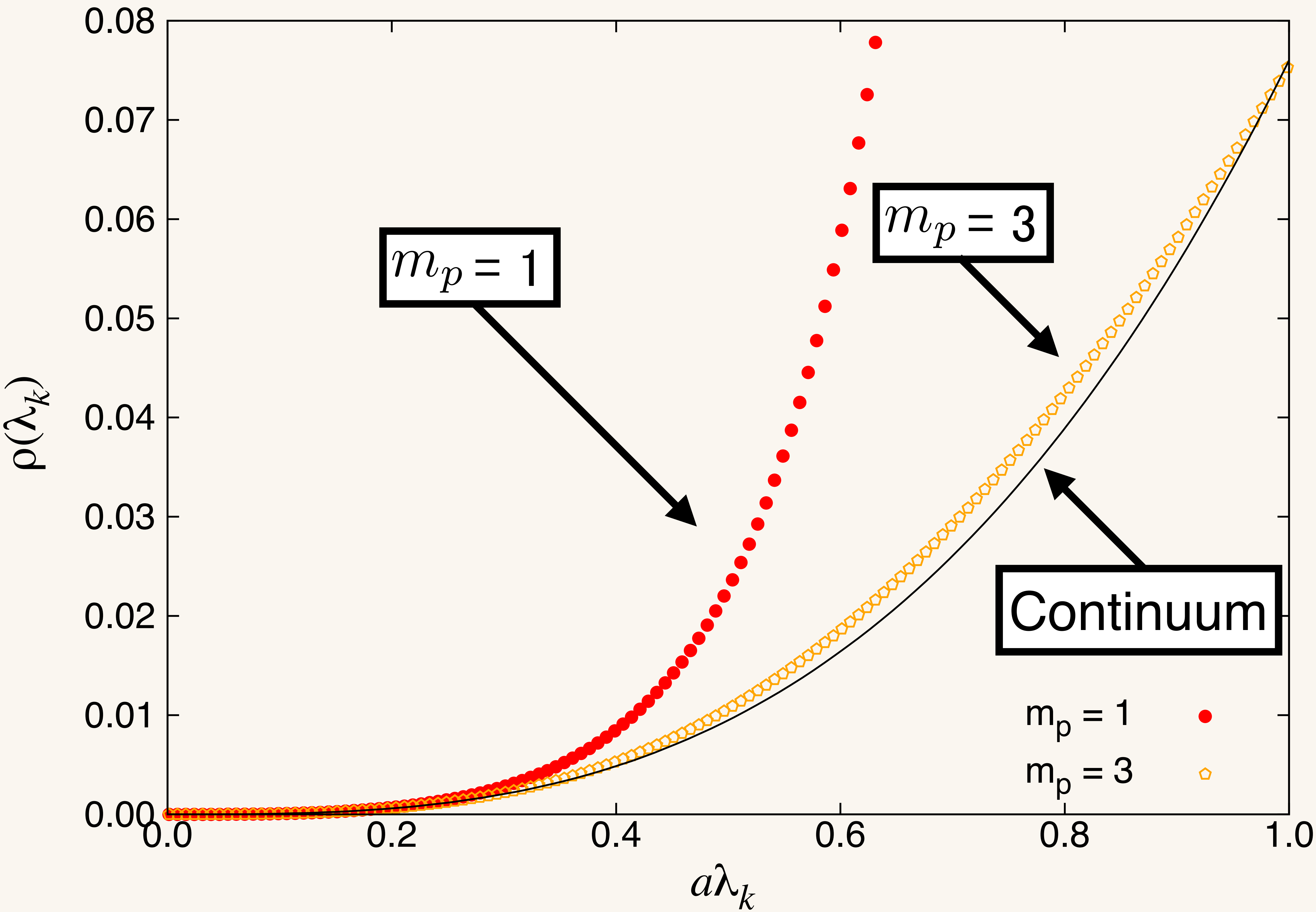
↓ General Pauli−Villars mass

$$\{D_{\mathrm{OV}}^{-1}, \gamma_5\} = \frac{2a\gamma_5}{m_p}$$

◇ Locality → Exponential locality is maintained

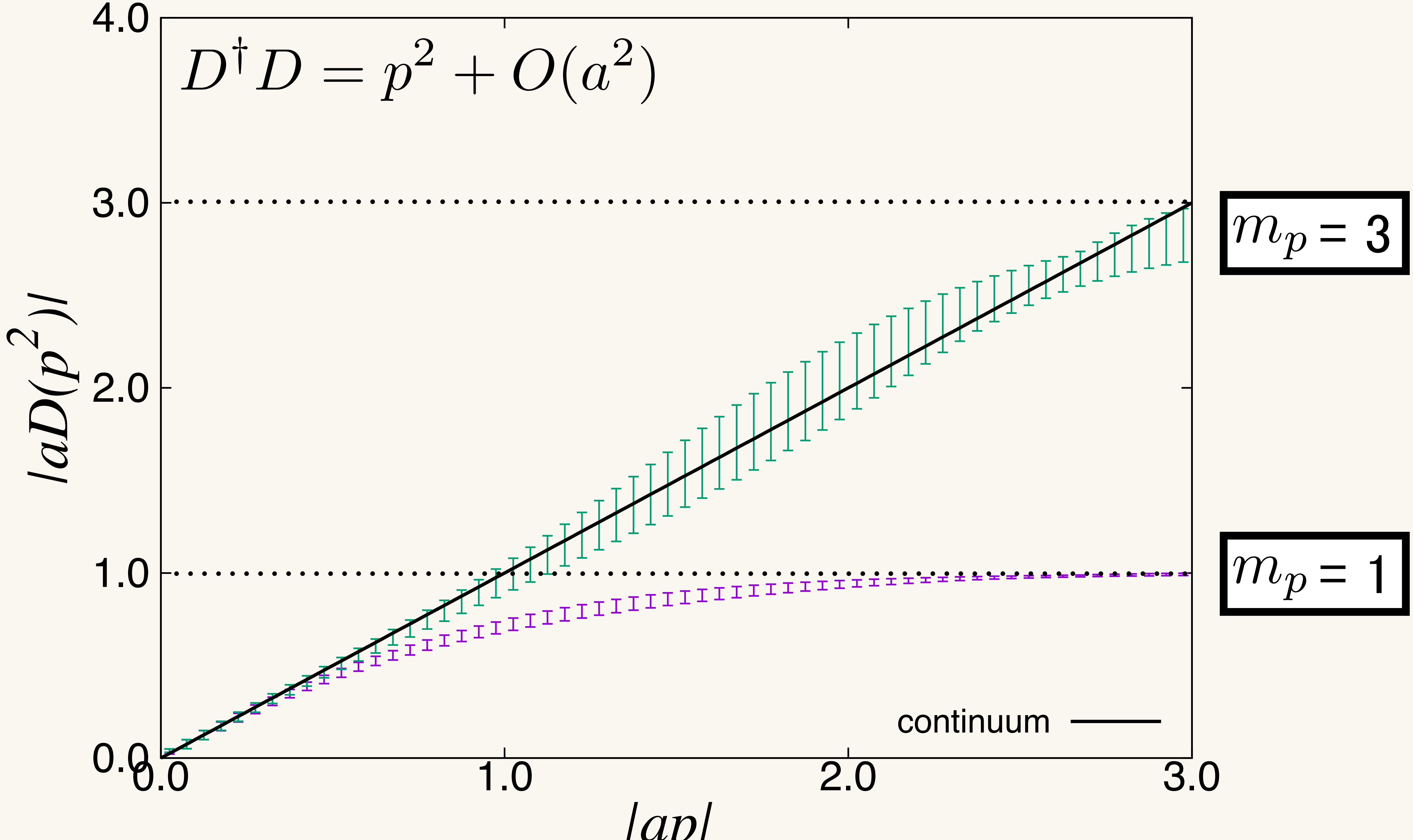
(Exponential locality of sign function) …… [P.Hernandez et.al.(1999)]

◉ Lattice eigenvalue spectrum (Free theory)



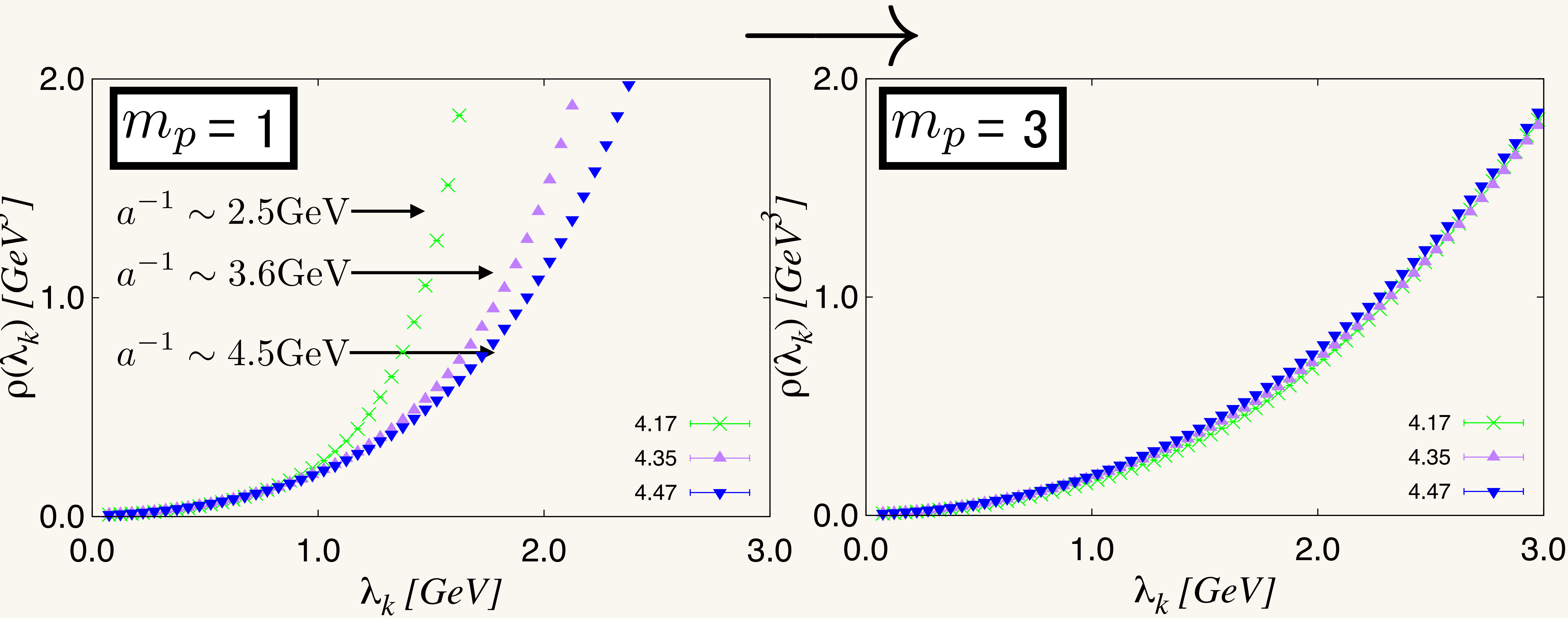
◉ Pauli-Villars mass as an eigenvalue cutoff scale (Tree)

$$|aD_{\mathrm{OV}}| \leq m_p$$



Reducing discretization effect

a dependence is reduced by $m_p = 1 \rightarrow 3$



$$a^2\lambda^2(m_p) = \frac{a^2\lambda^2(m_p=1)}{1-(1-1/m_p^2)a^2\lambda^2(m_p=1)}$$

Perturbative calculation

◉ Perturbative calculation up to $O(\alpha_s^3)$

◇ Eigenvalue density from chiral condensate

$$\rho(\lambda) = -\frac{1}{2\pi}\langle \bar{q}q(m = i\lambda + \epsilon)\rangle - \langle \bar{q}q(m = i\lambda - \epsilon)\rangle$$

◇ Known up to $O(\alpha_s^3)(n_f = 3)$

[J.Kneur, A.Neveu arXiv:1506.07506]

$$\rho(\mu = \lambda) = \frac{3\lambda^3}{4\pi^2}\left(1 + 1.06\alpha_s - 2.14\alpha_s^2 - 5.98\alpha_s^3 + O(\alpha_s^4)\right)$$

◇ Exponent $\frac{\mathrm{dlog}\rho(\lambda)}{\mathrm{dlog}\lambda}$ can be predicted up to $O(\alpha_s^4)$.

◉ Perturbation up to $O(\alpha_s^4)$ for the exponent of $\rho(\lambda)$

◇ $O(\alpha_s^4)$ term of $\dfrac{\mathrm{dlog}\rho(\lambda)}{\mathrm{dlog}\lambda}$ can be derived from RG eq.

$$0 = \left(\frac{\partial}{\partial \log\mu} - \gamma_m \left(1 + \lambda\frac{\partial}{\partial\lambda}\right) + \beta\frac{\partial}{\partial\alpha_s}\right)\rho(\lambda)$$

β, γ_m are known to $O(\alpha_s^5)$,

$$\left(\frac{\mathrm{dlog}\rho(\lambda)}{\mathrm{dlog}\lambda}\right) = 3 - \frac{8}{\pi}\alpha_s - 2.97\alpha_s^2 + 7.51\alpha_s^3 + 13.8\alpha_s^4 + \ldots$$

NOTE: Exponent is related to mass anomalous dimension in the conformal theory

$$\frac{4}{1+\gamma_m} = 3 - \frac{8}{\pi}\alpha_s - 1.45\alpha_s^2 - 0.32\alpha_s^3 - 0.27\alpha_s^4 + \ldots$$

[K.Cichy arXiv:1311.3572]

Setup

β	a^{-1} [GeV]	$L^3 \times T (\times L_5)$	N_{src}	#meas	am_{ud}	am_s	m_π [MeV]	$m_\pi L$
4.17	2.453(4)	$32^3 \times 64 (\times 12)$	8	100	0.0035	0.040	230(1)	3.0
					0.007	0.030	310(1)	4.0
					0.007	0.040	309(1)	4.0
	$a^{-1} \sim 2.5\mathrm{GeV}$				0.012	0.030	397(1)	5.2
					0.012	0.040	399(1)	5.2
					0.019	0.030	498(1)	6.5
					0.019	0.040	499(1)	6.5
		$48^3 \times 96 (\times 12)$	8	100	0.0035	0.040	226(1)	4.4
4.35	3.610(9)	$48^3 \times 96 (\times 8)$	12	50	0.0042	0.0180	296(1)	3.9
					0.0042	0.0250	300(1)	3.9
					0.0080	0.0180	407(1)	5.4
	$a^{-1} \sim 3.6\mathrm{GeV}$				0.0080	0.0250	408(1)	5.4
					0.0120	0.0180	499(1)	6.6
					0.0120	0.0250	501(1)	6.6
4.47	4.496(9)	$64^3 \times 128 (\times 8)$	8	39	0.0030	0.015	284(1)	4.0

$a^{-1} \sim 4.5\mathrm{GeV}$

◇ Mobius DW fermion + valence $m_p = 3$

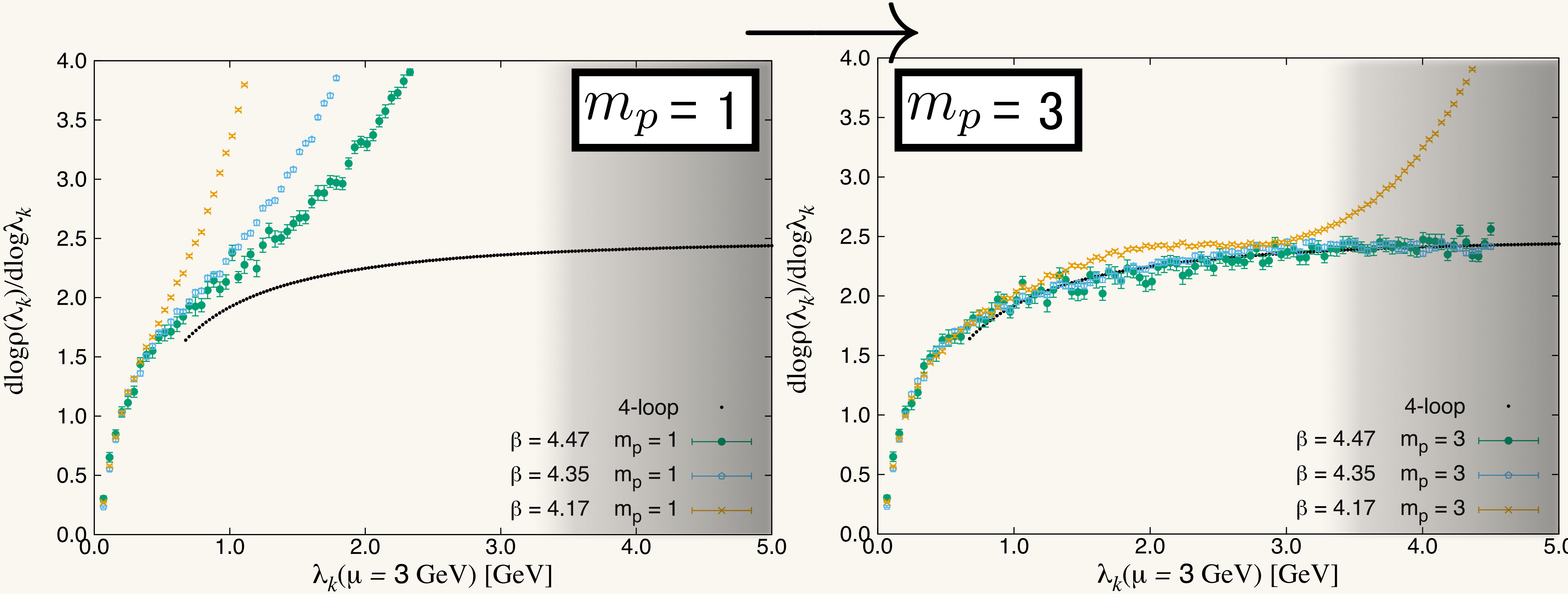
◇ 2 + 1 flavour

◇ $\overline{\mathrm{MS}}$ mass renormalization const. from short-distance correlator analysis

[M.Tomii (JLQCD) et.al. arXiv:1604.08702]

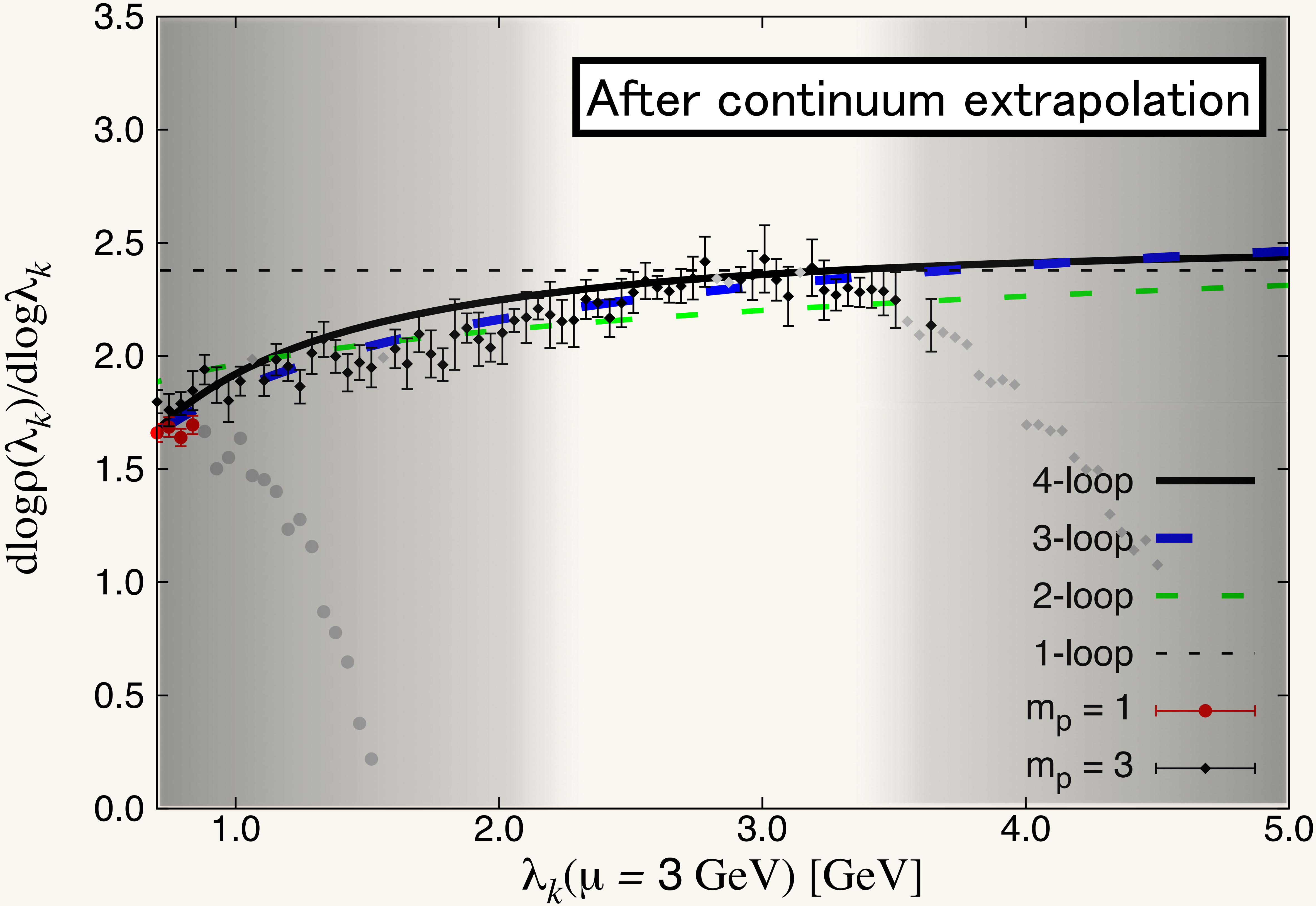
◉ Exponent $\dfrac{\mathrm{dlog}\rho(\lambda)}{\mathrm{dlog}\lambda}$ on the lattice

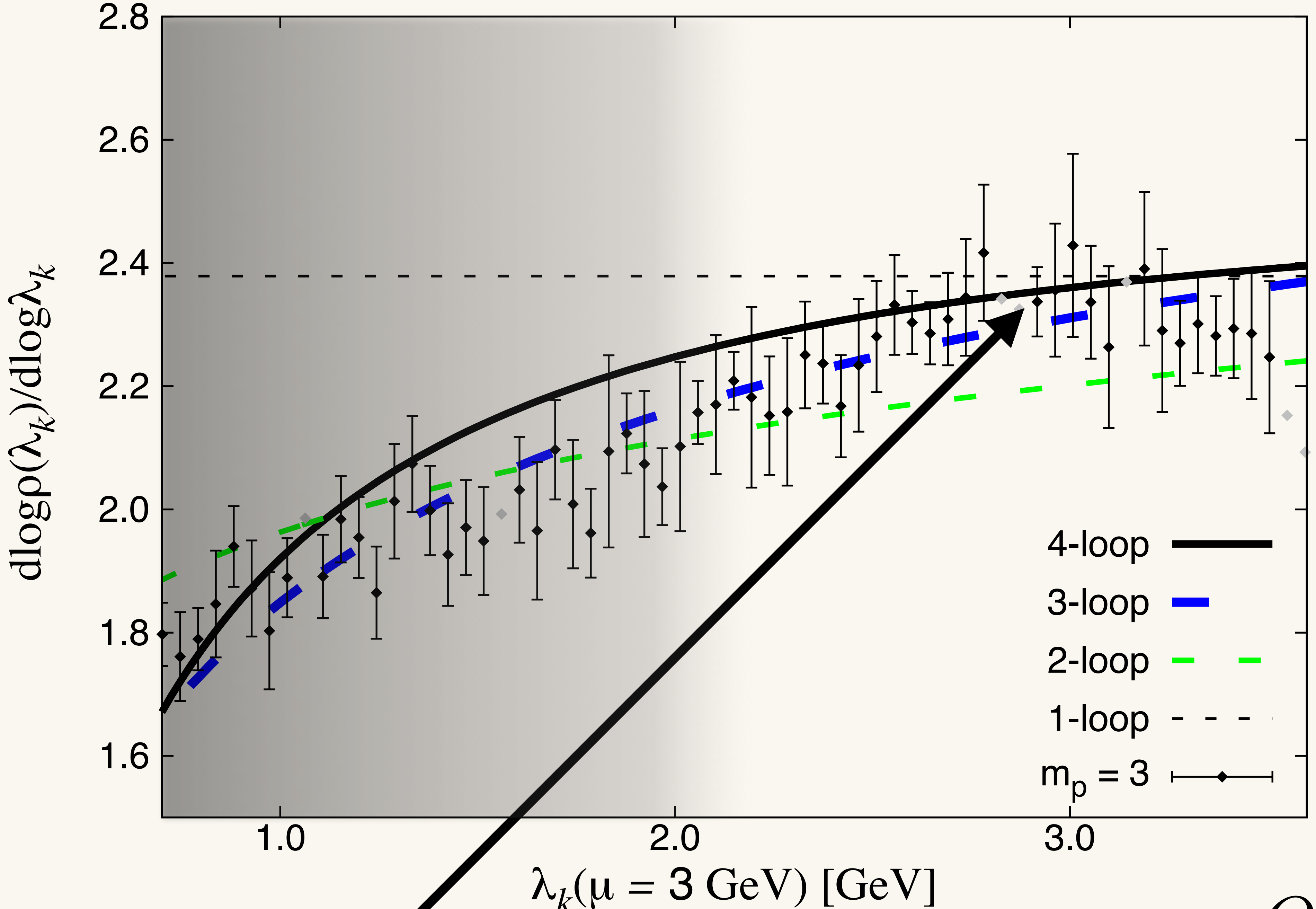
a dependence is reduced with $m_p = 3$



$$a^2\lambda^2(m_p) = \frac{a^2\lambda^2(m_p=1)}{1-(1-1/m_p^2)a^2\lambda^2(m_p=1)}$$

◉ Exponent $\dfrac{\mathrm{dlog}\rho(\lambda)}{\mathrm{dlog}\lambda}$ on the lattice $(a \to 0)$





(e.g.)

$\lambda(\mu=3GeV)$	$\mathrm{dlog}\rho(\lambda)/\mathrm{dlog}\lambda$	$(\mathrm{stat.})(\delta t_0)(\delta Z_m)$	(Δm_p)
2.92 GeV	2.34(6)	(4)(2)(3)	(9)

$O(\alpha_s^4)$ PT ⟺ 2.35

(1) Agreement with $O(\alpha_s^4)$ perturbative calculation at ~4%

(2) Extraction of : $\alpha_s(3\mathrm{GeV}) = 0.253(41)$ ⟺ PDG: $\alpha_s(3\mathrm{GeV}) = 0.244(5)$

$(\lambda = 2.92\mathrm{GeV},$ as an example$)$

◉ Summary

◇ We calculate Dirac spectral density by using a stochastic method, with Mobius domain-wall fermion.

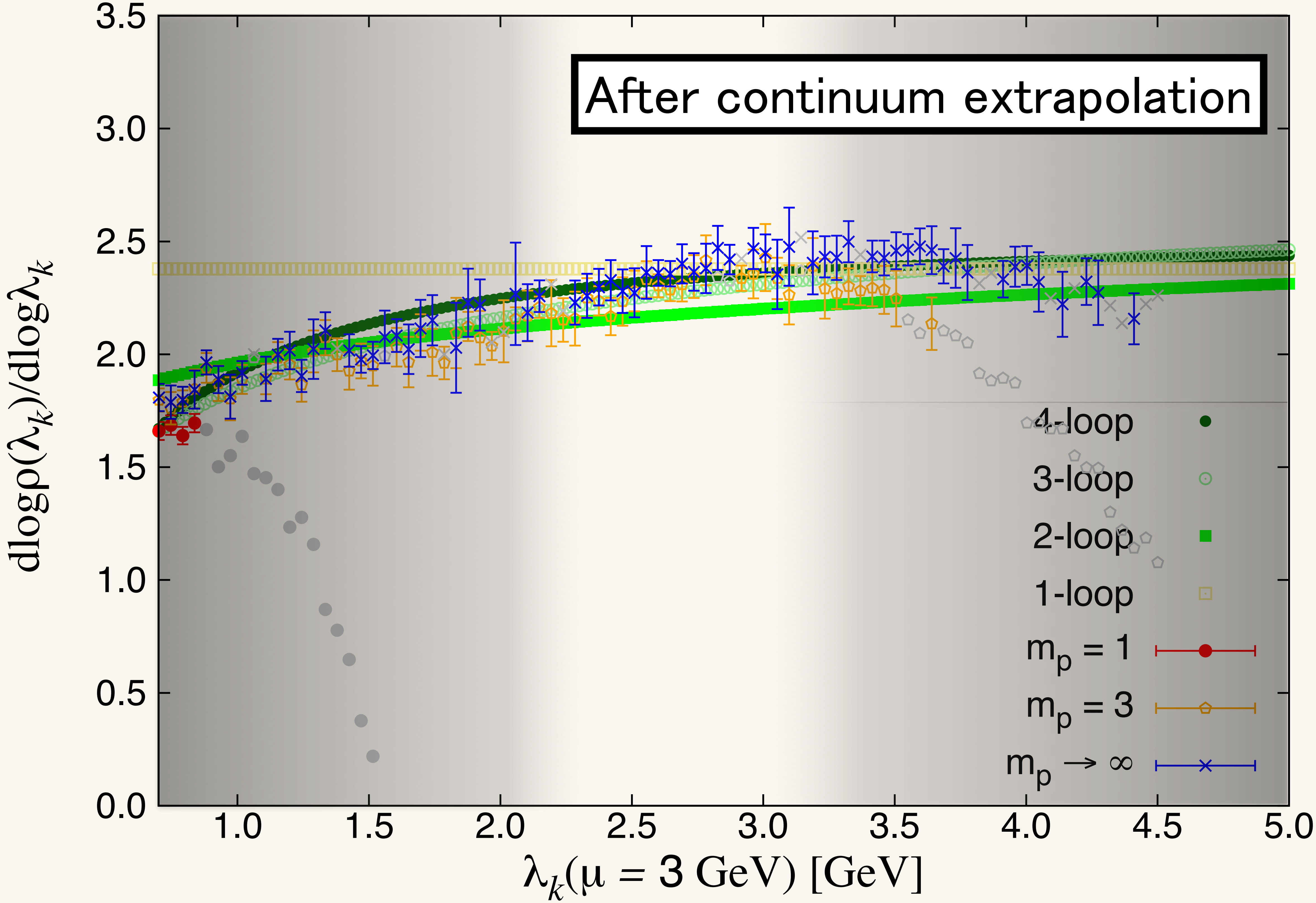
◇ With Pauli-Villars mass $m_p = 3$ the discretization error is reduced without loss of chirality and locality.

◇ Perturbation theory available up to $O(\alpha_s^3)$for $\rho(\lambda)$, $O(\alpha_s^4)$ for its exponent.

◇ Our calculation validate the perturbative calculation with ~5% uncertainty.

Appendix

Exponent $\frac{\mathrm{dlog}\rho(\lambda)}{\mathrm{dlog}\lambda}$ on the lattice $(a \to 0)$



◉ Exponent $\dfrac{\mathrm{dlog}\rho(\lambda)}{\mathrm{dlog}\lambda}$ on the lattice ($a \to 0$)

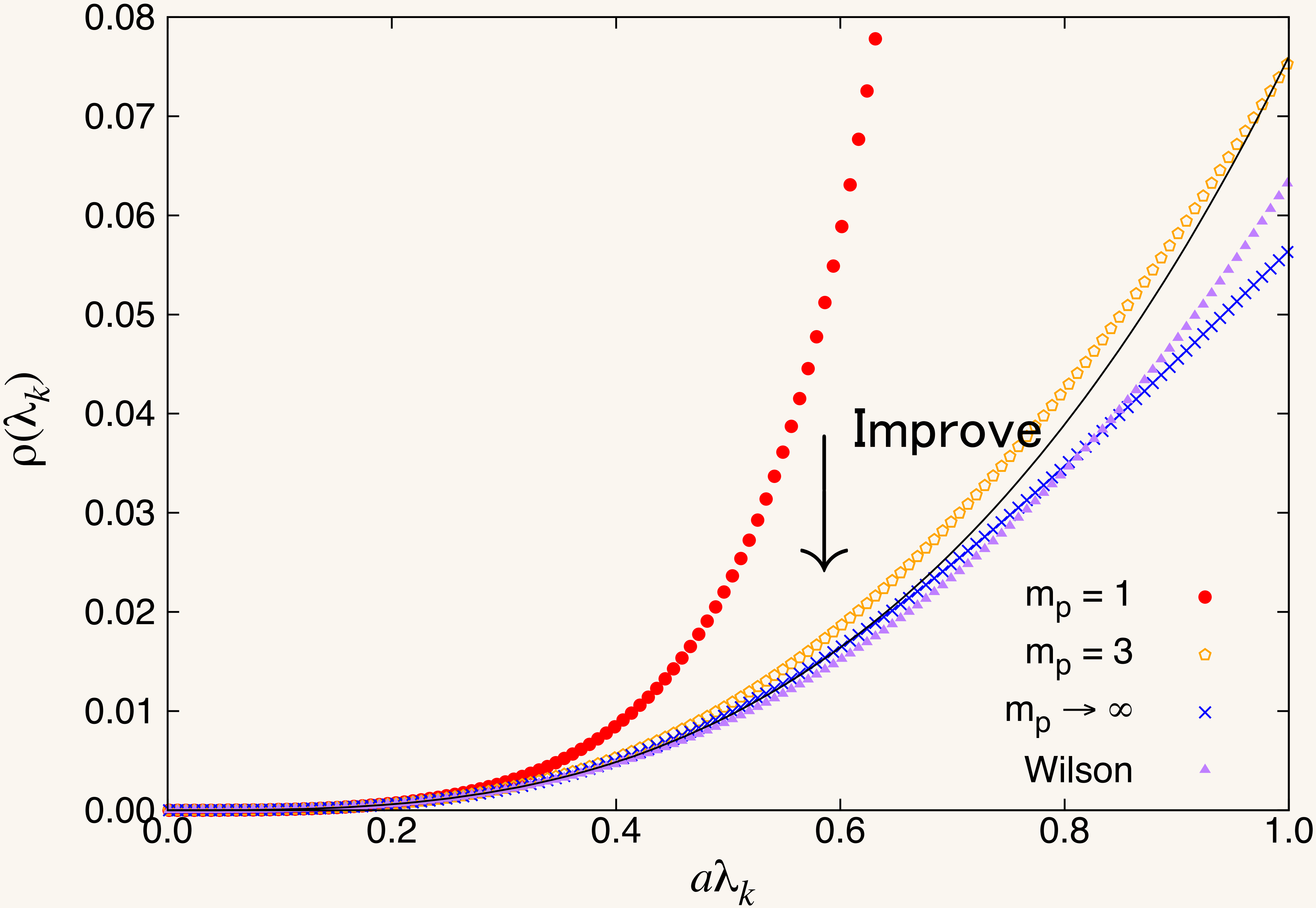
λ(3GeV) [GeV]	Scaling	(stat.)(δt_0)(δZ_m)	(Δm_p)	χ^2/d.o.f.
2.51	2.28(9)	(4)(5)(6)	(1)	0.05
2.56	2.33(8)	(4)(3)(6)	(3)	0.78
2.60	2.30(5)	(4)(3)(1)	(6)	0.99
2.65	2.29(5)	(4)(2)(2)	(7)	0.31
2.69	2.31(8)	(4)(3)(6)	(9)	1.96
2.74	2.34(10)	(4)(8)(2)	(2)	0.65
2.78	2.42(11)	(4)(8)(7)	(4)	0.20
2.92	2.34(6)	(4)(2)(3)	(9)	1.40
2.96	2.36(11)	(4)(10)(1)	(11)	0.69
3.05	2.34(9)	(4)(7)(5)	(2)	0.02

(1) ~5% test with $O(\alpha_s^4)$ perturbation

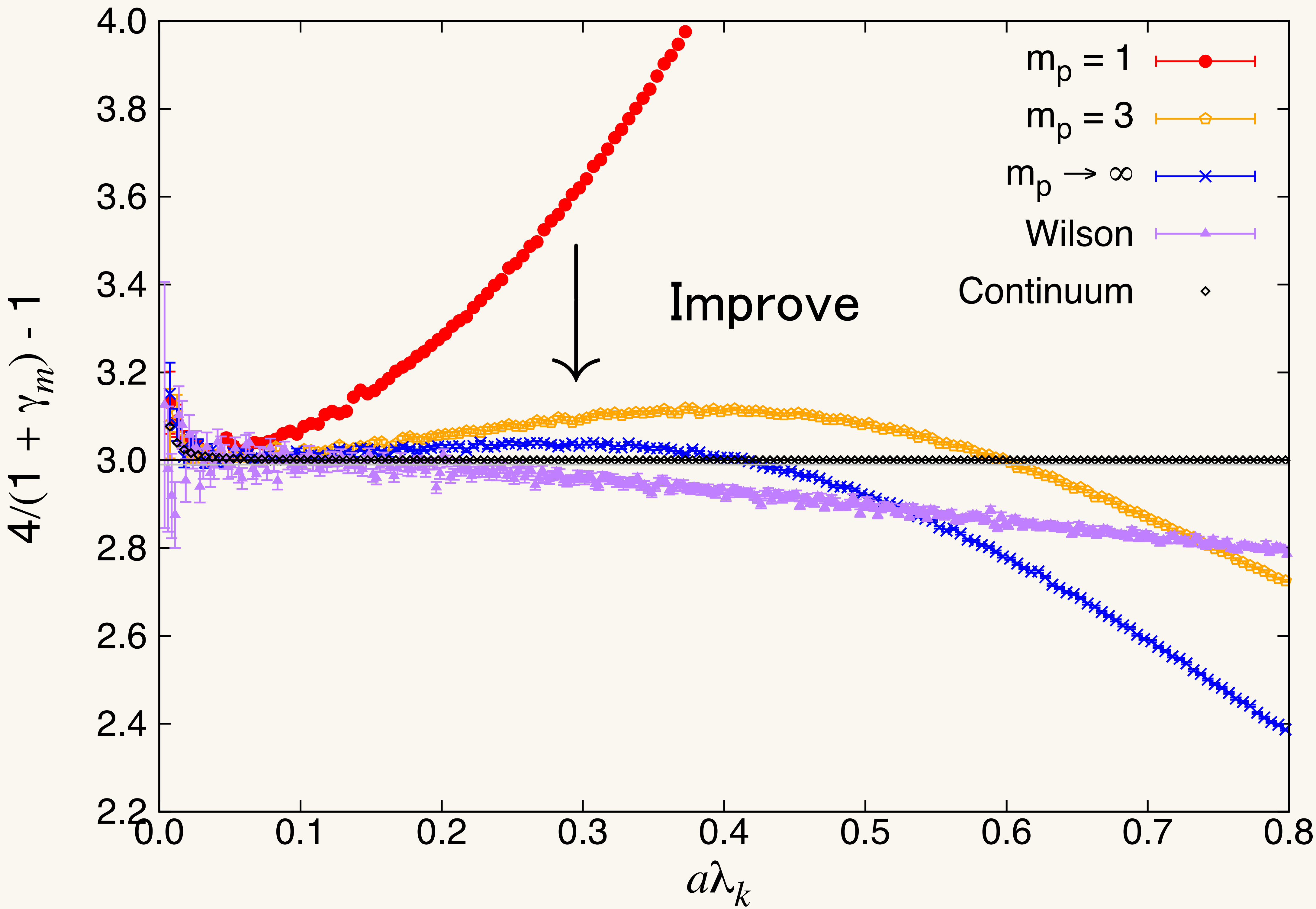
(2) Extraction of : $\alpha_s(3\mathrm{GeV}) = 0.253(41)$ ⬌ PDG: $\alpha_s(3\mathrm{GeV}) = 0.244(5)$

($\lambda = 2.92$GeV, as an example)

◉ Lattice eigenvalue density (Tree level)

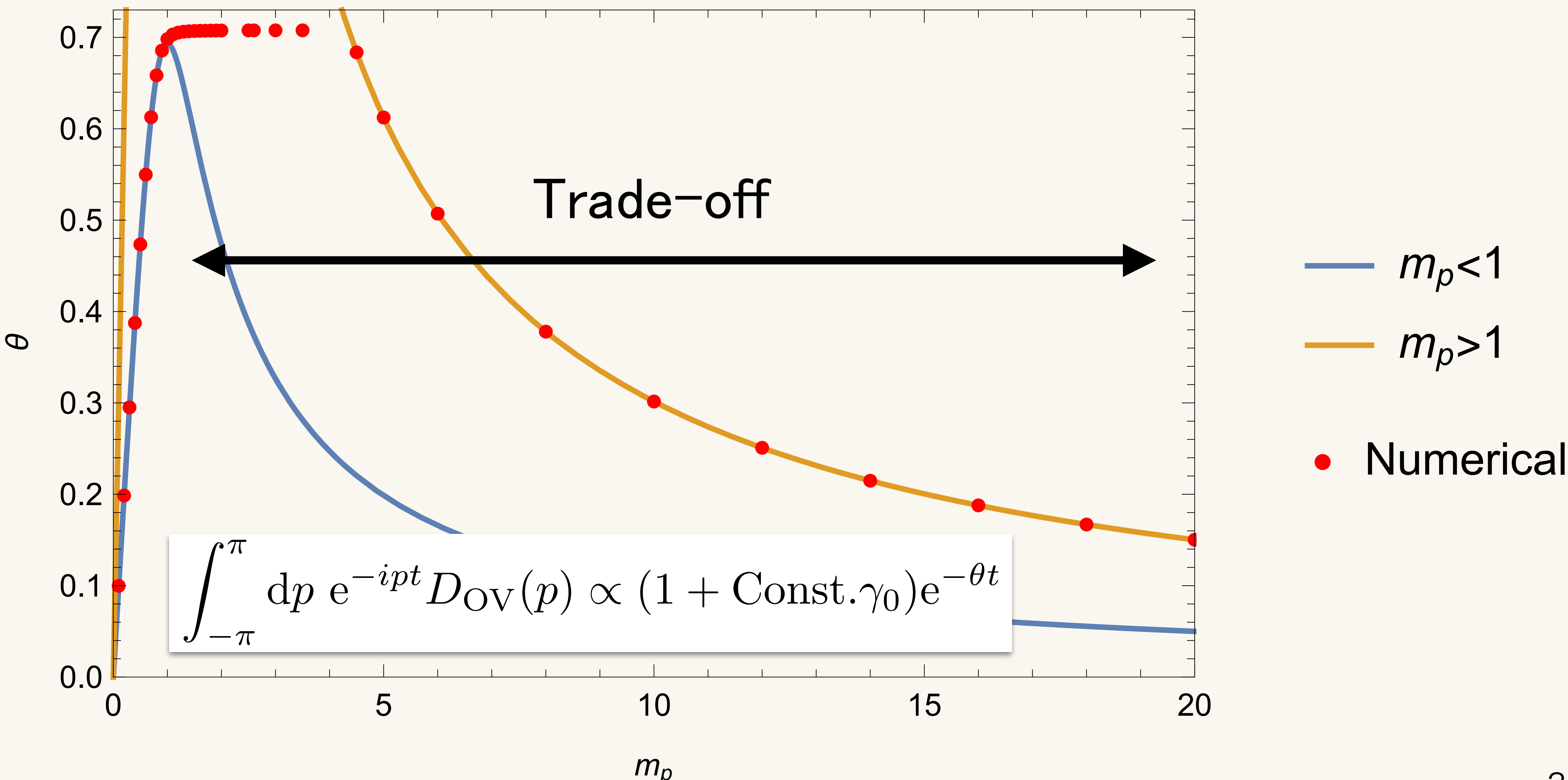


● Lattice power of eigenvalue density (Tree level)



◉ Chirality-Locality Trade-off

$$\{D_{\mathrm{OV}}^{-1}, \gamma_5\} = \frac{2a\gamma_5}{m_p}$$



◉ Chirality-Locality Trade-off

$$\{D_{\rm OV}^{-1}, \gamma_5\} = \frac{2a\gamma_5}{m_p}$$

◇ Complete (Fixed point) fermion

$$\{D_{\rm FP}^{-1}, \gamma_5\} = \frac{2a\gamma_5}{\kappa_f}$$

◉ Overlap fermion by DW fermion

◇ Overlap fermion with Generalized Pauli-Villars mass m_p

$$aD_{\mathrm{OV}} \equiv (2 - cM_0)M_0 m_p \frac{(1+m_f)+(1-m_f)\mathrm{sign}(\gamma_5 D_{\mathrm{Mobius}})}{(1+m_p)+(1-m_p)\mathrm{sign}(\gamma_5 D_{\mathrm{Mobius}})}$$

◇ 4-kind of tunable parameters (M_0, r, c, m_p)

→For more better construnction.

◇ Scale parameter can be determined by continuum limit.

$$\lim_{a \to 0} D_{\mathrm{OV}}^\dagger D_{\mathrm{OV}} = p^2$$

◉ Cutoff scale and fermion mass

◇ Maximum eigenvalue

$$(2 - cM_0)M_0 m_p \quad \rightarrow \text{depend on } m_p$$

◇ Scaled fermion mass m_f^{scaled}

$$aD_{\text{OV}} = \left(ia\not{p} + \underbrace{(2 - cM_0)M_0 m_f}_{m_f^{\text{scaled}}} \right) \left(1 - ia\frac{\not{p}}{2m_p M_0(2 - cM_0)} \right) + O(a^3)$$

→ independent of m_p

◇ Only Pauli-Villas mass can change cutoff scale WITHOUT changing scaled fermion mass.