A new method for the beta function in the chiral symmetry broken phase

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with Lattice Higgs Collaboration:

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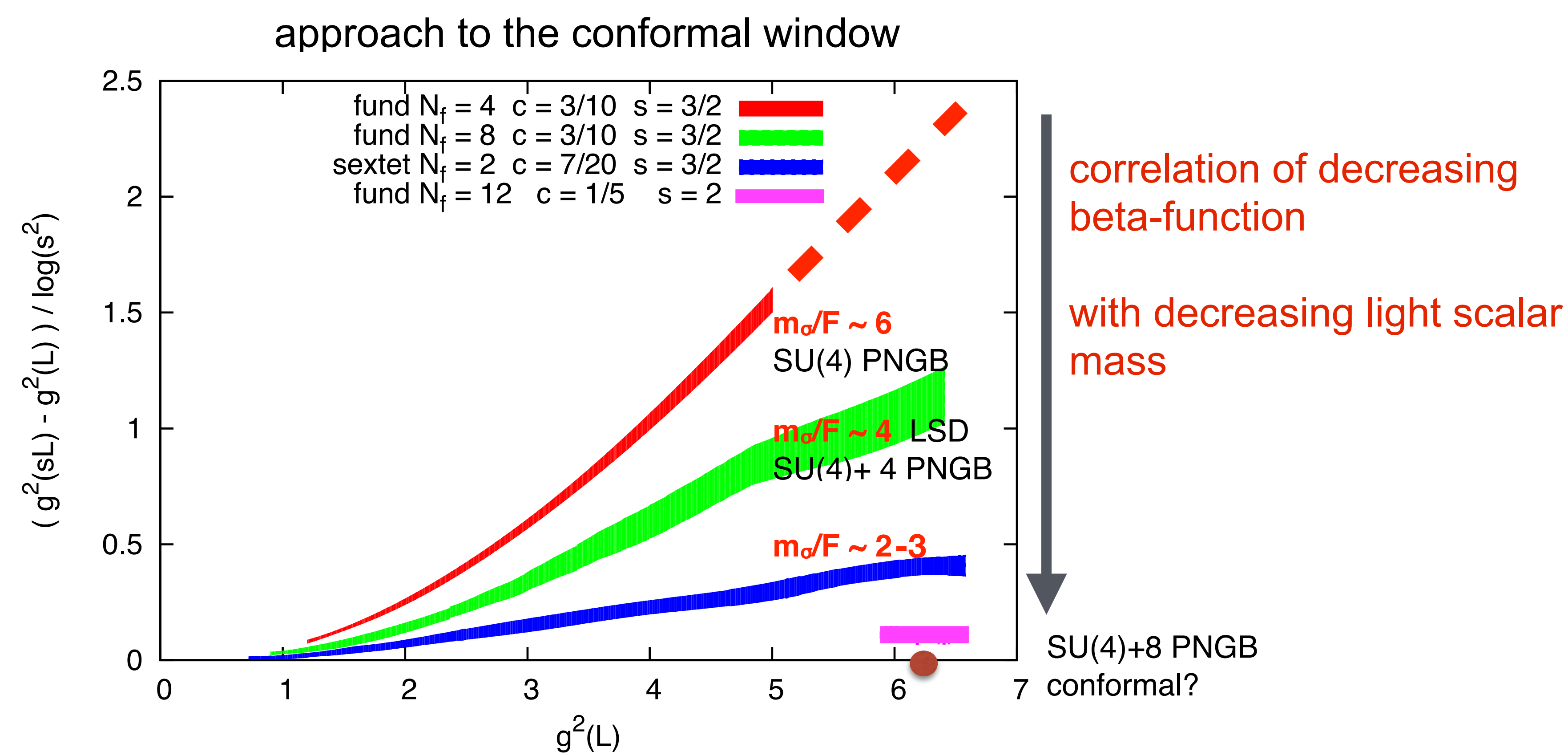
context



lattice simulations dedicated to beta function measurements are feasible but expensive — often work directly at massless fermion limit, with some choice of boundary conditions

complementary work to studies of the p- and/or epsilon-regimes, with their own independent and costly simulations — but does the renormalized coupling reach the relevant range?

could the beta function be measured directly via p-regime simulations?
— “cheap” reusing of the same ensembles
— determination at the relevant renormalized coupling
— consistency with other beta function methods

Yes: will show results for the sextet SU(3) model at $g^2 = 6.7$

gradient flow of gauge fields *Lüscher arXiv:1006.4518, Narayanan & Neuberger hep-th/0601210*

$$\frac{dA_\mu}{dt} = D_\nu F_{\nu\mu}$$ flow "time" t $$A_\mu(t), \quad E = \frac{1}{4}(F^a_{\mu\nu})^2$$

perturbation theory $$\overline{\mathrm{MS}} : \langle E \rangle = \frac{3(N^2-1)g^2}{128\pi^2 t^2}\{1 + \bar{c}_1 g^2 + \mathcal{O}(g^4)\}$$ $\mathrm{SU(N)}$

RG scale $\mu = 1/\sqrt{8t}$

non-perturbative definition $$g^2(t) \equiv \frac{1}{\mathcal{N}}\left(\frac{128\pi^2}{3(N^2-1)}\right) t^2 \langle E \rangle_{\mathrm{latt}}$$ normalization factor $\mathcal{N}$
e.g. boundary conditions

goal: determine $\mu^2 \dfrac{dg^2}{d\mu^2}$ at targeted values of renormalized coupling g^2

finite or infinite volume? infinitesimal or discrete step? $g^2(t') - g^2(t)$

one approach: **step scaling** use finite volume to your advantage

physical volume L lattice volume L/a

adjust flow time to lattice volume such that $c = \sqrt{8t}/L$ held fixed

RG scale is now $\mu = 1/(cL)$

in practice e.g. target $g_c^2(L/a) = 6, \quad c = 0.25$ directly at zero fermion mass

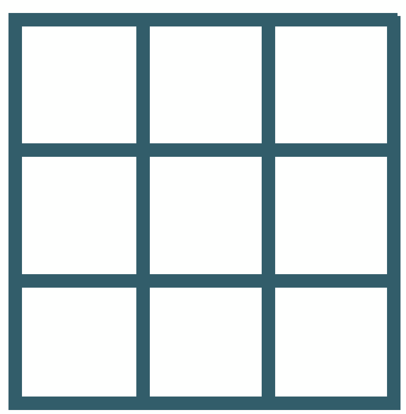
choose lattice volume e.g. $L/a = 16$ adjust bare coupling (lattice spacing) until target reached - tuning

next: keep bare coupling (lattice spacing) fixed, change lattice volume,
c held fixed i.e. change RG scale

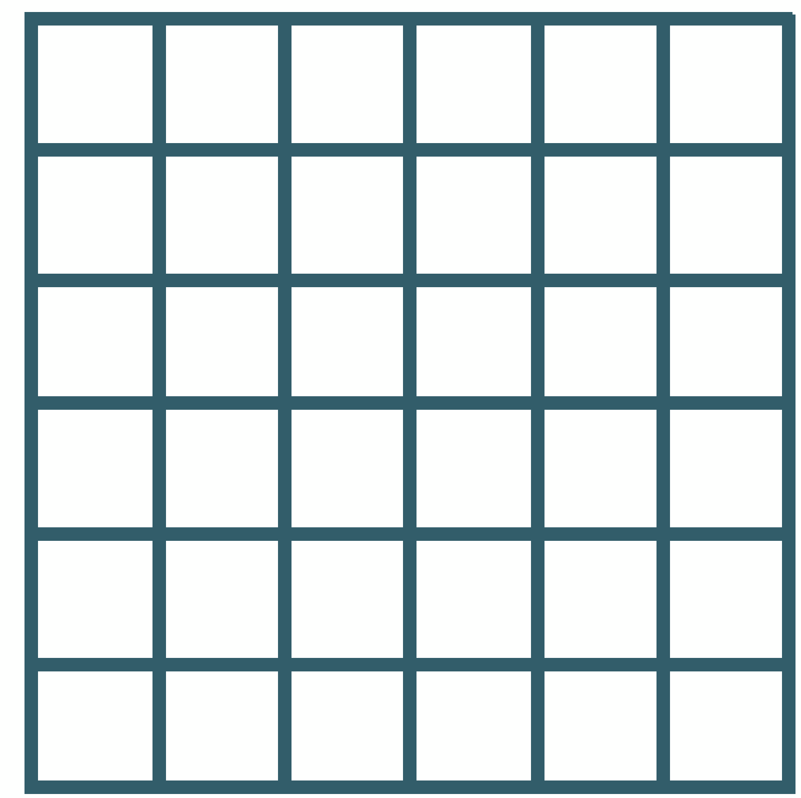
$$\beta(g_c^2) = \frac{g_c^2(sL/a) - g_c^2(L/a)}{\log(s^2)}$$

e.g. $s = 2$

$g_c^2(L/a)$



$g_c^2(2L/a)$



repeat for sequence of lattice volumes e.g.

$$L/a = 16, 18, 20, 24, 28 \longrightarrow 2L/a = 32, 36, 40, 48, 56$$

tune bare coupling such that each satisfies $g_c^2(L/a) = 6, \quad c = 0.25$

continuum limit of $\beta(g_c^2)$ taking $a/L \to 0$

ESSENTIAL AND EXPENSIVE

alternative: work in infinite volume limit, at non-zero fermion mass

control of flow: can measure directly not only $g^2(t)$ but also $dg^2(t)/dt$

goal: measure $t \cdot dg^2/dt = -\mu^2 \cdot dg^2/d\mu^2$ on p-regime ensembles at targeted values of g^2

will require infinite volume extrapolation, followed by chiral extrapolation — have theoretical guidance

continuum limit: lattice scale t_0/a^2 set by fixed choice of renormalized coupling e.g.

$g^2(L/a, t_0/a^2) = 6.7$ *this does not correspond to Lüscher choice* $t^2\langle E\rangle|_{t_0} = 0.3$

once t_0/a^2 known, measure $\beta(g^2) = t \cdot (dg^2/dt)|_{t_0}$

infinite volume and chiral limit extrapolations required for both t_0/a^2 and β

continuum limit: $a^2/t_0 \to 0$

benefits:
— gauge configuration ensembles already available from extensive mass spectrum simulations
— the possible renormalized coupling values are directly related to the correct phase of the theory
— complements step-scaling method of measuring the beta function

will show this for the two flavor two-index symmetric (sextet) SU(3) model **composite Higgs BSM**

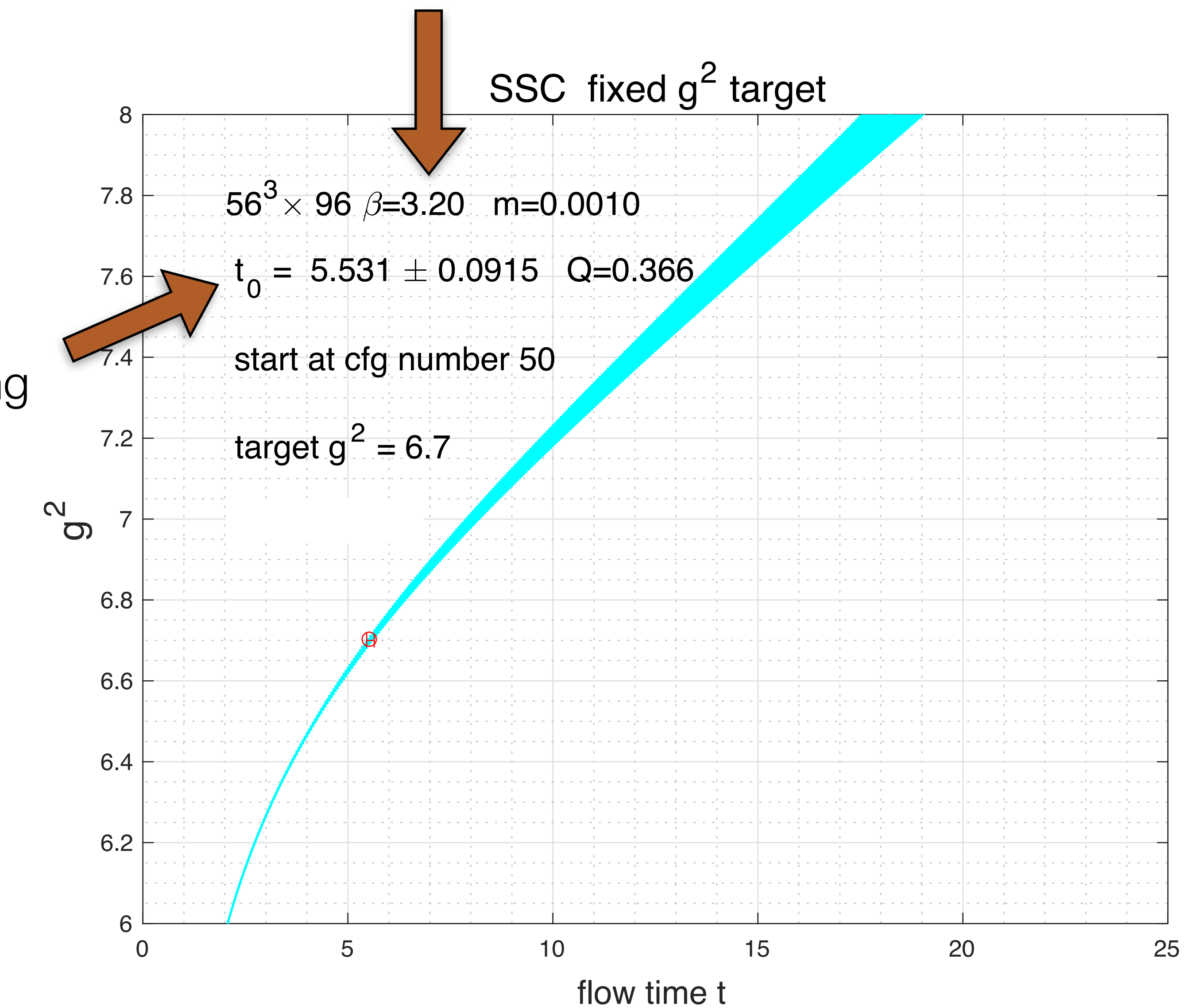
method is general, can apply to other gauge theories

target: $g^2(L/a, t_0/a^2) = 6.7$

must be attainable across all ensembles

bare coupling $\frac{6}{g_0^2} = 3.20$ coarsest lattice spacing

SSC:
— simulation gauge action: Symanzik
— flow gauge action: Symanzik
— discretization of E : Clover



staggered fermions, 2 steps of stout improvement

for this particular ensemble: $56^3 \times 96, \; ma = 0.001$

$$t_0/a^2 = 5.531 \pm 0.0915$$

error in scale determination increases along flow, but don't want flow too short either - cutoff effects same ensemble: measure derivative at $t_0/a^2 = 5.531 \pm 0.0915$

approximate derivative

$$\frac{1}{12\epsilon}\{-F(t+2\epsilon)+8F(t+\epsilon)-8F(t-\epsilon)+F(t-2\epsilon)\} = \frac{dF}{dt}|_t + \mathcal{O}(\epsilon)^4$$

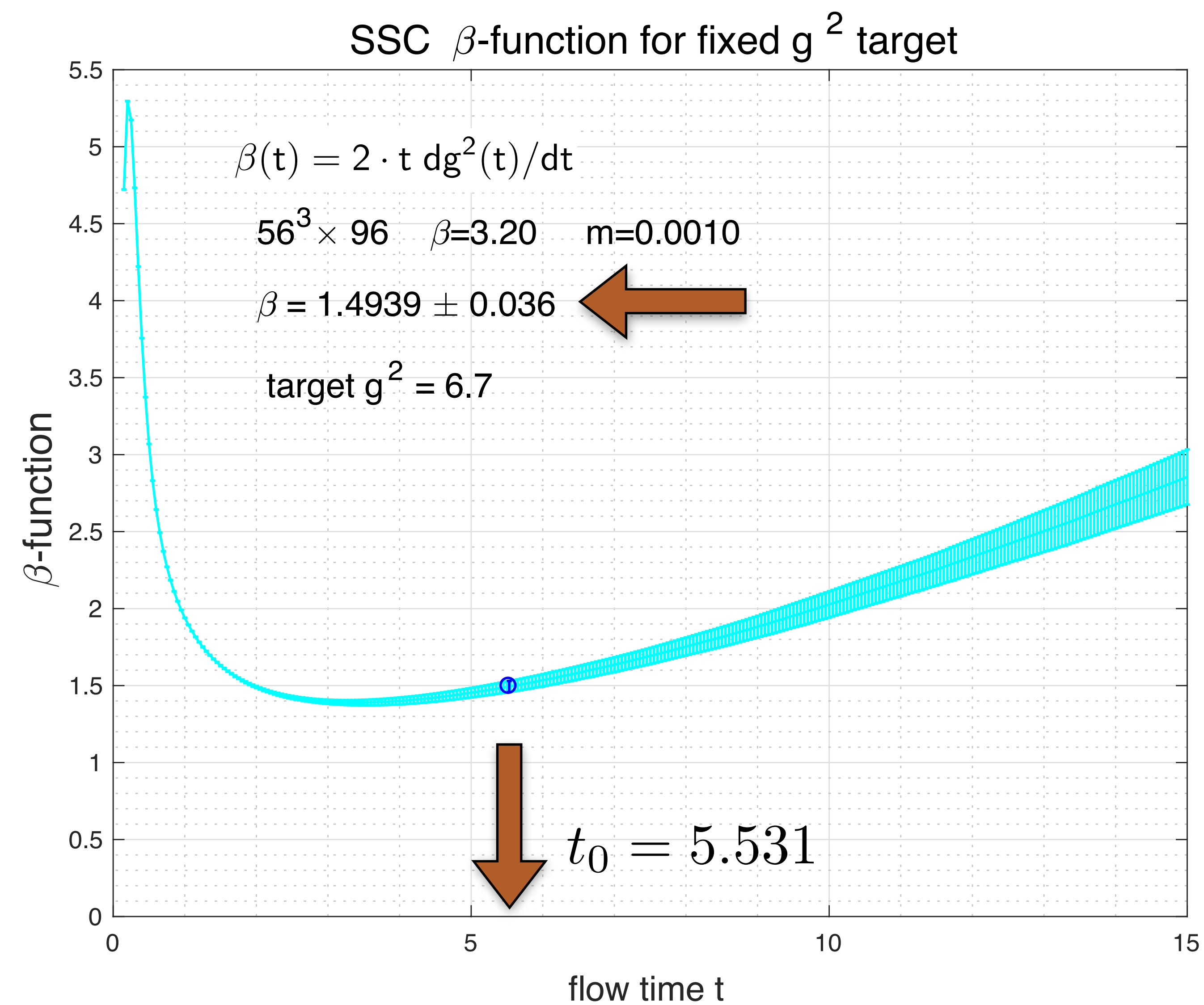
same ensemble as before:

$$56^3 \times 96,\ ma = 0.001$$

accurately determined

$$2t\left(\frac{dg^2}{dt}\right)|_{t_0} = 1.4939 \pm 0.036$$

repeat for a sequence of ensembles at the same lattice spacing, and the same fermion mass, but different lattice volumes



infinite volume limit for Goldstone boson at this lattice spacing and fermion mass

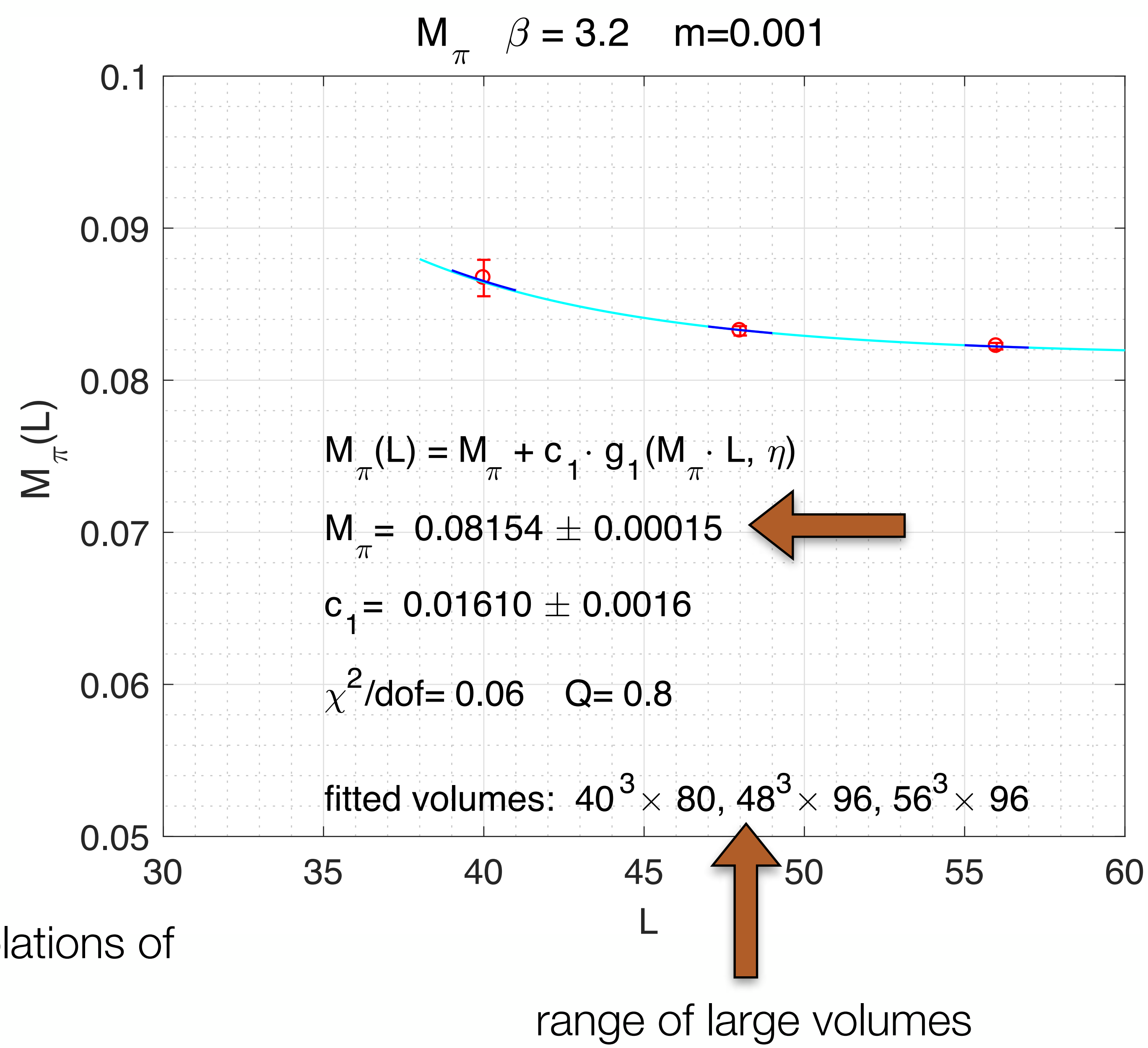
finite-volume corrections from Goldstone boson wrapping around the finite lattice

functional form: $g_1(M_\pi \cdot L, \eta)$

infinite sum of Bessel functions with aspect ratio $\eta = L_t/L_s$

result: infinite volume mass

$$M_\pi \cdot a = 0.08154 \pm 0.00015$$

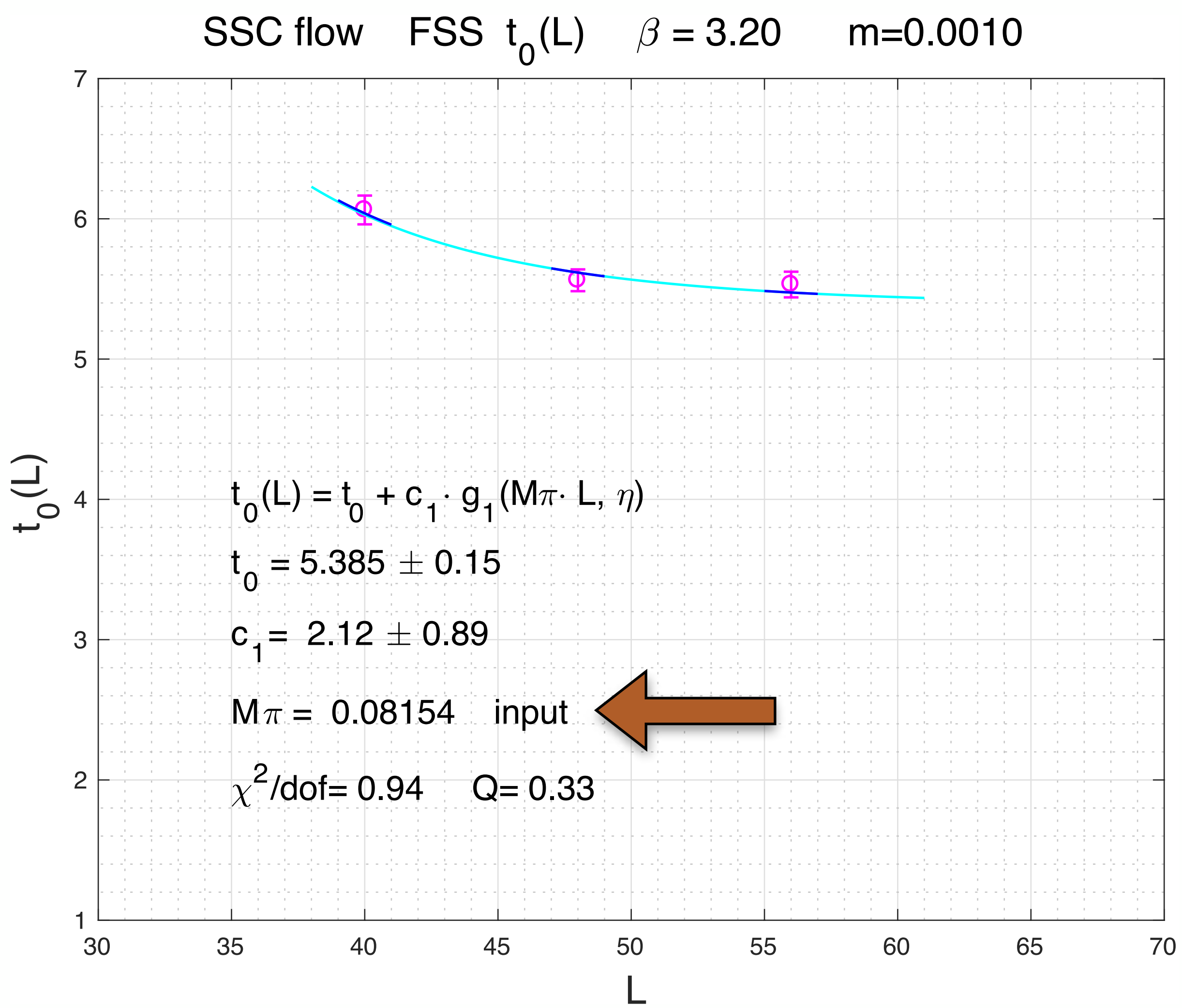


this will be used as input for infinite volume extrapolations of

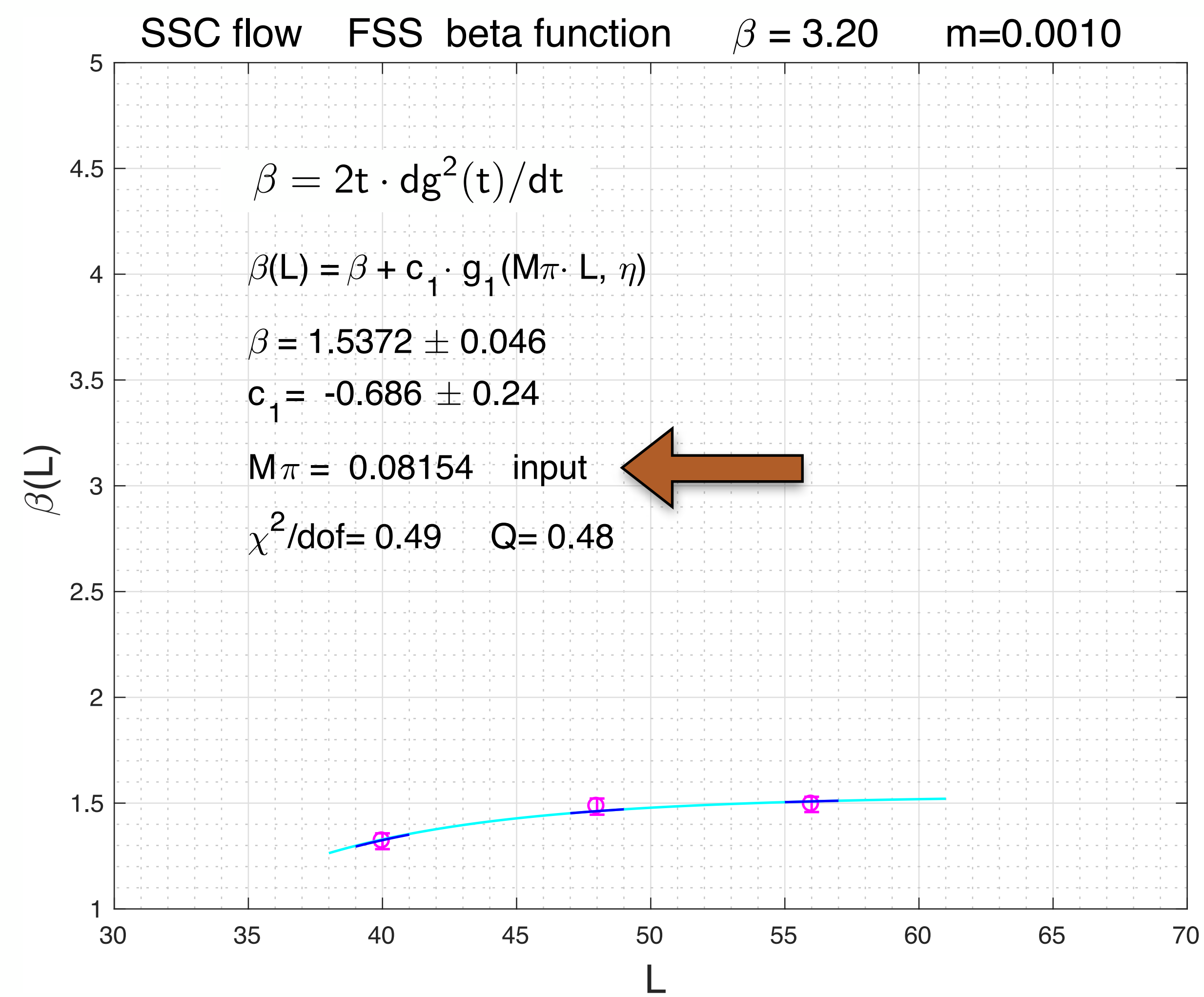
t_0/a^2 and $t \cdot (dg^2/dt)|_{t_0}$

want footprint of flow smaller than Goldstone boson correlation length $\sqrt{8t_0}/a \ll 1/(M_\pi \cdot a)$

influences choice of target $g^2(L/a, t_0/a^2)$



scale t_0/a^2



beta function $\beta = 2t \cdot (dg^2/dt)|_{t_0}$

same functional form $g_1(M_\pi \cdot L, \eta)$

gives infinite volume values at one lattice spacing and one fermion mass

next step: repeat at same lattice spacing and several fermion masses

generate a set of $t_0(M_\pi^2)$ and corresponding $\beta(M_\pi^2)$

how to extrapolate to zero Goldstone boson mass?

Bär & Golterman, arXiv:1312.4999

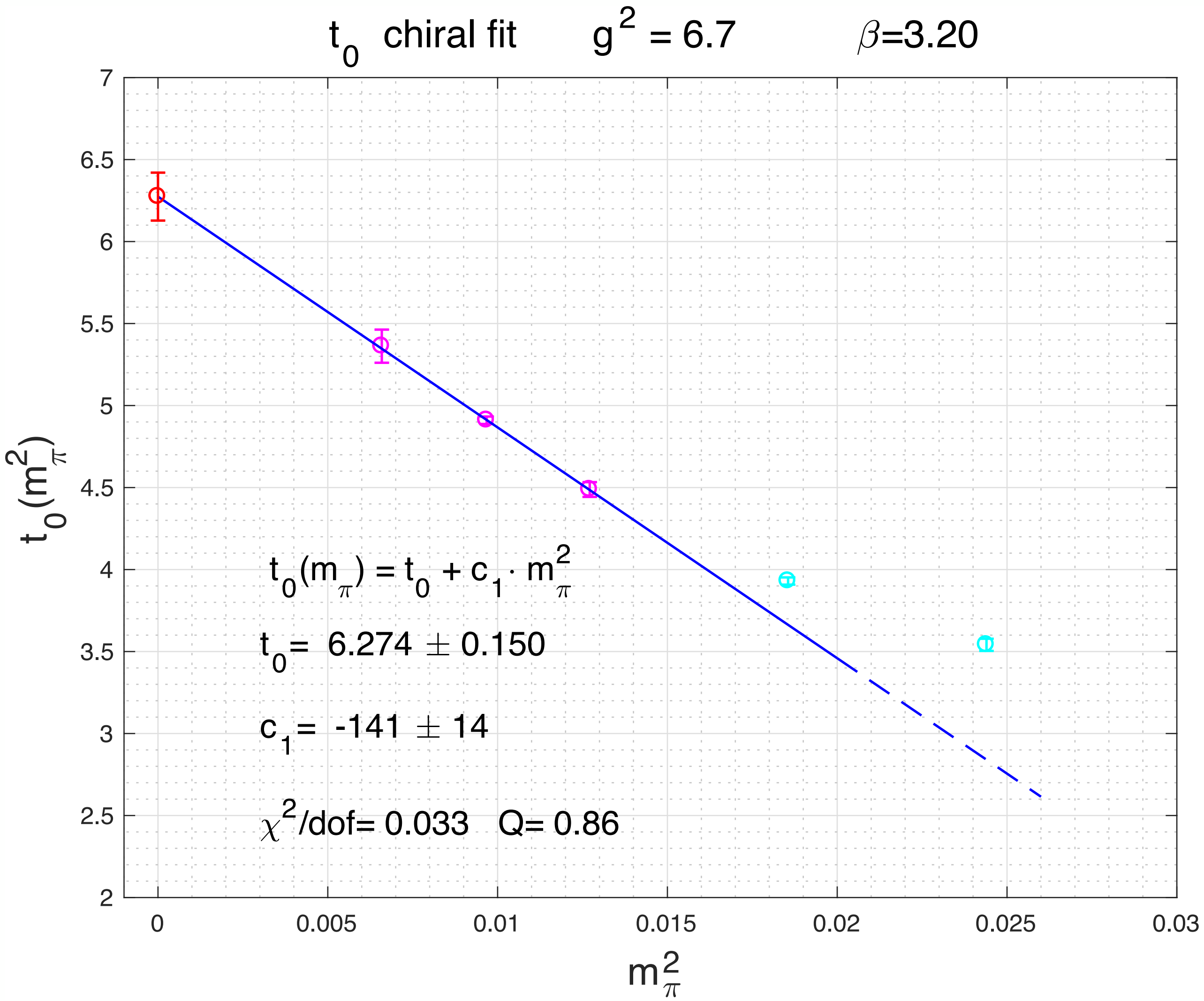
chiral expansion for quantities related to gradient flow, such as the scale

$$t_0 = t_{0,\mathrm{ch}} \left(1 + k_1 \frac{M_\pi^2}{(4\pi f)^2} + k_2 \frac{M_\pi^4}{(4\pi f)^4} \log\left(\frac{M_\pi^2}{\mu^2}\right) + k_3 \frac{M_\pi^4}{(4\pi f)^4} \right)$$

expected next-to-leading order term is linear in M_π^2

look for this behavior in infinite-volume results

we do not attempt to fit chiral logarithms or $\mathcal{O}(M_\pi^4)$ terms



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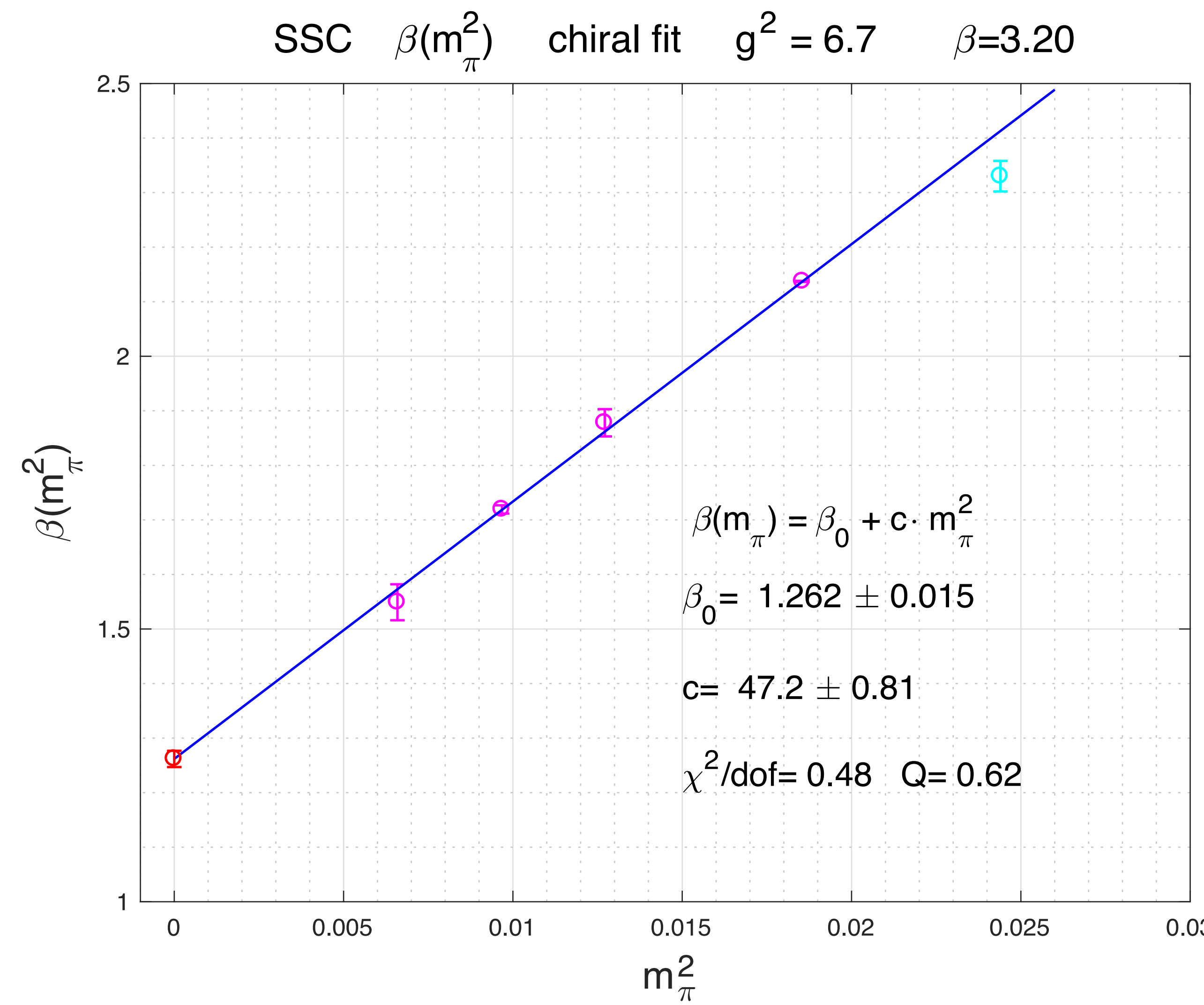
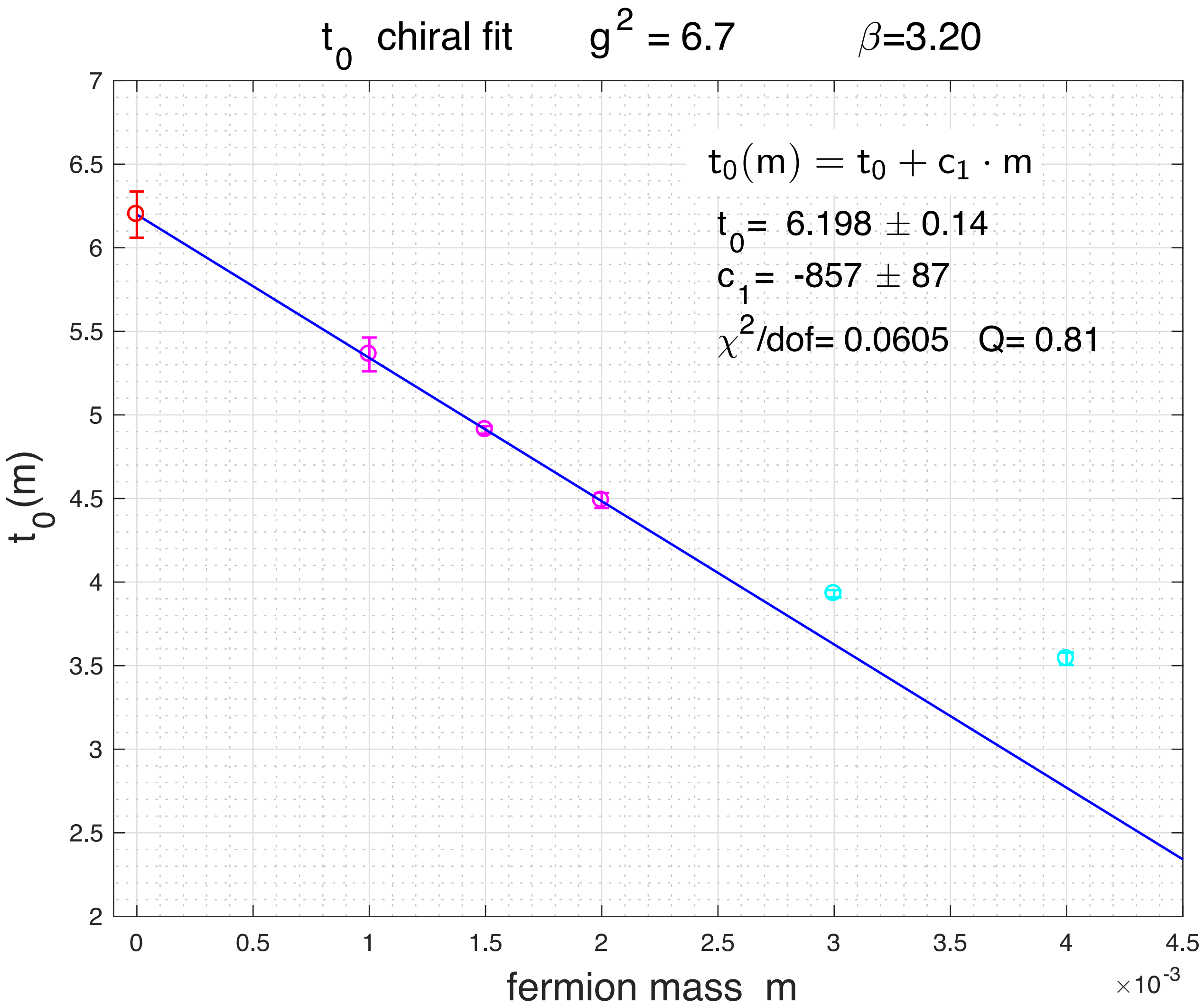


beta function

linear in M_π^2 behavior visible in both $t_0(M_\pi^2)$ and $\beta(M_\pi^2)$ at the lightest masses

obtain chiral limit values at one lattice spacing — repeat entire procedure at other lattice spacings with same target $g^2(L/a, t_0/a^2) = 6.7$

results above from the coarsest lattice spacing



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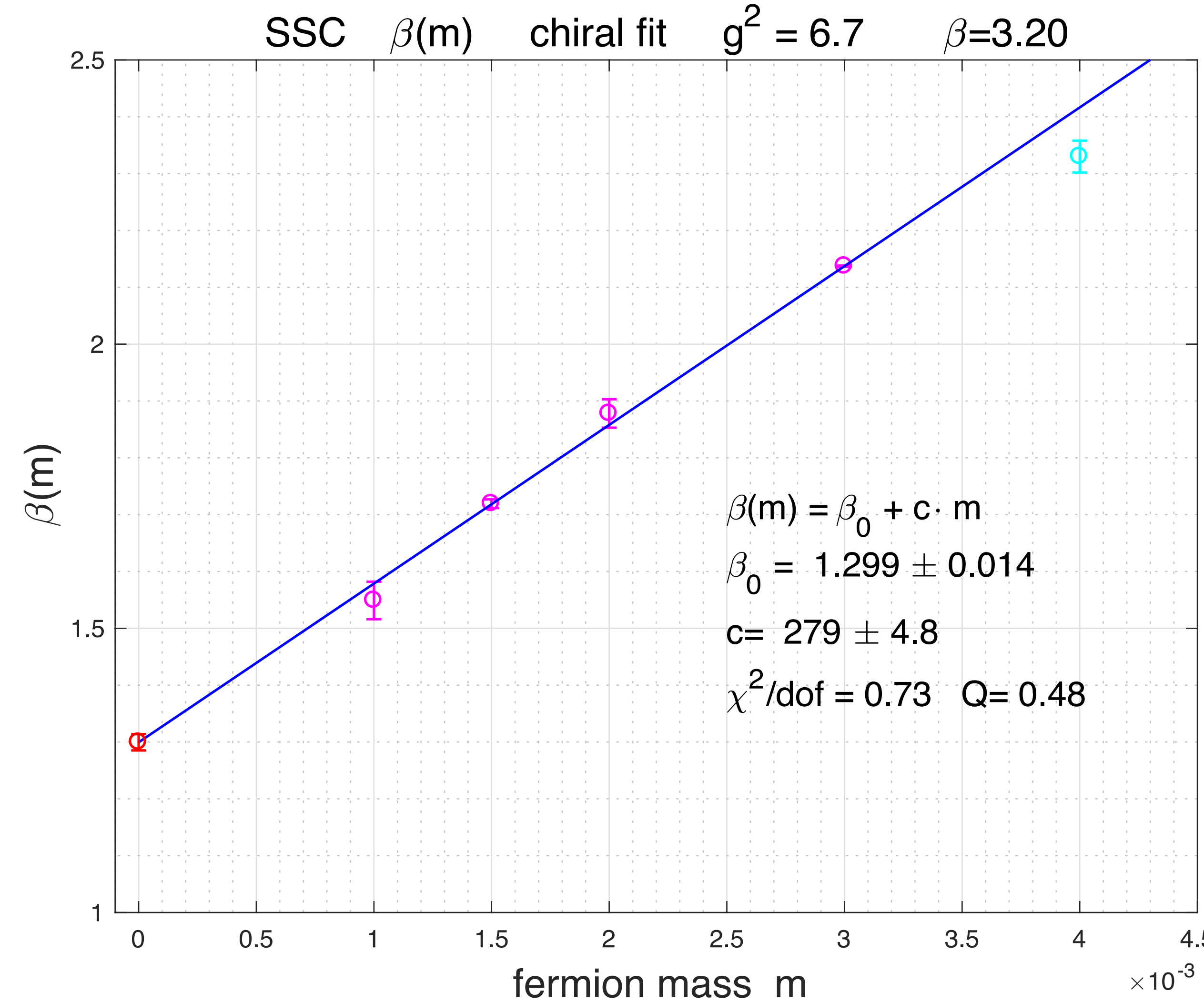


beta function

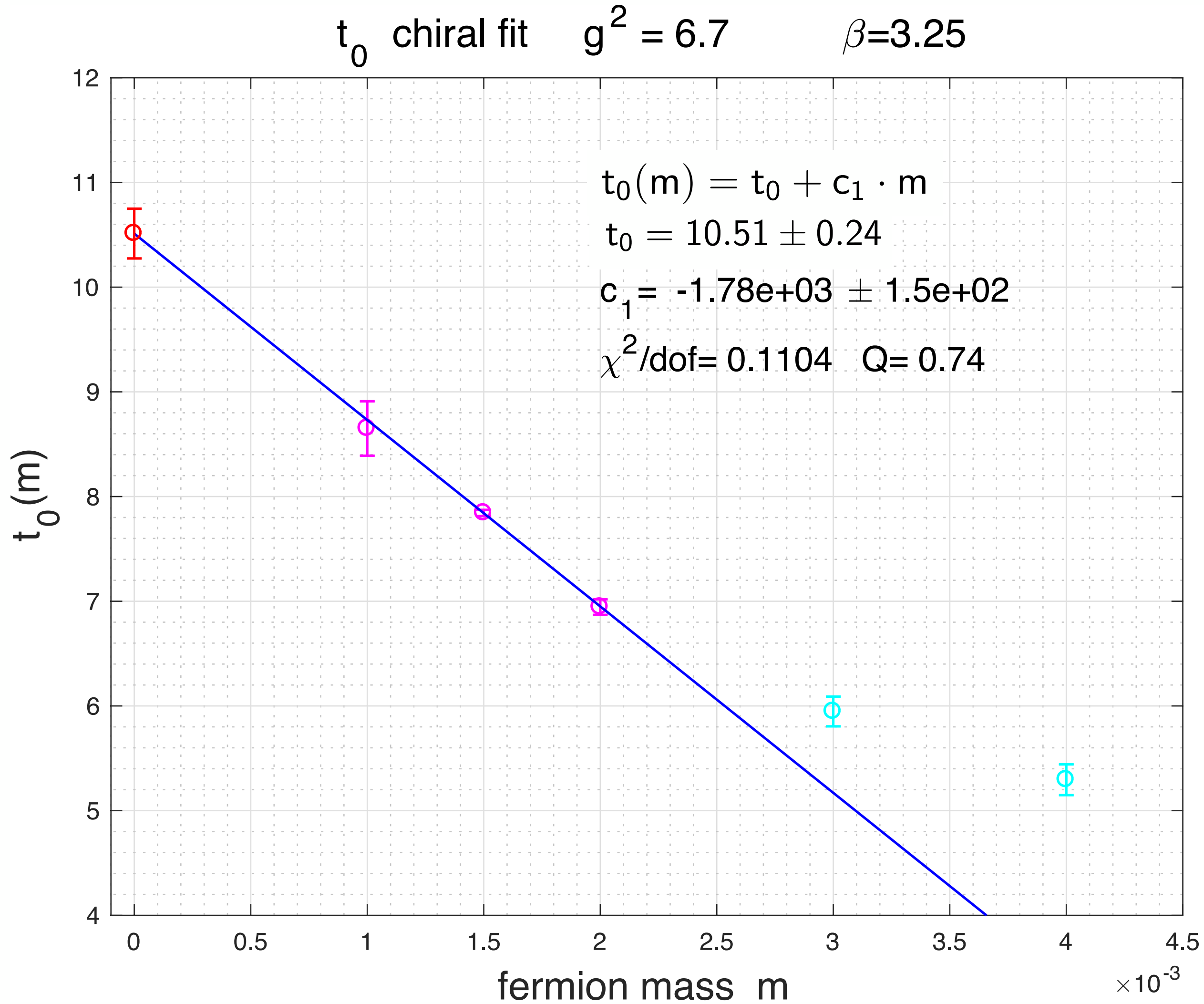
alternative: chiral extrapolation directly in fermion mass

linear in M_π^2 corresponds to linear in m

above: results at same coarsest lattice spacing

also a good description of the data

rest of talk: will use this fitting method



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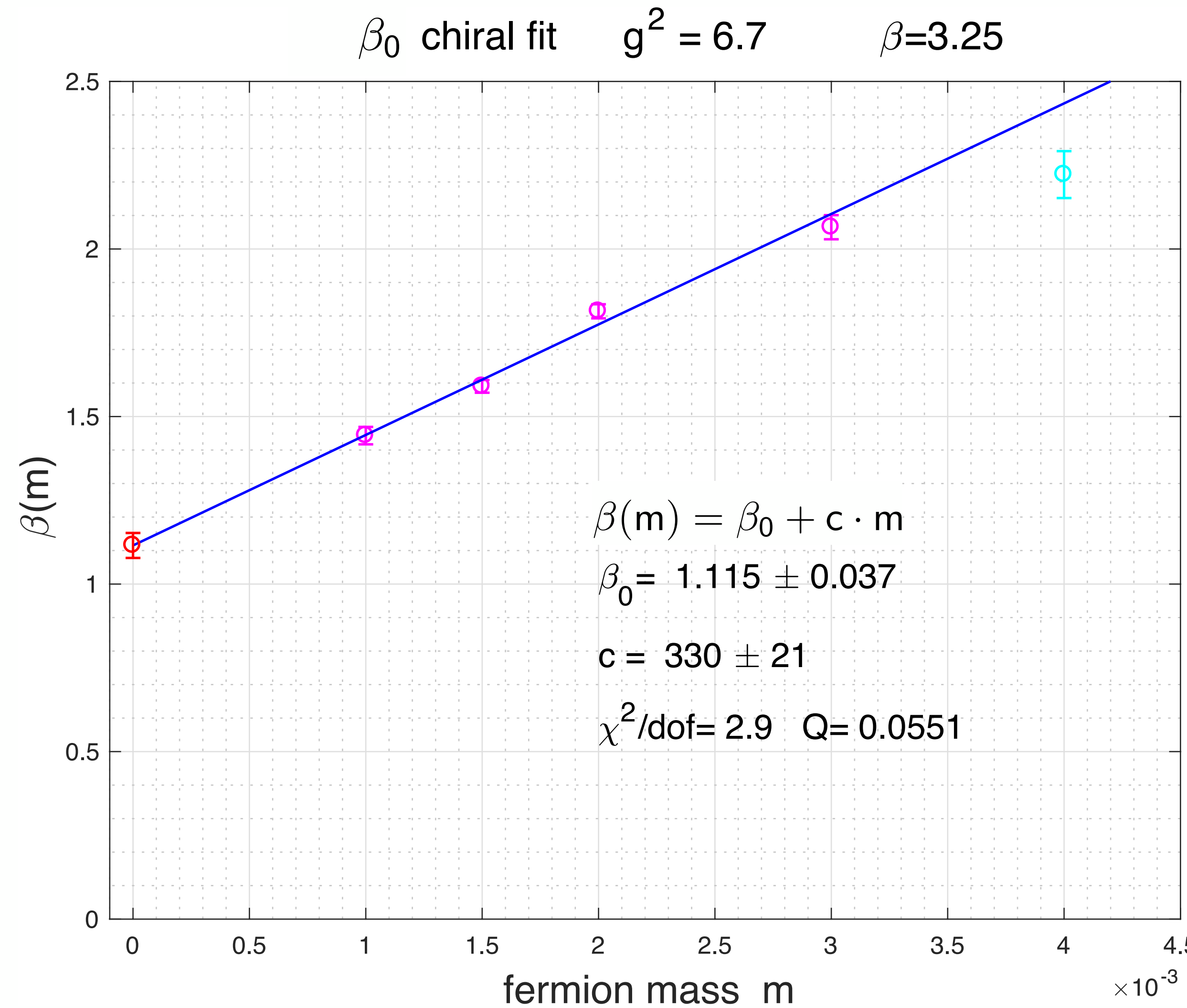


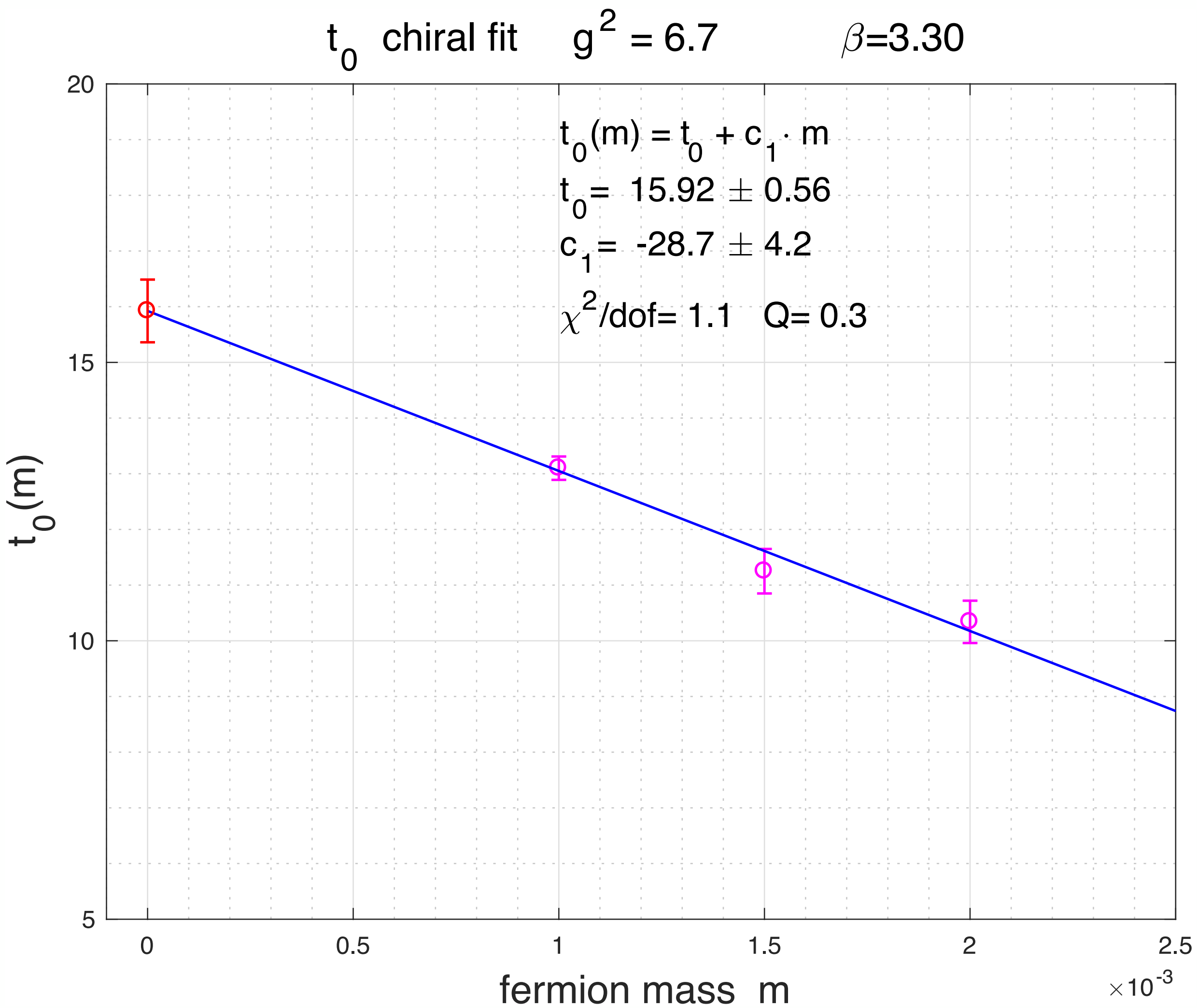
beta function

these results correspond to a bare coupling $\frac{6}{g_0^2} = 3.25$ intermediate lattice spacing

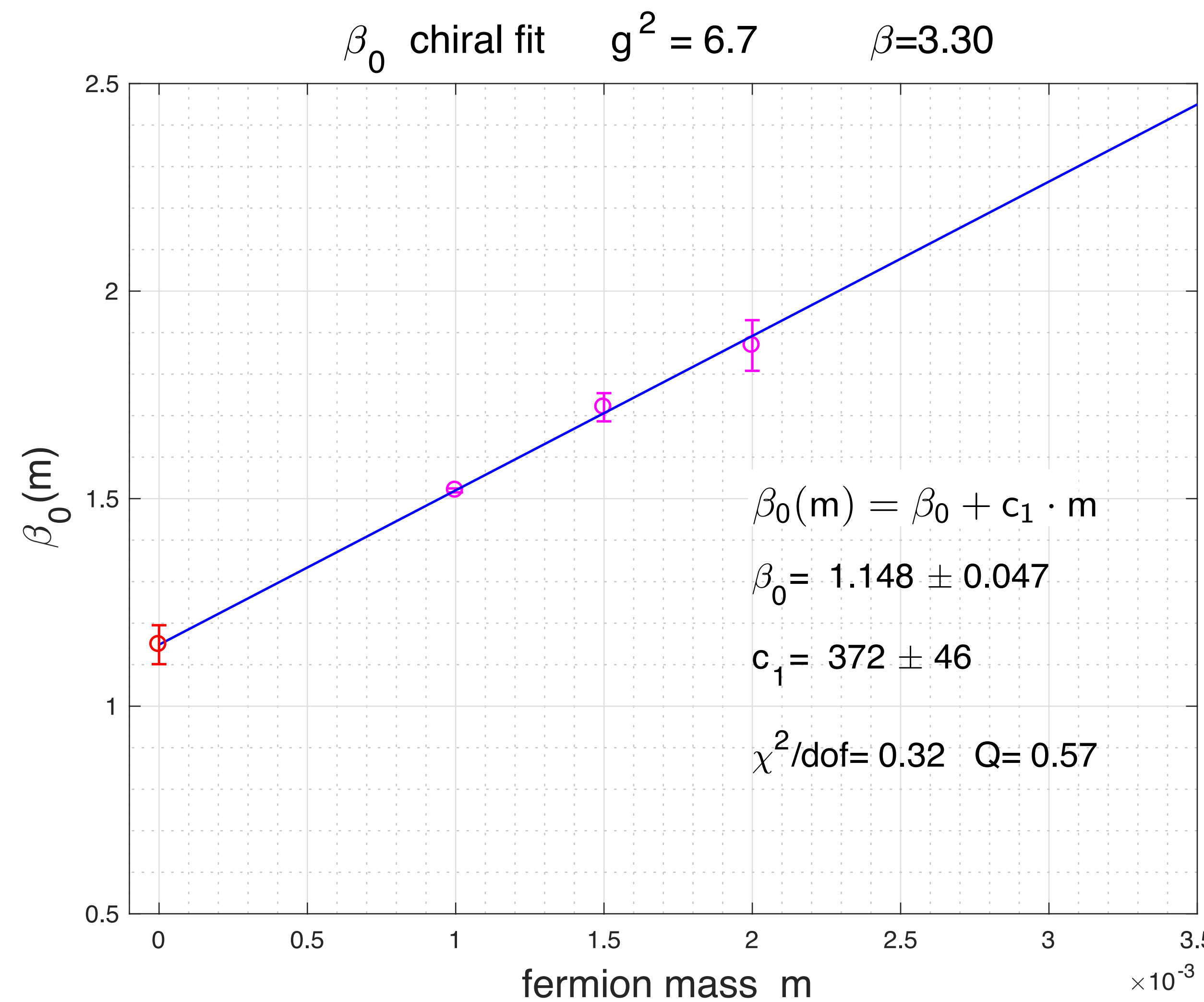
lattice volumes: $32^3 \times 64, \;\; 40^3 \times 80, \;\; 48^3 \times 96, \;\; 56^3 \times 96, \;\; 64^3 \times 96$

larger value of t_0/a^2 in chiral limit corresponding to smaller lattice spacing

~ 15% change in chiral limit of $\beta(\mathrm{m})$ compared to coarser lattice spacing — modest cutoff effect



scale



beta function

third set of results corresponding to bare coupling $\frac{6}{g_0^2} = 3.30$ finest lattice spacing

lattice volumes: $40^3 \times 80, \ 48^3 \times 96, \ 56^3 \times 96, \ 64^3 \times 96$

from chiral limit scale t_0/a^2 lattice spacing changes by factor ~1.6 from coarsest to finest ensembles

at each lattice spacing, repeated the full procedure: infinite volume limit, then chiral extrapolation last step: continuum extrapolation

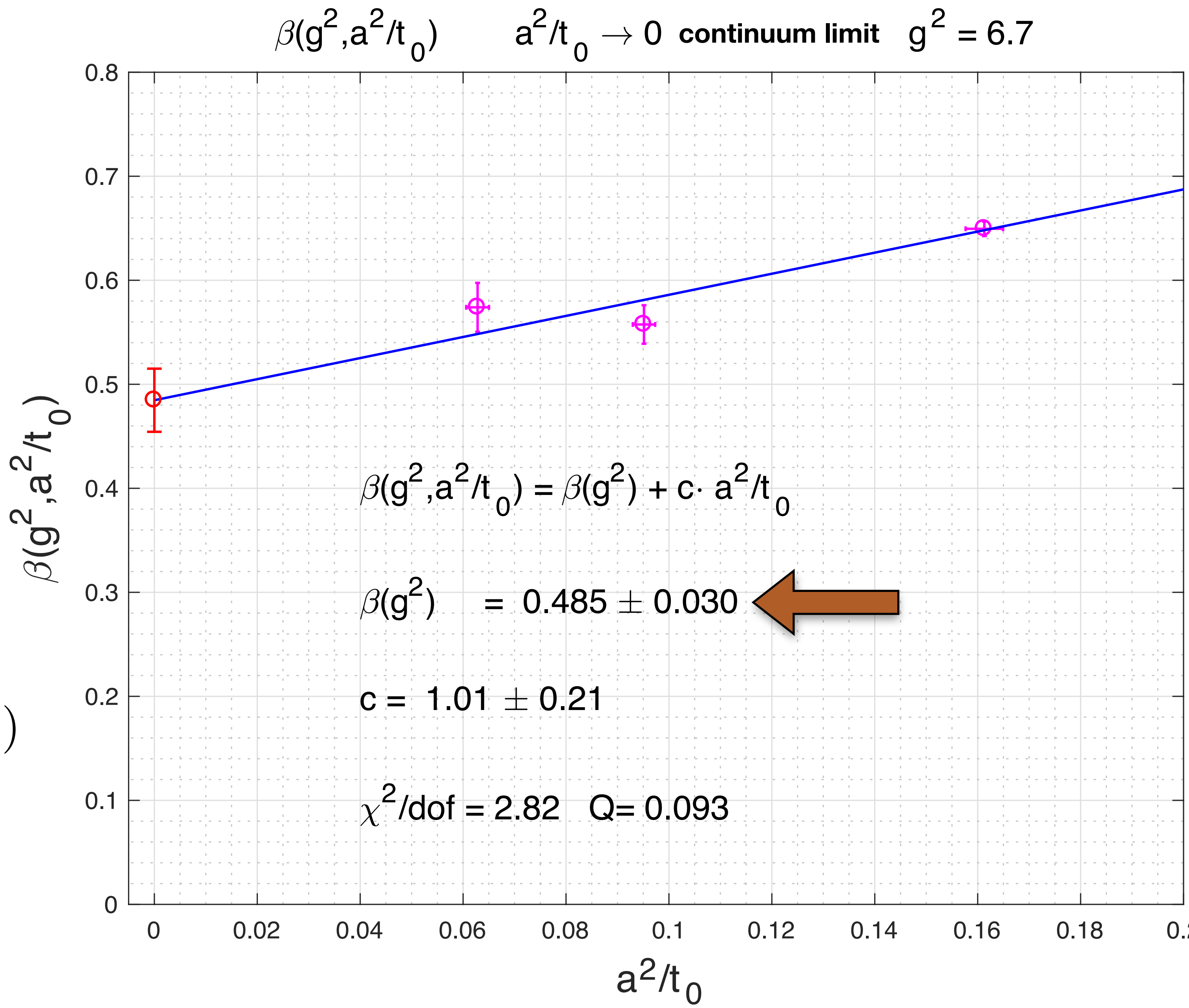
expect cutoff effects are $\mathcal{O}(a^2)$

linear extrapolation in a^2/t_0

cutoff effects apparently mild

errors in both t_0/a^2 and $\beta(g^2, a^2/t_0)$

included in continuum extrapolation



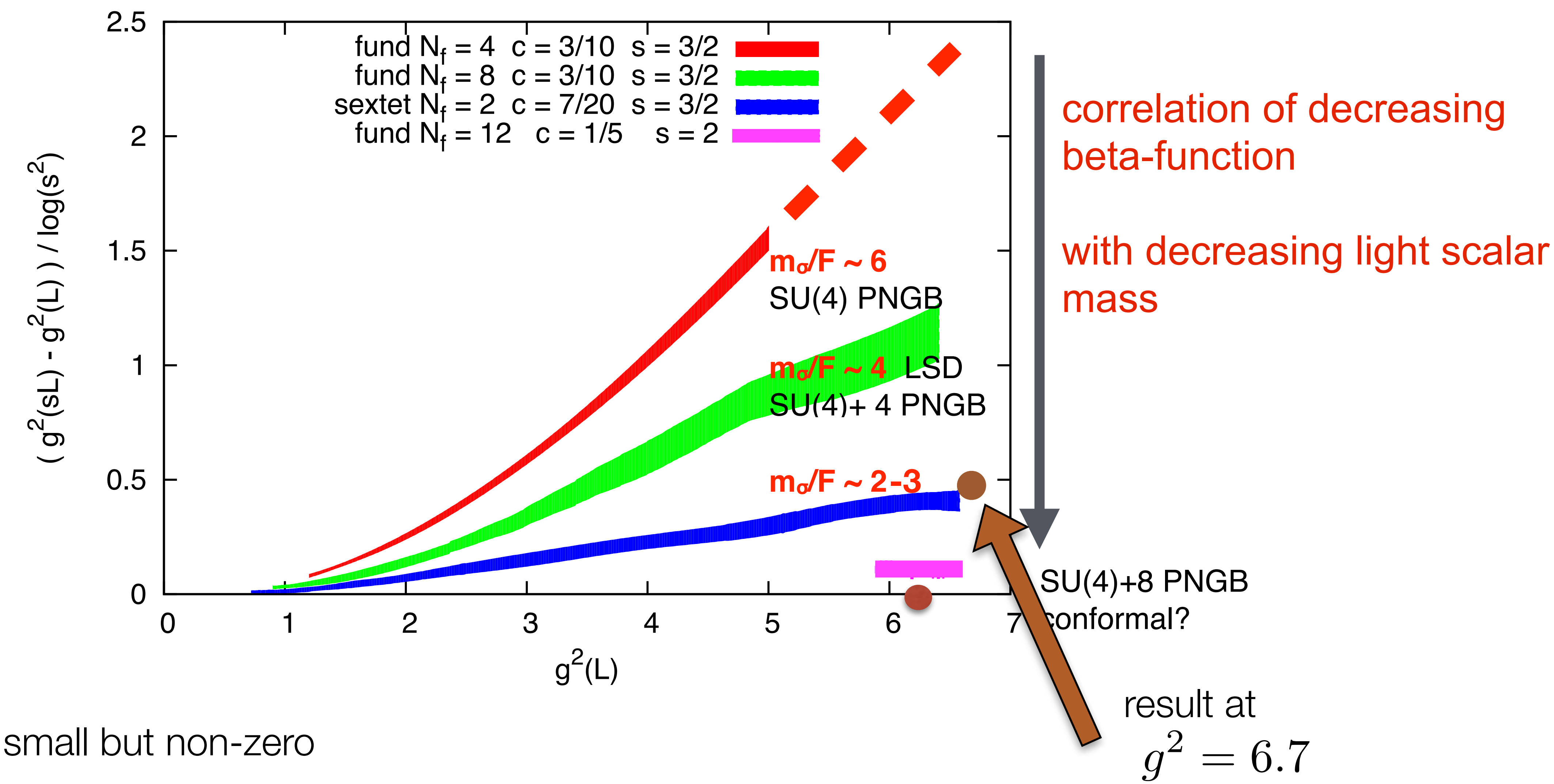
result for beta function at renormalized coupling $g^2 = 6.7$

$$\mathrm{t} \cdot \mathrm{dg}^2/\mathrm{dt} = -\mu^2 \cdot \mathrm{dg}^2/\mathrm{d}\mu^2 = 0.485 \pm 0.030$$

with infinite volume, $M_\pi \cdot a \to 0, \quad a^2/t_0 \to 0$ limits comparison with step-scaling determination of the beta function for the 2 flavor sextet SU(3) model

consistency between **infinitesimal** and **finite-step s=3/2** beta functions   scheme-dependence?

approach to the conformal window



beta function small but non-zero

no more “room” for an Infra Red Fixed Point corresponding to a zero of the beta function

— bridged the gap between perturbative and strong coupling

result: sextet model appears near-conformal, remains interesting as composite Higgs BSM theory

Thank you for your attention