Including electromagnetism in $K \rightarrow \pi\pi$ decay calculations

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N.H. Christ* and X. Feng

RBC and UKQCD Collaborations

The RBC & UKQCD collaborations

BNL and RBRC

Mattia Bruno
Tomomi Ishikawa
Taku Izubuchi
Luchang Jin
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni
Sergey Syritsyn

Columbia University

Ziyuan Bai
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
David Murphy
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang

University of Connecticut

Tom Blum
Dan Hoying
Cheng Tu

Edinburgh University

Peter Boyle
Guido Cossu
Luigi Del Debbio
Richard Kenway
Julia Kettle
Ava Khamseh
Brian Pendleton
Antonin Portelli
Tobias Tsang
Oliver Witzel
Azusa Yamaguchi

KEK

Julien Frison

University of Liverpool

Nicolas Garron

Peking University

Xu Feng

University of Southampton

Jonathan Flynn
Vera Guelpers
James Harrison
Andreas Juettner
Andrew Lawson
Edwin Lizarazo
Chris Sachrajda

York University (Toronto)

Renwick Hudspith

Outline

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- Overview
  - $K^0 - \overline{K^0}$ mixing
  - $K \rightarrow \pi\pi$
- Lattice methods
  - $K^0 - \overline{K^0}$ mixing
  - $K \rightarrow \pi\pi$
- Coulomb effects in finite volume.
- Conclusion

Neutral Kaons at order α_{EM}

- Tests of standard-model CP violation will need higher precision.
- Indirect CP violation (ε)
  - Can be computed at the percent level
  - Larger, parametric uncertainty in V_{cb} will be reduced by Belle II
- Direct CP violation (ε')
  - $\Delta I = 1/2$ rule, $A_2/A_0 = 1/22$, suggests A_2 may acquire EM corrections ~ 22 α_{EM}
  - Similar effect on ε' :

$$\epsilon' = \frac{i\,e^{\delta_2-\delta_0}}{\sqrt{2}}\left|\frac{A_2}{A_0}\right|\left(\frac{\mathrm{Im}\,A_2}{\mathrm{Re}A_2}-\frac{\mathrm{Im}\,A_0}{\mathrm{Re}A_0}\right)$$

$K^0 - \overline{K}^0$ mixing + E&M

- The usual theoretical framework is consistent with E&M and $m_u \neq m_d$
- Usual Wigner-Weiskopf treatment is quite general:

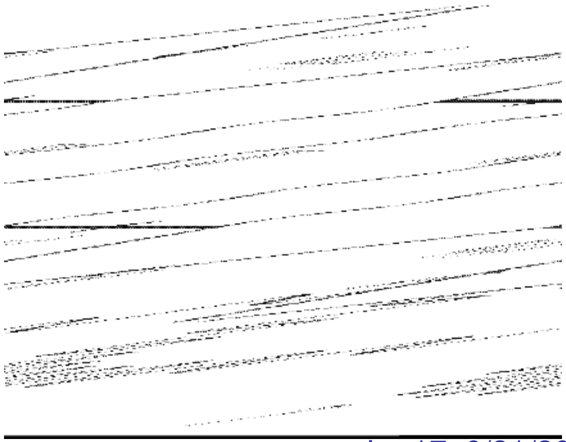
$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left\{ \begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix} \right\} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

$$\Gamma_{ij} = 2\pi \sum_\alpha \int_{2m_\pi}^\infty dE \langle i | H_W | \alpha(E) \rangle \langle \alpha(E) | H_W | j \rangle \delta(E - m_K)$$

$$M_{ij} = \sum_\alpha \mathcal{P} \int_{2m_\pi}^\infty dE \frac{\langle i | H_W | \alpha(E) \rangle \langle \alpha(E) | H_W | j \rangle}{m_K - E}$$

$K^0 - \overline{K}^0$ mixing at order α_{EM}

- Add electroweak corrections to standard Inami-Lim calculation.
- Both short- (perturbative) and long-distance (non-perturbative) are needed.
- Infra-red divergence cancels when long-distance lattice photon is included.
- Possibly interesting project after the ~5% long-distance effects have been computed.



$K^0 \to \pi\pi$ with E&M

- Many challenges
  - Without isospin $\pi\pi$ system becomes a general 2-channel problem
  - Infrared problems (use Lubicz *et al.* [arXiv:1611.08497] ?)
  - 3-particle lattice states: $|\pi\pi\gamma>$
- Earlier continuum treatment: Cirgliano, *et al.* [arXiv:hep-ph/008290] – try to improve using lattice methods

$K^0 \to \pi\,\pi$ – Lattice strategy

- Exploit perturbation theory in α_{EM}
- Identify three independent components
  1) $\pi^+ \pi^-$ Coulomb force – discussed here
  2) transverse radiation – not yet considered
  3) $m_{\pi^+} - m_{\pi^0}$ – easy, I=0 and 2 are not mixed
- Distinguish 1) and 2) by using Coulomb gauge:

$$L_{\rm int} = \int d^3r \left\{ \sum_{q=u,d,s} e_q \vec{A}_{\rm tr}(\vec{r})\, \overline{q}\vec{\gamma} q(\vec{r}) \right\}$$

$$+ \int\int d^3r\, d^3r' \Big(\sum_{q=u,d,s} e_q \overline{q}\gamma^0 q(\vec{r}) \Big) \frac{1}{|\vec{r}-\vec{r}\,'|} \Big(\sum_{q'=u,d,s} e_{q'} \overline{q}\gamma^0 q(\vec{r}\,') \Big)$$

$K^0 \rightarrow \pi\,\pi$ – Lattice strategy

- Address 3) by using $m_q = m_q^{(0)} + m_q^{(1)}$ for $q=\underline{u},d$
  - $m_q^{(0)}$ removes E&M mass shift: $m_{\pi^+}(m_q^{(0)}) = m_{\pi^0}(m_q^{(0)})$
  - $m_q^{(1)}$ restores physical masses:

    $m_{\pi+/0}(m_q^{(0)} + m_q^{(1)}) = 139.6/135.0$ MeV
- Focus in this talk on only the Coulomb force.

$K^0 \rightarrow \pi\,\pi$ decay without E&M

- Define: $\,^{\rm out}\langle(\pi\pi)_I|H_W|K^0\rangle = e^{i\delta_I}A_I \quad I=0,2$
- *T*-invariance:

$$T|(\pi\pi)_I\rangle^{\rm out} = |(\pi\pi)_I\rangle^{\rm in} = e^{2i\delta_I}|(\pi\pi)_I\rangle^{\rm out}$$

- Requires A_I to be real.
- CPT implies:

$$^{\rm out}\langle(\pi\pi)_I|H_W|K^0\rangle = -e^{2i\delta_I}\;^{\rm out}\langle(\pi\pi)_I|H_W|\overline{K}^0\rangle^*$$

- Giving a CP violating matrix element:

$$^{\rm out}\langle\pi^+\pi^-|H_W\Big[\frac{|K^0\rangle+|\overline{K}^0\rangle}{\sqrt{2}}\Big] = \sqrt{\frac{1}{6}}\Big\{\sqrt{2}e^{i\delta_0}(A_0-A_0^*)+e^{i\delta_2}(A_2-A_2^*)\Big\}$$

- Gives $\quad \epsilon' = \dfrac{i\,e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\dfrac{{\rm Re}A_2}{{\rm Re}A_0}\left(\dfrac{{\rm Im}A_2}{{\rm Re}A_2}-\dfrac{{\rm Im}A_0}{{\rm Re}A_0}\right)$

$K^0 \rightarrow \pi\,\pi$ decay with E&M

- Now the *S* matrix is not diagonalized by isospin eigenstates:

$$S_{c'c} = {}^{\text{out}}\langle(\pi\pi)^{\gamma}_{c'}|(\pi\pi)^{\gamma}_{c}\rangle^{\text{in}} \quad c, c' = +-, 00$$

- Diagonalize $S^D = \Omega^\dagger S \Omega$, use *T* to choose Ω orthogonal and define:

$$|(\pi\pi)^{\gamma}_{s}\rangle^{\text{in/out}} = \sum_{c=+-,00} \Omega_{cs}|(\pi\pi)^{\gamma}_{c}\rangle^{\text{in/out}} \quad s = 0, 2$$

with

$${}^{\text{out}}\langle(\pi\pi)^{\gamma}_{s'}|(\pi\pi)^{\gamma}_{s}\rangle^{\text{in}} = e^{2i\delta'_s}\delta_{s's} \quad s', s = 0, 2$$

$K^0 \to \pi\,\pi$ decay with E&M

- Use these new out states to represent the Watson theorem:

$$ {}^{\text{out}}\langle(\pi\pi)^{\gamma}_{s}|H_W|K^0\rangle = e^{i\delta^{\gamma}_{s}} A^{\gamma}_{s} $$

CP conservation implies A_s^{γ} is real

- For physical out states:

$$ {}^{\text{out}}\langle(\pi\pi)^{\gamma}_{c}|H_W|K^0\rangle = \sum_{s=0,2} \Omega_{cs} e^{i\delta^{\gamma}_{s}} A^{\gamma}_{s} \quad c = +-, 00 $$

- Which in turn determines ε':

$$ \epsilon' = \frac{1}{3}(\eta_{+-} - \eta_{00}) = -\frac{i\,\det(\Omega)\, e^{i(\delta^{\gamma}_{2}-\delta^{\gamma}_{0})}}{\Omega_{+-,0}\Omega_{00,0}} \frac{\mathrm{Re}A^{\gamma}_{2}}{\mathrm{Re}A^{\gamma}_{0}} \left(\frac{\mathrm{Im}A^{\gamma}_{2}}{\mathrm{Re}A^{\gamma}_{2}} - \frac{\mathrm{Im}A^{\gamma}_{0}}{\mathrm{Re}A^{\gamma}_{0}}\right) $$

- Determine A_s^{γ} in δ_s^{γ} and Ω_{cs} using lattice QCD

Finite quantization + Coulomb I

- Following Luscher introduce Helmholtz equation

$$\left(\vec{\nabla}^2 + k^2 - 2\mu V_L(\vec{r})\right) G_C(\vec{r}) = \delta(\vec{r})$$

- Use finite volume Fourier modes for $V_L(\vec{r})$

$$V_L(\vec{r}) = 4\pi e^2 \sum_{\vec{n}\neq\vec{0}} \frac{1}{L^3} \frac{e^{i\vec{r}\cdot\vec{n}\frac{2\pi}{L}}}{\left(\frac{2\pi}{L}\vec{n}\right)^2}$$

- Assume L sufficiently large that there exists R_C obeying:

$$R_{QCD} < R_C << L \quad \text{and} \quad \left[V_L(\vec{r}) \approx 1/|\vec{r}|\right]_{|\vec{r}|= R_C}$$

- At R_C equate the strong + Coulomb solution with the s-wave component of G_C
- Determine $G_C(r)$ to first order in α_{EM}

Finite quantization + Coulomb I

- Finite volume energy is determined by matching the strong + Coulomb and the Helmholtz solutions:

$$\left\{\cos(\delta_0')F_0(|\vec{r}|) + \sin(\delta_0')G_0(|\vec{r}|) = \left[G_C(\vec{r})\right]_{l=0}\right\}_{|\vec{r}|=R_C}$$

- Following Luscher this can be done by comparing the left and right sides as $|\vec{r}| \rightarrow 0$
- Result is $\phi_C(E) + \delta_0'(E) = n\pi$ with $\eta = e^2/v$ and

$$\cot(\phi_C(E)) = -(1+\pi\eta)\frac{1}{kL}\sum_{\vec{n}}\frac{1}{\left(-\vec{n}^2+\frac{kL}{2\pi}\right)^s}\Bigg|_{s=1}$$
$$+\lim_{r\to 0} 8\pi\eta\left\{\sum_{\vec{n}}\sum_{\vec{m}\neq 0}\frac{e^{i\vec{n}\cdot\vec{r}\frac{2\pi}{L}}}{\pi(2\pi)^4}\frac{1}{\vec{n}^2+\left(\frac{kL}{2\pi}\right)^2}\frac{1}{\vec{m}^2}\frac{1}{(\vec{n}-\vec{m})^2+\left(\frac{kL}{2\pi}\right)^2} - \frac{1}{4\pi}\ln(1/kr)\right\}$$

(See also Bean and Savage arXiv:1407.4846)

- However, V_L introduces large $(1/L)^n$ corrections

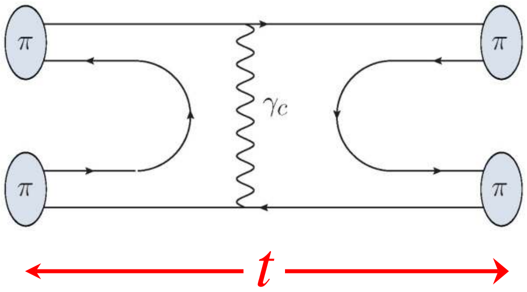
Finite quantization + Coulomb II

- A better approach:
  - Use $$V_T(r) = \begin{cases} \frac{e^2}{r} & r \le R_T \\ 0 & r > R_T \end{cases}$$
  - Choose $R_T > R_{\text{strong}}$ and $R_T < L/2$
- Usual single- and double-channel finite-volume quantization can be employed.
- Resulting "truncated" phase shift determines the physical Coulomb scatter phase analytically, including $-\eta \ln(2kr)$ or $-\eta \ln(2k/\mu)$ term.
- Correction is not needed for η_{+-}, η_{00} or ε':

$$\eta_{+-} \equiv \frac{{}^{\text{out}}\langle(\pi\pi)^{\gamma}_{+-}|H_W|K_L\rangle}{{}^{\text{out}}\langle(\pi\pi)^{\gamma}_{+-}|H_W|K_S\rangle} \qquad \eta_{00} \equiv \frac{{}^{\text{out}}\langle(\pi\pi)^{\gamma}_{00}|H_W|K_L\rangle}{{}^{\text{out}}\langle(\pi\pi)^{\gamma}_{00}|H_W|K_S\rangle}$$

Lattice calculation

- Add explicit, 1st-order Coulomb interaction:



$$\langle 0|(\pi\pi)_I(t)(\pi\pi)_{I'}(0)|0\rangle = \sum_{s=0,2} \langle 0|(\pi\pi)_I(t)|(\pi\pi)_s^\gamma\rangle e^{-E_s^\gamma t}\langle(\pi\pi)_s^\gamma|(\pi\pi)_{I'}(0)|0\rangle$$

$$= \begin{pmatrix} e^{-E_0 t}(1+\Delta N_{00}+\Delta E_0 t) & \Delta N_{02}(e^{-E_0 t}+e^{-E_2 t}) \\ \Delta N_{20}(e^{-E_0 t}+e^{-E_2 t}) & e^{-E_2 t}(1+\Delta N_{22}+\Delta E_2 t) \end{pmatrix}_{I,I'}$$

- ΔE_s determines the shift in the phase shift δ_s^γ, $s = 0, 2$
- ΔN_{II} are $O(\alpha_{EM})$ corrections giving the new S matrix eigenstates allowing us to compute $A_s^\gamma = \langle(\pi\pi)_s^\gamma|H_W|K^0\rangle$ s=0, 2 and ε'

Conclusion

- Many important issues not yet understood.
- Determine if a lattice treatment of the transverse photons is possible.
- If "*yes*", then a lattice calculation of the Coulomb corrections might be developed over the next three years.