18 - 24 JUNE 2017
Granada - SPAIN Lattice2017

HQE parameters from unquenched lattice data on pseudoscalar and vector heavy-light meson masses

Based on arXiv:1704.06105

Paolo Gambino[1] Aurora Melis[2](speaker) Silvano Simula[3]

[1]Università di Torino and INFN
[2]IFIC and Universitat de Valencia
[3]INFN sezione di RomaTre

Motivations

Extraction of $|V_{cb}|$: a long lasting 3σ-tension between the two main available determinations

$$\underbrace{|V_{cb}| = (39.05 \pm 0.75) \times 10^{-3}}$$

Exclusive:
$B \to D^{(*)}\ell\nu$ results + latticeQCD computation of form factors

$$\underbrace{|V_{cb}| = (42.00 \pm 0.65) \times 10^{-3}}$$

Inclusive:
OPE expansion depending on few HQET parameters extracted from experiment

Improving the precision will

- (dis)favor new physics interpretation
- help the search of new physics in rare decays (FCNC decays like $B_s \to \mu^+\mu^-$, $K^+ \to \pi^+\nu\overline{\nu}$, $K_L \to \pi^0\nu\overline{\nu}$ as well as ϵ_K)
- be particularly interesting in view of the anomalies in $B \to D^{(*)}\tau\nu$

Which HQE parameters?

The B meson expectation values of the dimension-5 and -6 local operators appearing in the OPE expansion of the total semileptonic width[1]

$$\Gamma[\overline{B} \to X_c e \overline{\nu}] = \frac{G_F m_b^5}{192\pi^3} |\mathbf{V_{cb}}|^2 g(r) A_{ew} \Bigg[1 - \frac{(\mu_\pi^2 - \mu_G^2)|_B}{2m_b^2} - \frac{(\rho_D^3 + \rho_{LS}^3)|_B}{2m_b^3} + \\ - \frac{\mu_G^2|_B - \frac{(\rho_D^3 + \rho_{LS}^3)|_B}{m_b}}{m_b^2} \frac{2(1-r)^4}{g(r)} + \frac{d(r)}{g(r)} \frac{\rho_D^3|_B}{m_b^3} + \\ + \sum_n \left(\frac{\alpha_s(m_b)}{\pi} \right)^n p_c^{(n)}(r,\mu) + \mathcal{O}\left(\frac{1}{m_b^4} \right) \Bigg], \qquad r = \frac{m_c^2}{m_b^2}$$

where

$$\mu_\pi^2|_B = \frac{\langle B|\overline{h}_v (i\vec{D})^2 h_v|B\rangle}{2\langle B|B\rangle} \text{ (Kinetic)} \qquad \mu_G^2|_B = \frac{\langle B|\overline{h}_v (G_{\mu\nu}\sigma^{\mu\nu}) h_v|B\rangle}{2\langle B|B\rangle} \text{ (Chromomagnetic)}$$

$$\rho_D^3|_B = \frac{\langle B|\overline{h}_v (-\frac{1}{2}\vec{D}\cdot\vec{E}) h_v|B\rangle}{2\langle B|B\rangle} \text{ (Darwin)} \qquad \rho_{LS}^3|_B = \frac{\langle B|\overline{h}_v (\vec{\sigma}\cdot\vec{E}\times\vec{\pi}) h_v|B\rangle}{2\langle B|B\rangle} \text{ (Conv.Current)}$$

[1] **A recent inclusive computation of V_{cb}: [arXiv:1411:6560v2[hep-ph]]**

Which HQE parameters?

We study the $1/\widetilde{m}_h$ dependence of two meson mass combinations:

$$\mathbf{M_{av}(\widetilde{m}_h) \equiv \frac{M_{PS}(\widetilde{m}_h) + 3M_V(\widetilde{m}_h)}{4}} \qquad \mathbf{\Delta M(\widetilde{m}_h) \equiv M_V(\widetilde{m}_h) - M_{PS}(\widetilde{m}_h)}.$$

(Spin averaged) **(Hyperfine splitting)**

Their HQET expansions read as

$$\frac{M_{av}(\widetilde{m}_h)}{\widetilde{m}_h} = 1 + \frac{\overline{\Lambda}|_\infty}{\widetilde{m}_h} + \frac{\mu_\pi^2|_\infty}{2\widetilde{m}_h^2} + \frac{(\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3)|_\infty}{4\widetilde{m}_h^3} + \frac{\sigma^4|_\infty}{\widetilde{m}_h^4} ,$$

$$\widetilde{m}_h \Delta M(\widetilde{m}_h) = \frac{2}{3} c_G(\widetilde{m}_h, \widetilde{m}_b)\mu_G^2(\widetilde{m}_b)|_\infty + \frac{(\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3)|_\infty}{3\widetilde{m}_h} + \frac{\Delta\sigma^4|_\infty}{\widetilde{m}_h^2} .$$

$\overline{\Lambda}$: energy of the light quark and gluons,

$\rho_{\pi\pi}^3, \rho_{\pi G}^3, \rho_S^3, \rho_A^3$: non local zero momentum transfer correlators,

$\widetilde{m}_h = m_h^{kin}(\mu_{soft})$: the heavy quark mass in the kinetic scheme at $\mu_{soft} = 1$GeV.

Simulation details

ETMC gauge configurations with $N_f = 2 + 1 + 1$ dynamical quarks

ensemble	β	V/a^4	N_{cfg}	$a\mu_\ell$	$a\mu_c$	$a\mu_h > a\mu_c$
$A30.32$	1.90	$32^3 \times 64$	150	0.0030	{0.21256, 0.25000, 0.29404}	{0.34583, 0.40675, 0.47840, 0.56267, 0.66178, 0.77836, 0.91546},
$A40.32$			150	0.0040		
$A50.32$			150	0.0050		
$A40.24$		$24^3 \times 48$	150	0.0040		
$A60.24$			150	0.0060		
$A80.24$			150	0.0080		
$A100.24$			150	0.0100		
$B25.32$	1.95	$32^3 \times 64$	150	0.0025	{0.18705, 0.22000, 0.25875}	{0.30433, 0.35794, 0.42099, 0.49515, 0.58237, 0.68495, 0.80561}
$B35.32$			150	0.0035		
$B55.32$			150	0.0055		
$B75.32$			75	0.0075		
$B85.24$		$24^3 \times 48$	150	0.0085		
$D15.48$	2.10	$48^3 \times 96$	90	0.0015	{0.14454, 0.0150, 0.19995}	{0.23517, 0.27659, 0.32531, 0.38262, 0.45001, 0.52928, 0.62252}
$D20.48$			90	0.0020		
$D30.48$			90	0.0030		

In Physical units[a]: $a \sim (0.06 - 0.09)\text{fm}$ $M_\pi \sim (210 - 450)\text{MeV}$ $m_\ell \sim (3 - 12) m_{u/d}^{\text{phys}}$

$m_c \sim (0.7 - 1.1) m_c^{\text{phys}}$ $m_h \sim (1.1 - 3.3) m_c^{\text{phys}} \leq 0.8 m_b^{\text{phys}}$

[a] **Determination of quark masses and RCs: [arXiv:1403.4504 [hep-lat]]**

Kinetic mass scheme

We adopt the kinetic mass $\widetilde{m}_h$ instead of the pole mass.
The pole masses m_h^{pole} suffer in perturbation theory from IR renormalons of order $\mathcal{O}(\Lambda_{QCD})$.

$$\mathbf{\widetilde{m}_h = m_h^{pole} - \delta m_h(IR\ sensitive\ part)}$$

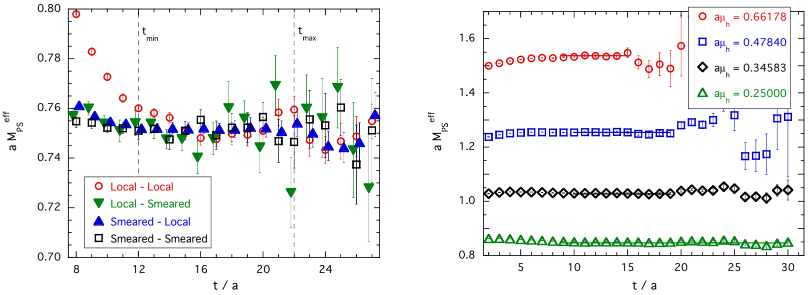
The relation[2] between the kinetic mass and the $\overline{\text{MS}}$ masse $\overline{m}_h$ is known up to N2LO

$$\begin{aligned}
\widetilde{m}_h &= \overline{m}_h(\overline{m}_h)\left\{1+\frac{4}{3}\frac{\alpha_s(\overline{m}_h)}{\pi}\left[1-\frac{4}{3}x-\frac{1}{2}x^2\right]+\left(\frac{\alpha_s(\overline{m}_h)}{\pi}\right)^2\right. \\
&\cdot \left[\frac{\beta_0}{24}(8\pi^2+71)+\frac{35}{24}+\frac{\pi^2}{9}\ln(2)-\frac{7\pi^2}{12}-\frac{\zeta_3}{6}\right. \\
&+ \frac{4}{27}x\left(24\beta_0\ln(2x)-64\beta_0+6\pi^2-39\right) \\
&+ \frac{1}{18}x^2\left(24\beta_0\ln(2x)-52\beta_0+6\pi^2-23\right) \\
&- \left.\left.\frac{32}{27}x^3-\frac{4}{9}x^4\right]+\mathcal{O}(\alpha_s^3)\right\}, \qquad x=\frac{\mu_{soft}}{\overline{m}_h(\overline{m}_h)}
\end{aligned}$$

In charm-bottom region $\widetilde{m}_h/\overline{m}_h(\overline{m}_h)$ varies in the range $0.8-1.1$.

[2] **Kinetic-$\overline{\text{MS}}$ mass relation: [arXiv:1107.3100[hep-ph]]**

Extraction of ground state meson masses



$M_{PS(V)}$ are determined from the plateau of the effective mass

$$M^{eff}_{PS(V)}(t) \equiv \text{arcosh}\left[\frac{C^{SL}_{PS(V)}(t-1)+C^{SL}_{PS(V)}(t+1)}{2C^{SL}_{PS(V)}(t)}\right] \xrightarrow[t\geq t^{PS(V)}_{\min}]{} M_{PS(V)} \ .$$

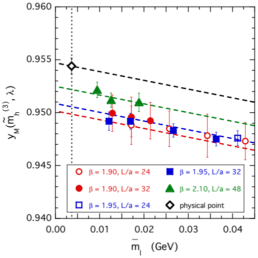
We adopt also Gaussian-smeared interpolating quark fields to suppress faster the excited states (smearing parameters: $k_G = 4$, $N_G = 30$).

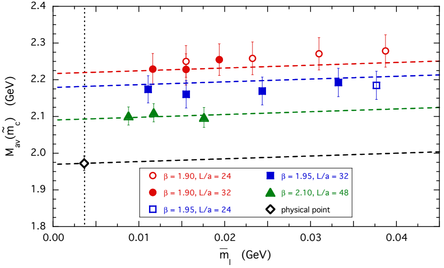
ETMC ratio method to $\mathbf{M_{av}(\widetilde{m}_h)}$ and determination of $\mathbf{\widetilde{m}_b}$

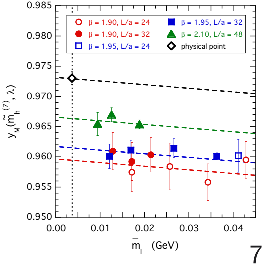
1. Interpolate the lattice data to a sequence of heavy-quark masses $\widetilde{m}_h^{(n)} = \lambda \widetilde{m}_h^{(n-1)}$
2. Construct ratios that go to 1 in the static limit $\widetilde{m}_h \to \infty$

$$y_M(\widetilde{m}_h^{(n)}, \lambda) = \frac{M_{av}(\widetilde{m}_h^{(n)})}{M_{av}(\widetilde{m}_h^{(n-1)})} \underbrace{\frac{\widetilde{m}_h^{(n-1)}}{\widetilde{m}_h^{(n)}}}_{\lambda} \qquad \left(\lim_{\widetilde{m}_h \to \infty} \frac{M_{av}}{\widetilde{m}_h} = 1 \right).$$

3. Extrapolate to the physical point (combined linear fit in $\overline{m}_\ell$, a^2).

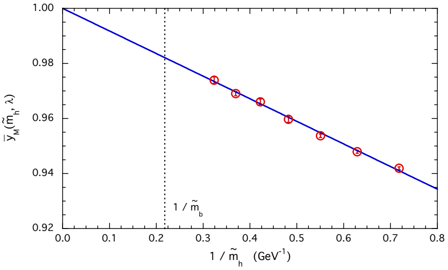






The ratios start at $\widetilde{m}_h^{(1)} = \widetilde{m}_c = 1.253(29)$GeV where we get $\mathbf{M_{av}(\widetilde{m}_c) = 1.967(25)}$**GeV** [PDG:$(M_D + 3M_{D^*})/4 = 1.973$GeV]

ETMC ratio method to $M_{av}(\widetilde{m}_h)$ and determination of $\widetilde{m}_b$



4. Interpolation in $1/\widetilde{m}_h$ imposing the static limit,

$$\overline{y}_M(\widetilde{m}_h, \lambda) = 1 + \frac{\epsilon_1}{\widetilde{m}_h} + \frac{\epsilon_2}{\widetilde{m}_h^2} + \mathcal{O}\left(\frac{1}{\widetilde{m}_h^3}\right)$$

applied taking into account the correlations between lattice points.

5. Chain equation : $\underbrace{M_{av}(\widetilde{m}_c)}_{\text{triggering point}} \quad \overline{y}_M(\widetilde{m}_h^{(2)}, \lambda)\, \overline{y}_M(\widetilde{m}_h^{(3)}, \lambda) \ldots \overline{y}_M(\widetilde{m}_h^{(K+1)}, \lambda) = \lambda^K M_{av}(\widetilde{m}_h^{(K+1)})$.

$\widetilde{m}_b = \lambda^K \widetilde{m}_c$ determined iteratively imposing $M_{av}(\widetilde{m}_h^{(K+1)}) = (M_B + 3M_{B*})/4$ after $K = 10$ steps.

$$\mathbf{\widetilde{m}_b = 4.605\ (120)_{\rm stat}\ (57)_{\rm syst}\ GeV = 4.605\ (132)\ GeV} \qquad [\mathbf{\overline{m}_b(\overline{m}_b) = 4.257(120)\, GeV}].$$

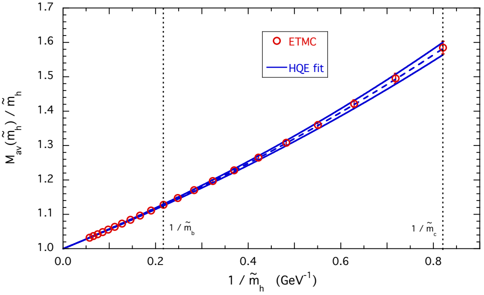
Compatible with the determination $\overline{m}_b(\overline{m}_b) = 4.26(10)$GeV[3] (using the running mass $\overline{m}_h$(2GeV), different ratios involving B-B_s meson masses)

[3] **Determination of m_b in $\overline{\rm MS}$ scheme: [arXiv:1107.1441[hep-lat]]**

HQE expansion parameters from $M_{av}(\widetilde{m}_h)$

We extend the chain equation beyond the b quark point, we choose $n \sim 20$ ($\widetilde{m}_h \simeq 4\widetilde{m}_b$),

$$\frac{M_{av}(\widetilde{m}_h^{(n)})}{\widetilde{m}_h^{(n)}} = \frac{M_{av}(\widetilde{m}_c)}{\widetilde{m}_c} \prod_{i=2}^{n} \overline{y}_M(\widetilde{m}_h^{(i)}, \lambda).$$



Correlated fit based on the HQE expansion

$$\frac{M_{av}(\widetilde{m}_h)}{\widetilde{m}_h} = 1 + \frac{\overline{\Lambda}}{\widetilde{m}_h} + \frac{\mu_\pi^2}{2\widetilde{m}_h^2} + \frac{\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3}{4\widetilde{m}_h^3} + \frac{\sigma^4}{\widetilde{m}_h^4}$$

Results of the dim-6 fit $\widetilde{m}_h > \widetilde{m}_c$

$$\overline{\Lambda} = 0.551\ (13)_{stat}\ (2)_{syst}\ \text{GeV}$$
$$\mu_\pi^2 = 0.314\ (14)_{stat}\ (2)_{syst}\ \text{GeV}^2$$
$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 = 0.174\ (12)_{stat}\ (2)_{syst}\ \text{GeV}^3$$

Results of the dim-6 fit $\widetilde{m}_h > 2\widetilde{m}_c$

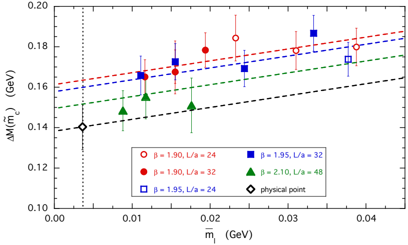
$$\overline{\Lambda} = 0.552\ (13)\ \text{GeV}$$
$$\mu_\pi^2 = 0.323\ (16)\ \text{GeV}^2$$
$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 = 0.153\ (24)\ \text{GeV}^3$$

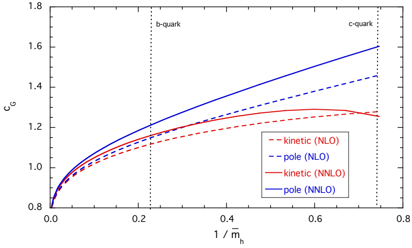
Results of the dim-7 fit

$$\overline{\Lambda} = 0.552\ (13)_{stat}\ (2)_{syst}\ \text{GeV}$$
$$\mu_\pi^2 = 0.325\ (17)_{stat}\ (3)_{syst}\ \text{GeV}^2$$
$$\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 = 0.133\ (34)_{stat}\ (6)_{syst}\ \text{GeV}^3$$
$$\sigma^4 = 0.0071\ (55)_{stat}\ (10)_{syst}\ \text{GeV}^4$$

ETMC ratio method to $\Delta M(\widetilde{m}_h)$

Hyperfine mass splitting at the triggering point: $\mathbf{\Delta M(\widetilde{m}_c) = 140(11)MeV}$ $[M_{D^*} - M_D = 141.4\text{MeV}]$.





In the static limit $\widetilde{m}_h \to \infty$ HQE predicts

$$\lim_{\widetilde{m}_h \to \infty} \widetilde{m}_h \frac{\Delta M(\widetilde{m}_h)}{c_G(\widetilde{m}_h, \widetilde{m}_b)} = \frac{2}{3}\mu_G^2(\widetilde{m}_b),$$

thus, the ratio method can be applied to the ratios:

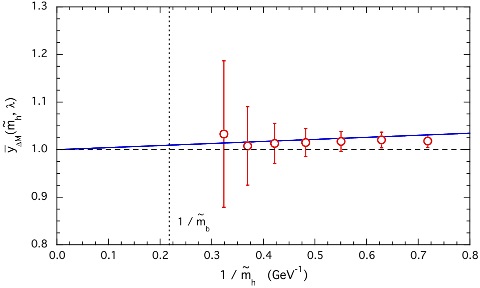
$$y_{\Delta M}(\widetilde{m}_h^{(n)}, \lambda) \equiv \underbrace{\frac{\widetilde{m}_h^{(n)}}{\widetilde{m}_h^{(n-1)}}}_{\lambda} \frac{\Delta M(\widetilde{m}_h^{(n)})}{\Delta M(\widetilde{m}_h^{(n-1)})} \frac{c_G(\widetilde{m}_h^{(n-1)}, \widetilde{m}_b)}{c_G(\widetilde{m}_h^{(n)}, \widetilde{m}_b)}.$$

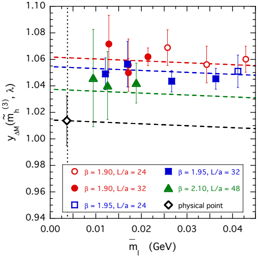
c_G: short distance Wilson coefficient,

$$c_G = \underbrace{\left[1 + \frac{13}{6}\frac{\alpha_s}{\pi} + (11.4744\beta_0 - 9.6584)\left(\frac{\alpha_s}{\pi}\right)^2\right] \cdot \mathcal{R}}_{\overline{c}_G(\overline{m}_h)} \cdot \frac{\widetilde{m}_h}{m_h^{pole}}$$

$\mathcal{R}(\widetilde{m}_h, \widetilde{m}_b)$ is the evolution factor in the $\overline{MS}$ scheme
$\widetilde{m}_h / m_h^{pole}$ improves the convergence.

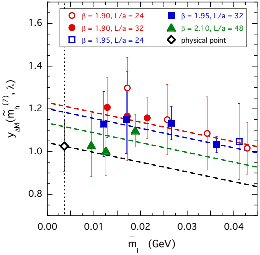
ETMC ratio method to $\Delta M(\widetilde{m}_h)$





Interpolation in $1/\widetilde{m}_h$ imposing the static limit and taking into account the correlations between lattice points

$$\overline{y}_{\Delta M}(\widetilde{m}_h, \lambda) = 1 + \frac{\Delta\epsilon_1}{\widetilde{m}_h} + \frac{\Delta\epsilon_2}{\widetilde{m}_h^2} + \mathcal{O}\left(\frac{1}{\widetilde{m}_h^3}\right) .$$

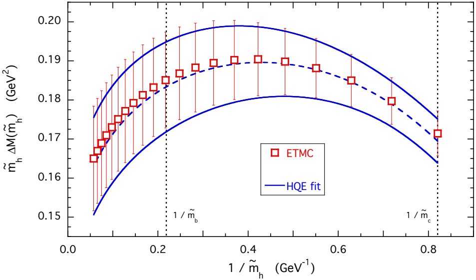


Using the chain equation, at the *b*-quark mass we obtain $\mathbf{\Delta M(\widetilde{m}_b) = 40.2(2.1)MeV}$ [$M_{B*} - M_B = 45.42(26)$MeV].

HQE expansion parameters from $\Delta M(\widetilde{m}_h)$

The chain equation can be extended beyond the b quark point, we choose $n \sim 20$ ($\widetilde{m}_h \simeq 4\widetilde{m}_b$),

$$\widetilde{m}_h^{(n)} \frac{\Delta M(\widetilde{m}_h^{(n)})}{c_G(\widetilde{m}_h^{(n)}, \widetilde{m}_b)} = \widetilde{m}_c \frac{\Delta M(\widetilde{m}_c)}{c_G(\widetilde{m}_c, \widetilde{m}_b)} \prod_{i=2}^{n} \overline{y}_{\Delta M}(\widetilde{m}_h^{(i)}, \lambda) \,.$$



We apply a correlated fit based on the HQE expansion

$$\widetilde{m}_h \Delta M(\widetilde{m}_h) = \frac{2}{3} c_G(\widetilde{m}_h, \widetilde{m}_b) \boldsymbol{\mu}_{\mathbf{G}}^2(\widetilde{m}_b) + \frac{\boldsymbol{\rho}_{\boldsymbol{\pi}\mathbf{G}}^3 + \boldsymbol{\rho}_{\mathbf{A}}^3 - \boldsymbol{\rho}_{\mathbf{LS}}^3}{3\widetilde{m}_h} + \frac{\boldsymbol{\Delta\sigma}^4}{\widetilde{m}_h^4} \,.$$

Results of the dim-6 fit $\widetilde{m}_h > \widetilde{m}_c$

$$\mu_G^2(\widetilde{m}_b) = 0.250 \ (18)_{\rm stat} \ (8)_{\rm syst} \ \text{GeV}^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.143 \ (57)_{\rm stat} \ (21)_{\rm syst} \ \text{GeV}^3$$

Results of the dim-6 fit $\widetilde{m}_h > 2\widetilde{m}_c$

$$\mu_G^2(\widetilde{m}_b) = 0.254 \ (22) \ \text{GeV}^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.158 \ (70) \ \text{GeV}^3$$

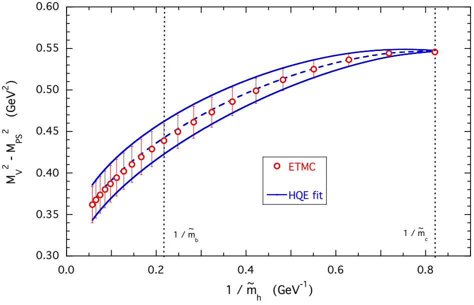
Results of the dim-7 fit

$$\mu_G^2(\widetilde{m}_b) = 0.254 \ (20)_{\rm stat} \ (9)_{\rm syst} \ \text{GeV}^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.173 \ (74)_{\rm stat} \ (25)_{\rm syst} \ \text{GeV}^3$$
$$\Delta\sigma^4 = 0.0092 \ (58)_{\rm stat} \ (14)_{\rm syst} \ \text{GeV}^4$$

Check: HQE expansion parameters from $M_V^2 - M_{PS}^2$

As a consistency check, we repeat the analysis for the quantity $M_V^2 - M_{PS}^2$, whose HQE expansion reads

$$M_V^2 - M_{PS}^2 = \frac{4}{3} c_G(\widetilde{m}_h, \widetilde{m}_b)\mu_G^2(\widetilde{m}_b) + \frac{2}{3} \frac{\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 + 2\overline{\Lambda}\mu_G^2(\widetilde{m}_b)}{\widetilde{m}_h} + \frac{\Delta\tilde{\rho}^4}{\widetilde{m}_h^2} \; .$$



Results of the ΔM analysis

$$\mu_G^2(\widetilde{m}_b) = 0.250\ (18)_{\rm stat}\ (8)_{\rm syst}\ \text{GeV}^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.143\ (57)_{\rm stat}\ (21)_{\rm syst}\ \text{GeV}^3$$

Results of the $M_V^2 - M_{PS}^2$ analysis

$$\mu_G^2(\widetilde{m}_b) = 0.270\ (17)\ \text{GeV}^2$$
$$\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 + 2\overline{\Lambda}\mu_G^2(\widetilde{m}_b) = 0.164\ (46)\ \text{GeV}^3$$
$$\Delta\tilde{\rho}^4 = 0.010\ (8)\ \text{GeV}^4$$
$$\Longrightarrow \rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 = -0.134\ (67)\ \text{GeV}^3$$

Final Results and conclusions

- A precise lattice computation of the pseudoscalar and vector masses up to $4\widetilde{m}_b$ using the gauge configurations of the ETMC with $N_f = 2+1+1$ at $a \simeq (0.06 - 0.09)$fm and $M_\pi \simeq (210 - 450)$MeV.
- We adopt the ETMC ratio method in the kinetic mass scheme.
- The first unquenched lattice determination of the HQE parameters.

$$
\begin{aligned}
\widetilde{m}_b &= 4.605\ (132)\ (150)_{conv}\ \text{GeV} = 4.605\ (201)\ \text{GeV}\ , \\
\overline{\Lambda} &= 0.552\ (13)\ (22)_{conv}\ \text{GeV} = 0.552\ (26)\ \text{GeV}\ , \\
\mu_\pi^2 &= 0.321\ (17)\ (27)_{conv}\ \text{GeV}^2 = 0.321\ (32)\ \text{GeV}^2\ , \\
\mu_G^2(m_b) &= 0.253\ (21)\ (13)_{conv}\ \text{GeV}^2 = 0.253\ (25)\ \text{GeV}^2\ , \\
\rho_D^3 - \rho_{\pi\pi}^3 - \rho_S^3 &= 0.153\ (30)\ (17)_{conv}\ \text{GeV}^3 = 0.153\ (34)\ \text{GeV}^3\ , \\
\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 &= -0.158\ (71)\ (45)_{conv}\ \text{GeV}^3 = -0.158\ (84)\ \text{GeV}^3\ .
\end{aligned}
$$

$()_{conv}$: conversion from the $\overline{MS}$ to the kinetic scheme at the charm mass.

Final Results and conclusions

In the BPS limit $\mu_\pi^2 = \mu_G^2$ and $\rho_D^3 + \rho_{LS}^3 = 0$,

- $(\mu_\pi^2 - \mu_G^2) = 0.064(19)\mathrm{GeV}^2$, deviation of 20 – 25% from the BPS limit,
- $(\rho_D^3 + \rho_{LS}^3) \geq 0.317(65)\mathrm{GeV}^3$, $4.9 - 3.6\sigma$ from the BPS limit.

In this work μ_π^2 and $\mu_G^2(m_b)$ refer to asymptotic matrix elements, while semileptonic fits are sensitive to

$$\mu_\pi^2|_B = \mu_\pi^2|_\infty - \frac{\rho_{\pi\pi}^3 + \frac{1}{2}\rho_{\pi G}^3}{\widetilde{m}_b} + \mathcal{O}(1/\widetilde{m}_b^2) \,,$$

$$\mu_G^2(m_b)|_B = \mu_G^2(m_b)|_\infty + \frac{\rho_S^3 + \rho_A^3 + \frac{1}{2}\rho_{\pi G}^3}{\widetilde{m}_b} + \mathcal{O}(1/\widetilde{m}_b^2) \,.$$

Recent determinations:[4] $\boldsymbol{\mu_\pi^2|_B = 0.432(68)\mathrm{GeV}^2}$, $\boldsymbol{\mu_\pi^2|_B = 0.465(68)\mathrm{GeV}^2}$ **imposing $\boldsymbol{\mu_G^2|_B = 0.35(7)\mathrm{GeV}^2}$.**

We can say,

$$\rho_{\pi\pi}^3 + \frac{1}{2}\rho_{\pi G}^3 = -0.51(35)\mathrm{GeV}^2, \quad \rho_S^3 + \rho_A^3 + \frac{1}{2}\rho_{\pi G}^3 + \overbrace{\frac{1}{2}\rho_{\pi G}^3 + \rho_{\pi\pi}^3} > 0, \quad \rho_S^3 + \rho_A^3 + \frac{1}{2}\rho_{\pi G}^3 > 0.51(35)\mathrm{GeV}^2$$

$$\Rightarrow \boldsymbol{\mu_G^2|_B = \mu_G^2|_\infty + 0.11(8)\mathrm{GeV}^2 = 0.36(8)\mathrm{GeV}^2}.$$

Thank you for your attention.

[4] **[arXiv:1606.06174[hep-ph]],[arXiv:1411.6560[hep-ph]]**

Correlations

	$\widetilde{m}_b$	$\overline{\Lambda}$	μ_π^2	ρ^3	$\mu_G^2(\widetilde{m}_b)$	$\Delta\rho^3$
$\widetilde{m}_b$	1.0	0.905	0.910	0.886	0.572	-0.488
$\overline{\Lambda}$	0.905	1.0	0.999	0.999	0.497	-0.420
μ_π^2	0.910	0.999	1.0	-0.998	0.501	-0.423
ρ^3	0.886	0.999	-0.998	1.0	0.484	-0.408
$\mu_G^2(\widetilde{m}_b)$	0.572	0.497	0.501	0.484	1.0	-0.995
$\Delta\rho^3$	-0.488	-0.420	-0.423	-0.408	-0.995	1.0

Correlation matrix among the *b*-quark mass and the HQE parameters of the dim-6 analysis

	$\widetilde{m}_b$	$\overline{\Lambda}$	μ_π^2	ρ^3	σ^4	$\mu_G^2(\widetilde{m}_b)$	$\Delta\rho^3$	$\Delta\sigma^4$
$\widetilde{m}_b$	1.0	0.910	0.811	0.394	0.196	0.538	-0.440	0.312
$\overline{\Lambda}$	0.910	1.0	0.886	0.439	0.223	0.466	-0.375	0.260
μ_π^2	0.811	0.886	1.0	0.082	0.568	0.443	-0.362	0.258
ρ^3	0.394	0.439	0.082	1.0	-0.693	0.151	-0.108	0.057
σ^4	0.196	0.223	0.568	-0.693	1.0	0.155	-0.137	0.111
$\mu_G^2(\widetilde{m}_b)$	0.538	0.466	0.443	0.151	0.155	1.0	-0.993	0.961
$\Delta\rho^3$	-0.440	-0.375	-0.362	-0.108	-0.37	-0.993	1.0	-0.986
$\Delta\sigma^4$	0.312	0.260	0.258	0.057	0.111	0.961	-0.986	1.0

ˇCorrelation matrix among the *b*-quark mass and the HQE parameters of the dim-7 analysis