Perturbative matching of continuum and lattice quasi-distributions

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Quasi distribution approach [X.-D. Ji (2013)]

- Normal distributions through quasi distributions

$$q(x,\mu) = \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle \mathcal{N}(P)|O(\xi^-)|\mathcal{N}(P)\rangle,$$
$$O(\xi^-) = \overline{\psi}(\xi^-)\gamma^+ U_+(\xi^-,0)\psi(0)$$

- $\xi^\pm = (t \pm z)/\sqrt{2}$ light-cone coordinate

⇕

Large Momentum Effective Theory

$$\widetilde{q}(x,\Lambda,P_z) = \int \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\Lambda}{P_z}, \frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2}, \frac{M^2}{P_z^2}\right)$$

- Z can be perturbativey obtained.

$$\widetilde{q}(\tilde{x},\mu,P_z) = \int \frac{d\delta z}{2\pi} e^{-i\tilde{x}P_z\delta z} \langle \mathcal{N}(P_z)|\widetilde{O}(\delta z)|\mathcal{N}(P_z)\rangle,$$
$$\widetilde{O}(\delta z) = \overline{\psi}(\delta z)\gamma^z U_z(\delta z,0)\psi(0)$$

- Separated in spatial z-direction. Calculable on lattice.

Matching: continuum and lattice

Matching between continuum and lattice operators

$$O^{\mathrm{cont}} = Z O^{\mathrm{latt}}$$

- The matching can be done by both perturbatively and non-perturbatively (RI/MOM).
- We have to care of unphysical operator mixings due to lack of symmetries in the lattice side.

Matching in coordinate space

$$O^{\mathrm{cont}}(\delta z) = Z(\delta z) O^{\mathrm{latt}}(\delta z)$$

- Matching before moving to momentum space.
- Simple.

[T.I, YM Ma, JW Qiu, S. Yoshida (2016)]

Renormalization

- Renormalization of Wilson lines

$$W_{\mathcal{C}} = Z_z e^{\delta m \ell(\mathcal{C})} W_{\mathcal{C}}^{\rm ren}$$

- Well-known. [Dotsenko, Vergeles, Arefeva, Craigie, Dorn, … ('80)]
- δm: mass renormalization of a test particle moving along $\mathcal{C}$

All the power divergence is contained.

- Auxiliary z-field (just like static heavy quark)

- By integrating out the z-field, the Wilson line is recovered.

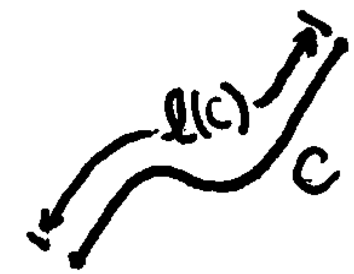
$$\int \mathcal{D}\overline{z}\mathcal{D}z e^{-\int_x \overline{z}(D_z+m)z} z(\delta z)\overline{z}(0) = \langle z(\delta z)\overline{z}(0)\rangle = U_z(\delta z, 0)$$

- Additive mass renormalization δm
- z-field wave function renormalization Z_z

Renormalization

Renormalization of non-local quark bilinear

$$O_{\mathcal{C}} = Z_{\psi,z} e^{\delta m \ell(\mathcal{C})} O_{\mathcal{C}}^{\mathrm{ren}}$$



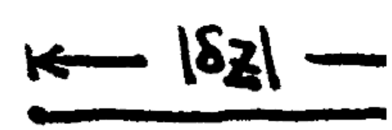
- $Z_{\psi,z}$: ψ , z-field wave function, ψ-z-field vertex renormalization
- Renormalizability has been shown only up to two-loop (HQET).

[XD Ji, JH Zhang (2015)]

Power divergence

- Power divergence makes the theory ill-defined.
  (e.g. no continuum limit on lattice.)
- The power divergence must be subtracted nonperturbatively.
- Power divergence subtracted non-local operator:

$$\widetilde{O}^{\mathrm{subt}}(\delta z) = e^{-\delta m |\delta z|} \widetilde{O}(\delta z)$$



[T.I, YM Ma, JW Qiu, S. Yoshida (2016)]

[JW Chen, XD Ji, JH Zhang (2016)]

Mixing pattern

- Analysis by symmetry

We can check operator mixing pattern and other properties by symmetry, which is non-perturbative argument.

- Chiral symmetry χ

  Wilson lattice fermion does not have the chiral symmetry.

- Charge conjugation $\mathcal{C}$
- (generalized) Parity $\mathcal{P}_\mu$

  Parity transformation can be generalized to any direction in Euclidean space.

- (generalized) Time-reversal $\mathcal{T}_\mu$

  Time-reversal can be generalized to any direction in Euclidean space.

Mixing pattern

- Usual local quark bilinear

$$O_\Gamma = \int_x \overline{\psi}(x)\Gamma\psi(x)$$

	$\Gamma = I$	γ_μ	γ_5	$\gamma_\mu\gamma_5$	$\sigma_{\mu\nu}$
$\mathcal{P}_\rho$	E	$\mathrm{E}_{(\rho=\mu)}$	O	$\mathrm{O}_{(\rho=\mu)}$	$\mathrm{O}_{(\rho=\mu,\nu)}$
		$\mathrm{O}_{(\rho\neq\mu)}$		$\mathrm{E}_{(\rho\neq\mu)}$	$\mathrm{E}_{(\rho\neq\mu,\nu)}$
$\mathcal{T}_\rho$	E	$\mathrm{O}_{(\rho=\mu)}$	O	$\mathrm{E}_{(\rho=\mu)}$	$\mathrm{O}_{(\rho=\mu,\nu)}$
		$\mathrm{E}_{(\rho\neq\mu)}$		$\mathrm{O}_{(\rho\neq\mu)}$	$\mathrm{E}_{(\rho\neq\mu,\nu)}$
$\mathcal{C}$	E	O	E	E	O
χ	P	N	P	N	P

- No mixing between different Γs.
- Chiral symmetry is not essential to prevent the mixing.

Mixing pattern

Non-local quark bilinear

$$O_\Gamma(\delta z) = \int_x \overline{\psi}(x+\delta z)\Gamma U_z(x+\delta z; x)\psi(x)$$

- Define operator basis: $O_{\Gamma\pm}(\delta z) = O_\Gamma(\delta z) \pm O_\Gamma(-\delta z)$

real or imaginary part of $O_\Gamma(\delta z)$

	$\Gamma = I_{+/-}$	$\gamma_{i+/-}$	$\gamma_{3+/-}$	$\gamma_{5+/-}$	$\gamma_i\gamma_{5+/-}$	$\gamma_3\gamma_{5+/-}$	$\sigma_{i3+/-}$	$\epsilon_{kij}\sigma_{ij+/-}$
$\mathcal{P}_3$	E	O	E	O	E	O	O	E
$\mathcal{P}_{l\neq 3}$	E/O	$\mathrm{E/O}_{(i=l)}$ $\mathrm{O/E}_{(i\neq l)}$	O/E	O/E	$\mathrm{O/E}_{(i=l)}$ $\mathrm{E/O}_{(i\neq l)}$	E/O	$\mathrm{O/E}_{(i=l)}$ $\mathrm{E/O}_{(i\neq l)}$	$\mathrm{E/O}_{(k=l)}$ $\mathrm{O/E}_{(k\neq l)}$
$\mathcal{T}_3$	E/O	E/O	O/E	O/E	O/E	E/O	O/E	E/O
$\mathcal{T}_{l\neq 3}$	E	$\mathrm{O}_{(i=l)}$ $\mathrm{E}_{(i\neq l)}$	E	O	$\mathrm{E}_{(i=l)}$ $\mathrm{O}_{(i\neq l)}$	O	$\mathrm{O}_{(i=l)}$ $\mathrm{E}_{(i\neq l)}$	$\mathrm{E}_{(k=l)}$ $\mathrm{O}_{(k\neq l)}$
$\mathcal{C}$	E/O	O/E	O/E	E/O	E/O	E/O	O/E	O/E
χ	P	N	N	P	N	N	P	P

- Without chiral symmetry, possible operator mixings remain:

$$\gamma_3 \longleftrightarrow I, \qquad \epsilon_{kij}\sigma_{ij} \longleftrightarrow \gamma_k\gamma_5 \qquad i,j,k \neq 3$$

- Chiral symmetry kills all the mixing.

Mixing pattern

- Non-local quark bilinear

$$O_\Gamma(\delta z) = \int_x \overline{\psi}(x+\delta z)\Gamma U_z(x+\delta z; x)\psi(x)$$

- e.g. Wilson -type fermion one-loop expression

$$\begin{aligned}\langle O_\Gamma(\delta z)\rangle &= (1+g^2\mathcal{A}_{\Gamma\Gamma}(\delta z, r^2))\langle O_\Gamma(\delta z)\rangle_0 \\ &\quad + g^2\mathcal{A}_{\Gamma\Gamma'}(\delta z, r^2)\,r\,\langle O_{\Gamma'}(\delta z)\rangle_0,\end{aligned}$$

$$\begin{aligned}\langle O_{\Gamma'}(\delta z)\rangle &= g^2\mathcal{A}_{\Gamma'\Gamma}(\delta z, r^2)\,r\,\langle O_\Gamma(\delta z)\rangle_0 \\ &\quad + (1+g^2\mathcal{A}_{\Gamma'\Gamma'}(\delta z, r^2))\langle O_{\Gamma'}(\delta z)\rangle_0.\end{aligned}$$

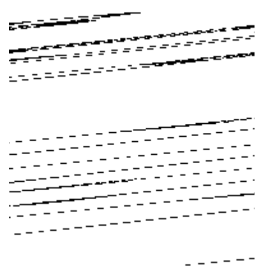
$$\boxed{\Gamma' = \gamma_3\Gamma + \Gamma\gamma_3}$$

[M. Constantinou, H. Panagopoulos (arXiv:1705.11193)]

Matching between continuum and lattice

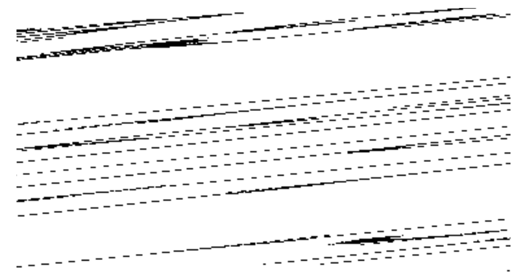
[T.I, YM Ma, JW Qiu, S. Yoshida (2016)]

One-loop in continuum (3d UV cutoff)



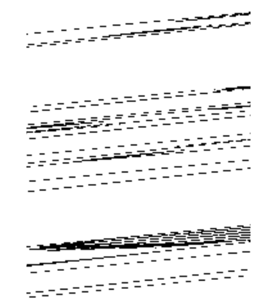
$\delta\Gamma_0$

vertex-type



$\delta\Gamma_1$

sail-type



$\delta\Gamma_2$

tadpole-type

$$\delta\Gamma_0(\delta z) = \frac{g^2 C_F}{8\pi^2}\left(\mathrm{Ei}(-k_{\perp z}) - (2+k_{\perp z})e^{-k_{\perp z}}\right)\Big|_{k_{\perp z}=\lambda|\delta z|}^{\mu|\delta z|} \xrightarrow[\delta z\to 0]{} \frac{g^2 C_F}{8\pi^2}\ln\frac{\mu}{\lambda},$$

$$\delta\Gamma_1(\delta z) = \frac{g^2 C_F}{4\pi^2}\left(\ln\frac{\mu}{\lambda} + \left(-\mathrm{Ei}(-k_{\perp z}) + e^{-k_{\perp z}}\right)\Big|_{k_{\perp z}=\lambda|\delta z|}^{\mu|\delta z|}\right) \xrightarrow[\delta z\to 0]{} 0,$$

$$\delta\Gamma_2(\delta z) = \frac{g^2 C_F}{4\pi^2}\left(\ln\frac{\mu}{\lambda} - \mathrm{Ei}(-k_{\perp z})\Big|_{k_{\perp z}=\lambda|\delta z|}^{\mu|\delta z|}\right) \xrightarrow[\delta z\to 0]{} 0.$$

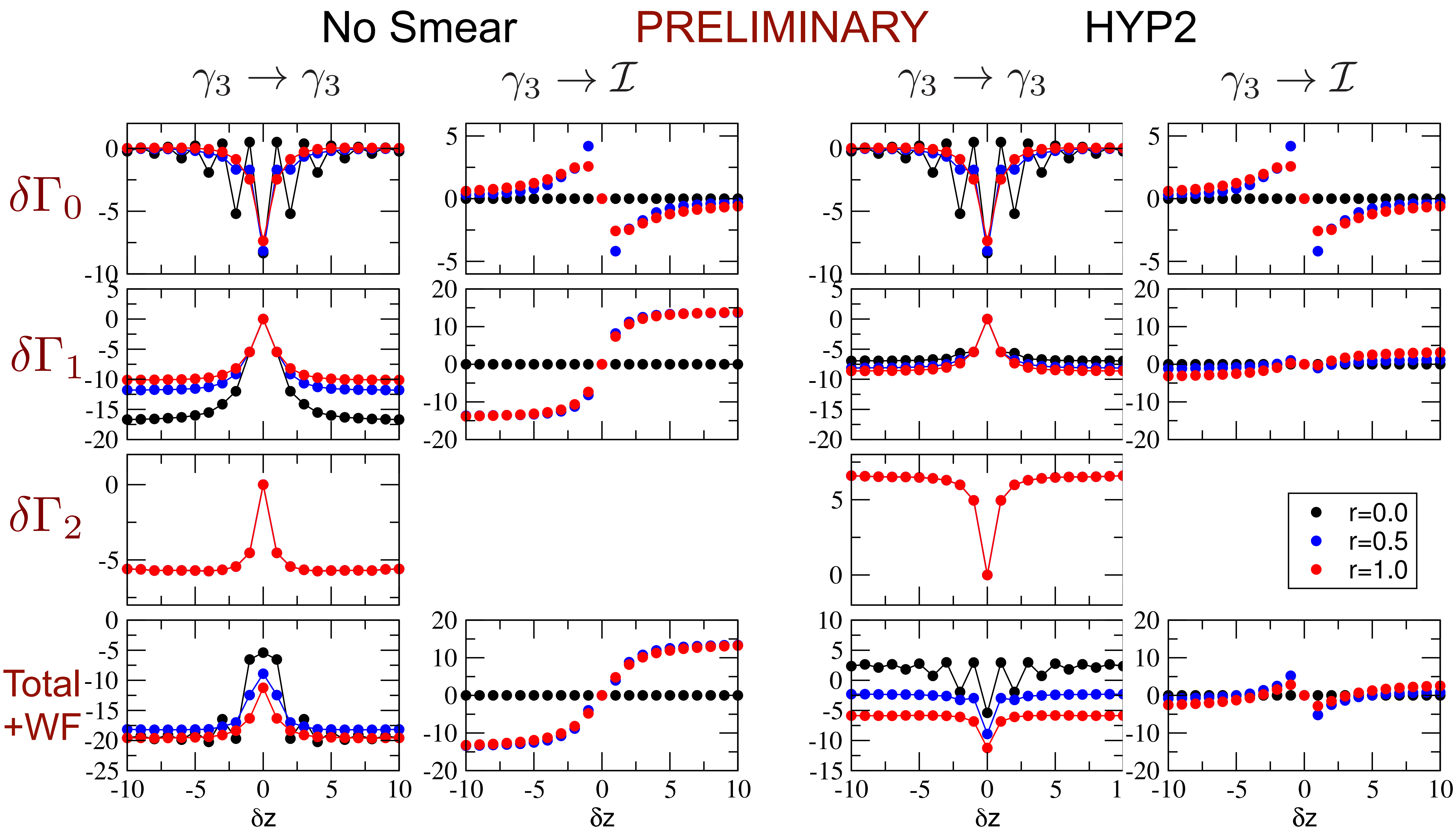
$\mathrm{Ei}(x) = -\int_{-x}^{\infty} dt\,\frac{e^{-t}}{t}$: exponential integral

- Local case ($\delta z \to 0$) can be reproduced.
- Linear divergence is already subtracted.
- UV(μ) and IR(λ) regulators are introduced in $\perp = (t, x, y)$ direction.
- 2d cutoff (transverse direction) is also possible, but no simple analytical expression.

Matching between continuum and lattice

- One-loop matching coefficients for Wilson fermion+Plaq gauge

$$Z(\delta z) = 1 + \frac{g^2}{(4\pi)^2} C_F c(\delta z) + O(g^4) \qquad \mu = a^{-1}$$



Discretization error analysis

- Regularization Independent Momentum Scheme for NPR

RI/MOM condition

$$Z_\Gamma^{\mathrm{RI/MOM}(\mu_R)} Z_f^{\mathrm{RI/MOM}(\mu_R)} \frac{1}{12} \mathrm{Tr}\Lambda_\Gamma(p)\Gamma\big|_{p^2=\mu_R^2} = \frac{1}{12} \mathrm{Tr}\Lambda_\Gamma^{\mathrm{tree}}(p)\Gamma\big|_{p^2=\mu_R^2}$$

- Application to quasi-PDFs.

[ETMC (2017), J.-W. Chen et al. (2017)]

- Window problem

$$\Lambda_{\mathrm{QCD}} \ll \mu_R \ll 1/a$$

to avoid PT error   to avoid disc error

Discretization error is some worry.

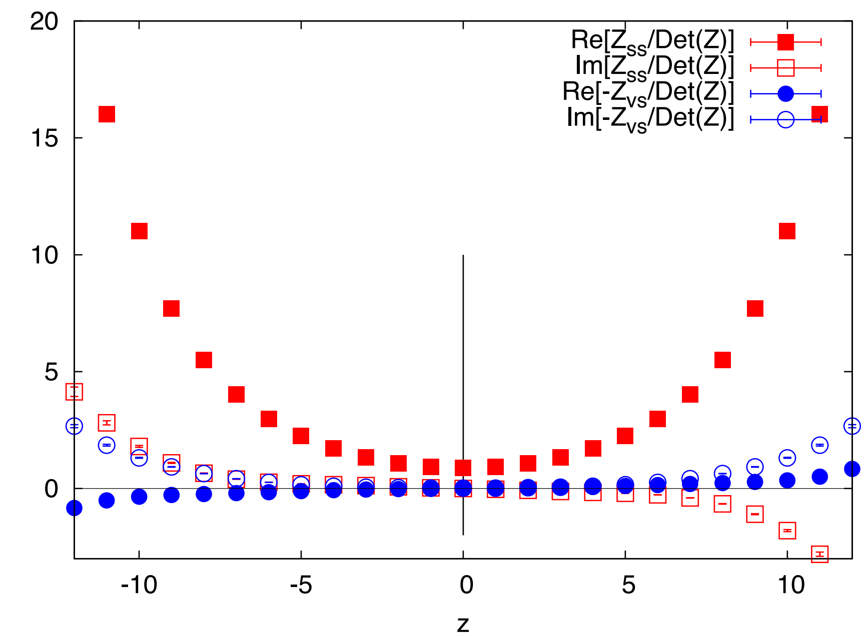


FIG. 2. The renormalization constant of the quasi-PDF operator $O_{\gamma_z}(z)$ (red boxes) and the mixing to the scalar quasi-PDF operator $O_{\mathcal{I}}(z)$(blue dots) with the momentum along the wilson link being $6\pi/L = 1.29$ GeV and $\mu_R^2 = p^2$=5.74 GeV^2. The size of the mixing coefficient is about an order of magnitude smaller than the renormalization factor in the large z region.

[J-W Chen et al. (arXiv:1706.0129)]

Discretization error analysis

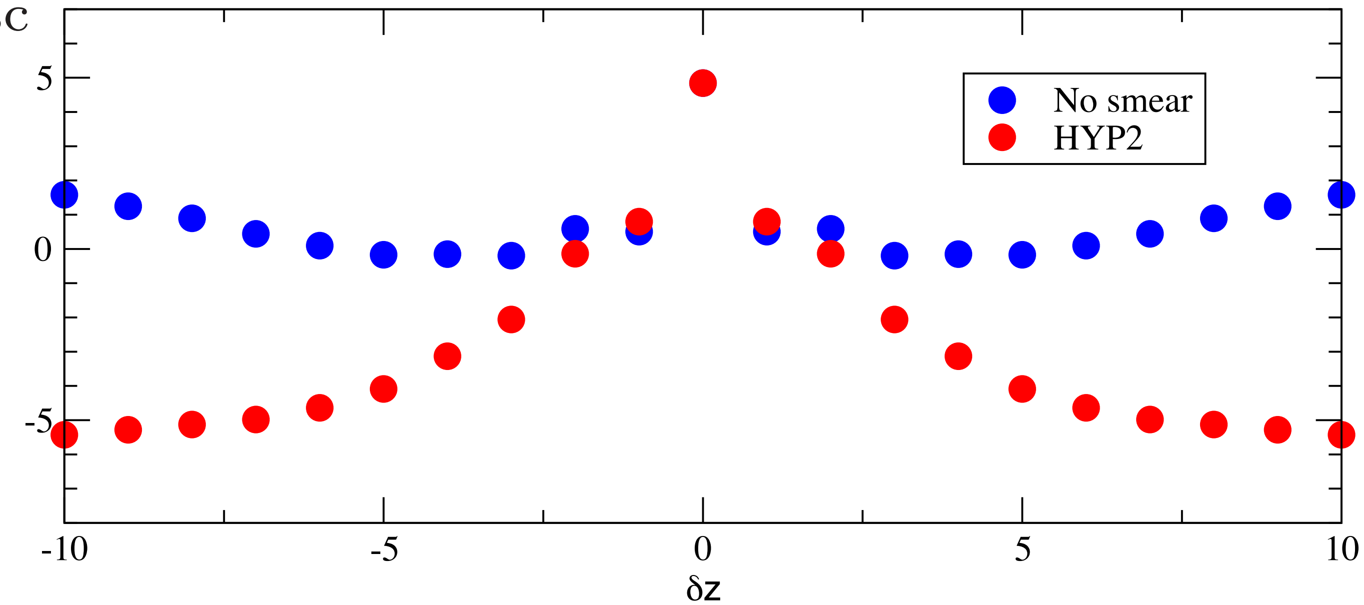
- Perturbative estimate of the discretization error

  - One-loop all order of pa error

$$\left[\frac{1}{12e^{-ip_3\delta z}}\mathrm{Tr}\left[\Lambda_{\gamma_3}(p)\gamma_3\right]\right]^{\mathrm{disc}}$$

1-loop coefficient

PRELIMINARY



$$\mathrm{Wilson}(r=1),\ \mathrm{Landau\ gauge},\ \vec{p}=(3,2,3)2\pi/L,\ L=24$$

- Perturbative subtraction of discretization error

  Discretization error in RI/MOM can be removed at one-loop level to help to reduce the uncertainty.

[QCDSF/UKQCD(2010), C. Alexandrou et. al. (2012)]

Summary and outlook

- PT matching with lattice perturbation:
  - Wilson fermion.
  - Calculations for Clover-Wilson and Domain-Wall fermion are also on-going.
- Application to RI/MOM NPR to reduce discretization error.
- O(pa) improvement for Cover-Wilson fermion to reduce possible large lattice artifact due to high momentum.