(COMMENTS ON) THE THETA-ANGLE, GLOBAL SYMMETRIES AND SIGN PROBLEM IN QED

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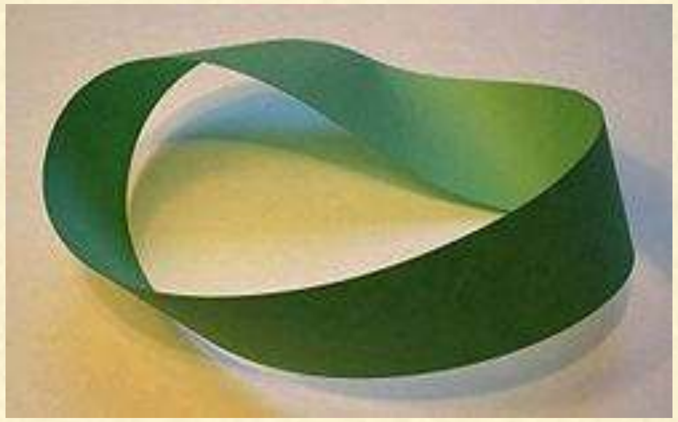
DISCLAMER

Most of what I will say is old wine in a new glass

What I will try to do is add a new twist and say what can, but hasn't been done on a lattice.



WHY ABELIAN THEORIES?

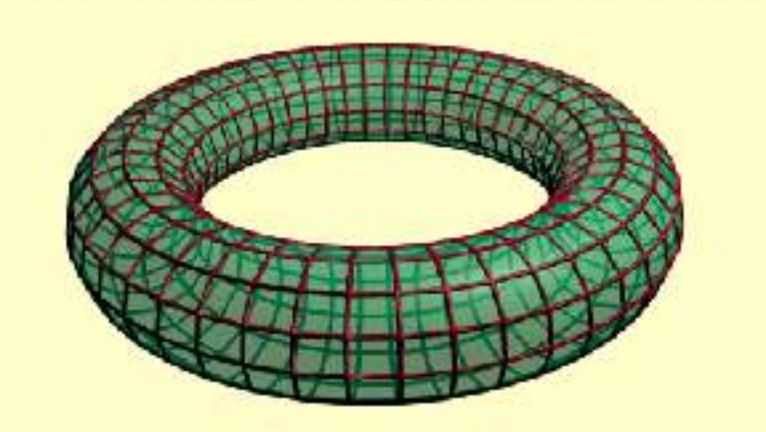
- Common excuse: they have confinement (trivially), theta-angle, and, in the case of CP(N-1) model, asymptotic freedom.
- This is a very tiny part of my motivation. In fact Abelian-Higgs systems have interesting applications in quantum magnets.
- Particularly 1(+1)D and 2(+1)D antiferromagnets are fascinating systems, which are, regrettably, not known much to high energy theorists.
  -TS, H. Shao, A. Sandvik, M. Unsal arXiv:1608.09011
  -Z. Komargodski, TS, M. Unsal arXiv:1706.05731

SOME COMMENTS AND LITERATURE

- The system I wish to focus is a U(1) Abelian-Higgs system in 1+1D with θ-angle.
- At arbitrary θ, the pure-gauge system is exactly solvable. It shows a level crossing at θ=π (1st order transition). θ=π theory breaks C-symmetry spontaneously.
- Gattringer et al. studied a sign-problem free variant of the abelian-Higgs theory with a θ-term and without the sign problem. They observe the 1st order transition (i.e. C-symmetry spontaneously broken).
- However at some critical mass m, the θ=π theory is expected to restore C-symmetry at some critical mass m (2D ising), while if more scalar flavors are added, the theory at θ=π will become a CFT— Komargodski et. al. arXiv: 1705.04786. In both cases the θ=π theory is non-confining!
- Furthermore in 2+1D there are interesting monopole phases with domain walls which act as 1+1D θ=π Abelian-Higgs system.

COMPACT V.S. NONCOMAPCT U(1): SYMMETRIES AND OPERATORS

$$\Delta\mathcal{L} = \frac{1}{4g^2} F_{\mu\nu}^2 \qquad A_\mu \to A_\mu + \partial_\mu \rho$$



What is the gauge group?

If the gauge group is the real line R, then ρ must be single-valued on the torus.

If this is the case then Polyakov loops are $e^{i\alpha \oint_1 A}$, $\alpha \in \mathbb{R}$ and are perfectly gauge invariant.

If the gauge group is U(1), then ρ~ρ+2π and can wind along an incontractible cycle, and only $e^{ik \oint_1 A}$, $k \in \mathbb{Z}$

Global symmetries (without matter):

$$A \to A + d\rho \;,\;\; \rho(L) = \rho(0) + \Delta$$ NOT A GAUGE SYMMETRY!!!

Non-compact

$$e^{i\alpha \oint A} \to e^{i\alpha\Delta} e^{i\alpha \oint A}$$

Compact

$$e^{ik \oint A} \to e^{ik\Delta} e^{ik \oint A}$$

$\Delta \sim \Delta + 2\pi$

A symmetry that acts on line-operators!

A 1-form symmetry, known as center symmetry

What about θ-terms?

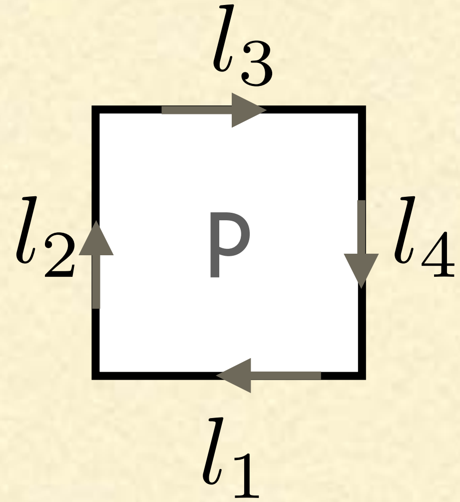
$$A(L) = A(0) \;, \text{– Non-compact case} \Rightarrow \int F = 0$$ No θ-term

$$A(L) = A(0) + d\phi \;, \text{– Compact case} \Rightarrow \int F \in 2\pi\mathbb{Z}$$

A 2π-periodic θ-term

Non-compact gauge theory on the lattice

$$S = J \sum_p F_p^2$$



$$F_p = A_{l_1} - A_{l_2} + A_{l_3} - A_{l_4}$$

$$F_p = \sum_{l \in \partial p} A_l$$

$A_l \in \mathbb{R}$ - gauge field living on links

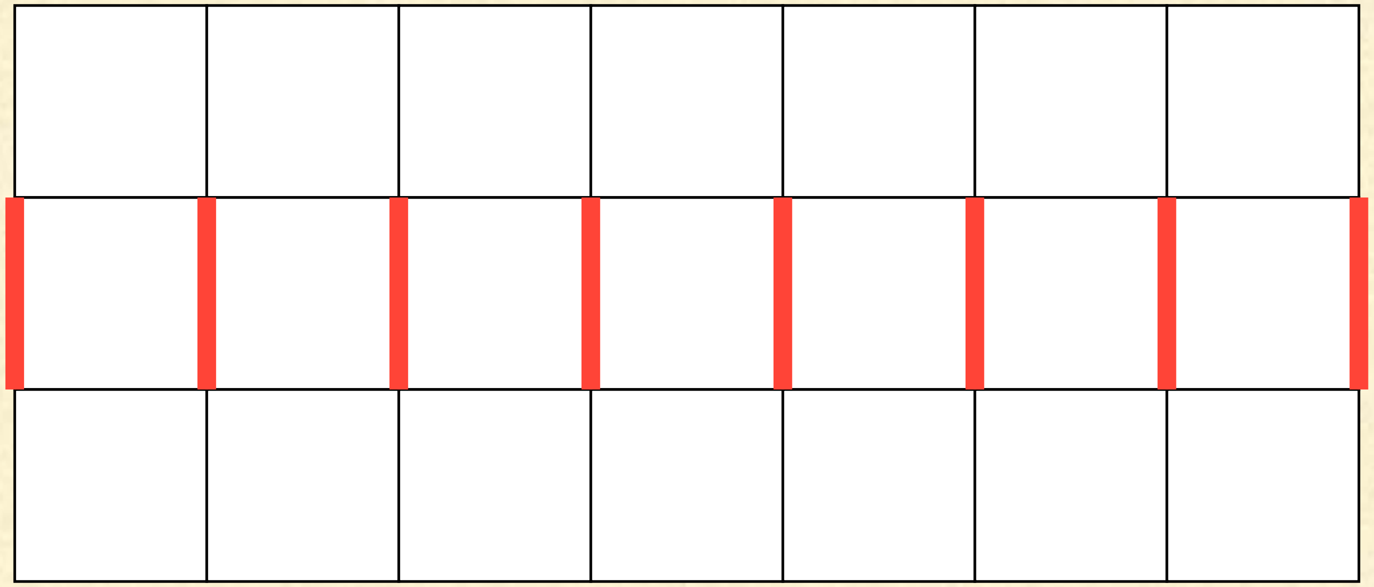
$A_l \to A_l + (\Delta\rho)_l$ - gauge symmetry

$(\Delta\rho)_{l(x,y)} = \rho_y - \rho_x$

(I will not discuss the measure of integration, which makes the definition of path integral problematic without gauge fixing. But I will comment on it later)

Global shift symmetry:

$A_l \to A_l + \delta$ for links which are parallel, on a given slice



THE CENTER SYMMETRY
(affects Polyakov loop in the relevant direction)

We wish to reduce this symmetry

$$\mathbb{R} \to U(1) = \mathbb{R}/\mathbb{Z}$$

Kapustin, Seiberg et. al, arXiv:1401.0740 , arXiv:1412.5148

$$F_p \to F_p + B_p \ , B_p \in 2\pi\mathbb{Z}$$

$$\left.\begin{aligned} A_l &\to A_l + 2\pi k_l \\ F_p &\to F_p + 2\pi(\Delta k)_p \\ B_p &\to B_p - 2\pi(\Delta k)_p \end{aligned}\right\}$$ discrete gauge symmetry!

B_p — 2-form gauge field!

$$S = J \sum_p (F_p + B_p)^2$$

Villain action (almost)!

The link variables A_l are still taking value on the infinite line.

The extra gauge field B allows an additional term in the action.

$$\frac{i\theta}{2\pi} \sum_p B_p$$

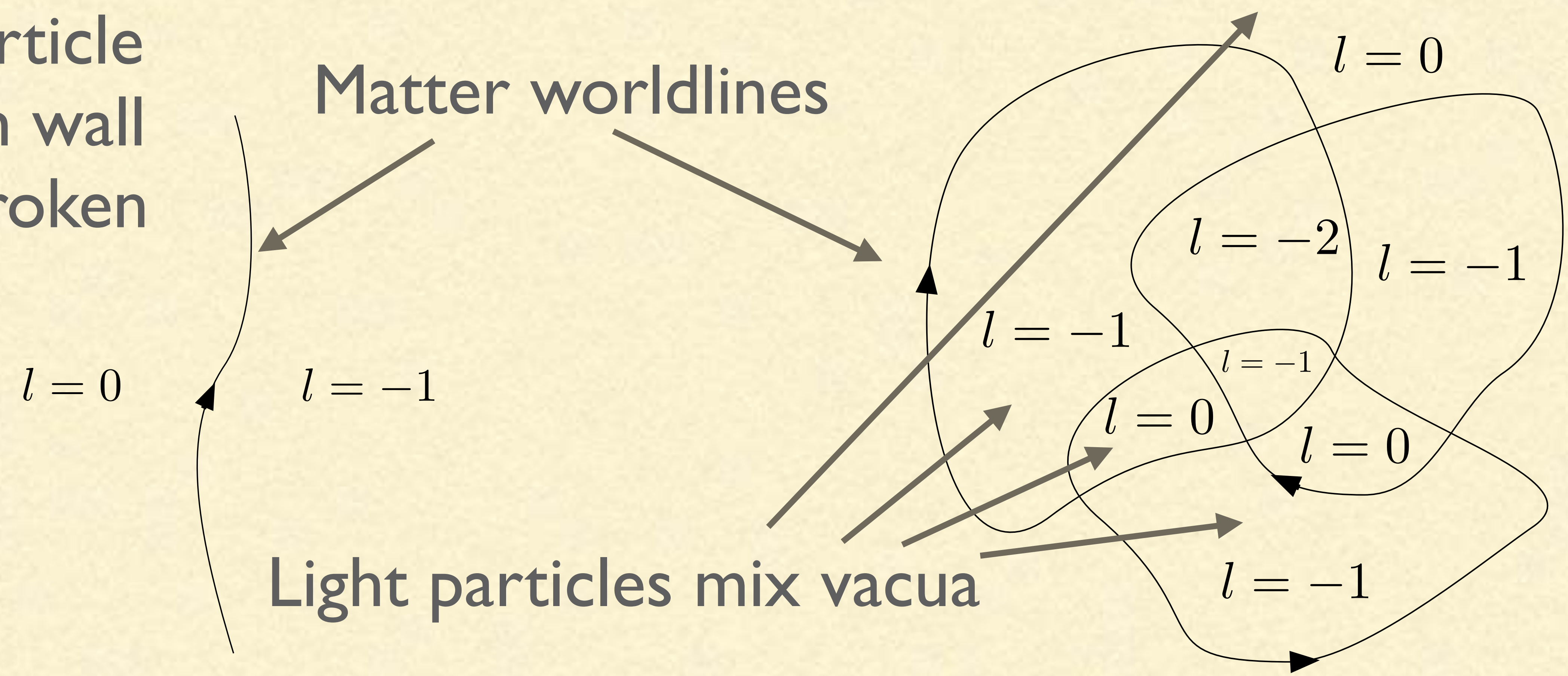
Poisson resumming over $B_{p,t}$ the partition function is just a summation of closed l_p=const.-fluxes wrapping around the torus

$$Z = \sum_l e^{-N_t N_x g^2 (l + \frac{\theta}{2\pi})^2}$$

At $\theta=\pi$, the $l=0,-1$ are degenerate: C-symmetry breaking

$$C_{\theta=\pi} : l \to -l - 1$$

Massive particle is a domain wall of the C-broken phase



What about higher dimensions?

In higher dimensions there is an additional symmetry:
the U(1) topological symmetry

3D: $j_\mu = \epsilon_{\mu\nu\rho} F_{\nu\rho} \qquad \partial_\mu j_\mu = \epsilon_{\mu\nu\rho} \partial_\mu F_{\nu\rho} = 0$

4D: $j_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma} \quad \partial_\mu j_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} \partial_\mu F_{\rho\sigma} = 0$

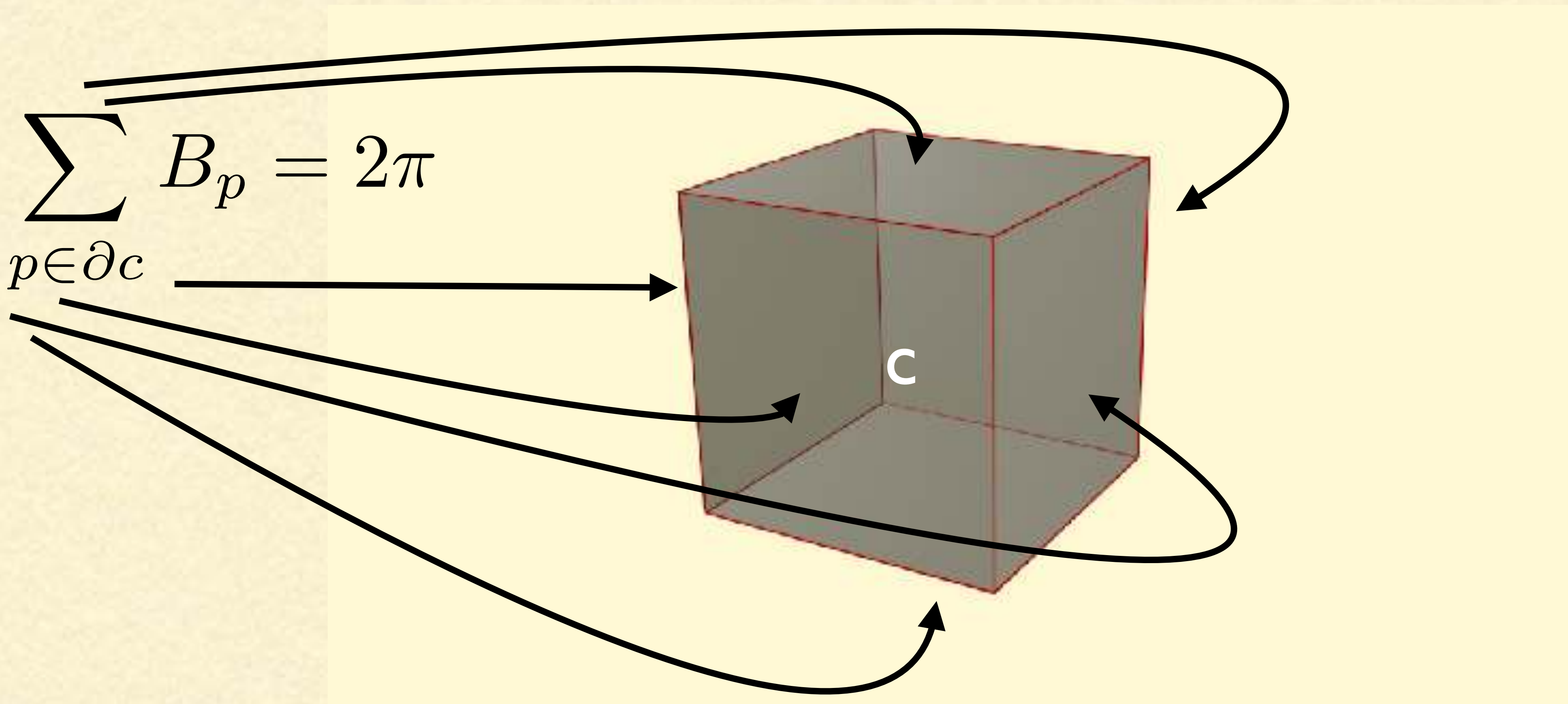
Both of these currents are EXPLICITLY broken by monopoles.
If only n-monopoles are allowed, there is a remaining topological symmetry

$$U(1)_T \to Z_n^T$$

In 3D, the system with n=4 describes a quantum antiferromagnet (QAF) with S=1/2 on a square lattice, while n=2 is the QAF with S=1/2 on the rectangular lattice or S=1 on a square lattice

WHAT ARE MONOPOLES?

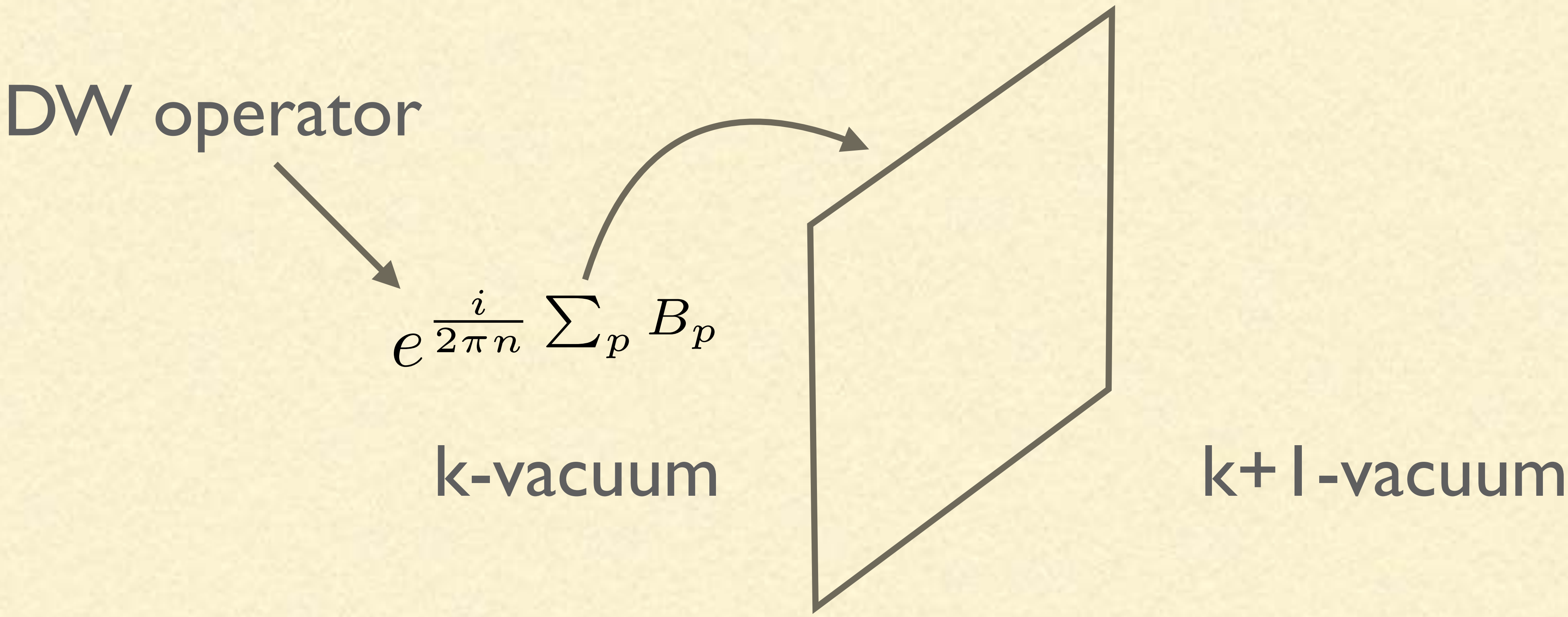
$$(\Delta B)_c = \sum_{p \in \partial c} B_p = 2\pi$$



MONOPOLE!

Constraining this to be either zero or a particular integer constructs a theory with a control of which part of the U(1)-topological symmetry is preserved

The discrete topological symmetry breaks spontaneously in a monopole phase and supports domain walls.



Again has a sign-problem, but it can be fixed by dualiazation

The domain wall for n=2 has interesting properties, and supports deconfined excitations and CFT. These phases have not been explored on the lattice! -TS, H. Shao, A. Sandvik, M. Unsal arXiv:1608.09011, -Z. Komargodski, TS, M. Unsal arXiv:1706.05731

CONCLUSIONS AND COMMENTS

- Interesting phases of the Abelian-Higgs systems were still not studied on the lattice
- These include the 1+1D theories with the θ angle as well as 2+1D theories with even monopole events
- These are effective theories of quantum magnets, so they have real-world application
- I haven't discussed the CP(N-1) model, but it can be done (but it's subtle).
- In addition 3+1D theories with 2-monopole particles are also immensely interesting: nontrivial dual ANO vortices
- Open questions: chern-simons in 3D and theta-term in 4D.

Thank you!

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