Decay constants and semileptonic form factors of B_c mesons

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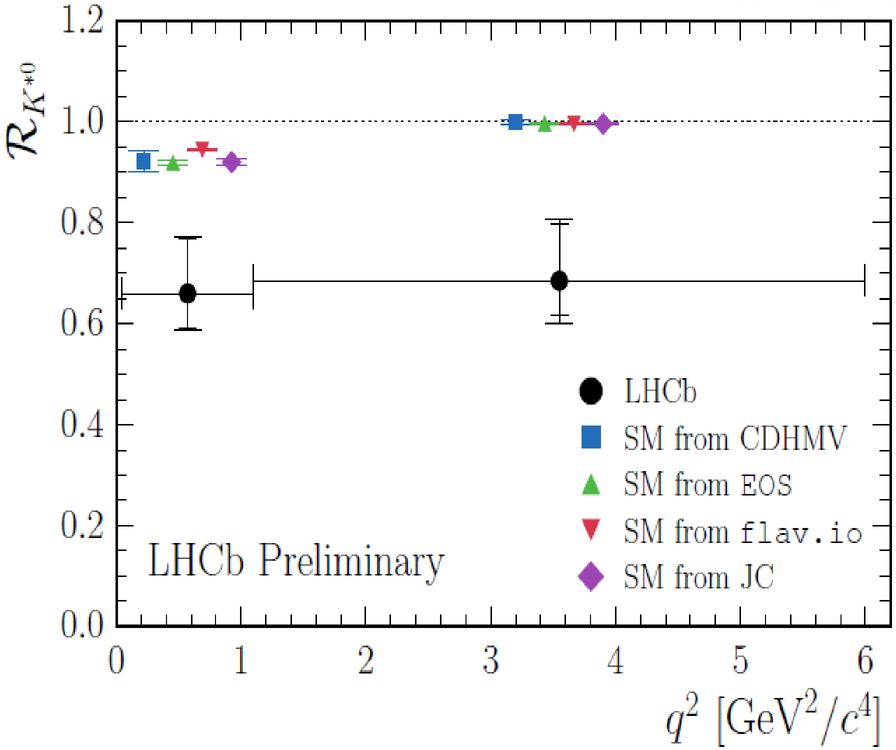
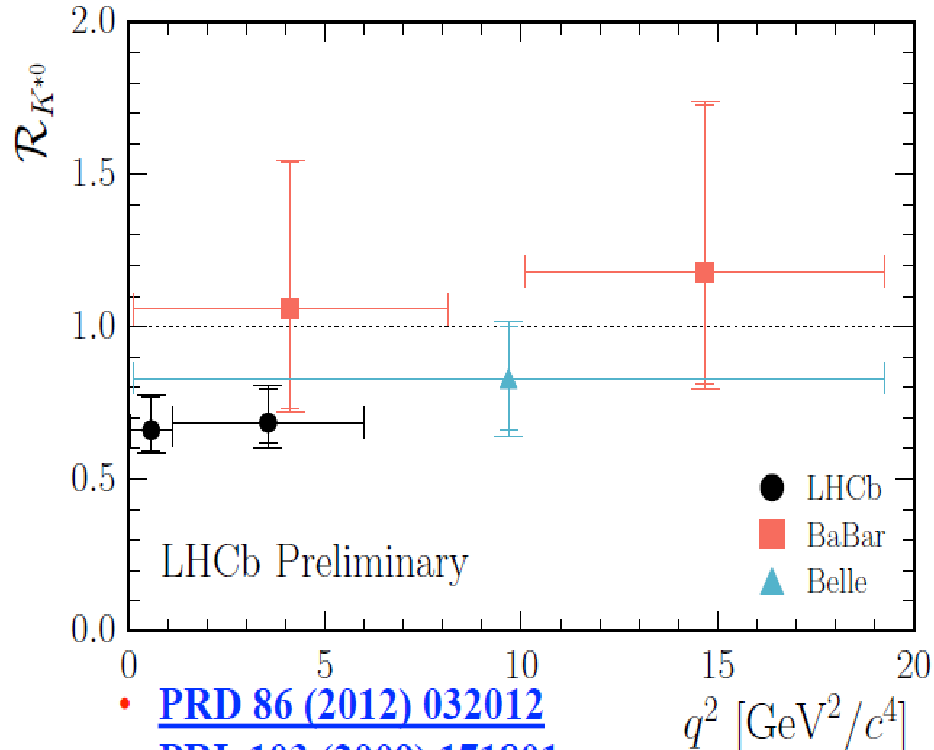
$$\mathcal{R}_{K^{*0}} = \frac{\mathcal{B}(B^0 \to K^{*0}\mu^+\mu^-)}{\mathcal{B}(B^0 \to K^{*0}J/\psi(\to \mu^+\mu^-))} \bigg/ \frac{\mathcal{B}(B^0 \to K^{*0}e^+e^-)}{\mathcal{B}(B^0 \to K^{*0}J/\psi(\to e^+e^-))}$$

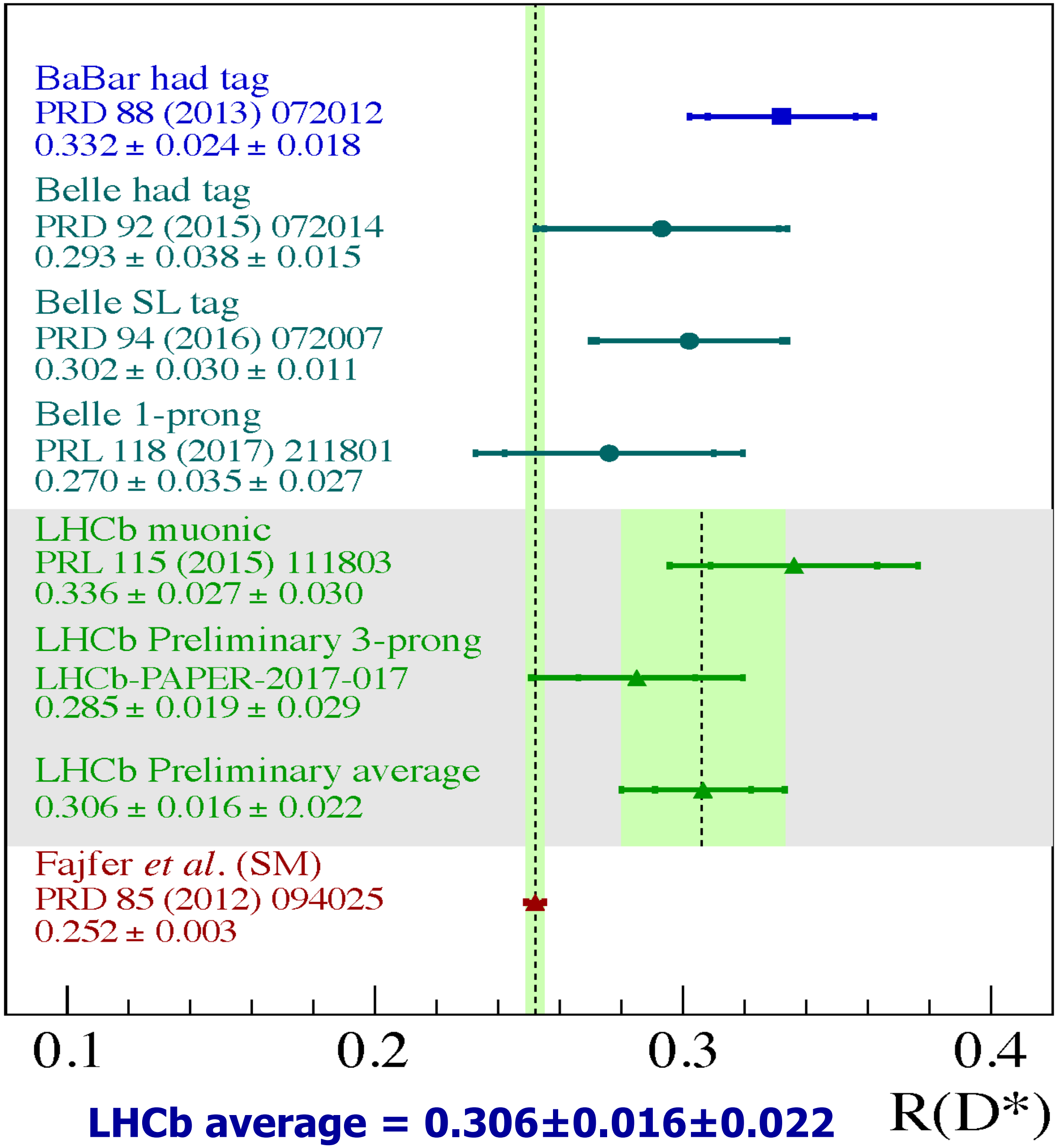




- PRD 86 (2012) 032012
- PRL 103 (2009) 171801

LHCb Preliminary	low-q^2	central-q^2
$\mathcal{R}_{K^{*0}}$	$0.660\ ^{+\ 0.110}_{-\ 0.070} \pm 0.024$	$0.685\ ^{+\ 0.113}_{-\ 0.069} \pm 0.047$
95% CL	[0.517–0.891]	[0.530–0.935]
99.7% CL	[0.454–1.042]	[0.462–1.100]

S. Bifani@Cern seminar



LHCb average = 0.306±0.016±0.022

Motivation

- **$|V_{cb}|$** from $b \to c$ transition in semileptonic decay
- Compare $R(B_c)$ and $R(B_c^*)$ with standard model prediction

- Expt :

  ✓ $B_C \to J/\psi\mu\nu$ R. Aaij et al. [LHCb Collaboration], Eur. Phys. J. C 74, 2839 (2014)

  $B_C \to J/\psi\tau\nu$ Soon?

- Lattice :
  - $B_C \to \eta_c l\nu$
  - $B_C \to J/\psi l\nu$

  Colquhoun *et al* **HPQCD : 1611.01987**

  A. Lytle : CKM2016

Set up

– Gauge Configurations: HISQ 2+1+1

– Valence quark propagators :

  -- Light to charm : Overlap

  -- Bottom : NRQCD with improved coefficients

Overlap fermions on 2+1+1 Flavors HISQ Configurations

- Lattices used for this study :

  HISQ gauge configurations from MILC

  24^3 x 64 , a = 0.12 fm, m_l/m_s = 1/5, $m_\pi L$ = 4.54, m_π = 305 MeV

  32^3 x 96 , a = 0.089 fm, m_l/m_s = 1/5, $m_\pi L$ = 4.5, m_π = 312 MeV

  48^3 x 144 , a = 0.058 fm, m_l/m_s = 1/5, $m_\pi L$ = 4.51, m_π = 319 MeV

  PHYSICAL REVIEW D 87, 054505 (2013) (MILC)

- **HYP smearing on gauge fields**
- **Both point source and coulomb gauge fixed wall source are used**
- **No of eigenvectors projected : 350 (a = 0.012 fm)**

  : 350 (a = 0.09 fm)

  : 75 (a = 0.058 fm)

Overlap Fermions

- Some desirable features:
  - No $O(a)$ error.
  - The effective propagator :

$$(1-\tfrac{1}{2}D)D(m)^{-1} = (D_c + ma)^{-1}$$

$D_c = D/(1 - D/2)$ is chirally symmetric, i.e., $\{\gamma_5, D_c\} = 0$.

  - $D_c + m$ is like in the continuum formalism.
  - Multi-mass algorithm (more than 20 masses -10-15% overhead
  - Renormalization may be relatively simple (e.g. with chiral Ward identity).

- Undesirable feature:
  -- *Cost*

Rest mass Vs Kinetic mass

Charm mass is tuned by meson kinetic mass and not from rest mass ……a la FermiLab formulation

El-khadra et al, PRD55, 3933(1997)

Expanding the energy momentum relation in powers of pa

$$E(p)^2 = M_1^2 + \frac{M_1}{M_2}\mathbf{p}^2 + O(\mathbf{p}^4)$$

$$= M_1^2 + c^2 p^2$$

$$|\mathbf{p}| << m_0, 1/a$$

Rest mass : $M_1 = E(\mathbf{0})$

Kinetic mass : $M_2 = M_1/c^2$

NRQCD Action

$$aH = aH_0 + a\delta H; \quad aH_0 = -\frac{\Delta^{(2)}}{2am_b},$$

$$a\delta H = -c_1\frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2\frac{i}{8(am_b)^2}(\nabla\cdot\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\cdot\nabla)$$
$$- c_3\frac{1}{8(am_b)^2}\sigma\cdot(\tilde{\nabla}\times\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\times\tilde{\nabla})$$
$$- c_4\frac{1}{2am_b}\sigma\cdot\tilde{\mathbf{B}} + c_5\frac{\Delta^{(4)}}{24am_b} - c_6\frac{(\Delta^{(2)})^2}{16n(am_b)^2}.$$

∇ is the symmetric lattice derivative

$\Delta^{(2)}$ and $\Delta^{(4)}$ ➡ $\sum_i D_i^2$ and $\sum_i D_i^4$

Tadpole improvement.

H_0 ➡ $\mathcal{O}(v^2)$

Coefficients are improved $\mathcal{O}(\alpha_s^2 a^2 v^4)$

δH ➡ $\mathcal{O}(v^4)$

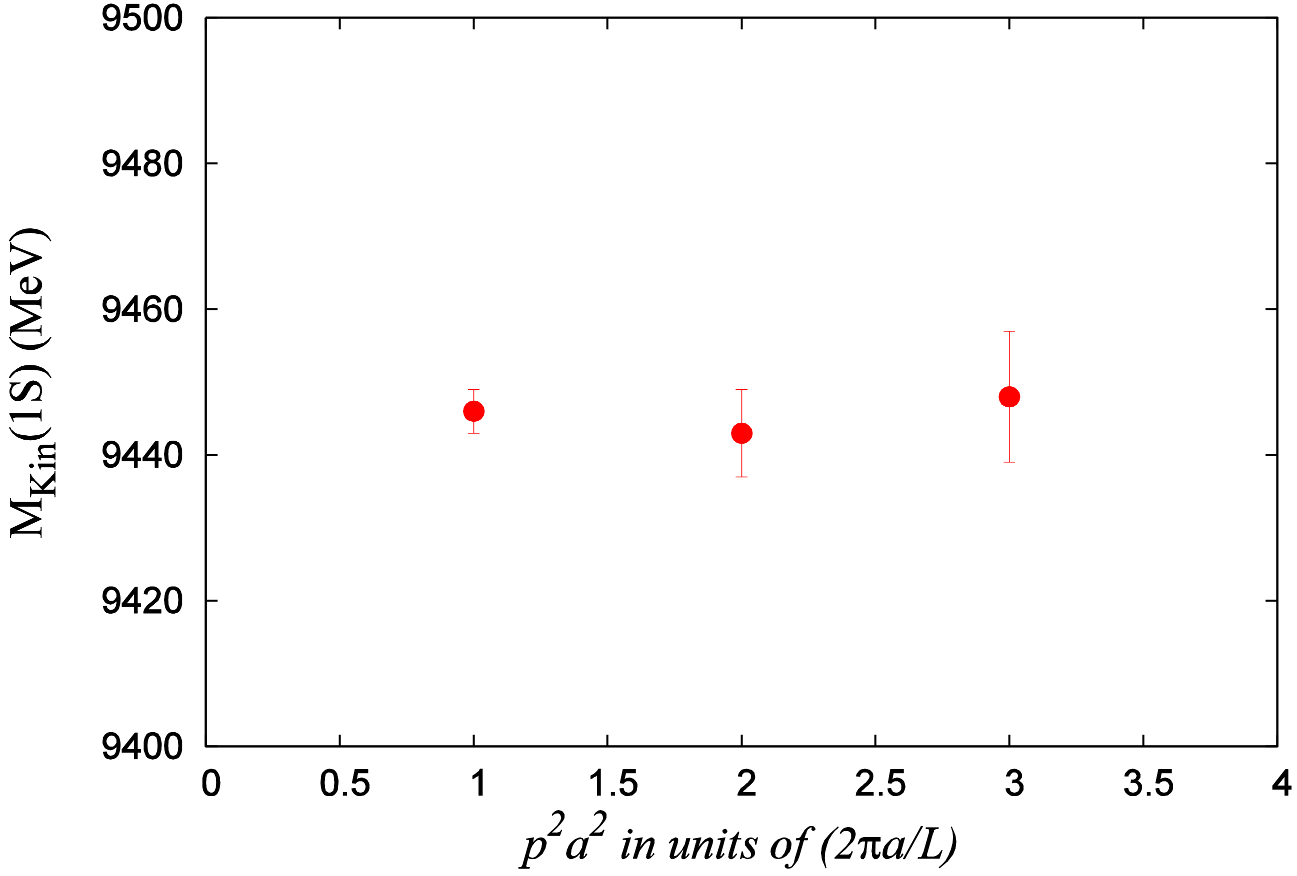
Dowdall et all, PRD 85,054509,2012
A. Hart et all, PRD 79, 074008 (2009)

$$aE(P) = aE(0) + \sqrt{a^2P^2 + a^2M_{\mathrm{Kin}}^2}$$

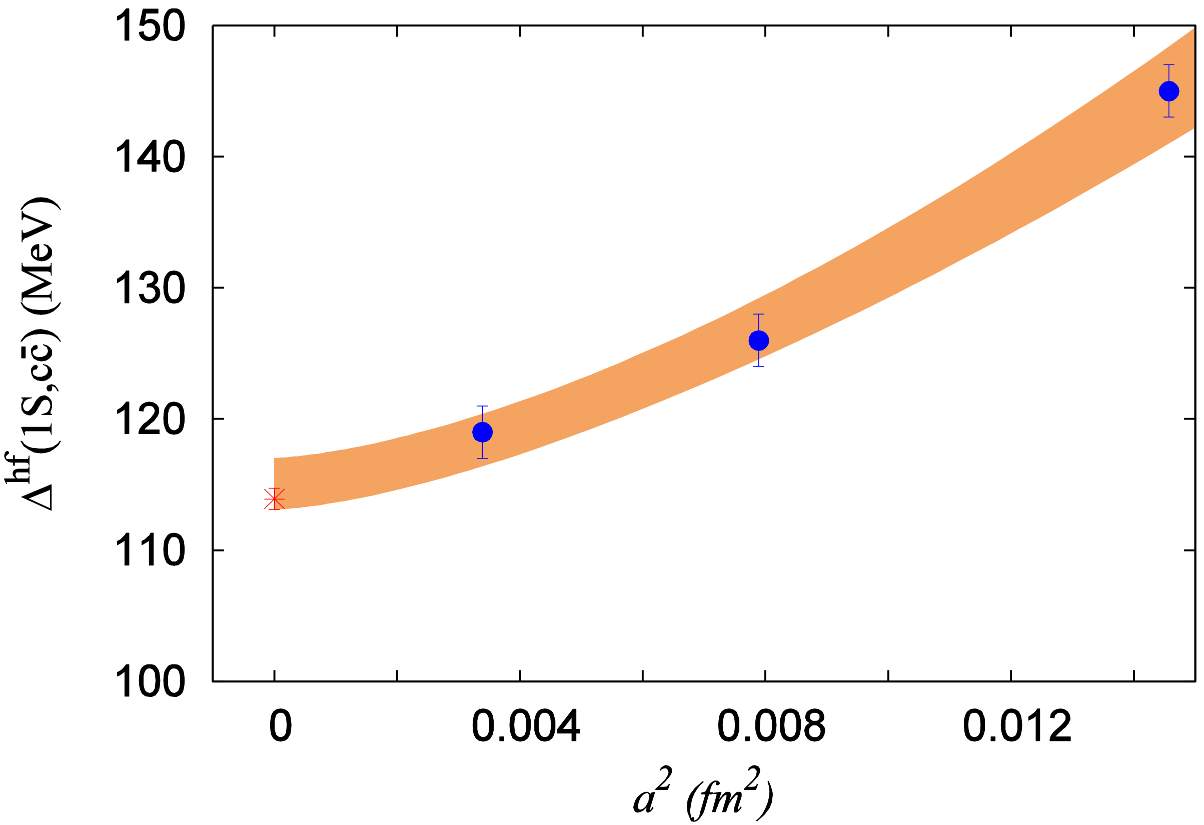
$$aM_{\mathrm{Kin}} = \frac{a^2P^2 - (a\Delta E)^2}{2a\Delta E}$$

$$\bar{M}_{\mathrm{Kin}}(1S) = \frac{(3M_{\mathrm{Kin}}(\Upsilon) + M_{\mathrm{Kin}}(\eta_b))}{4}$$

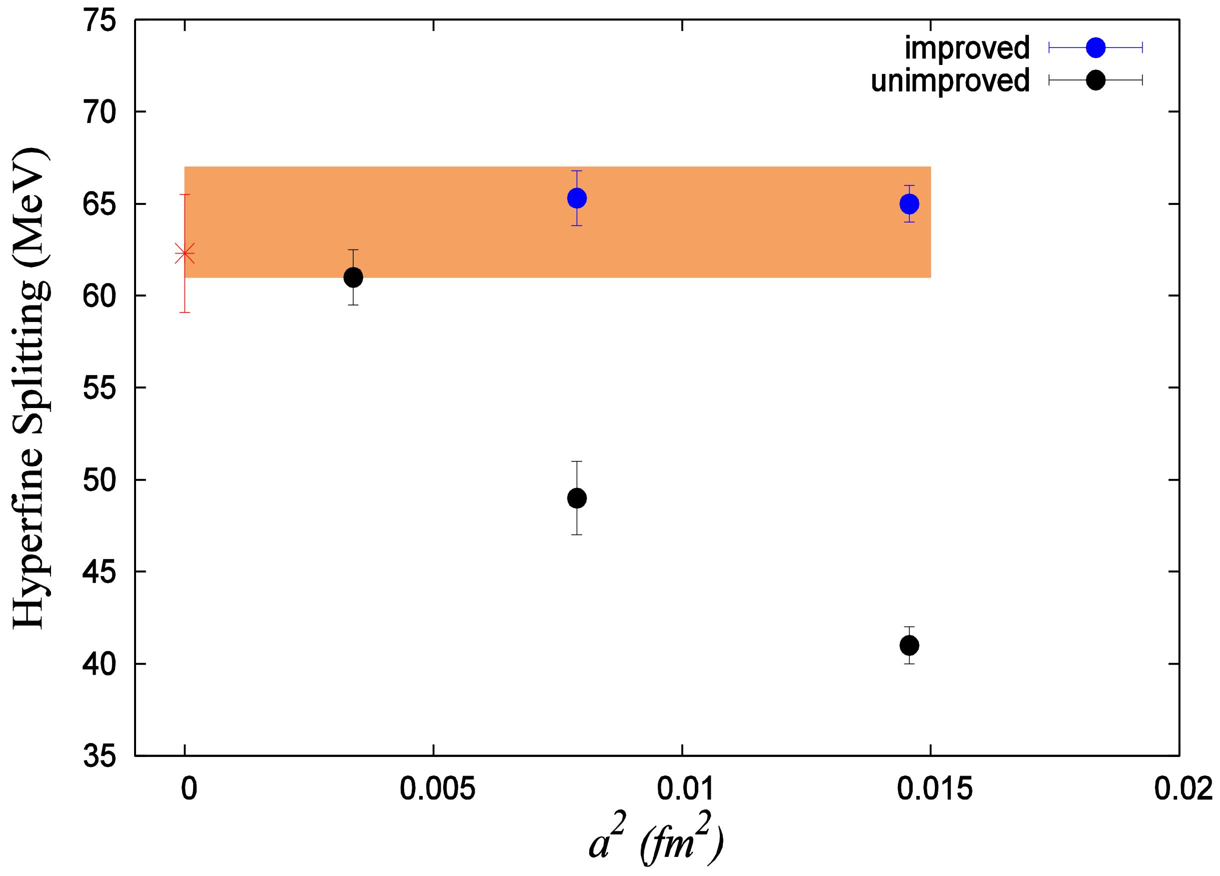
b-quark mass tunning



Hyperfine splitting in charmonia



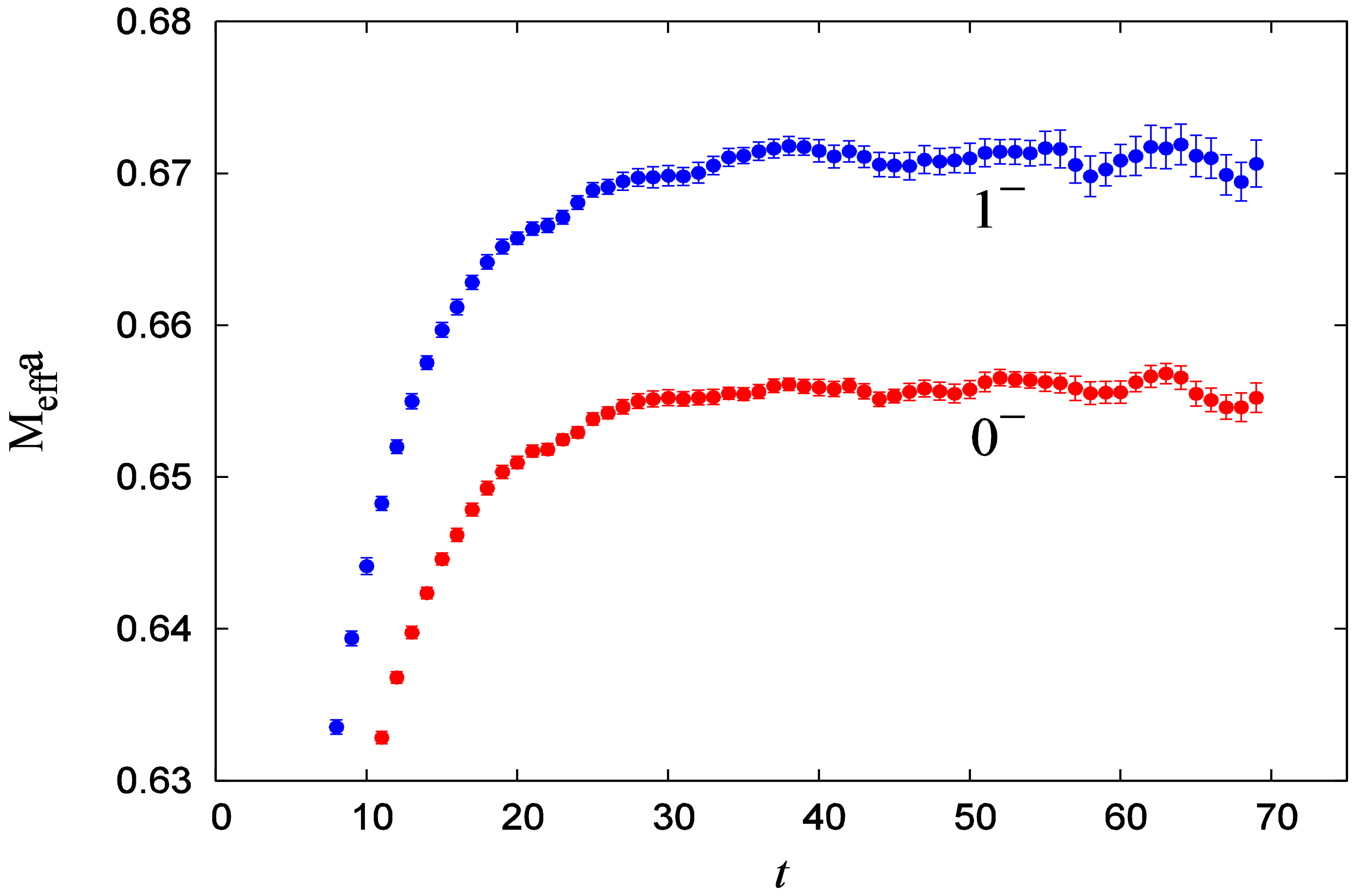
Hyperfine splitting in bottomonia



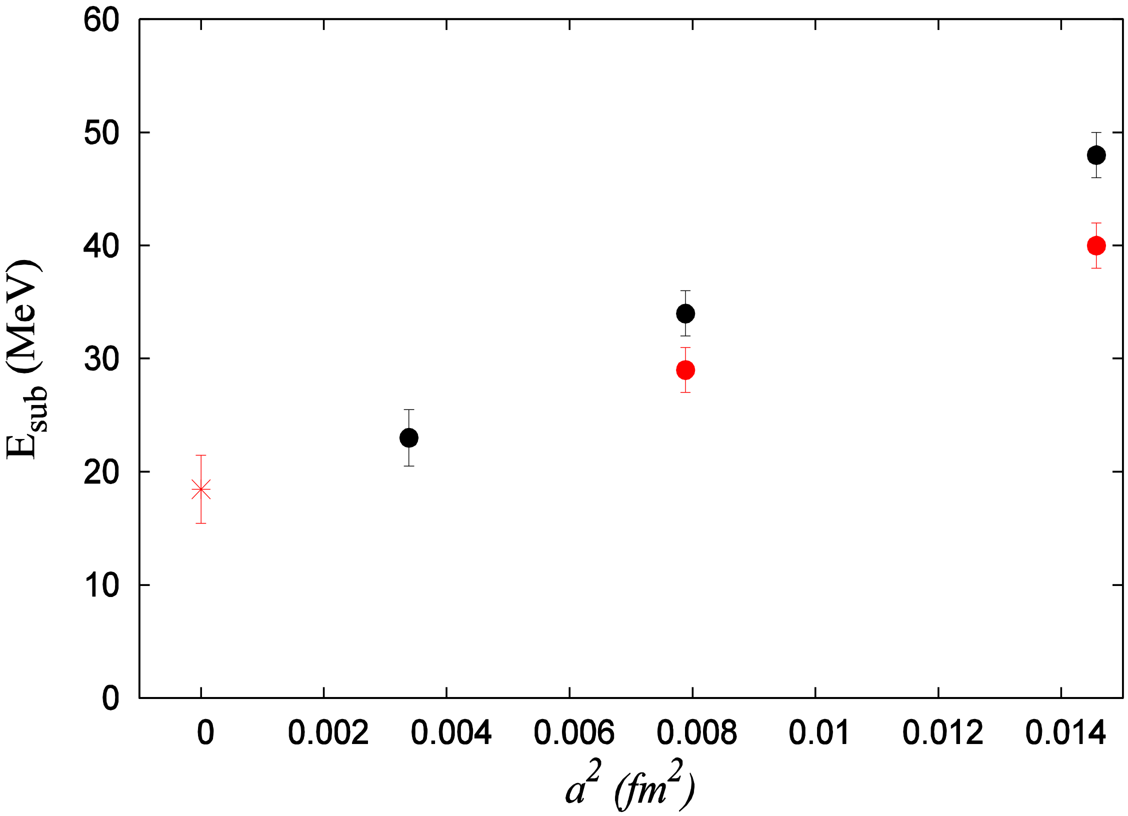
- We have calculated ground state energy spectra of mesons and baryons for all possible quantum numbers with ***l*, *s*, *c*** and ***b*** quarks (S.Mondal, talk Wednesday)

- Here we discuss decay constants and semileptonic form factors and present preliminary results.

Effective masses for B_c mesons

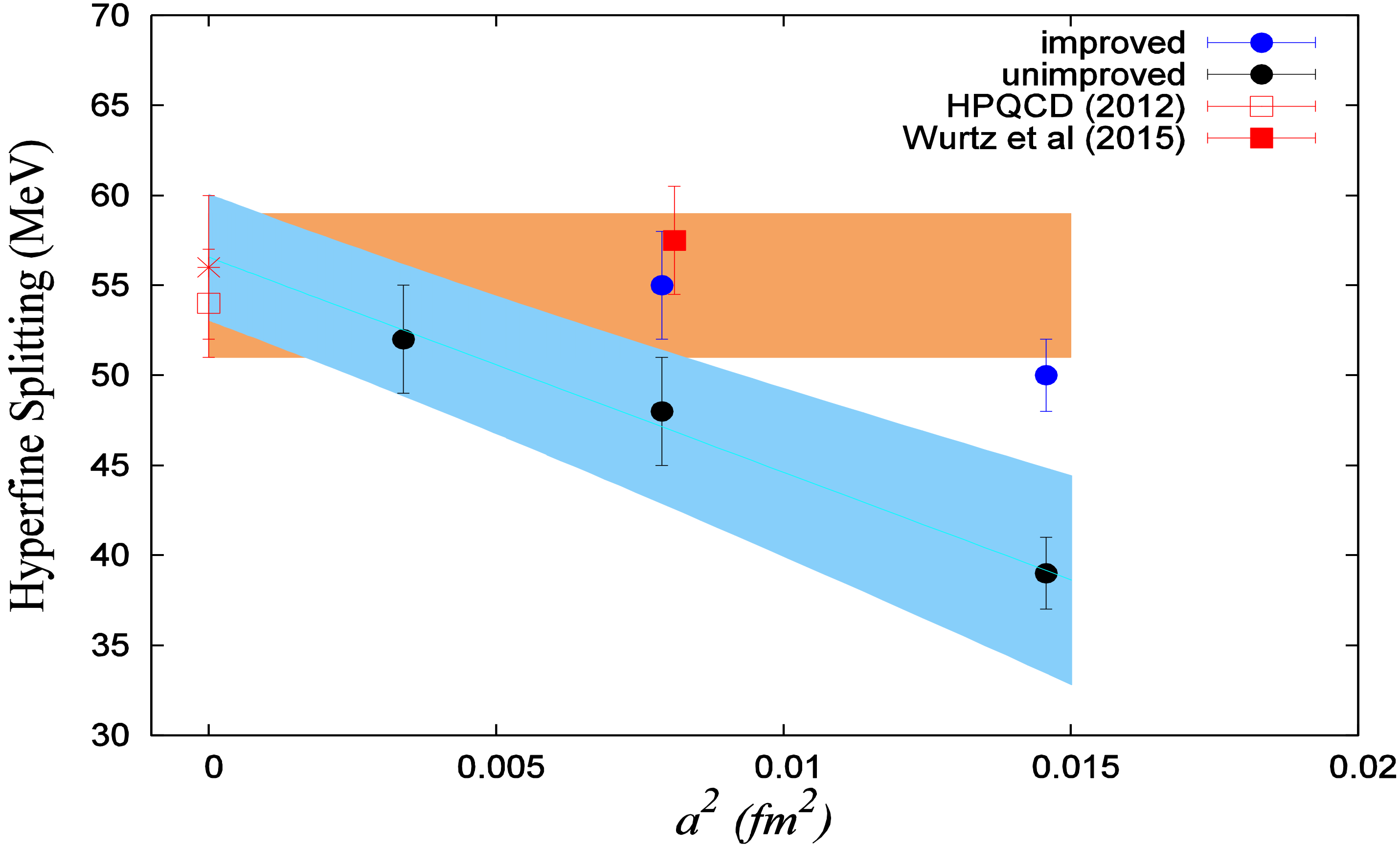


B_c (0^-)



$$E_X^{(\mathrm{sub})} = E_X - \frac{n_c}{2}\overline{E}_{c\bar{c}} - \frac{n_b}{2}\overline{E}_{b\bar{b}}$$

Prediction : B_c*



Currents and decay constants

Colquhoun *et al* HPQCD : Phys. Rev. D91 (2015), 114509

PS	V
$\langle 0 \vert J_{A_0} \vert B_q \rangle = f_{B_q} M_{B_q}$	$\langle 0 \vert J_{V_i} \vert B_q^j \rangle = f_{B_q^*} M_{B_q^*} \delta_{ij}$
$J_{A_0} = (1 + z_{A_0}\alpha_s)\left(J_{A_0}^{(0)} + J_{A_0}^{(1)}\right)$	$J_{V_i} = (1 + z_V \alpha_s)\left(J_{V_i}^{(0)} + J_{V_i}^{(1)}\right)$
$J_{A_0}^{(0)} = \bar{\Psi}_q \gamma_5 \gamma_0 \Psi_b$	$J_{V_i}^{(0)} = \bar{\Psi}_q \gamma_i \Psi_b$
$J_{A_0}^{(1)} = -\frac{1}{2m_b} \bar{\Psi}_q \gamma_5 \gamma_0 \vec{\gamma} \cdot \vec{\nabla} \Psi_b$	$J_{V_i}^{(1)} = -\frac{1}{2m_b} \bar{\Psi}_q \gamma_i \vec{\gamma} \cdot \vec{\nabla} \Psi_b$

$$C_J^i(t) = \sum_i \left| a_J^i \right|^2 e^{-E_i t} \qquad a_J^0 = \frac{\langle 0 | J | B_c \rangle}{\sqrt{2 M_{B_c}}}$$

Two point functions → Decay Constants

$$C_J^i(t) = \sum_i |a_J^i|^2 \, e^{-E_i t}$$

$$a_J^0 = \frac{\langle 0|J|B_c\rangle}{\sqrt{2M_{B_c}}}$$

Colquhoun *et al*

HPQCD :

Phys. Rev. D91 (2015), 114509

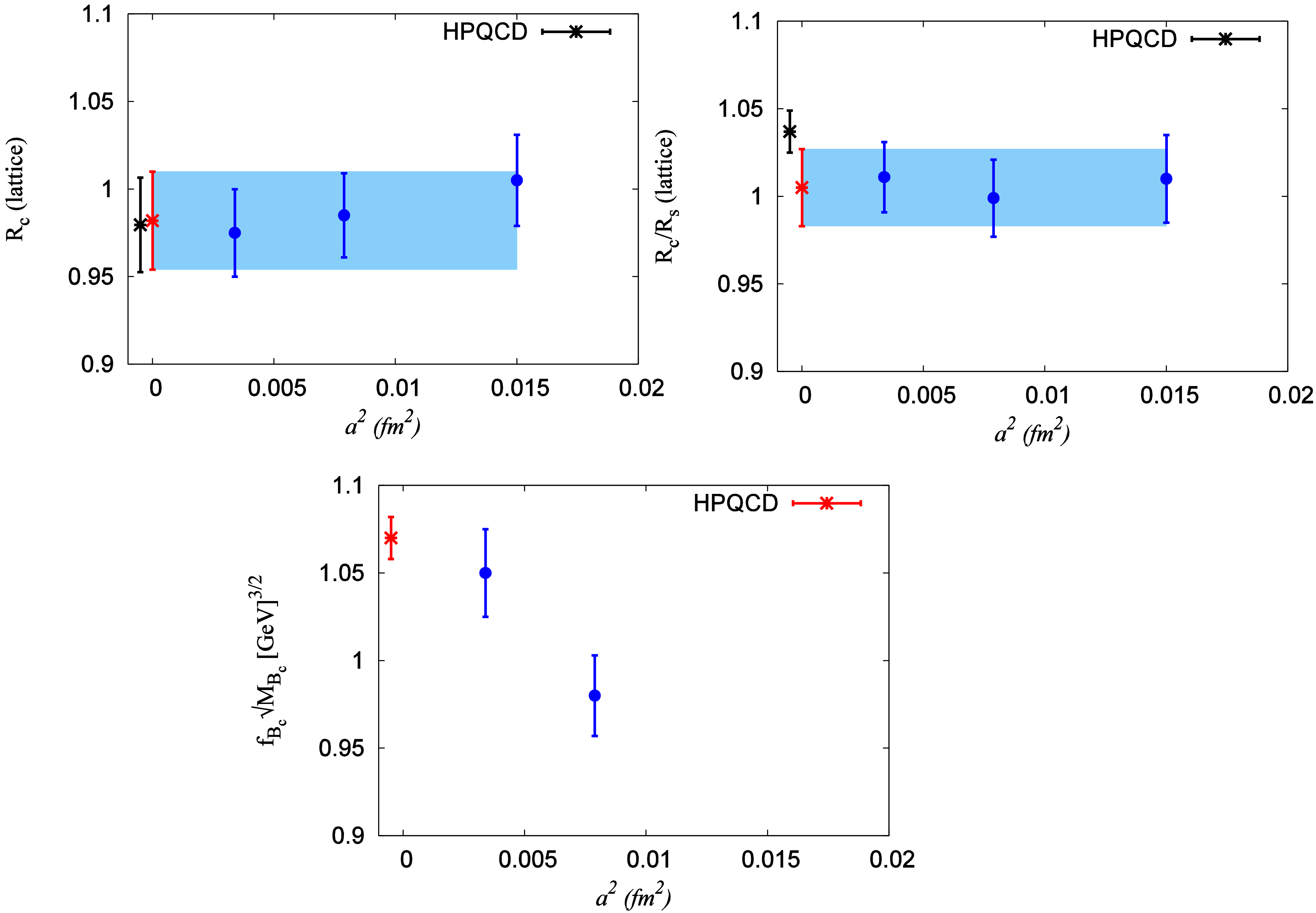
$$\Phi_J^{(0)} = \sqrt{2}\, a^0_{J^{(0)}} \qquad \Phi_J^{(1)} = \sqrt{2}\, a^0_{J^{(1)}}$$

$$f_{B_c}\sqrt{M_{B_c}} = (1 + z_{A_0}\alpha_s)\left(\Phi_{A_0}^{(0)} + \Phi_{A_0}^{(1)}\right)$$

$$f_{B_c^*}\sqrt{M_{B_c^*}} = (1 + z_V\alpha_s)\left(\Phi_{V_i}^{(0)} + \Phi_{V_i}^{(1)}\right)$$

$$R_c \equiv R_{B_c} \equiv \frac{f_{B_c^*}\sqrt{M_{B_c^*}}}{f_{B_c}\sqrt{M_{B_c}}} = (1 + \delta z\alpha_s)\frac{\left(\Phi_{V_i}^{(0)} + \Phi_{V_i}^{(1)}\right)}{\left(\Phi_{A_0}^{(0)} + \Phi_{A_0}^{(1)}\right)}$$

$$\frac{R_c}{R_s} = (1 + [\delta z_c - \delta z_s]\alpha_s)\frac{\left(\Phi_{B_c^*}^{(0)} + \Phi_{B_c^*}^{(1)}\right)}{\left(\Phi_{B_c}^{(0)} + \Phi_{B_c}^{(1)}\right)}\frac{\left(\Phi_{B_s^*}^{(0)} + \Phi_{B_s^*}^{(1)}\right)}{\left(\Phi_{B_s}^{(0)} + \Phi_{B_s}^{(1)}\right)}$$



HPQCD
R_c (lattice)
a^2 (fm^2)
1.1
1.05
1
0.95
0.9
0
0.005
0.01
0.015
0.02
HPQCD
R_c/R_s (lattice)
a^2 (fm^2)
HPQCD
$f_{B_c}\sqrt{M_{B_c}}$ $[\mathrm{GeV}]^{3/2}$
a^2 (fm^2)

Form factors

Colquhoun *et al* HPQCD : 1611.01987

A. Lytle : CKM2016

- $B_c \to \eta_c l\nu$

$$\langle \eta_c(p)|V^\mu|B_c(P)\rangle = f_+(q^2)\left[P^\mu + p^\mu - \frac{M^2-m^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M^2-m^2}{q^2}q^\mu$$

- $B_c \to J/\psi l\nu$

$$\langle J/\psi(p,\varepsilon)|V^\mu - A^\mu|B_c(P)\rangle = \frac{2i\varepsilon^{\mu\nu\rho\sigma}}{M+m}\varepsilon^*_\nu p_\rho P_\sigma V(q^2) - (M+m)\varepsilon^{*\mu}A_1(q^2) + \frac{\varepsilon^*\cdot q}{M+m}(p+P)^\mu A_2(q^2) + 2m\frac{\varepsilon^*\cdot q}{q^2}q^\mu A_3(q^2) - 2m\frac{\varepsilon^*\cdot q}{q^2}q^\mu A_0(q^2)$$

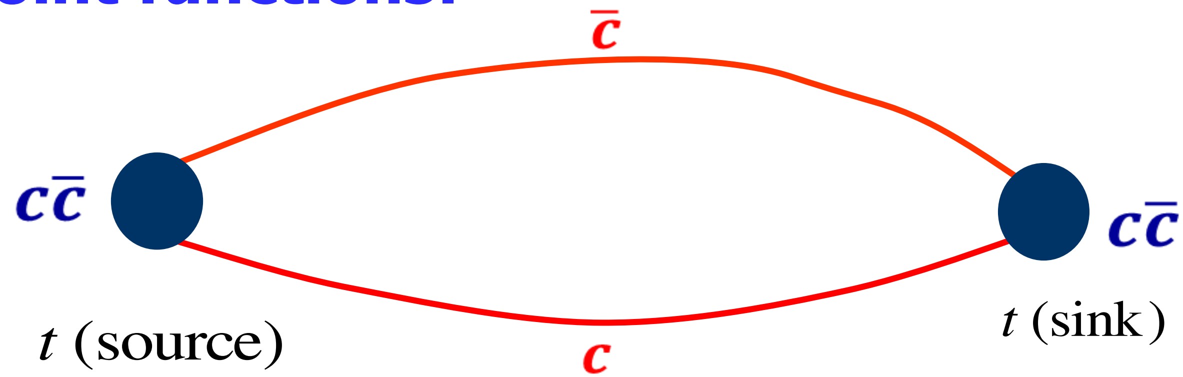
$$q = P - p$$

$q^2_{\max}$ **: Outgoing hadron at rest**

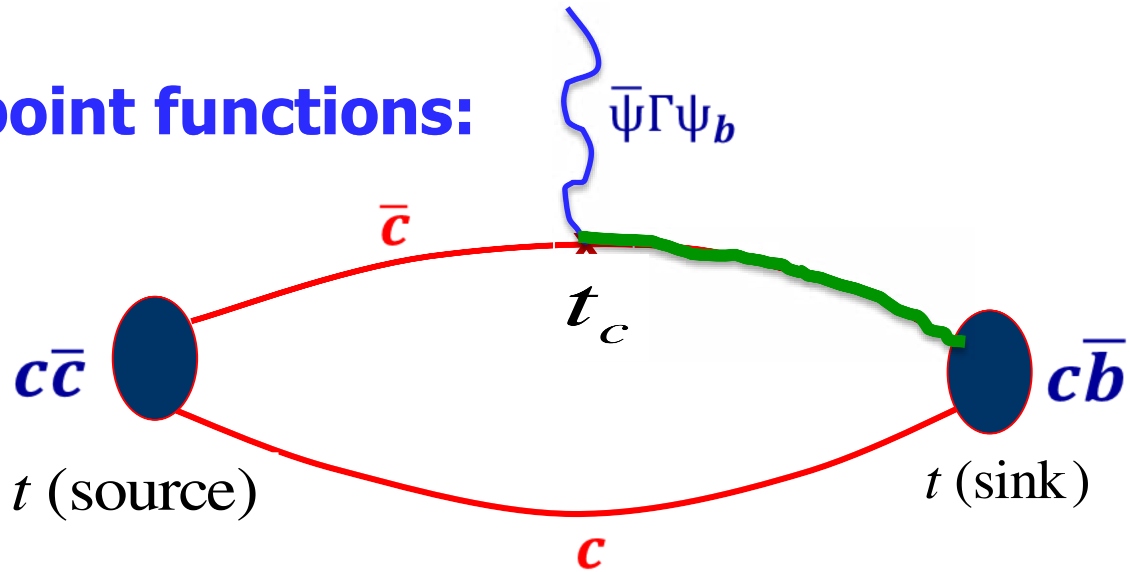
$q^2 = 0$ **: Maximum recoil**

Semileptonic Decay

- ❑ **Two point functions:**



- ❑ **Three point functions:**



Two and three point functions

$$C_2^{\eta_c}(t) = \sum_i \left(A_{\eta_c}^i\right)^2 e^{-E_{\eta_c}^i t}$$

$$C_2^{B_c}(\tau) = \sum_i \left(A_{B_c}^i\right)^2 e^{-E_{B_c}^i \tau}$$

$$C_{3,m}^{B_c \to \eta_c}(t,\tau) = \sum_{i,j} A_{\eta_c}^i \varphi^m A_{B_c}^j e^{-E_{\eta_c}^i t} e^{-E_{B_c}^i \tau}$$

φ^m: **Can be obtained by**

- **fitting these two and three point function simultaneously**
- **constructing appropriate ratios**

Ratios

$$R_1(t_i,\tau) = \frac{C_3(t,\tau)}{\left[C_2^{\eta_c}(t_i)C_2^{B_c}(t_i)\right]^{1/2}} e^{E_{\eta_c}(t-t_{i/2})} e^{E_{B_c}(\tau-t_{i/2})}$$

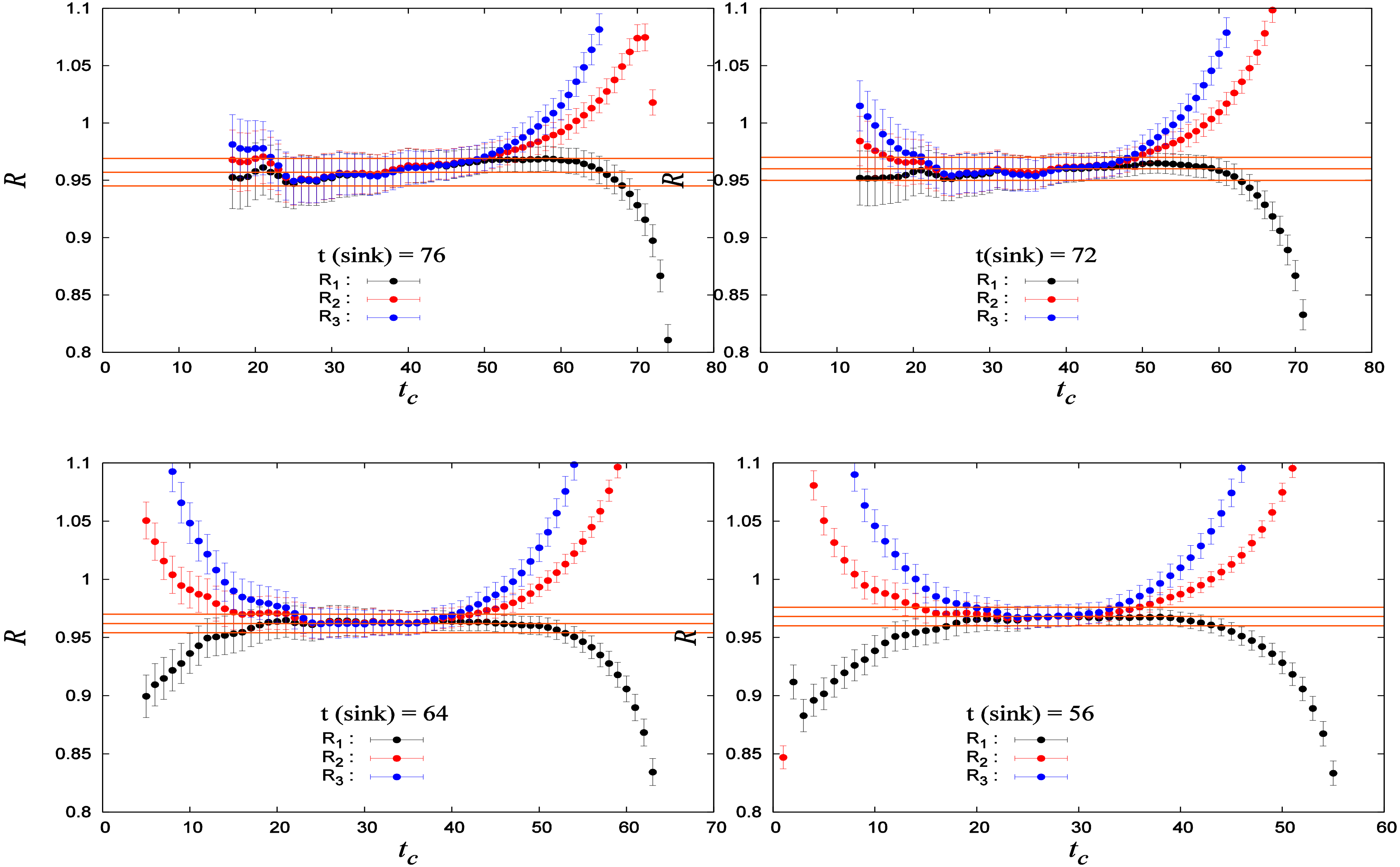
$$R_2(t,\tau) = \frac{C_3(t,\tau)}{\left[C_2^{\eta_c}(t)C_2^{B_c}(\tau)\right]^{1/2}} e^{E_{B_c}\tau/2} e^{E_{\eta_c}t/2}$$

$$R_3(t,\tau) = \frac{C_3(t,\tau)}{\frac{1}{A_{B_c}A_{\eta_c}} C_2^{\eta_c}(t)\, C_2^{B_c}(\tau)}$$

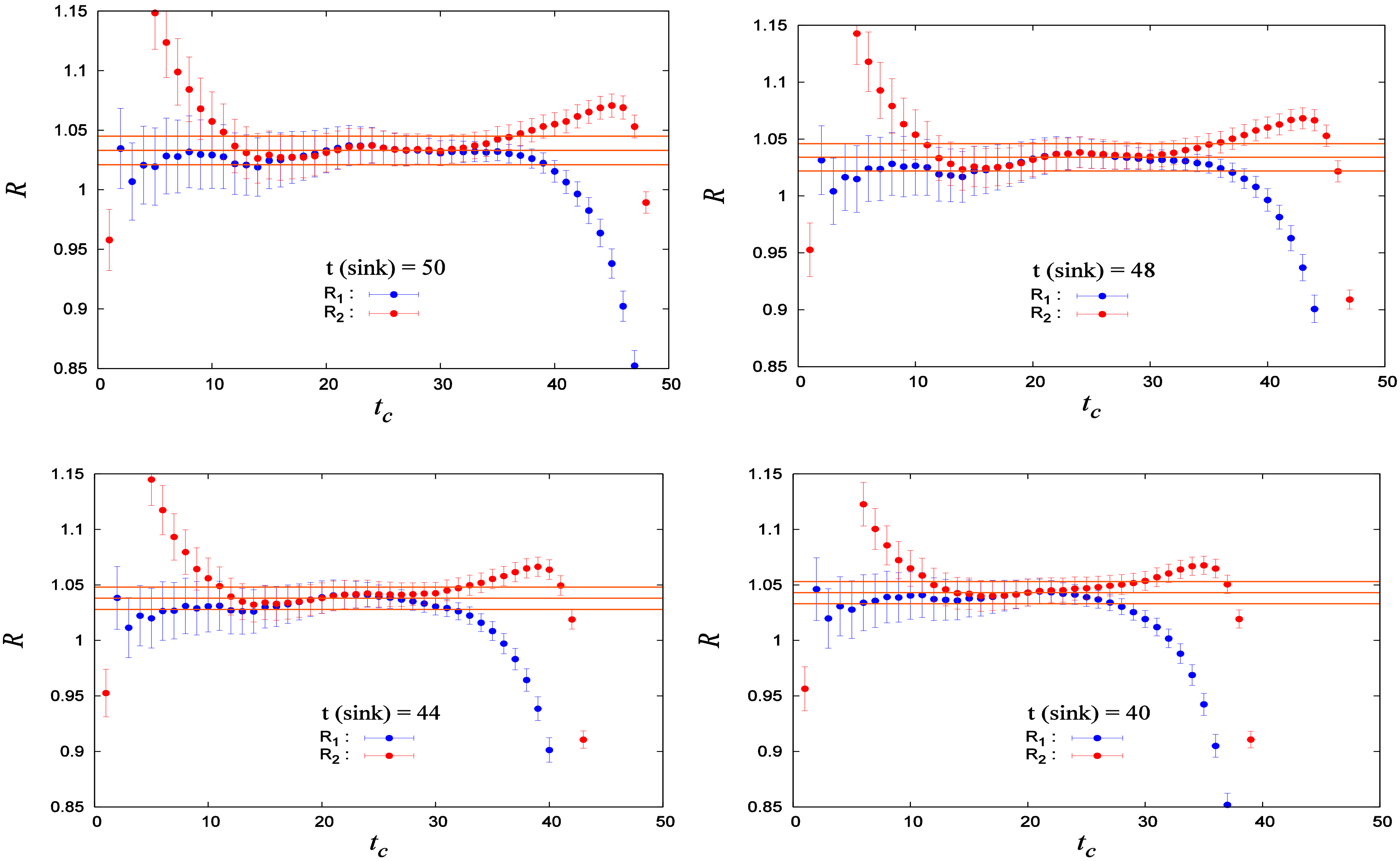
Bahr etal : 1701.03299

Banerjee et al PoS(LATTICE2016) 292

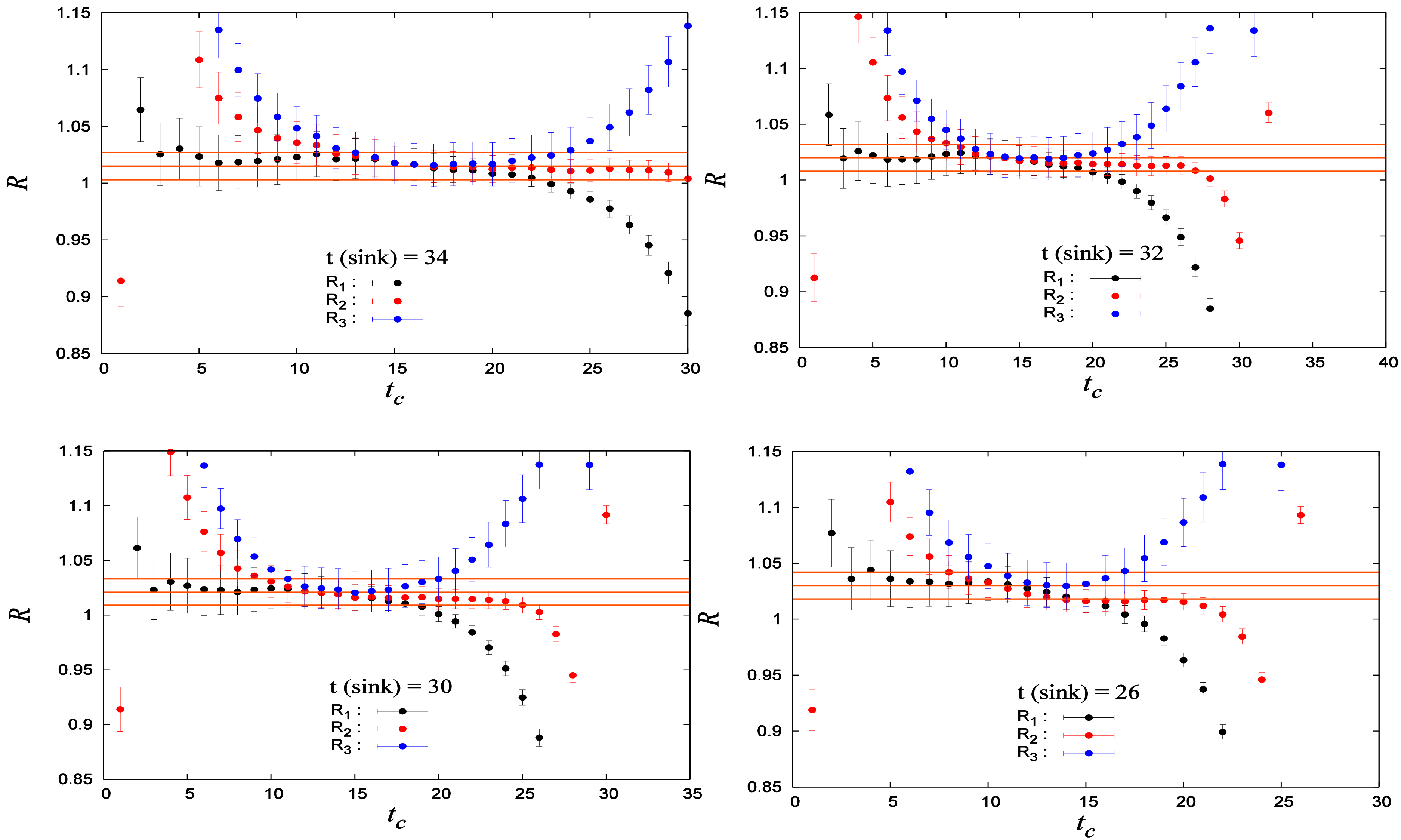
48^3x144 (a~$0.06fm$)

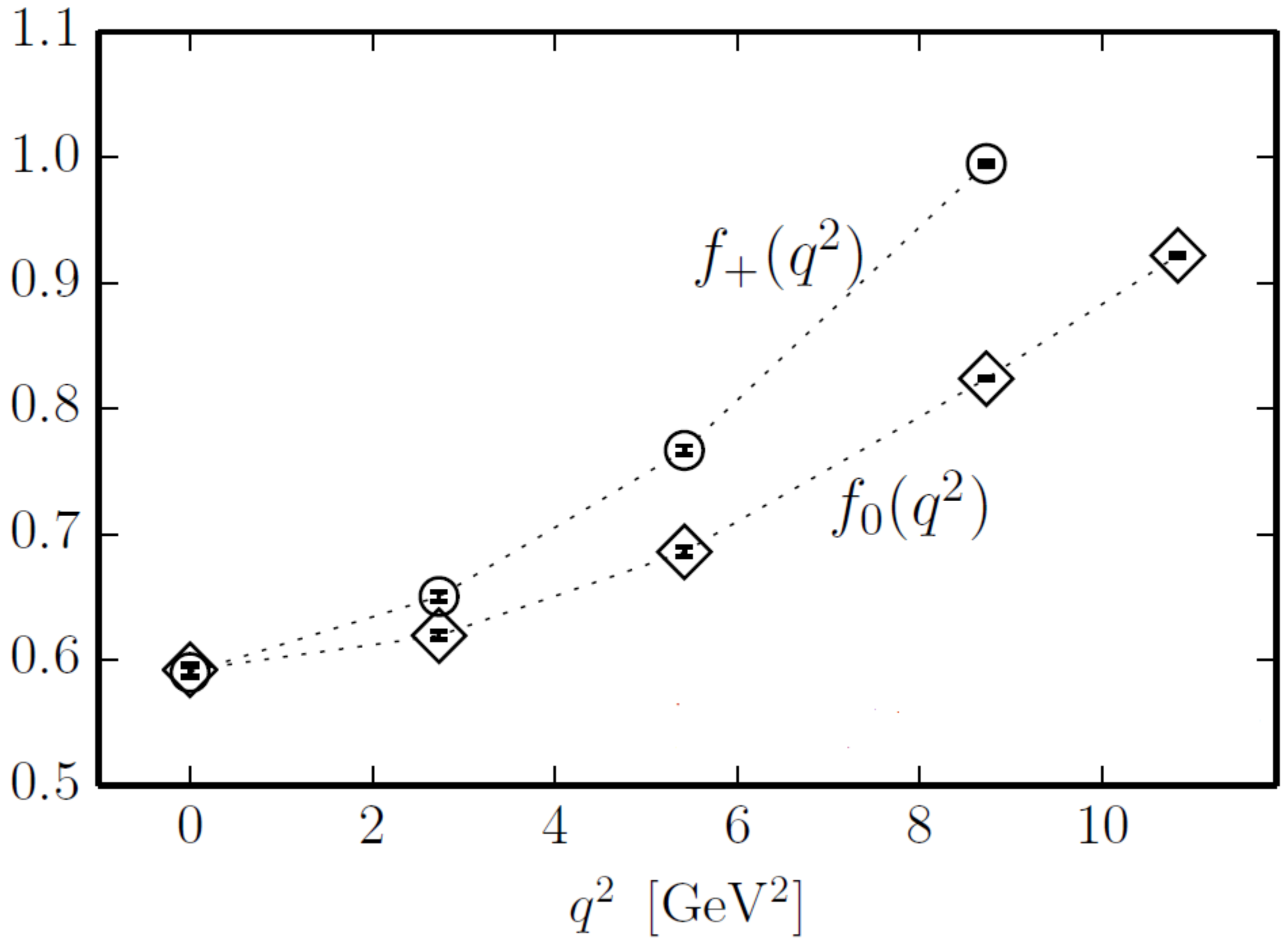


36^3x96 (a~0.09fm)



24^3x64 (a~0.12fm)

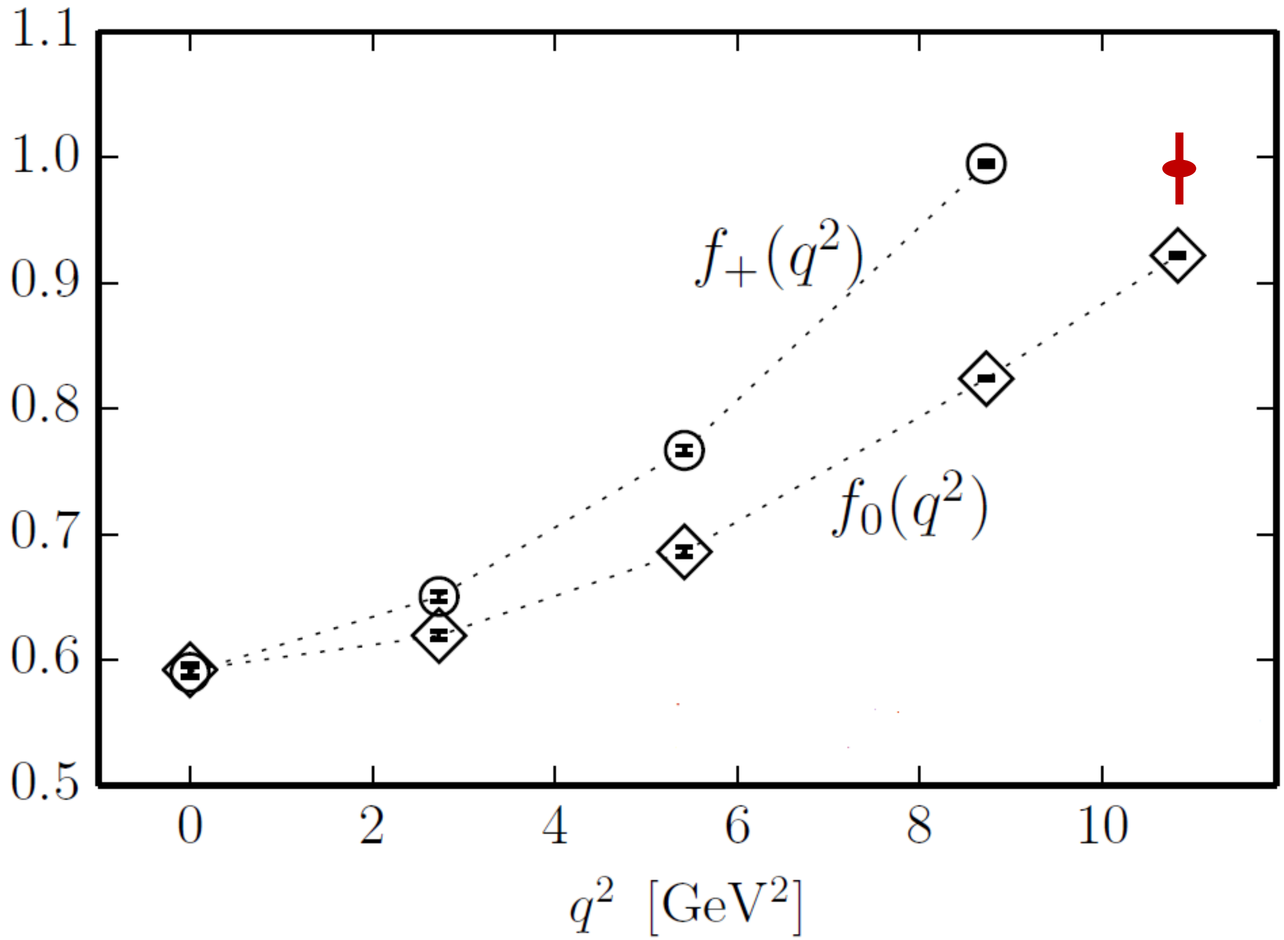




Colquhoun *et al* **HPQCD : 1611.01987**

A. Lytle : CKM2016

A. Lytle : Lattice2017



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A. Lytle : CKM2016

Conclusions and Outlooks

- **Study of decay constants and semileptonic decay of B_c mesons provides opportunity to obtain information about $b \rightarrow c$ transition, in particular important quantities like : $|V_{cb}|$ and $R(B_c)$.**
- **We have started to study these using NRQCD (*b*) and overlap (*l*, *s*, and *c*) valence quarks on HISQ gauge configurations.**
- **Preliminary results are presented here.**
- **Need to calculate renormalization constants to obtain physical results**
- **Use of fully relativistic bottom quarks on finer lattices will also be followed.**

- **Acknowledgements :** ILGTI, MILC (S. Gottlieb), R. Lewis, B. Colquhoun, A. Lytle and G. Bali

Lattice spacings and tuning of charm and strange masses

Lattice spacings are calculated by Omega(sss) mass = 1672 GeV

48^3 x 144 : 0.0582(5) fm
32^3 x 96 : 0.0877(10) fm
24^3 x 64 : 0.1192(14) fm
which are quite consistent with lattice spacings determined by MILC

- Strange mass is tuned by setting pseudoscalar $\underline{s}s$ mass at 685 MeV

$m_s a$ = 0.0738 (a = 0.0118fm)
= 0.0485 (a = 0.0888 fm)
= 0.028 (a = 0.0582fm)

Taking m_s = 100 MeV
$m_s a$ = 0.0450 (a = 0.0888fm),
= 0.0295(a = 0.0582fm)

- Charm mass is tuned by $\frac{1}{4}(m_{\eta_c} + 3 m_{J/\psi})$

$m_c a$ = 0.528 (a = 0.1192fm)
= 0.428 (a = 0.0888fm),
= 0.29 (a = 0.0582 fm)

Considering kinetic masses of mesons
(a la Fermilab formulation)