Observation of a Coulomb Flux Tube

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with Kai Chung, arXiv: 1704.08995

Topics

I will (try to) discuss two related topics:

1. The confining color Coulomb potential is associated with a color electric flux tube. (Work in collaboration with Kai Chung)
2. S-confinement: A confinement criterion (stronger than “color confinement”) for gauge theories with matter fields.

The Color Coulomb Potential

The color Coulomb potential $V_C(R)$ is the interaction energy of the state $\Psi_{\overline{q}q}$ generated by Coulomb gauge quark-antiquark creation operators acting on the ground state, i.e.

$$\begin{aligned} \mathcal{E}_C(R) &= \langle \Psi_{\overline{q}q} | H | \Psi_{\overline{q}q} \rangle \\ &= V_C(R) + \mathcal{E}_0 \end{aligned}$$

where, for heavy quark-antiquarks separated by $R = |\vec{R}_1 - \vec{R}_2|$

$$|\Psi_{\overline{q}q}\rangle = \mathcal{N} \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} b^{\dagger\sigma}(k_1, \lambda_1) d^{\dagger\sigma}(k_2, \lambda_2) e^{-i(\mathbf{k}_1 \cdot \vec{R}_1 + \mathbf{k}_2 \cdot \vec{R}_2)} |\Psi_0\rangle$$

The interaction energy is due to the fact that, in Coulomb gauge, creation of a charged color source is automatically accompanied by a longitudinal color electric field, due to the Gauss law constraint $D_i E_i = \rho_q$.

In Coulomb gauge, it works this way: separate the E-field into a transverse and longitudinal part $E = E^{tr} + E_L, \quad E_L = -\nabla\phi$, then

$$\partial_i D_i \psi = \rho_q + \rho_g$$

where

$$\rho_q^a = g\overline{q}T^a\gamma_0 q \quad , \quad \rho_g^a = gf^{abc}E_k^{tr,b}A_k^c$$

and defining the ghost operator

$$G^{ab}(\mathbf{x},\mathbf{y};A) = \left(\frac{1}{-\partial_i D_i(A)}\right)^{ab}_{\mathbf{xy}}$$

the solution of the Gauss law constraint is

$$\vec{E}_L^a(\mathbf{x},A,\rho) = -\vec{\nabla}_x \int d^3y\ G^{ab}(\mathbf{x},\mathbf{y};A)(\rho_q^b(y) + \rho_g^b(y))$$

and in the Hamiltonian $\int E_L^2$ gives rise to the non-local Coulomb interaction operator

$$\mathcal{E}_{coul} = \int d^3x d^3y d^3z \rho^a(x) \left(G^{ac}(\mathbf{x},\vec{z},A)(-\nabla^2)_{\vec{z}} G^{cb}(\vec{z},\mathbf{y},A) \right) \rho^b(y)$$

Spatial Distribution

We know from computer simulations that the Coulomb energy $\mathcal{E}_C(R)$ rises linearly with R. But what is the spatial distribution of E_L^2 due to the static color charges?

There is no obvious reason that it should be concentrated in a flux tube. If we consider only E_L due to the static quark-antiquark pair

$$\vec{E}^a_{L,q\overline{q}}(\mathbf{x}, A, \rho_q) = -\vec{\nabla}_x \int d^3y \; G^{ab}(\mathbf{x}, \mathbf{y}; A)\rho^b_q(y)$$

then squaring, summing over the color index, and taking the expectation value of the matter field color charge densities leads to

$$\begin{aligned} E^2_{L,q\overline{q}}(\mathbf{x}, A) \quad = \quad & \frac{g^2}{2N_c}\Big(\nabla_x G^{ab}(\mathbf{x}, 0; A) \cdot \nabla_x G^{ab}(\mathbf{x}, 0; A) \\ & +\nabla_x G^{ab}(\mathbf{x}, \vec{R}; A) \cdot \nabla_x G^{ab}(\mathbf{x}, \vec{R}; A) \\ & -2\nabla_x G^{ab}(\mathbf{x}, 0; A) \cdot \nabla_x G^{ab}(\mathbf{x}, \vec{R}; A)\Big) \end{aligned}$$

Does $\langle E^2_{L,q\overline{q}}(\mathbf{x}, A)\rangle$ form a flux tube?

It seems unlikely that $G^{ab}(\mathbf{x}, \mathbf{y}, A)$ would fall exponentially with $|\mathbf{x} - \mathbf{y}|$ for typical vacuum configurations. In that case it would be hard to see how the Coulomb potential could rise linearly with R.

Also the momentum-space ghost propagator $G^{ab}(\mathbf{k})$, has been computed in lattice Monte Carlo simulations with the result

$$G^{ab}(\mathbf{k}) = \langle G^{ab}(\mathbf{k}, A)\rangle \sim \frac{\delta^{ab}}{|\mathbf{k}|^{2.44}}$$

in the infrared $\rightarrow G^{ab}(r) \sim \delta^{ab}/r^{0.56}$ in position space.

So it is reasonable to assume some power-law falloff of $G^{ab}(\mathbf{x}, \mathbf{y}, A)$ with separation $|\mathbf{x} - \mathbf{y}|$, for typical vacuum fluctuations A. Then, unless there are very delicate cancellations, one would expect a power law falloff for $E^2_L(\mathbf{x}, A)$, as the distance of point $\mathbf{x}$ from the $\overline{q}q$ sources increases.

This would imply a long-range color Coulomb dipole field in the physical state $\Psi_{\overline{q}q}$.

Lattice Measurements

Let

$$L_t(\mathbf{x}) \equiv T \exp\left[ig \int_0^t dt' A_4(\mathbf{x}, t')\right]$$

Then the Coulomb energy is obtained from the logarithmic time derivative

$$\begin{aligned} \mathcal{E}_C(R) &= -\lim_{t\to 0} \frac{d}{dt} \log \langle \Psi_{\overline{q}q} | e^{-Ht} | \Psi_{\overline{q}q} \rangle \\ &= -\lim_{t\to 0} \frac{d}{dt} \log \left\langle \mathrm{Tr}[L_t(\mathbf{0}) L_t^\dagger(\mathbf{R})] \right\rangle \end{aligned}$$

while the minimal energy of static quark-antiquark state is obtained in the opposite limit

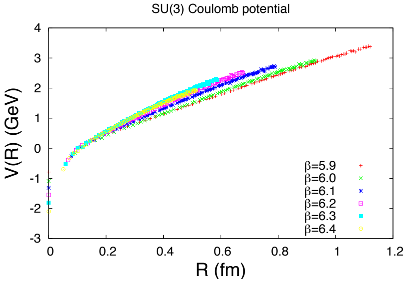
$$\mathcal{E}_{min}(R) = -\lim_{t\to\infty} \frac{d}{dt} \log \left\langle \mathrm{Tr}[L_t(\mathbf{0}) L_t^\dagger(\mathbf{R})] \right\rangle$$

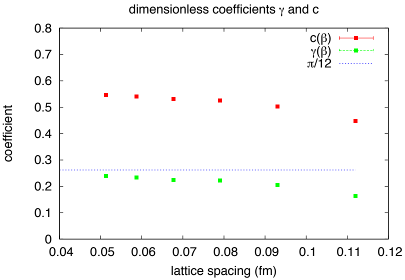
The lattice version is

$$\mathcal{E}_C(R_L) = -\log \left\langle \frac{1}{N} \mathrm{Tr}[U_0(\mathbf{0},0)U_0^\dagger(\mathbf{R}_L,0)] \right\rangle$$

which we measure, convert to physical units, and fit to

$$\mathcal{E}_C^{phys}(R) = \sigma_c(\beta)R - \frac{\gamma(\beta)}{R} + \frac{c(\beta)}{a(\beta)}$$





Results

- The Coulomb string tension σ_C is about four times greater than the asymptotic string tension σ. In the gluon chain model of string formation **(Thorn and JG)**, this value can in principle be reduced to σ by constituent gluons in the chain **(Szczepaniak and JG)**.

- An R-independent self-energy term $c/a(\beta)$ can be isolated and subtracted.

- There is a $-\gamma/R$ term in the potential with $\gamma \to \pi/12$ in the continuum limit. *This looks like a Lüscher term.* We find the same result for γ in SU(2).

  Coincidence? Or some connection to string theory?

The odd fact that $\gamma \approx \pi/12$ motivates us to look at the energy distribution of the Coulomb electric field.

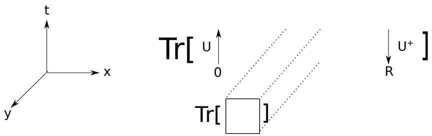
The Coulomb flux tube

The setup is standard: Let the quark-antiquark pair lie on the x-axis with separation R, and measure the $\text{Tr}E_x^2$ field a transverse distance y from the midpoint of the line joining the quarks

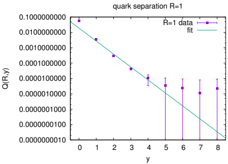
$$\langle \Psi_{\overline{q}q} | \text{Tr}E_x^2(\vec{p}) | \Psi_{\overline{q}q} \rangle - \langle \Psi_0 | \text{Tr}E_x^2 | \Psi_0 \rangle$$

On the lattice, in Coulomb gauge:

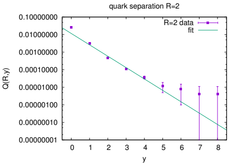
$$Q(R,y) = \frac{\langle \text{Tr}[U_0(\mathbf{0},0)U_0^\dagger(\mathbf{R}_L,0)]\frac{1}{2}\text{Tr}U_P(\vec{p},0)\rangle}{\langle \text{Tr}[U_0(\mathbf{0},0)U_0^\dagger(\mathbf{R}_L,0)]\rangle} - \langle \frac{1}{2}\text{Tr}U_P \rangle$$



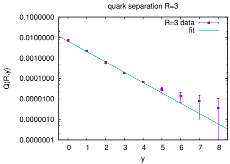
Result: exponential falloff in the transverse direction



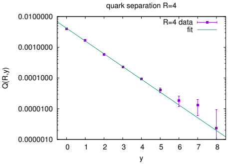
(a) $R = 1$



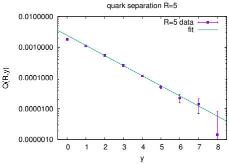
(b) $R = 2$



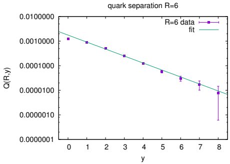
(c) $R = 3$



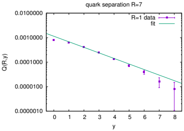
(d) $R = 4$



(e) $R = 5$

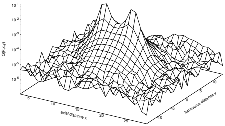


(f) $R = 6$

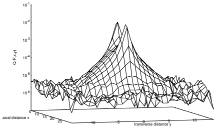


(g) $R = 7$

Here is a profile of the E_x^2 distribution at $R = 5$. Note the log scale on the y-axis.



(h)



(i)

We can compare the half-width of the Coulomb flux tube with that of the minimal energy flux tube (**Bali, Schlichter, and Schilling, 1994)** at the same coupling $\beta = 2.5$ and $R = 7$.

The half-width of the Coulomb flux tube is smaller by a factor of ≈ 1.7.

And it has a string tension $\sigma_C \approx 4\sigma$.

A confinement criterion for gauge theories with matter fields

Suppose we have an SU(N) gauge theory with matter fields in the fundamental representation, e.g. QCD. Wilson loops have perimeter-law falloff asymptotically, Polyakov lines have a non-zero VEV, what does it mean to say such theories (QCD in particular) are confining? Many people take it to mean "color confinement" or

C-confinement

There are only color neutral particles in the spectrum.

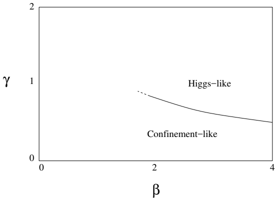
The problem with C-confinement is that it also holds true for gauge-Higgs theories, deep in the Higgs regime, where there are

- massive vector bosons *(W's)*
- only Yukawa forces,
- no linearly rising Regge trajectories.

If C-confinement is "confinement," then the Higgs phase is also confining.

C-confinement in gauge-Higgs theories

1. **Elitzur's** Theorem: No such thing as spontaneous symmetry breaking of a local gauge symmetry.
2. The **Fradkin-Shenker-Osterwalder-Seiler** (FSOS) Theorem: There is no transition in coupling-constant space which isolates the Higgs phase from a confinement-like phase.
3. **Frölich-Morchio-Strocchi** (FMS): gauge-invariant operators for particles (e.g. W's) in the physical spectrum in the Higgs region.
4. **Maas**: further development of FMS perturbation theory.



If the confinement-like (QCD-like) region has a color neutral spectrum, then so does the Higgs-like region.

On the other hand...

Charge separation confinement

On the other hand *QCD has linearly-rising Regge trajectories, and the Higgs regime does not.*

Linear Regge trajectories imply metastable color electric flux tubes; their absence means no flux tube formation.

Can this property somehow be elevated to a criterion which distinguishes qualitatively between a Higgs and a confinement phase? If so, how would it fit in with the FSOS theorem?

As in pure gauge theory, introduce massive static quark fields ψ, and consider gauge-invariant operators and physical states

$$\begin{aligned} Q_V(R) &= \psi^{\dagger a}(x) V^{ab}(x,y;A) \psi^b(y) \\ |\Psi_V\rangle &= Q_V(R)|\Psi_0\rangle \end{aligned}$$

where $V^{ab}(x,y,A)$ is any operator which depends *only* on the gauge field A_μ at a fixed time, and transforming as

$$V(x,y;A) \rightarrow g(x) V(x,y;A) g^\dagger(y) \tag{1}$$

The energy is

$$E_V(R) = -\lim_{t\to 0} \frac{d}{dt} \log \left[\langle Q_V^\dagger(R,t) Q_V(R,0) \rangle \right]$$

We define the criterion of “charge separation confinement” as follows:

S-confinement

For any physical state $\Psi_V(R)$ with massive quark-antiquark sources separated by R, there exists some positive constant σ such that

$$\frac{dE_V}{dR} \geq \sigma$$

In my opinion, *a “proof of confinement” in QCD should be a proof of S-confinement, not C-confinement.*

(Of course, a proof of a mass gap would also be nice, at least for your bank account.)

Even numerically, S-confinement is hard to demonstrate, because there are infinite possibilities for $V(x, y, A)$, e.g.

$$\begin{aligned} V_1(x, y, a) &= P\left[i\int_{C_{xy}} dx^\mu A_\mu\right] \\ V_2(x, y, a) &= \int DC_{xy}\ a(C_{xy})P\left[i\int_{C_{xy}} dx^\mu A_\mu\right] \\ V_3(x, y; A) &= \left(\frac{1}{D_i D_i + m^2}\right)_{xy} \end{aligned}$$

I would focus on

$$V_4^{ab}(x, y) = G^{\dagger ac}(x; A)G^{cb}(y; A)$$

where $G(x; A)$ is the non-abelian gauge transformation which takes the gauge field to Coulomb gauge. The reason is that in an abelian theory, where

$$G(x; A) = \exp\left[i\int d^3x'\ A_i(x')\partial_i \frac{1}{4\pi|x - x'|}\right]$$

the corresponding state Ψ_V is the minimal energy state containing two static $+/-$ electric charges, and it violates S-confinement (as it should). In a non-abelian theory, Ψ_V is equivalent to $\Psi_{\overline{q}q}$ in Coulomb gauge, introduced above.

S-confinement (Lattice)

There exists a positive constant σ such that, for any operator $V^{ab}(x,y,t)$ depending only on the gauge field on a timeslice t, transforming covariantly at sites x,y

$$\begin{aligned} \frac{dE_V}{dR} &\equiv -\frac{d}{dR}\log\left[\frac{\left\langle Tr\left[U_0(x,t)V(x,y,t+1)U_0^\dagger(y,t)V(y,x,t)\right]\right\rangle}{\langle Tr[V(x,y,t)V(y,x,t)]\rangle}\right] \\ &\geq \sigma \end{aligned}$$

Conjectures

1. QCD is S-confining.
2. Gauge-Higgs theory is S-confining in the confinement-like region, but only C-confining in the Higgs region of coupling-constant space.
3. The transition from the S-confinement phase to the Higgs phase corresponds to the width of resonances on linear Regge trajectories going to infinity.

I have no proof, but at least the V_4 (and other) proposals can be tested numerically for S-confinement in gauge-matter theories, such as gauge-Higgs and QCD.

This is work in progress.