Topological Susceptibility in N_f=2 QCD at Finite Temperature

JLQCD collaboration:
Yasumichi Aoki
Sinya Aoki
Guido Cossu
Hidenori Fukaya
Shoji Hashimoto
Kei Suzuki

KEK & RIKEN BNL Research Center

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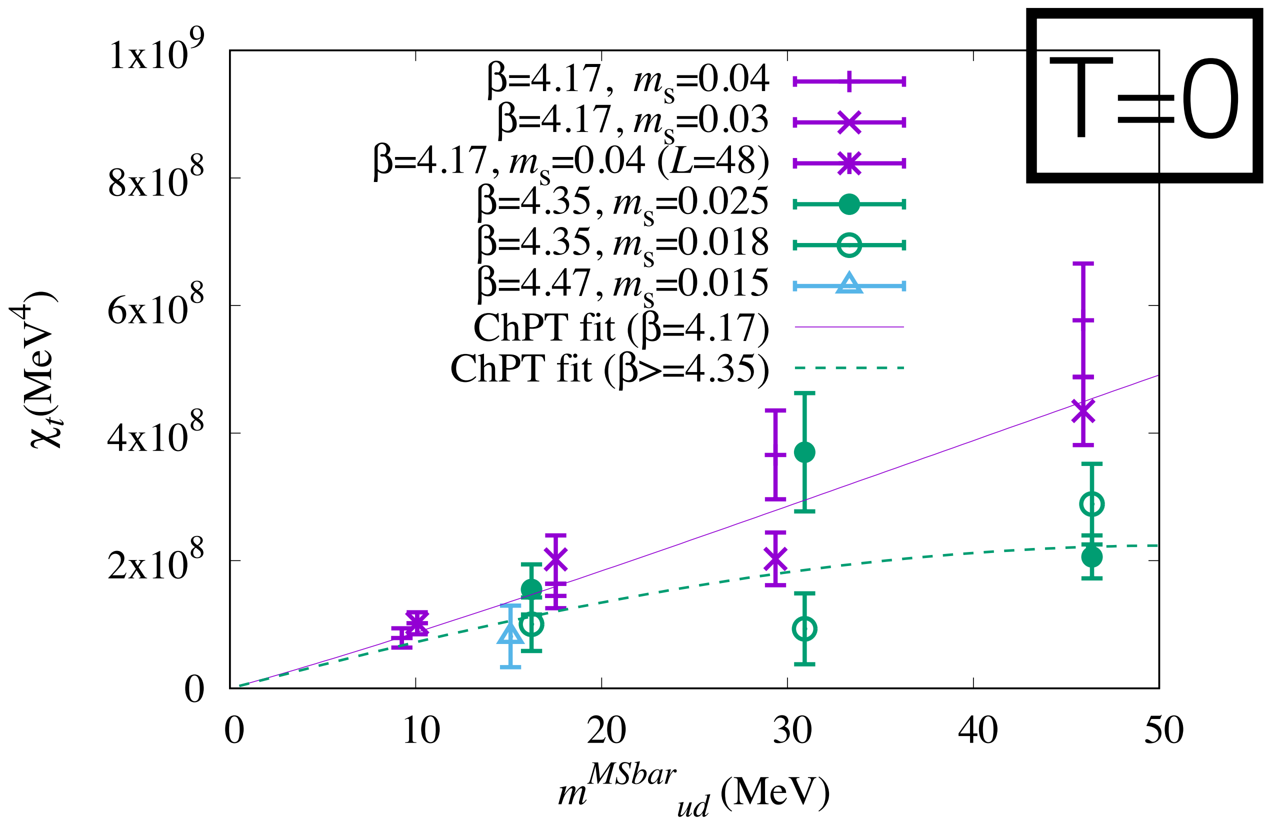
JLQCD finite temperature related

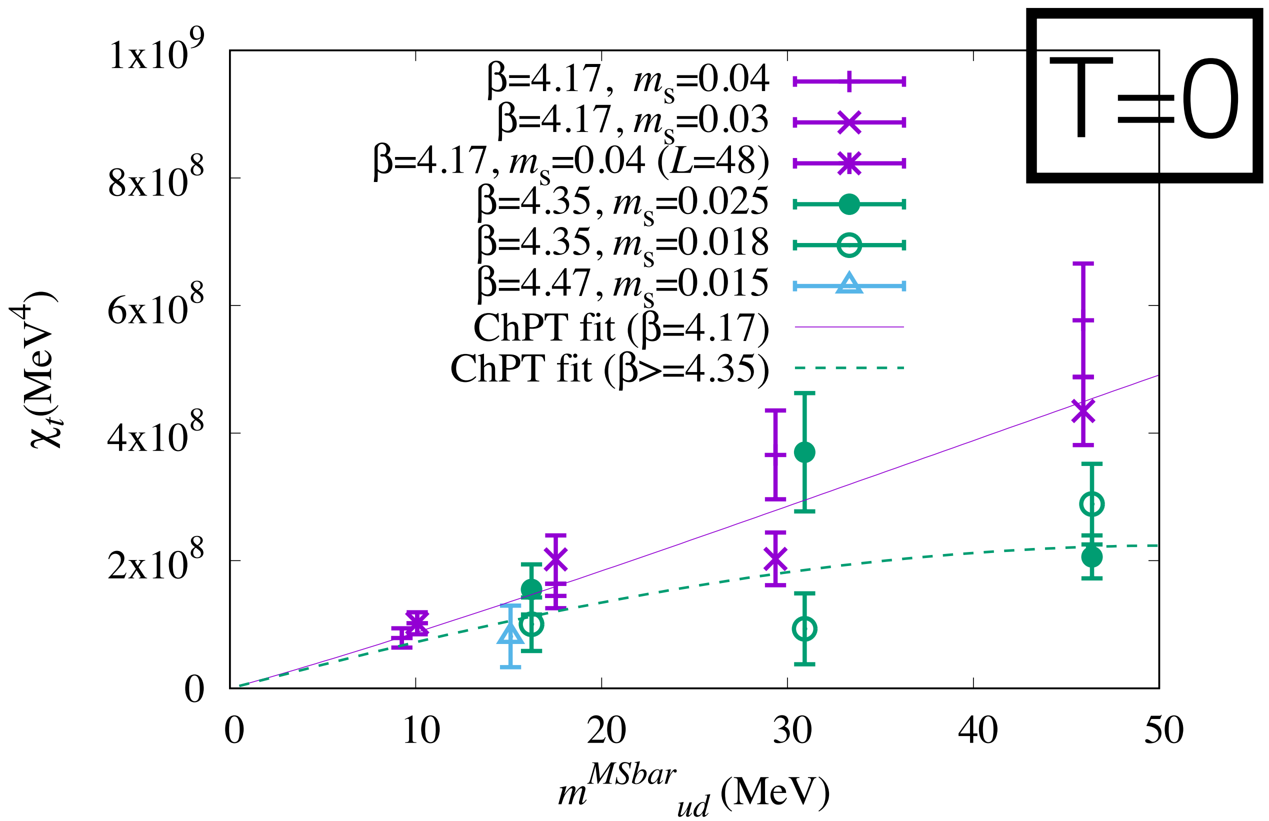
- Kei Suzuki: $U_A(1)$
- YA: this talk about the topological susceptibility
- Hidenori Fukaya
  - plenary panel discussion on $U_A(1)$
  - χ_t with slab method for T=0 (parallel talk)
- Christian Rohrhofer: vector channel correlators

topological susceptibility



[JLQCD: S.Aoki et al 2017, N_f=2+1 DWF
see talk by Fukaya]

topological susceptibility



[JLQCD: S.Aoki et al 2017, N_f=2+1 DWF
see talk by Fukaya]

- T > 0 ?
  - N_f=2
    - relation with $U(1)_A$ symm
    - relation with the order of PT
      - 2nd or 1st order ?
  - implication
    - axion : $\chi_t(T)$ @ phys.pt.
    - N_f=2+1 phase diagram

$U(1)_A$ and topology

- global symmetry of QCD: $U(N_f)_L \times U(N_f)_R$ @ $m_f \to 0$
- $T<T_c$: $\to SU(N_f)_V \times U(1)_V$
- $T>T_c$: $\to SU(N_f)_V \times U(1)_V \times SU(N_f)_A \times U(1)_A$?
  - $SU(N_f)_A$ is restored、$U(1)_A$?
  - able to be investigated through order parameters
- relation through fermionic (near) zero mode: $\rho(\lambda)$: density of eigenvalue

$SU(N_f)_A$ order parameter ↔ $U(1)_A$ order parameter

$$-\langle \bar{q}q \rangle = \pi\rho(0) = 0 \longleftrightarrow \Delta_{\pi-\delta} \sim \rho'(0) = 0?$$

$U(1)_A$ and topology

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[K.Suzuki]

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$SU(N_f)_A$ order parameter

$$-\langle \bar{q} q \rangle = \pi \rho(0) = 0$$

$U(1)_A$ order parameter

$$\Delta_{\pi-\delta} \sim \rho'(0) = 0$$

[K.Suzuki]

index theorem

topological charge Q_t

susceptibility $\chi_t = \frac{\langle Q_t^2 \rangle}{V}$

U(1)$_A$ and topology

- global symmetry of QCD: $U(N_f)_L \times U(N_f)_R$ @ $m_f \to 0$
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- T>
  -

signal should appear in topology

- able to be investigated through order parameters
- relation through fermionic (near) zero mode: ρ(λ) : density of eigenvalue

SU(N$_f$)$_A$ order parameter

$$-\langle \bar{q} q \rangle = \pi \rho(0) = 0$$

U(1)$_A$ order parameter

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[K.Suzuki]

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$U(1)_A$ and topology

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- $T<T_c$: → $SU(N_f)_V \times U(1)_V$
- T>
  -
  - able to be investigated through order parameters
- relati ... genvalue

signal should appear in topology

a senario: $\chi_t = 0$ for $0<m_f<m_c$

[S.Aoki, Fukaya, Taniguchi (2012)]

$$-\langle \bar{q}q \rangle = \pi\rho(0) = 0 \quad \longleftrightarrow \quad \Delta_{\pi-\delta} \sim \rho'(0) = 0$$ [K.Suzuki]

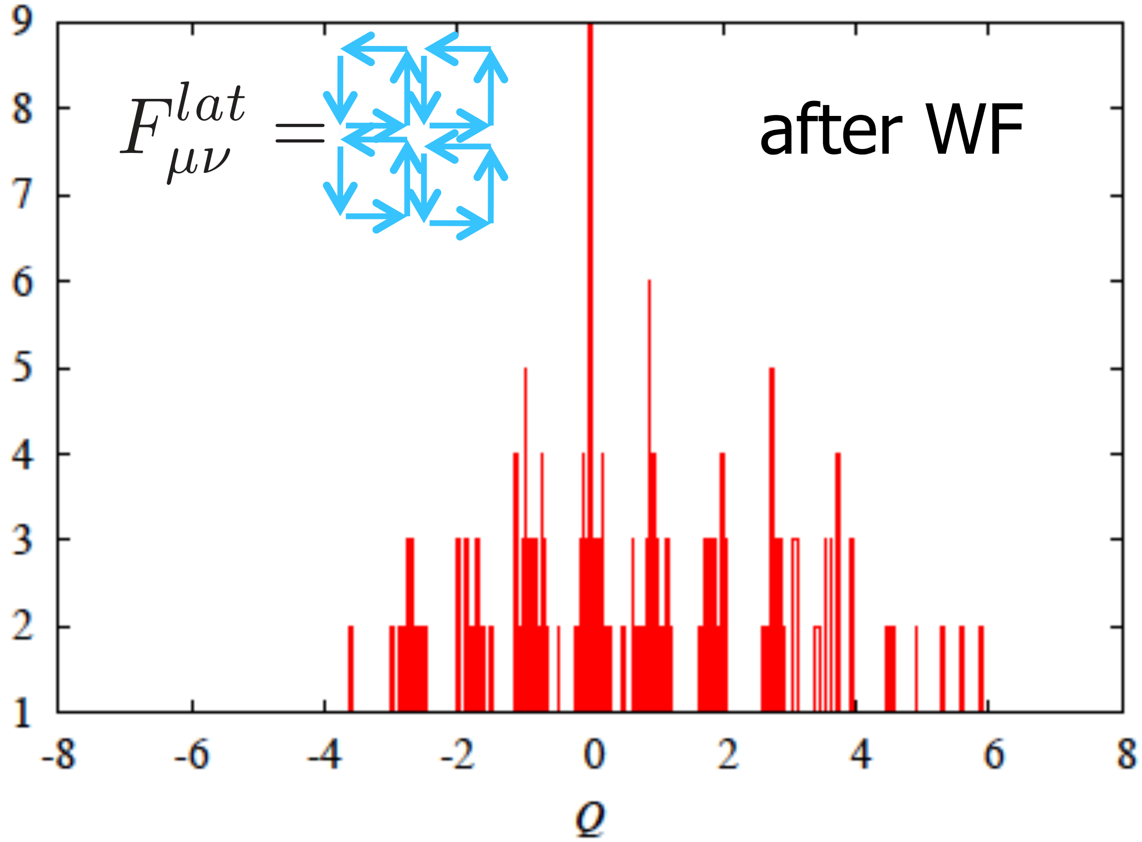
index theorem

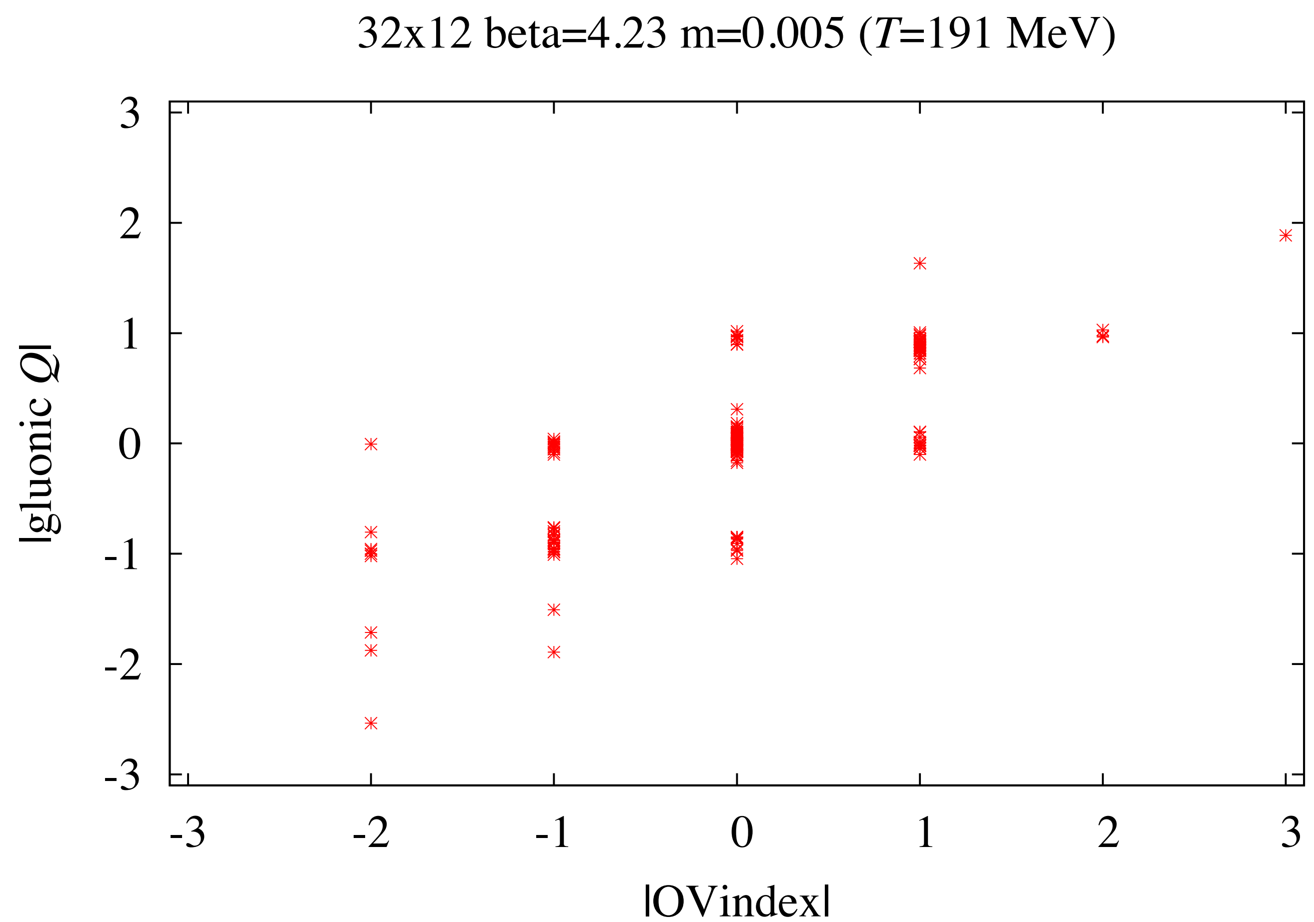
topological charge Q_t

susceptibility $\chi_t = \frac{\langle Q_t^2 \rangle}{V}$

topological charge → susceptibility

$$q(x) = \frac{1}{32\pi^2} \mathrm{Tr} \epsilon^{\mu\nu\rho\sigma} F^{lat}_{\mu\nu} F^{lat}_{\rho\sigma}$$





$$Q = \sum_x q(x)$$ charge

$$\chi_t = \frac{\langle Q^2 \rangle}{V}$$ susceptibility

N_f=2 DWF → Overlap

- DWF ensemble → reweighted to overlap
  - Möbius DWF: almost exact chiral symmetry: m_{res} = 0.05(3) MeV (β=4.3, L_s=16)
  - Overlap: exact chiral symmetry
- Q_t measurements
  - global sum of the gluonic charge density (clover) after Wilson Flow
  - Overlap Index
- reweighting: before / after and above 2 meas. yield 4 χ_t values
- current focus: 1/a = 2.6 GeV *** **PRELIMINARY** ***

N_f=2 DWF → Overlap

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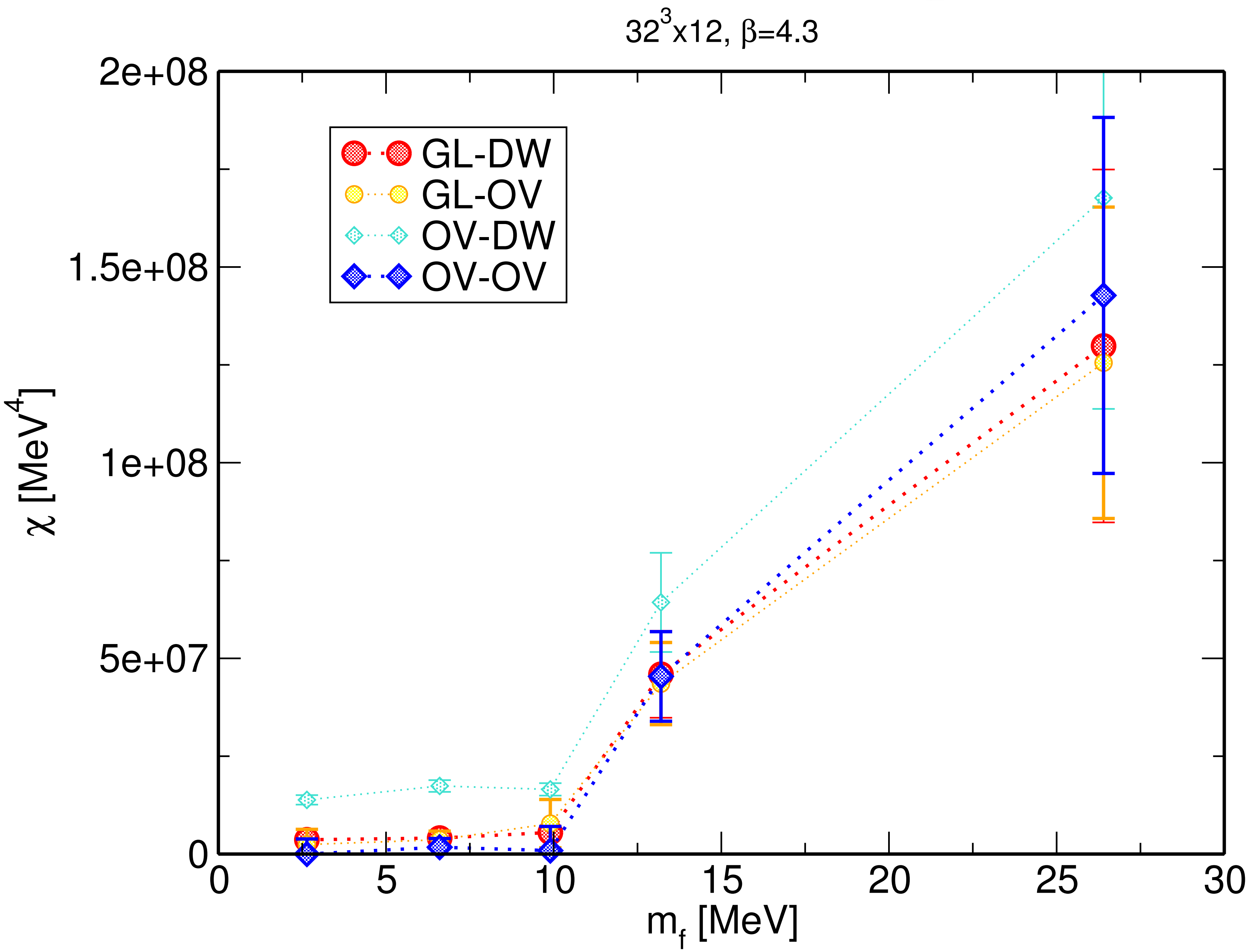
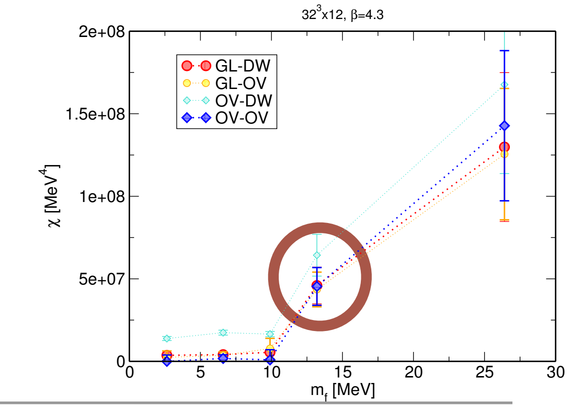
GL-DW	gluonic charge on DW ensemble
GL-OV	gluonic charge on OV ensemble
OV-DW	OV index on DW ensemble
OV-OV	OV index on OV ensemble

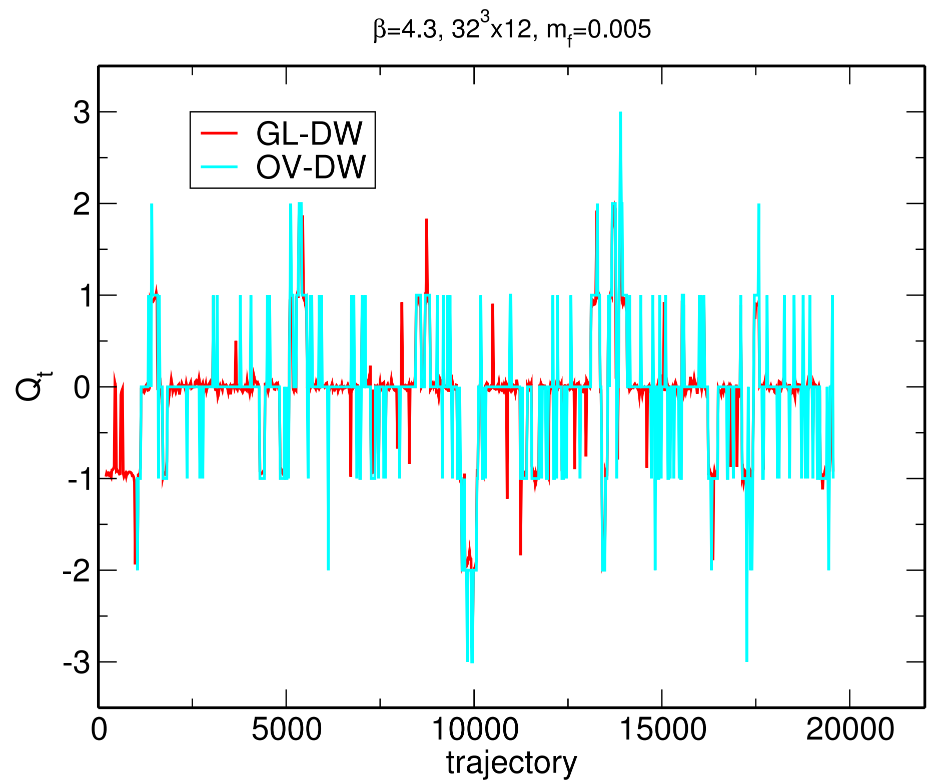
$\chi_t(m_f)$ for N_f=2 T=220 MeV

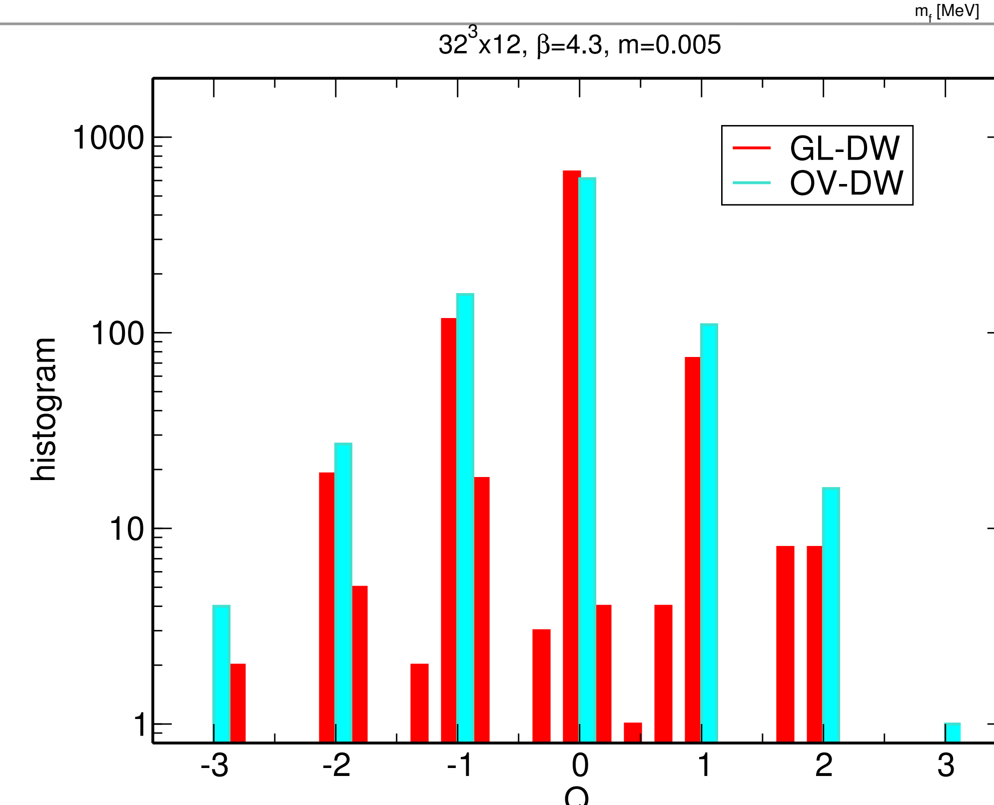
GL-DW	gluonic charge on DW
GL-OV	gluonic charge on OV
OV-DW	OV index on DW ensemble
OV-OV	OV index on OV ensemble



m=0.005 history and histogram

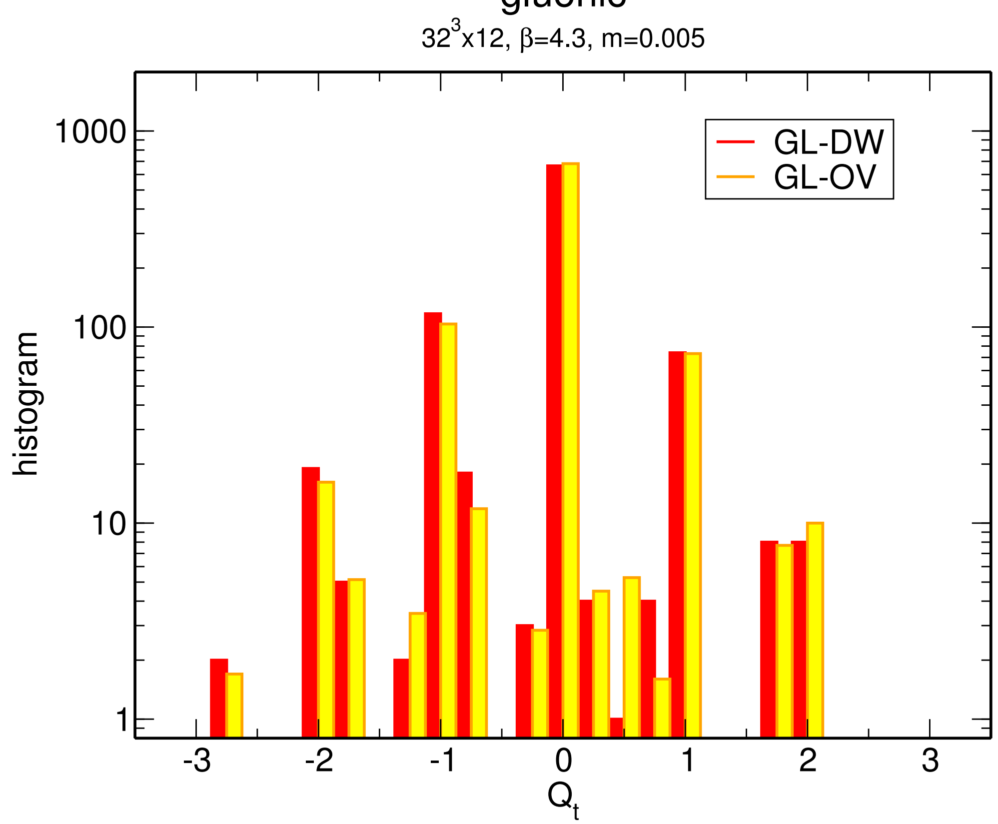


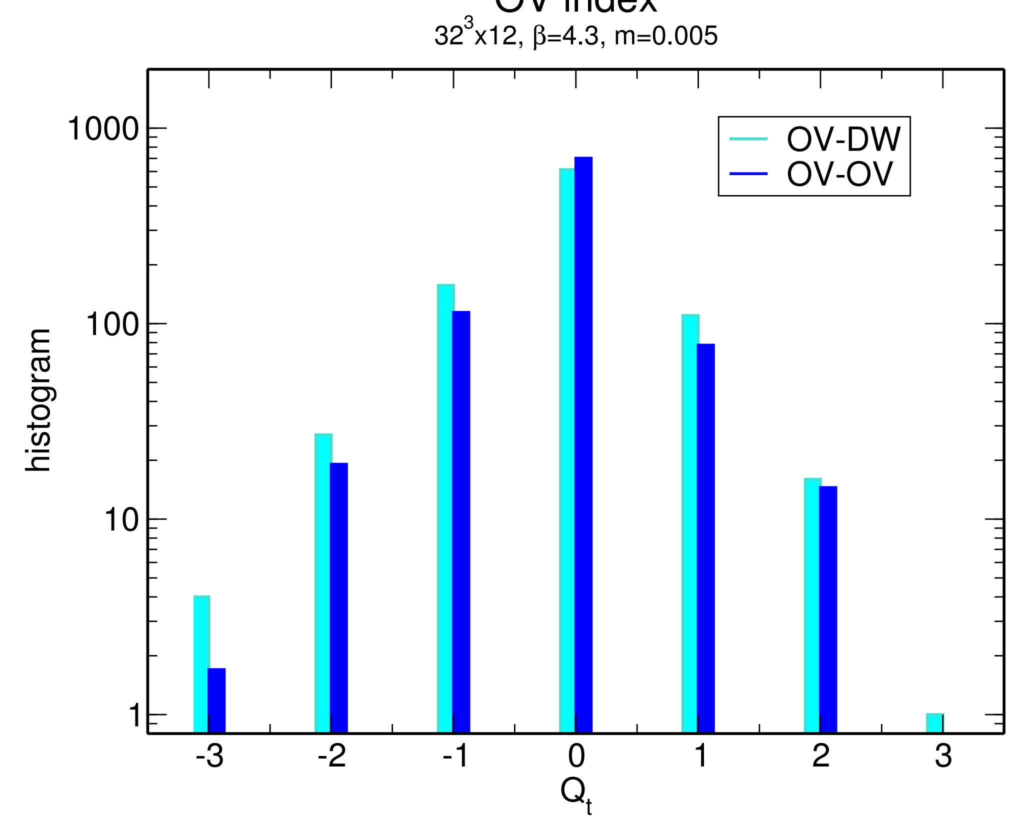




gluonic

OV index

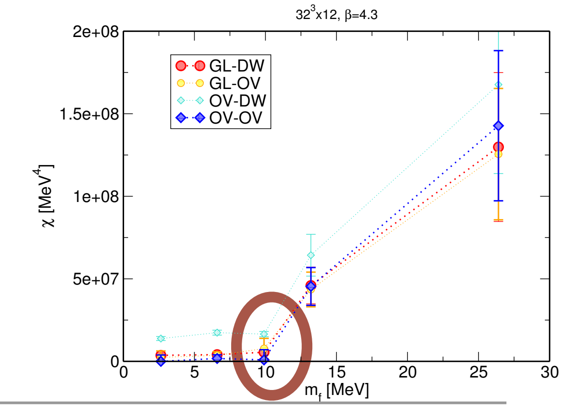


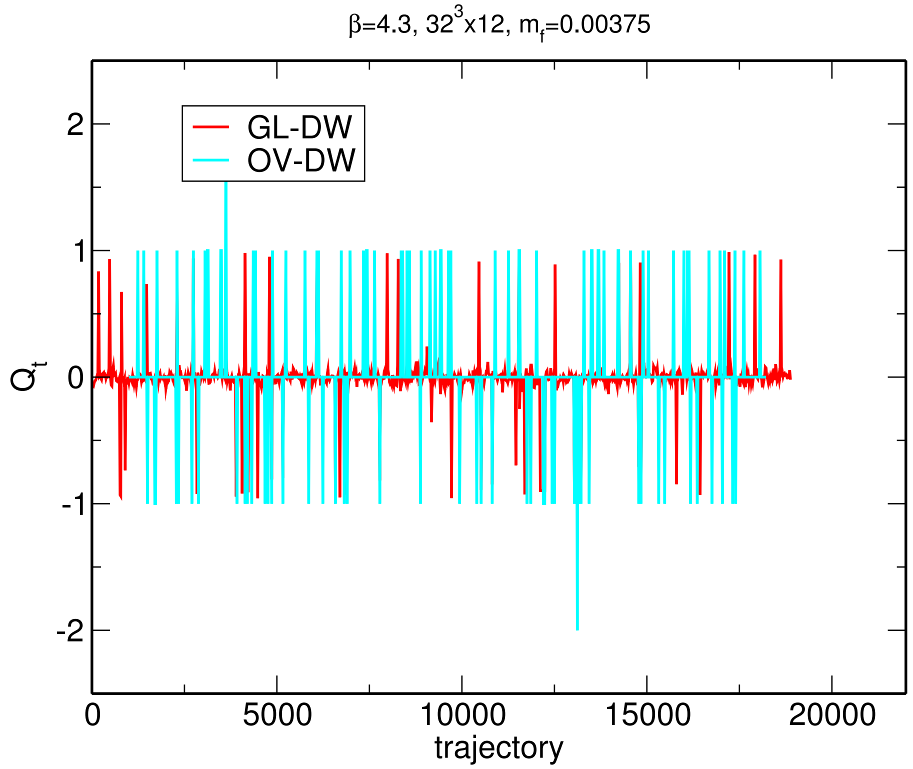


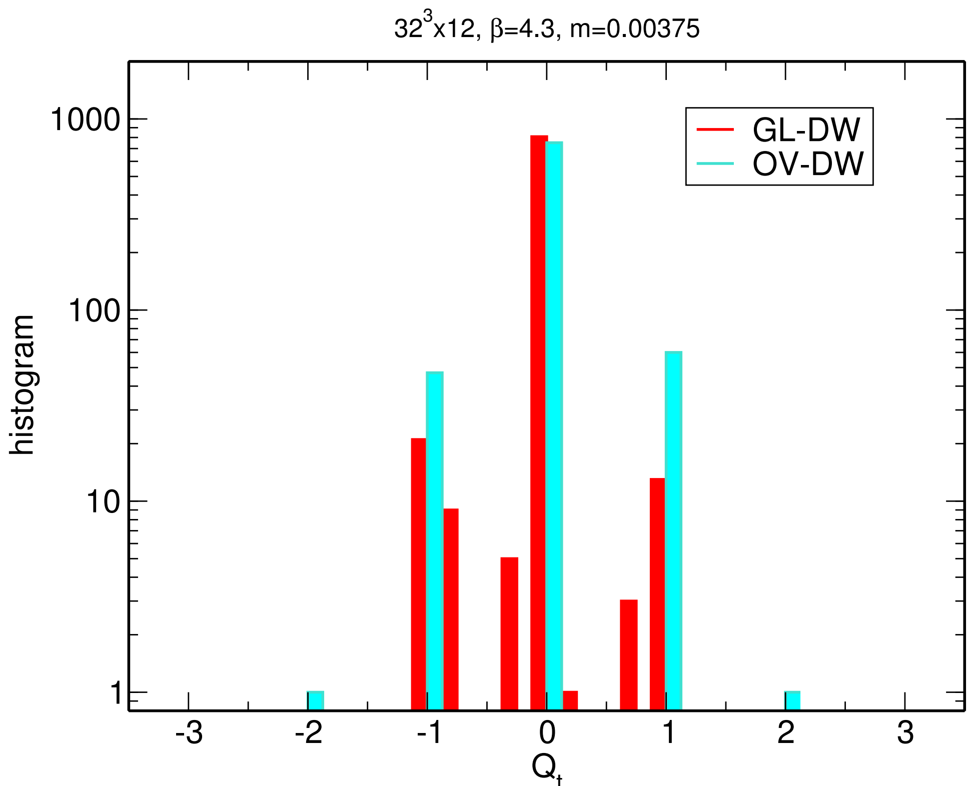
Out of 4 definitions of susceptibility

- OV index on DWF has pathology: overly enhanced artificial zero modes
  - should be excluded from the discussion
  - reweighting to OV ensemble suppresses the weight of fake zero modes
- with GLuonic Q: before / after reweighting does not change much
  - Wilson Flow: tends to erase artificial modes
  - OV reweighting: artificial zero modes have suppressed reweighting factor
  - shall take “before” reweighting (GL-DW), which is statistically better
- Good ones: GL-DW, OV-OV
- Discarded: GL-OV, OV-DW

m=0.00375 history and histogram

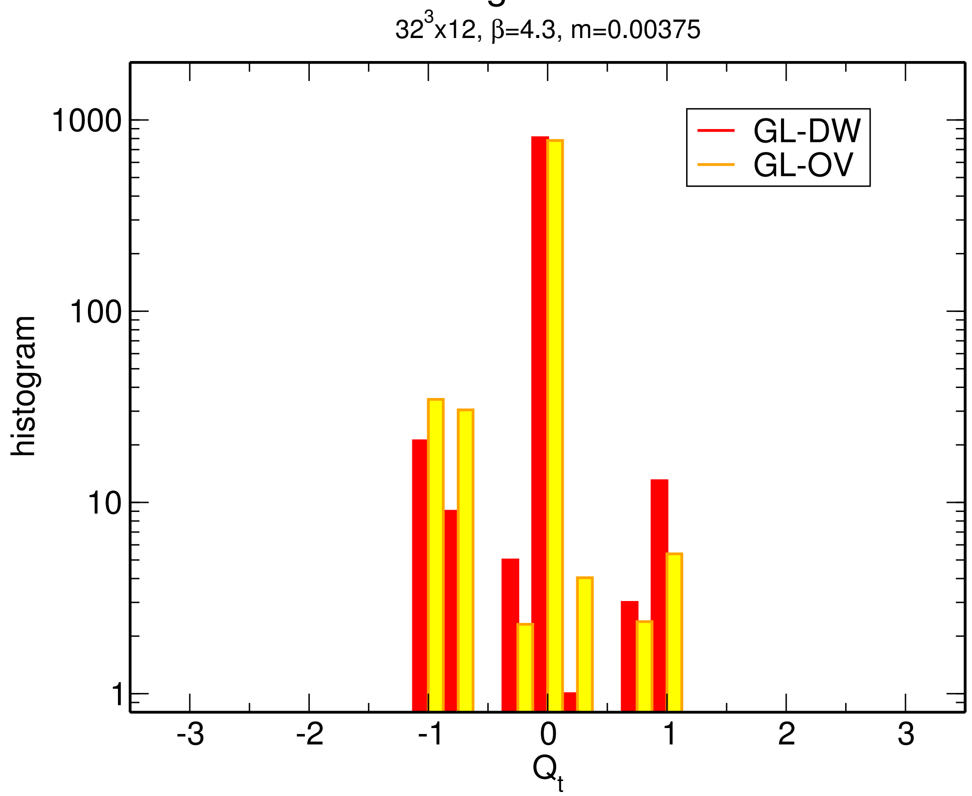


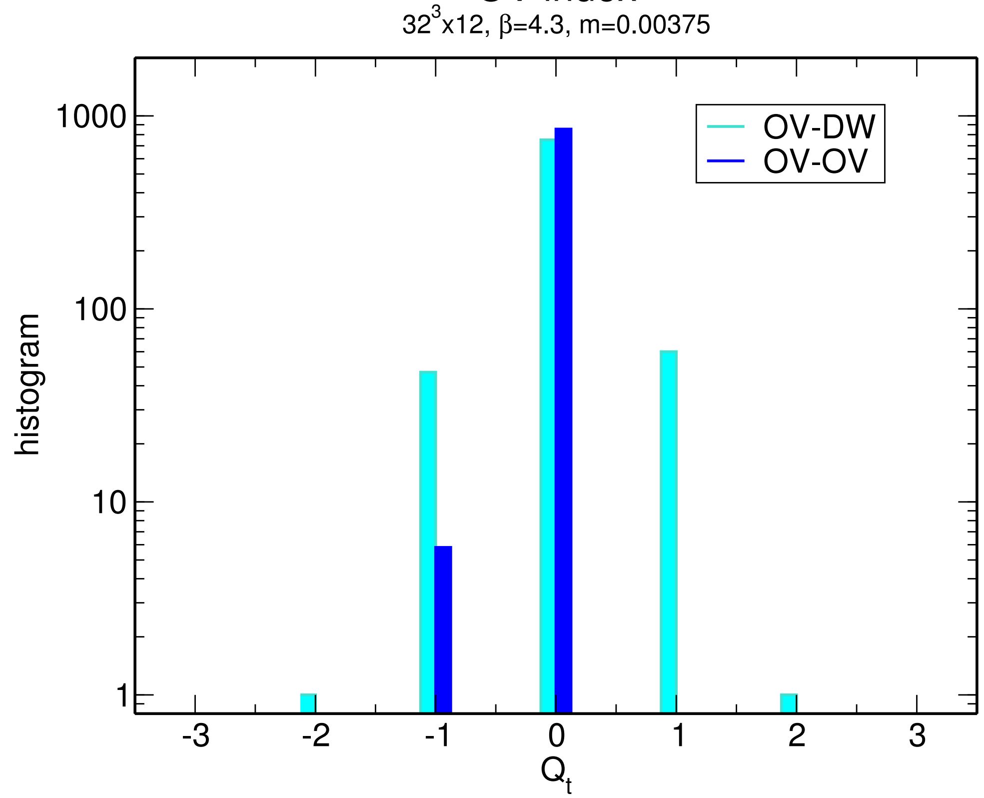




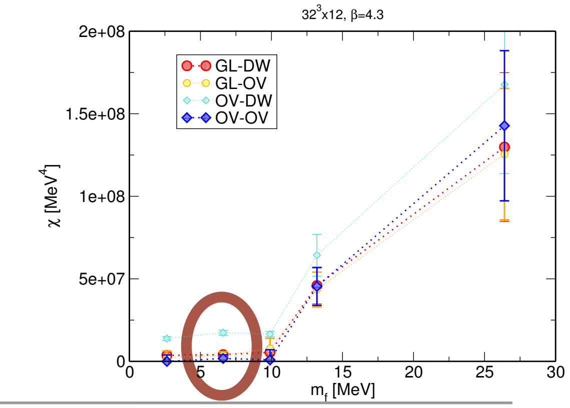
gluonic

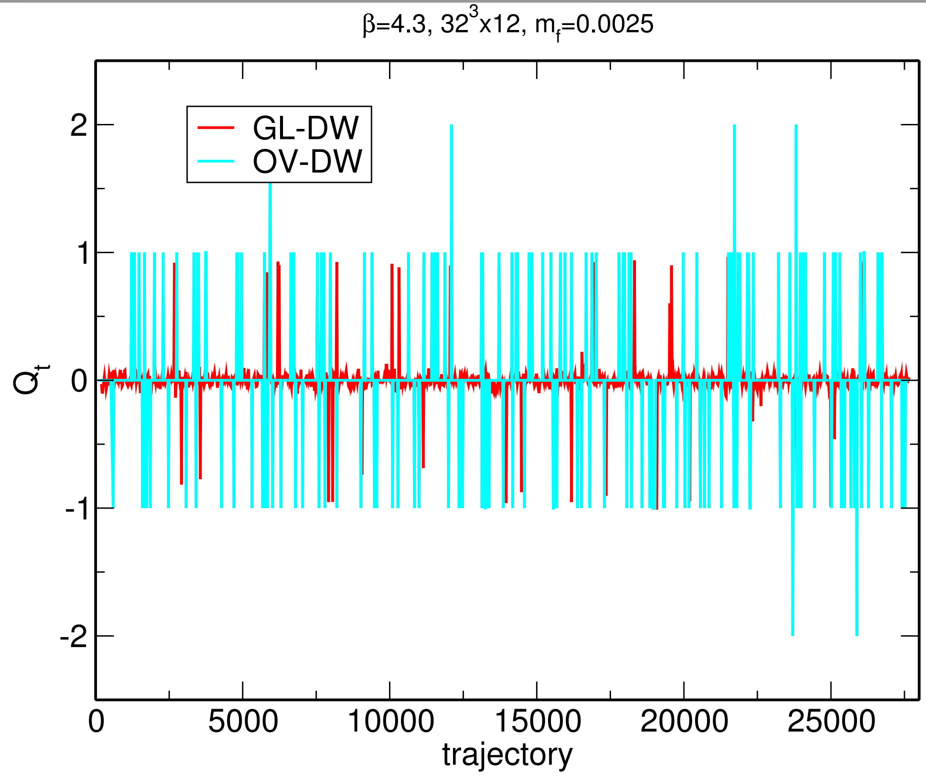
OV index

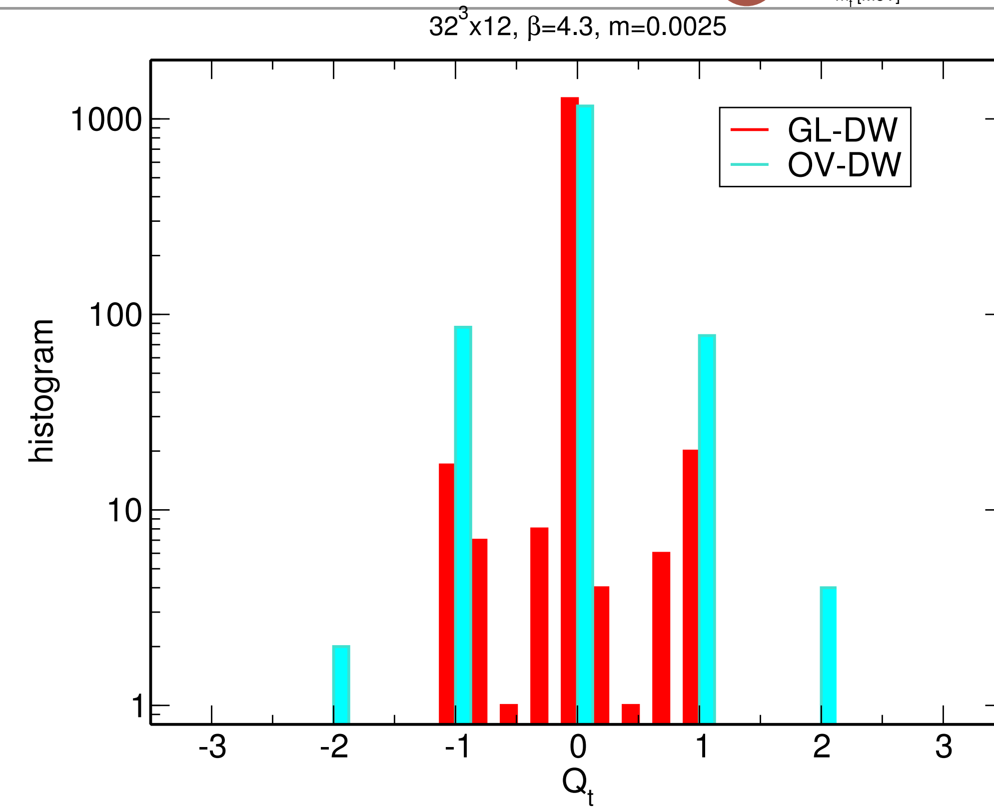




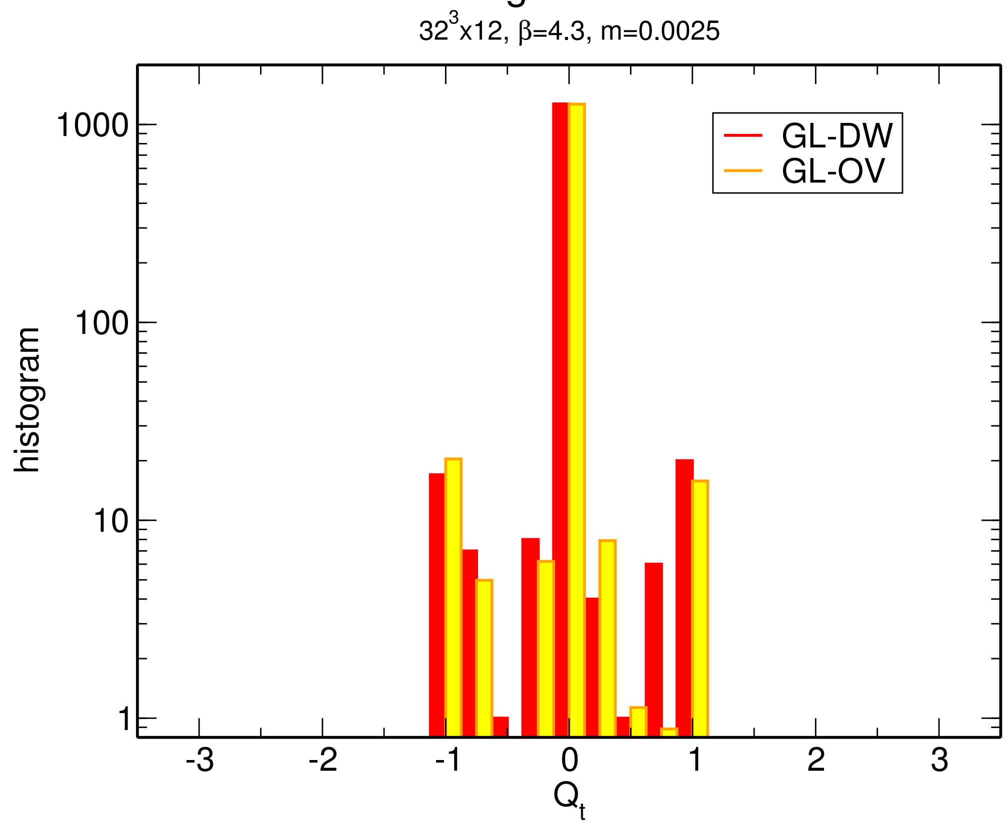
m=0.0025 history and histogram



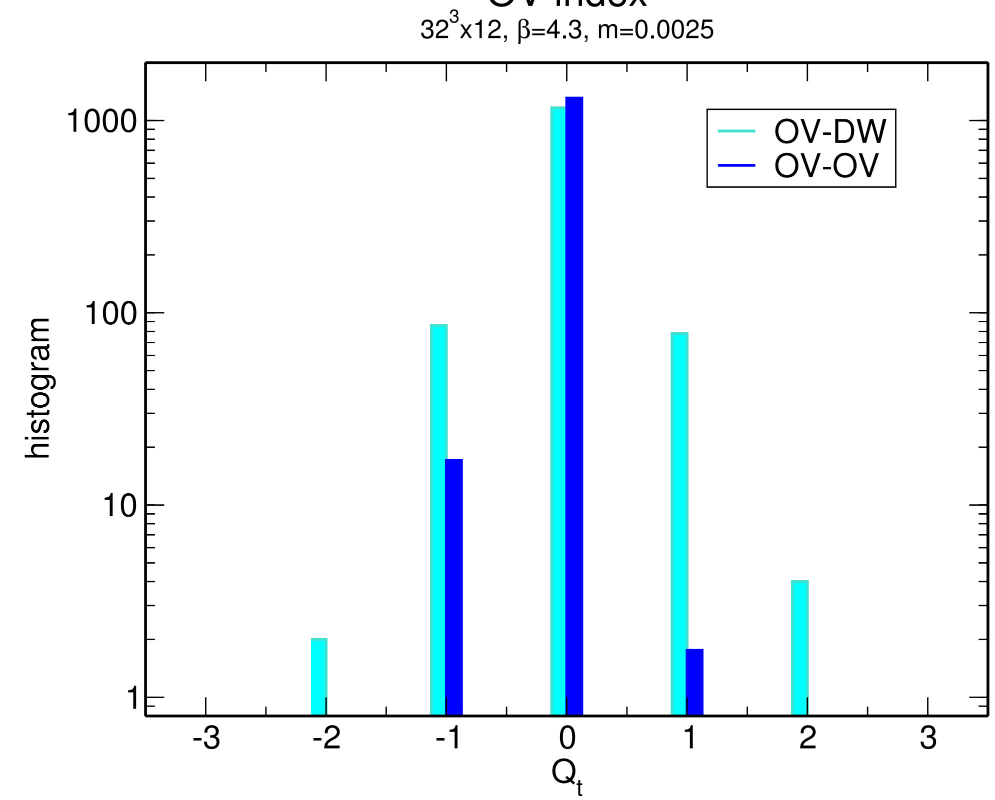




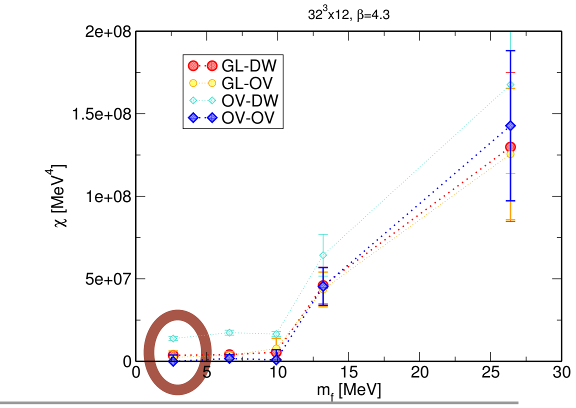
gluonic

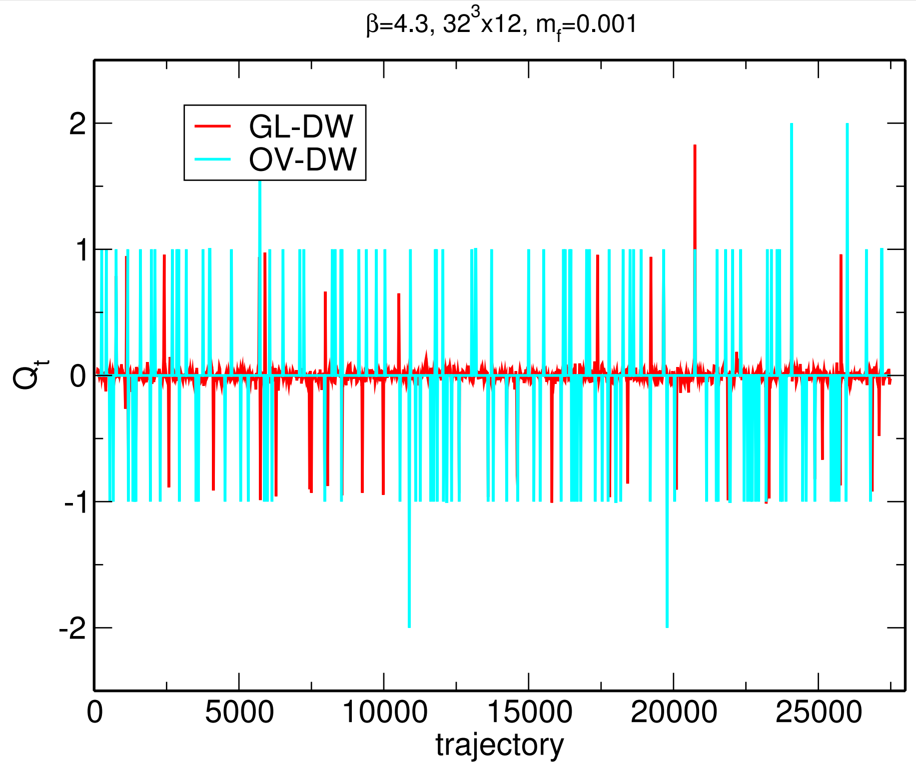


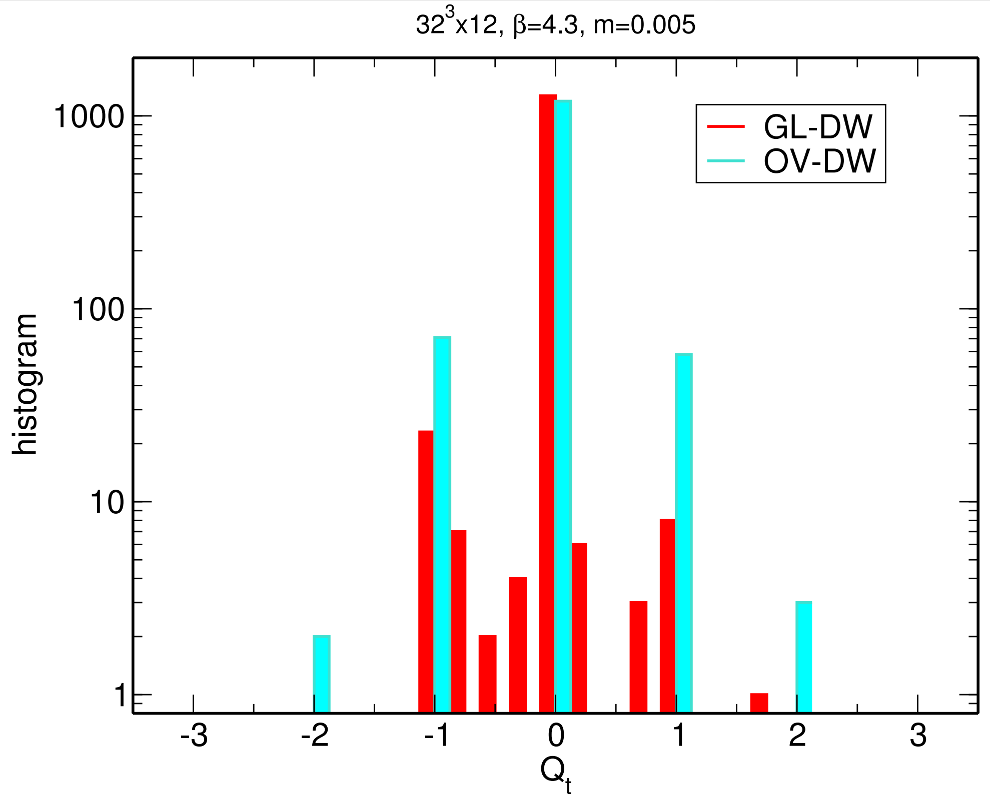
OV index

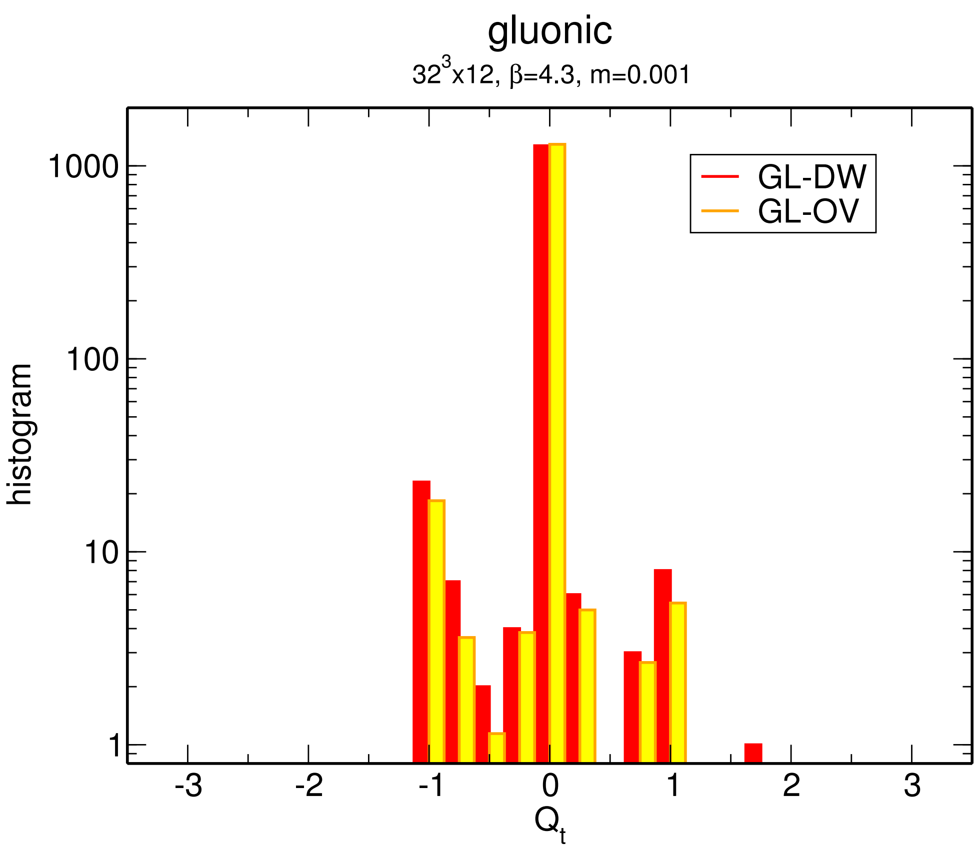


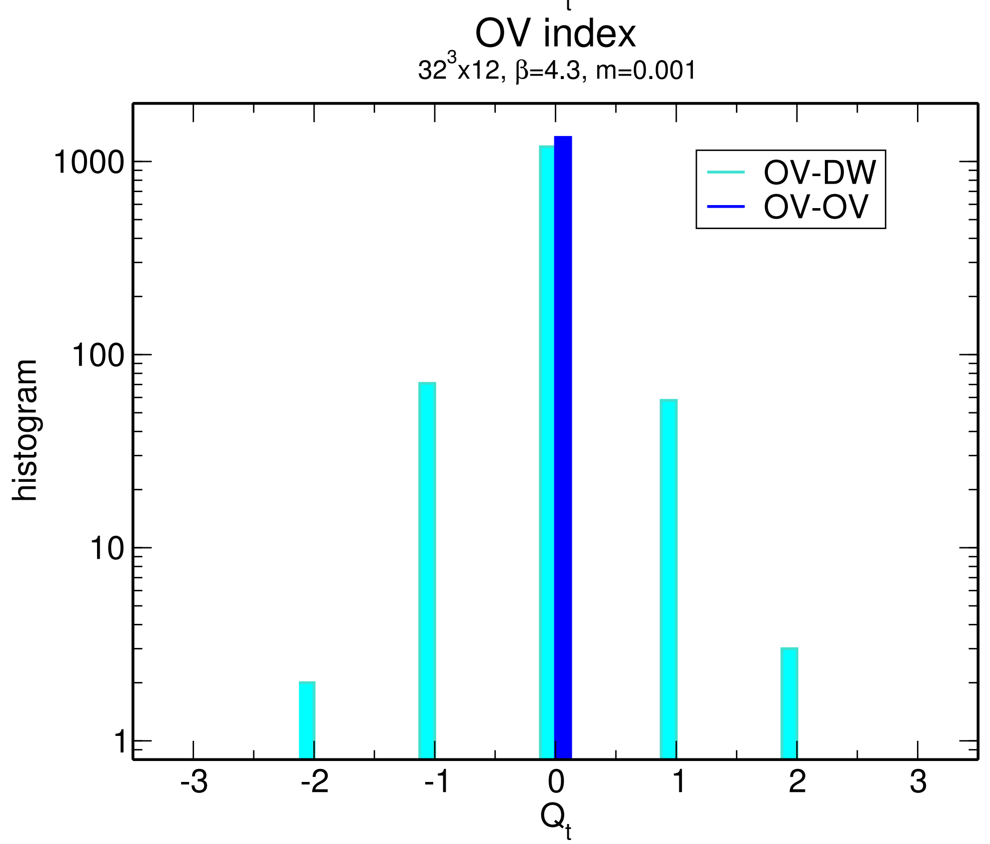
m=0.001 history and histogram



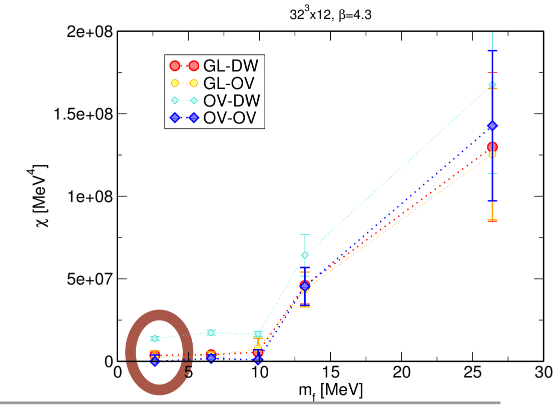


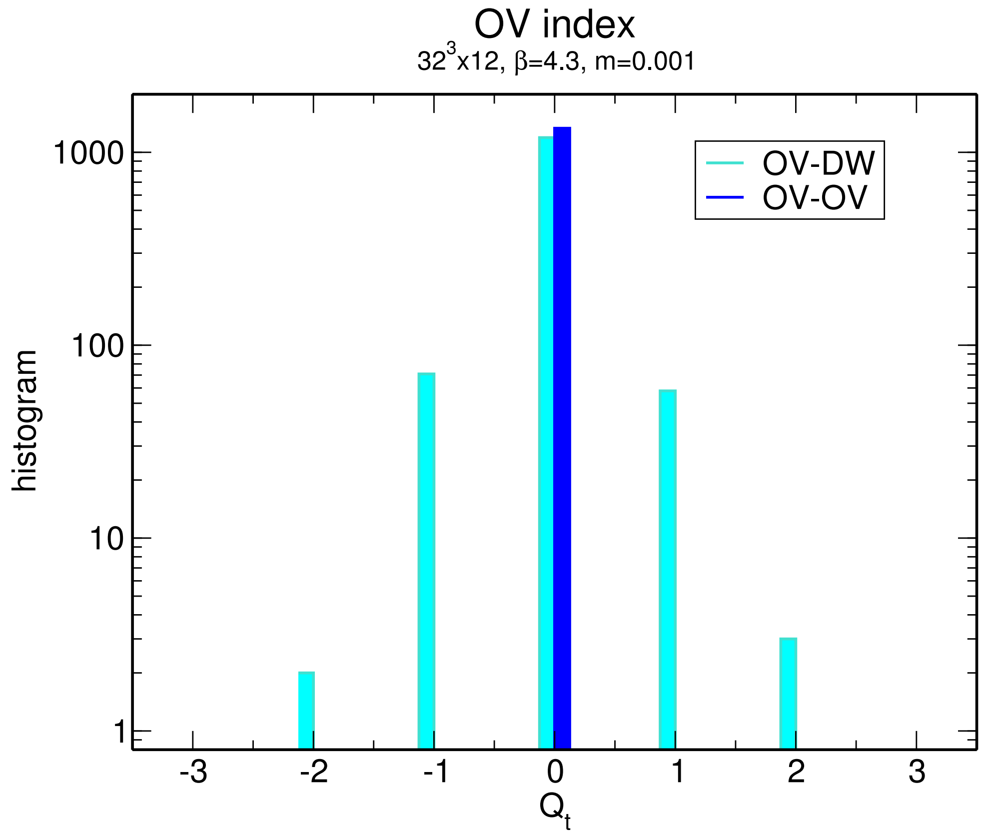






resolution of susceptibility (ex: m=0.001)





Effective number of statistics

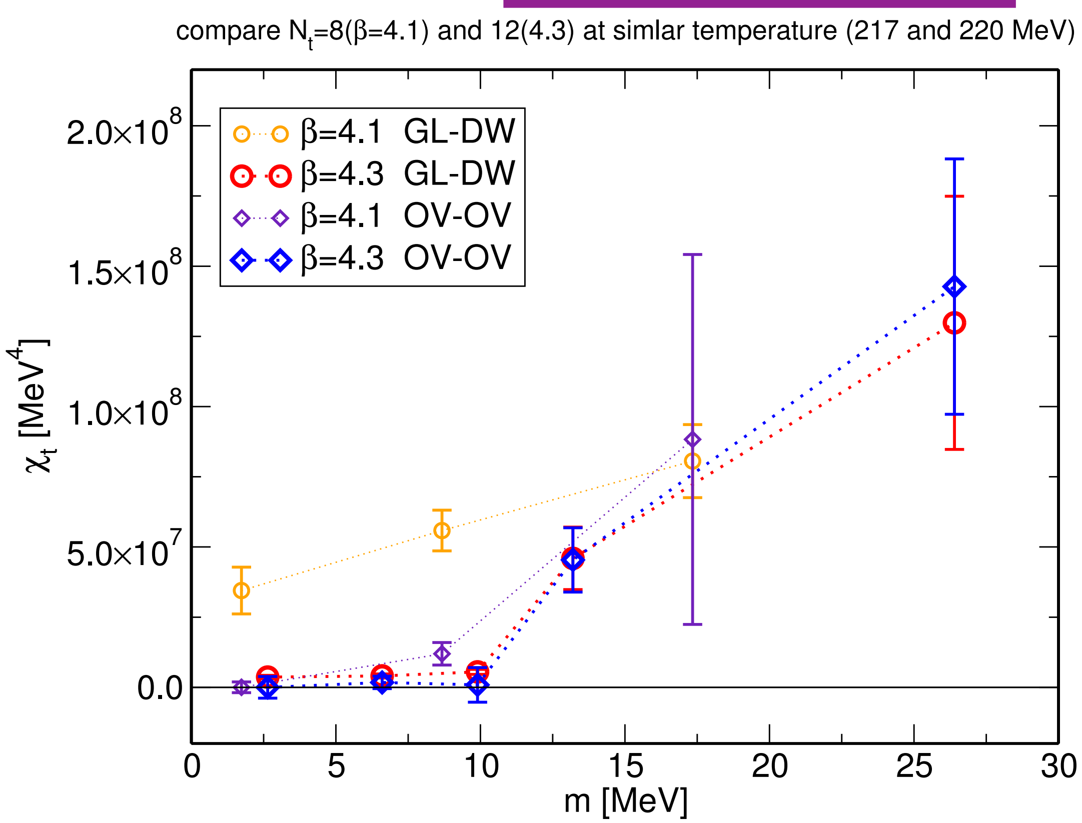
- decreases with reweighting
- $N_{eff}=N_{conf} <R>/R_{max}$
- $N_{conf}=1326 \rightarrow N_{eff} = 32$

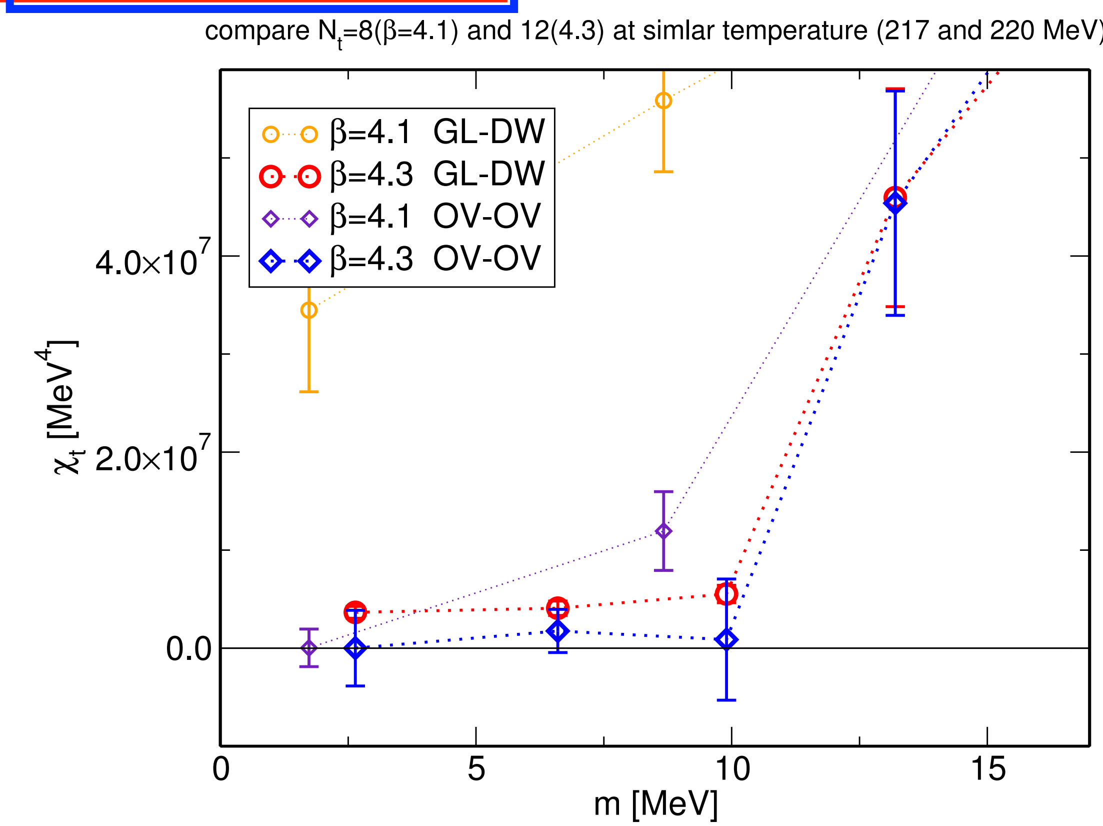
null measurement of topological excitation after reweighting

- does not readily mean $\chi_t=0$: (this case $<Q^2>=4(4) \times 10^{-6}$)
- there must be a resolution of χ_t under given statistics
  - [resolution of $<Q^2>$] = $1/N_{eff}$
- shall take the “statistical error” of $<Q^2> = \max(\Delta<Q^2>, 1/N_{eff})$

$\chi_t(m)$ T=~220 MeV discretization effect

comparing 1/a=1.7 GeV and 1/a=2.6 GeV ($(3.6\text{fm})^3$ and $(2.4\text{fm})^3$)



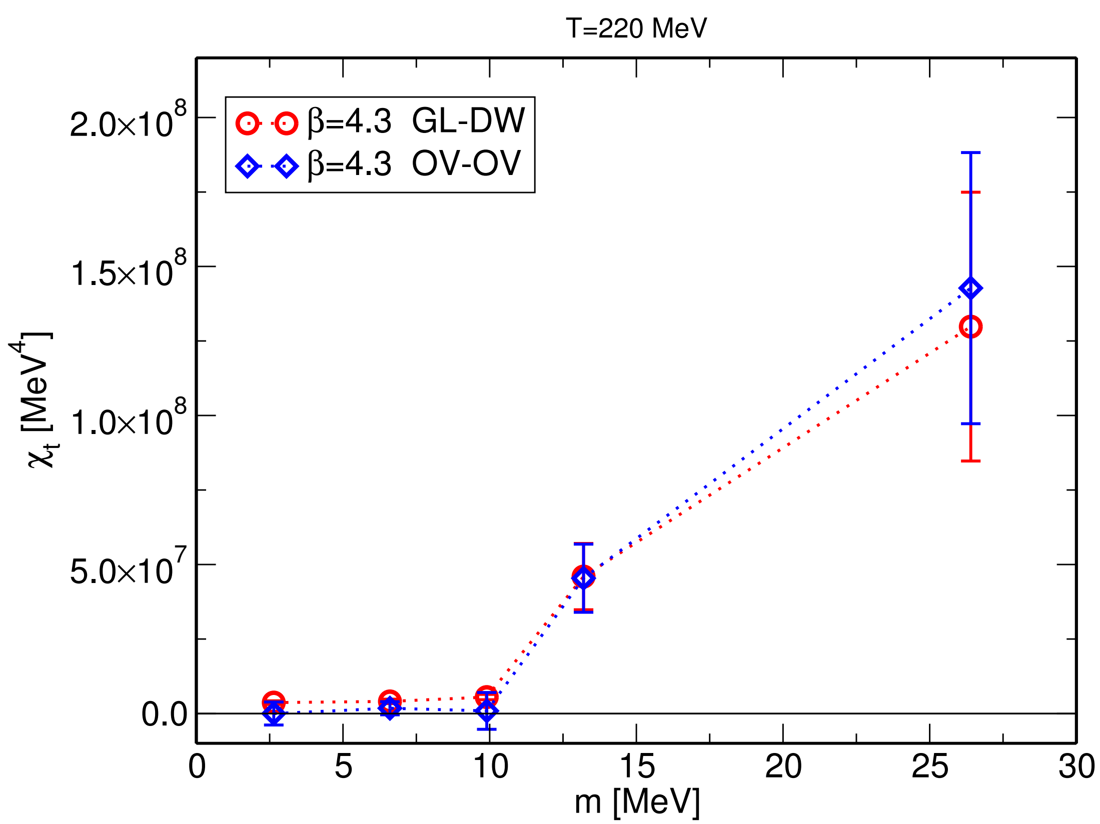


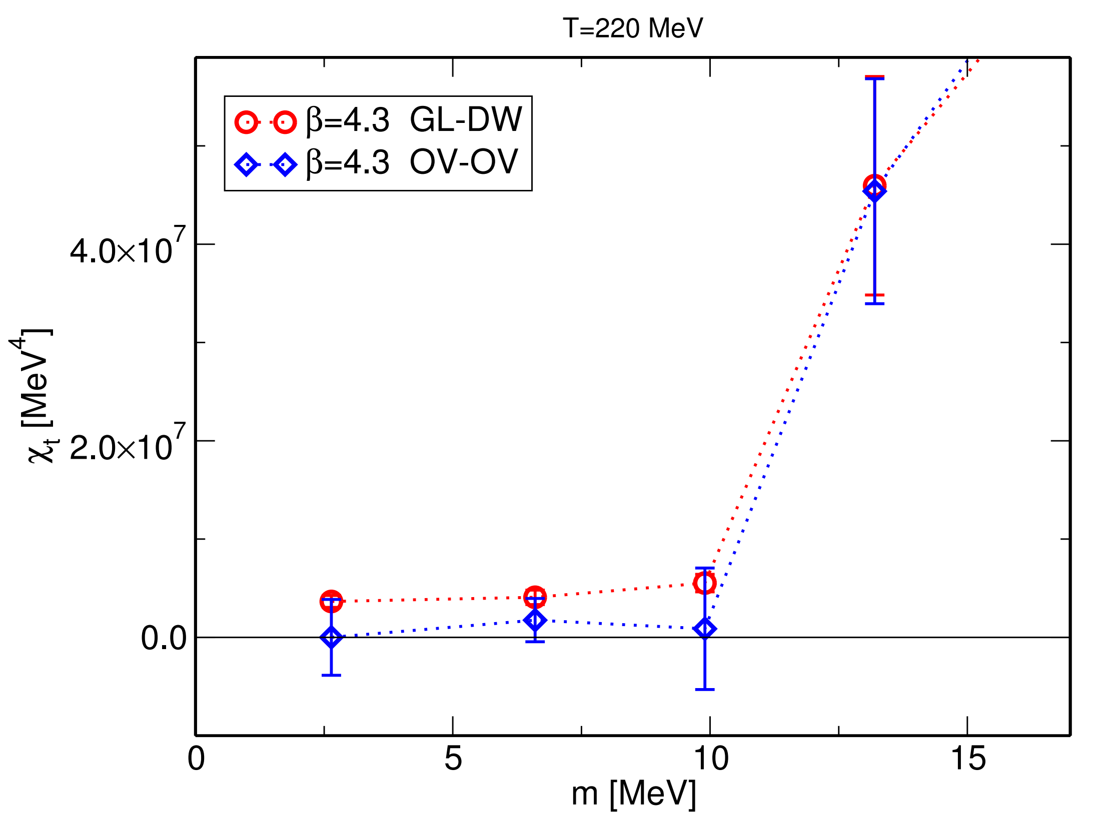
- OV-OV: better scaling
- GL-DW: large scaling violation for smaller m
- OV-OV: $\chi_t = 0$ (within error) for $0 \leq m \lesssim 10$ MeV
- GL-DW: $\chi_t > 0$, but, may well decrease as a

➡ *(consistent with OV-OV with large error of OV-OV)*

$\chi_t(m)$ T=~220 MeV   finer lattice

1/a=2.6 GeV





suggesting 2 regions

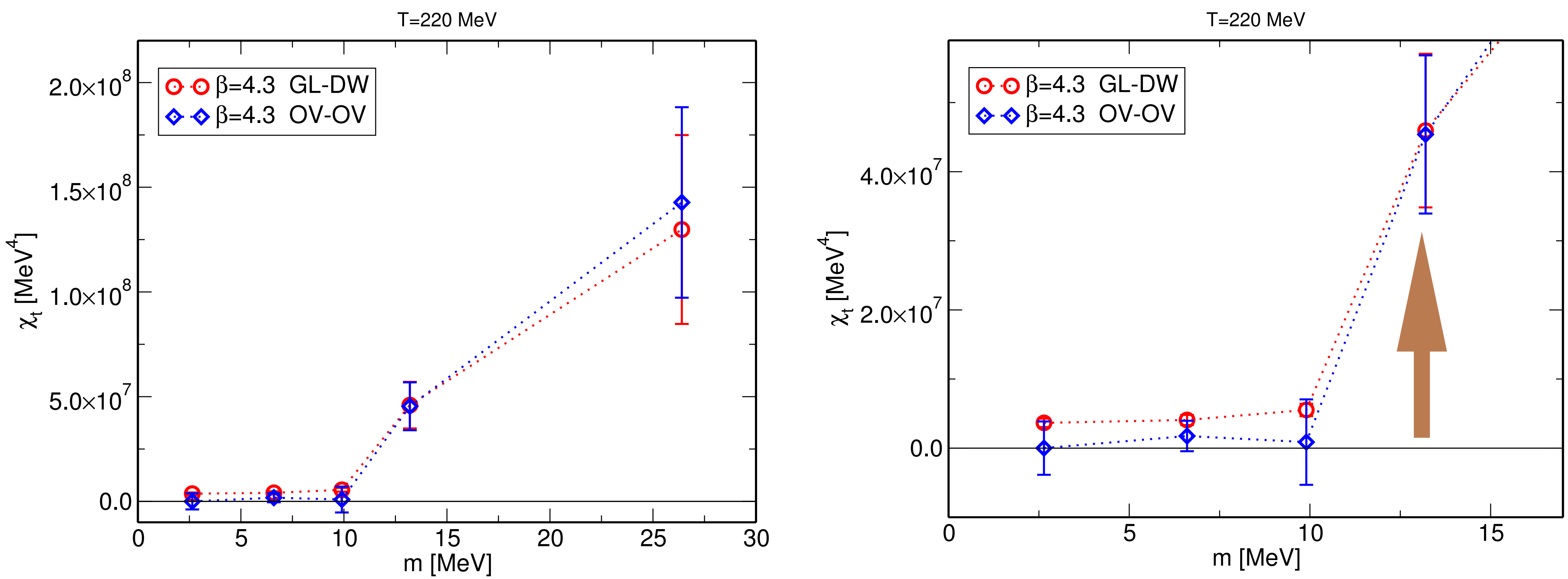
1: χ_t is very small (may vanish in $a \to 0$): $0 \leq m \lesssim 10$ MeV

2: sudden growth of χ_t : 10 MeV $\lesssim$ m

- physical ud mass point: m≃4 MeV

$\chi_t(m)$ T=~220 MeV   finer lattice

1/a=2.6 GeV



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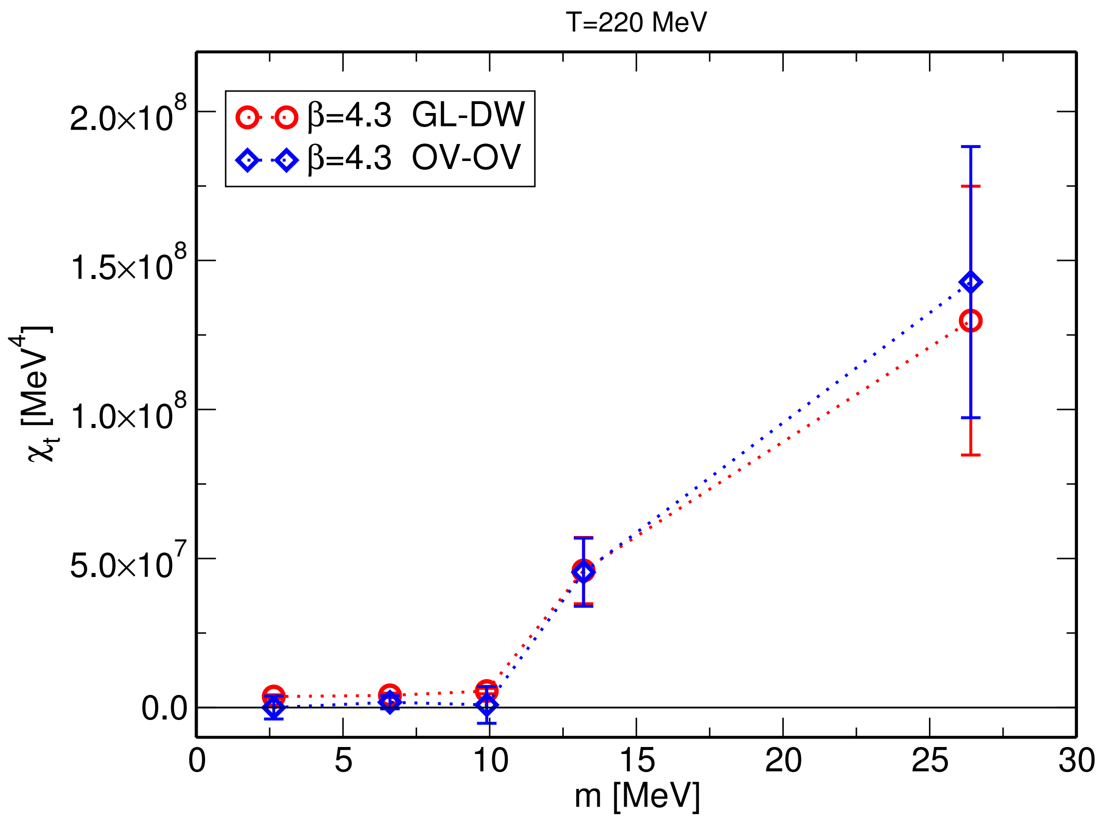
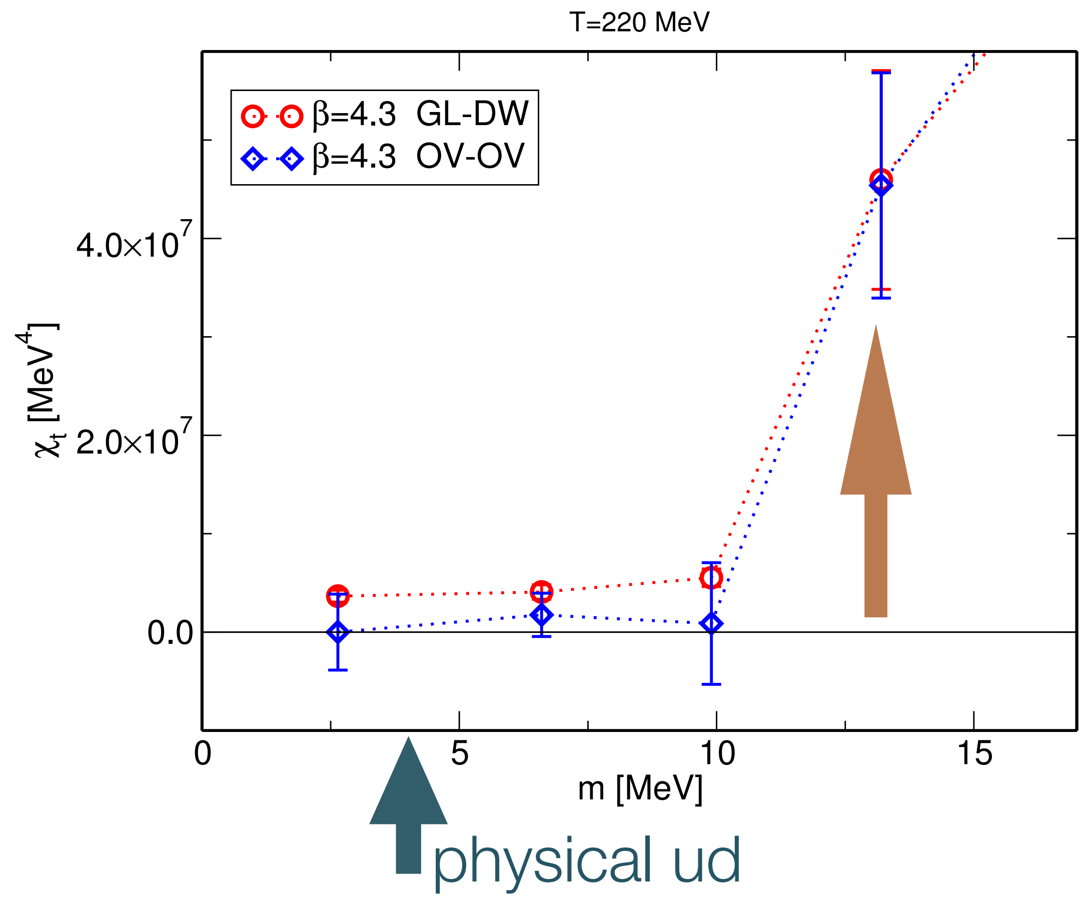




suggesting 2 regions

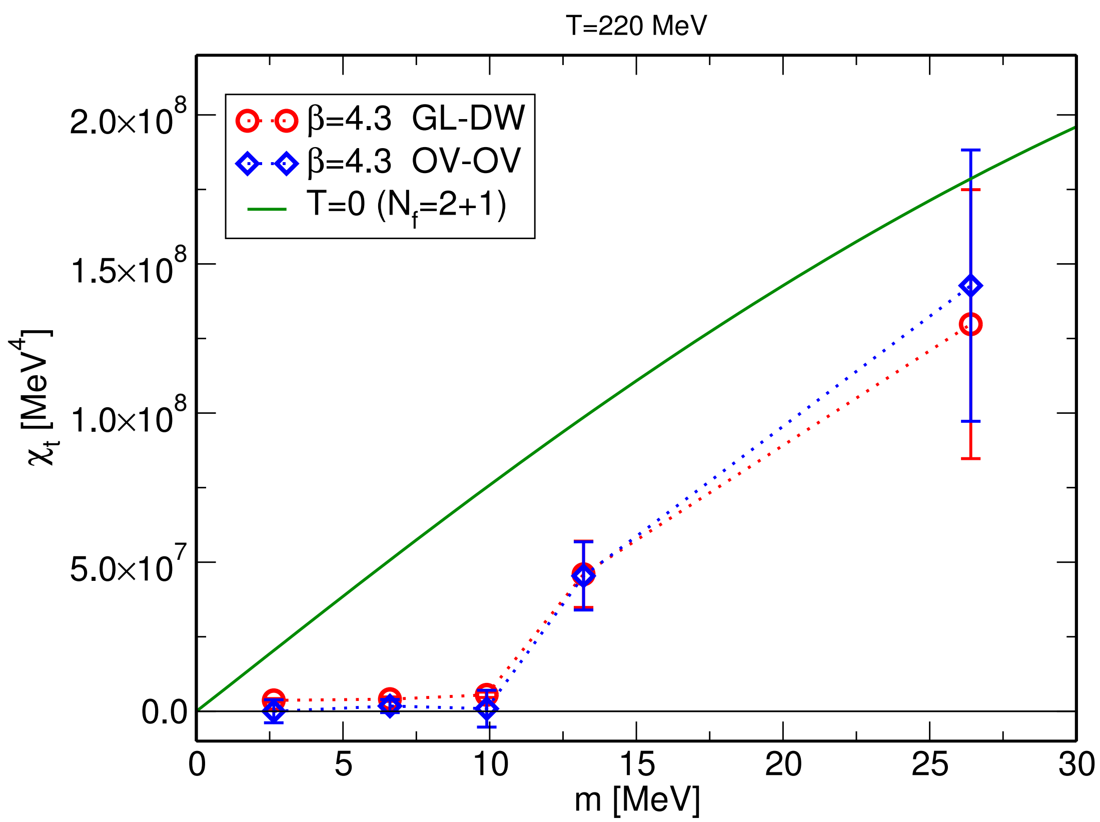
1: χ_t is very small (may vanish in $a \to 0$): 0 ≤ m ≲ 10 MeV

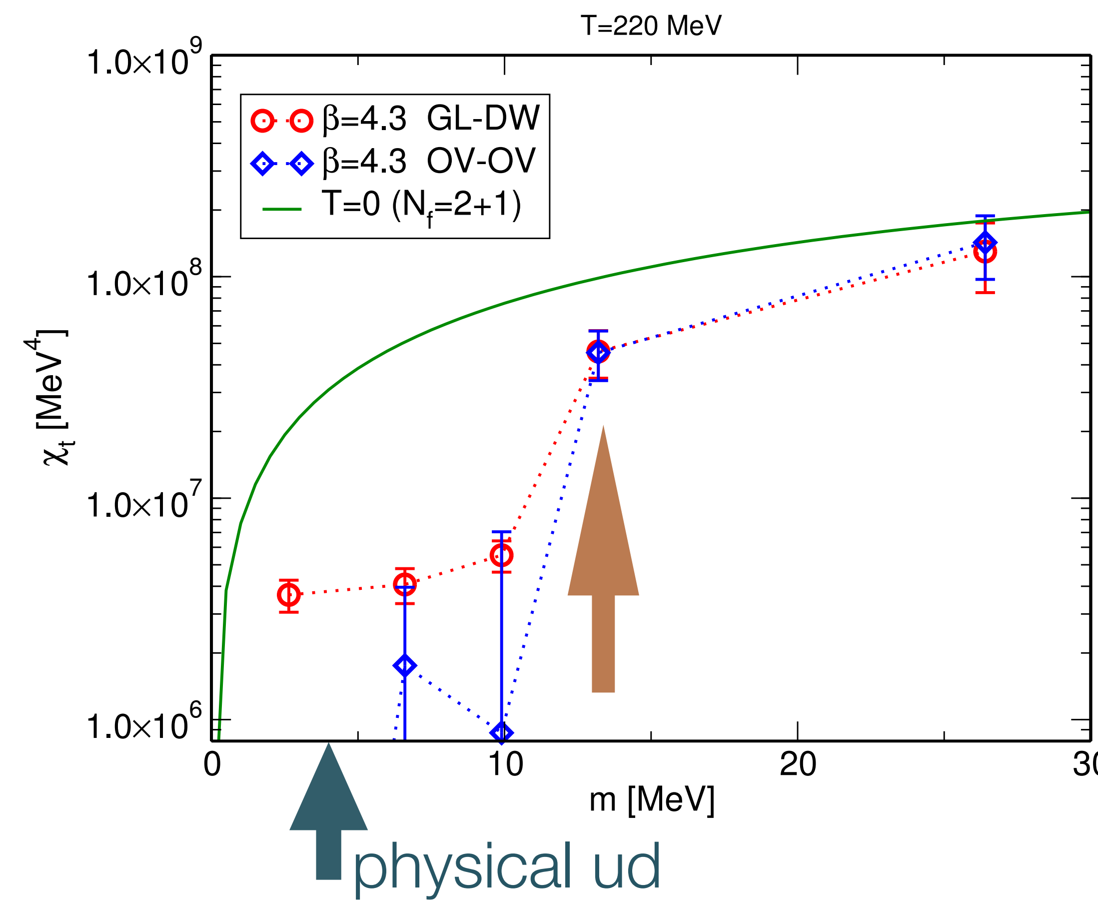
2: sudden growth of χ_t : 10 MeV ≲ m

- physical ud mass point: m≃4 MeV

$\chi_t(m)$ T=~220 MeV finer lattice against T=0

1/a=2.6 GeV





[T=0: JLQCD: S.Aoki et al 2017, N$_f$=2+1]

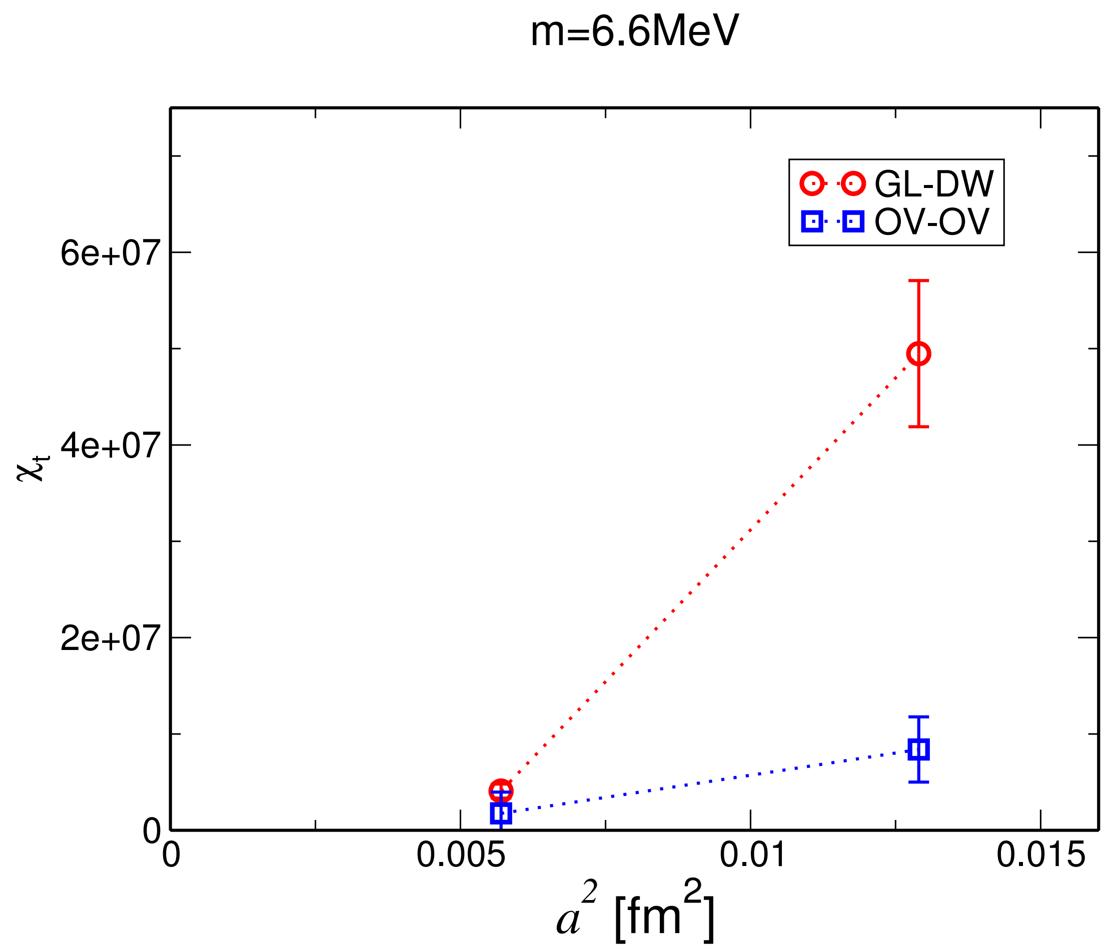
suggesting 2 regions

1: χ_t is very small (may vanish in $a \to 0$): $0 \leq m \lesssim 10$ MeV

2: sudden growth of χ_t : 10 MeV $\lesssim$ m

→ surprisingly close to T=0 value

$\chi_t(m)$ T=220 MeV a^2 scaling: m=6.6 MeV

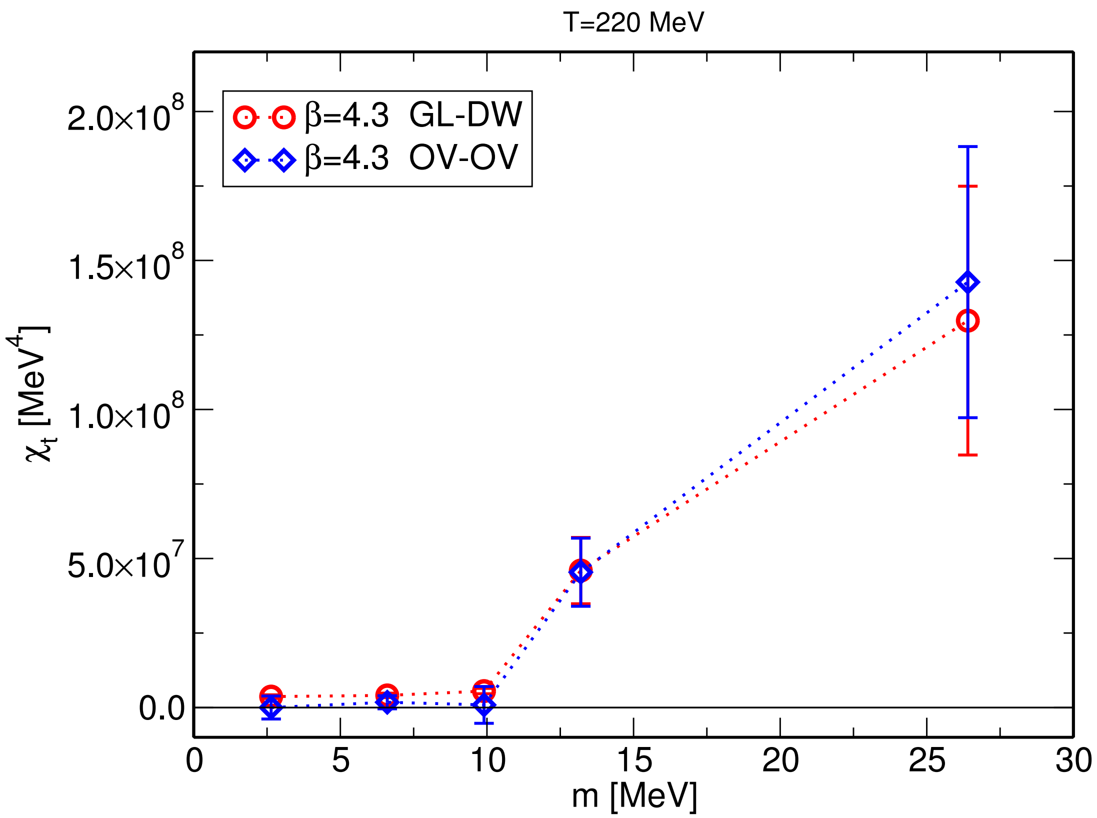


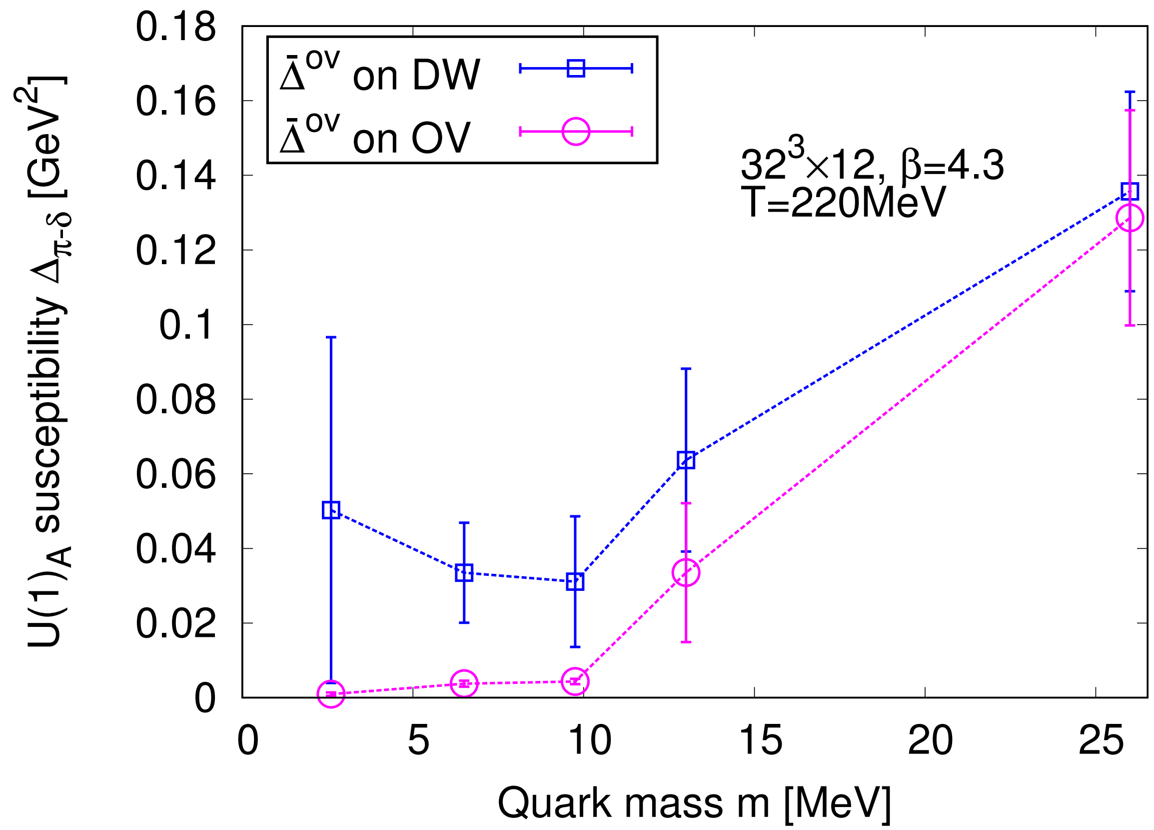
(V=(3.6fm)^3 and (2.4fm)^3)

continuum scaling in 1st region

- m=6.6 MeV
- vanishing towards continuum limit
- caveat: physical volume is different → needs further invest.

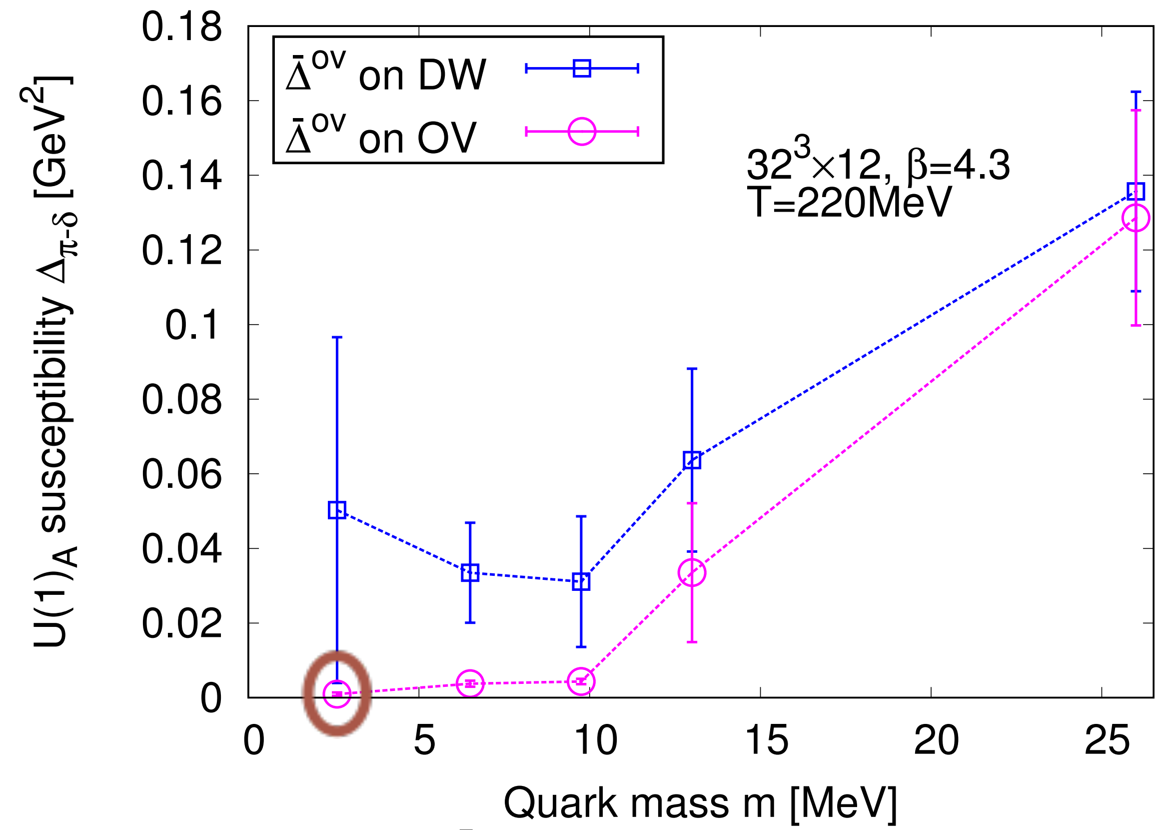
χ_t and $\Delta_{\pi\text{-}\delta}$ ($U_A(1)$ oder parameter) @ T=~220 MeV



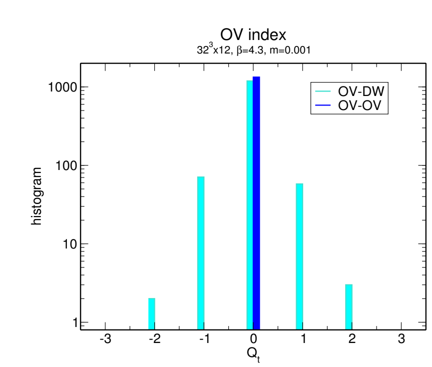


- $\Delta_{\pi\text{-}\delta}$ has a similar trend as function of m
- AFT scenario: [Aoki, Fukaya, Taniguchi]
  - $\chi = 0$ for $0<m<m_c$ → consistent with left figure
  - $\Delta_{\pi\text{-}\delta} = 0$ for m=0, but does not say anything for m>0: OK
- KY scenario…

competing scenarios for χ_t and $\Delta_{\pi\text{-}\delta}$ ($U_A(1)$ oder parameter) @ T=~220 MeV



- KY scenario [Kanazawa, Yamamoto 2016]
  - $\Delta_{\pi\text{-}\delta}$: including zero mode cont. is proper
  - $\Delta_{\pi\text{-}\delta}$ = const >0
  - $\Delta_{\pi\text{-}\delta} \simeq 8\ V\ f_A^2\ m^2$ for Q=0 sector (for $2V\ f_A m^2 < 1$)
- $\Delta_{\pi\text{-}\delta}$ @ lightest point only from Q=0
- $\chi_t = 2\ f_A\ m^2$
- inconsistent with m>10 MeV χ_t growth



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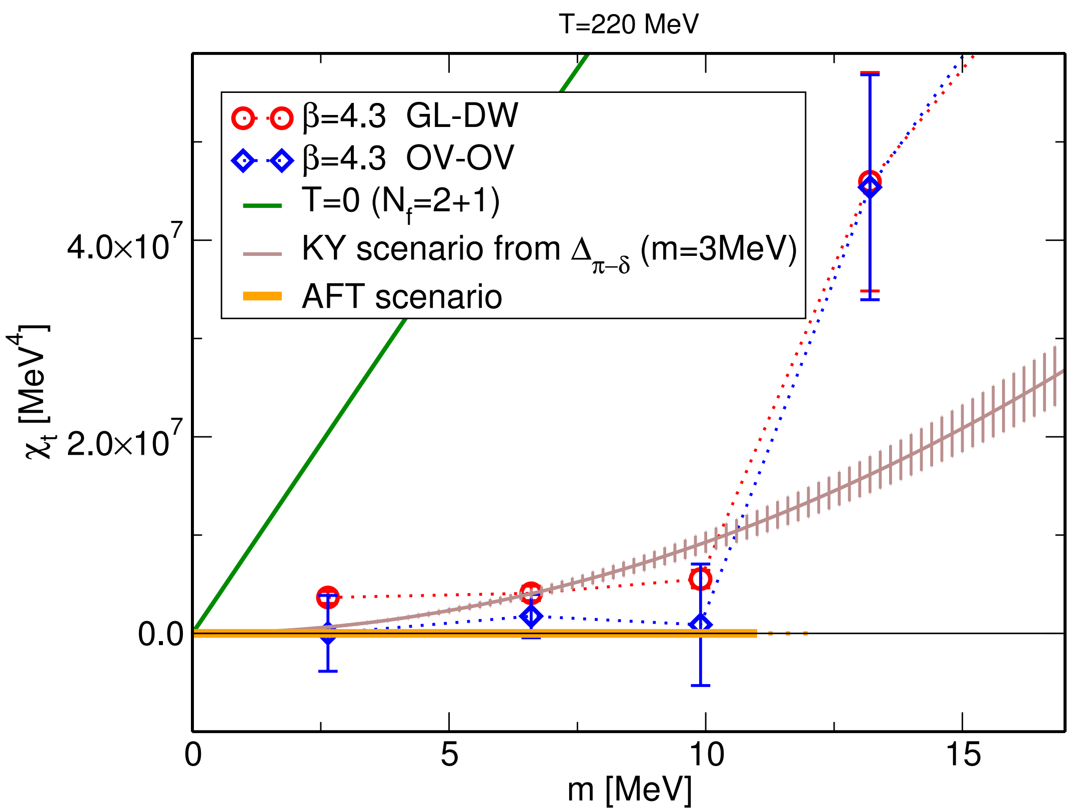

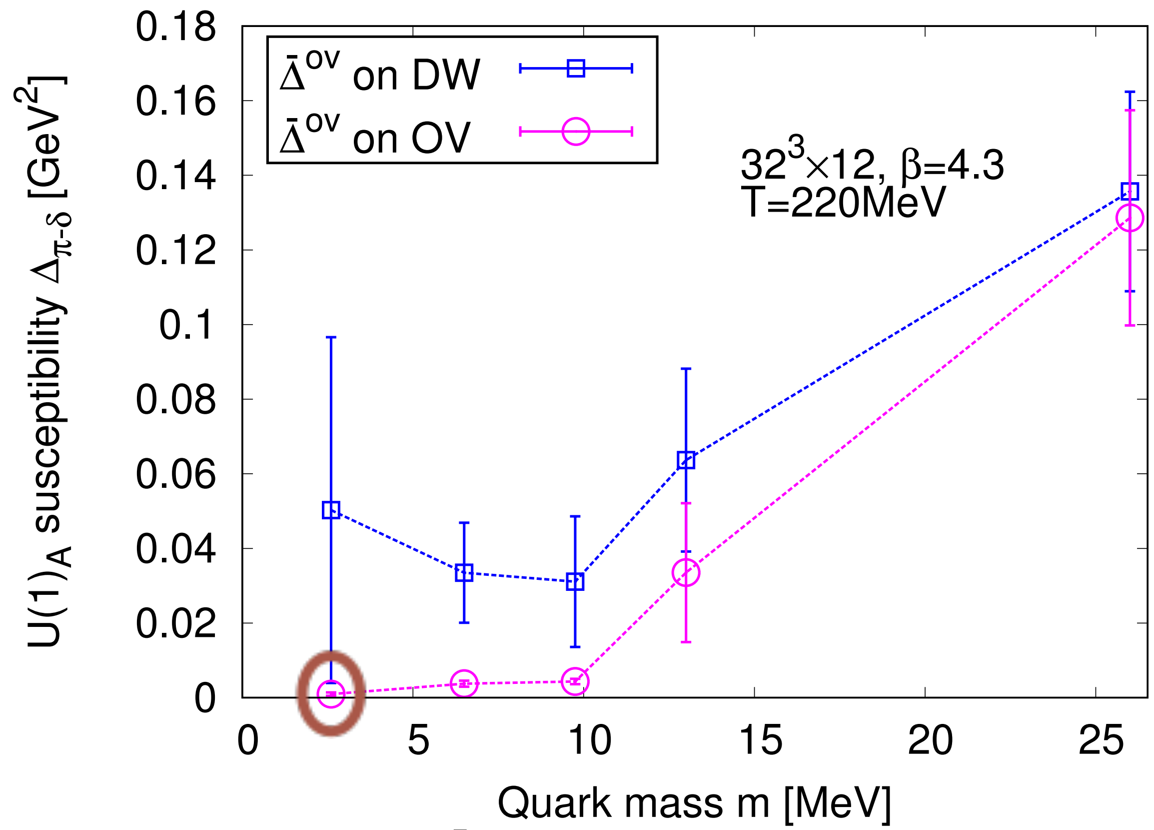

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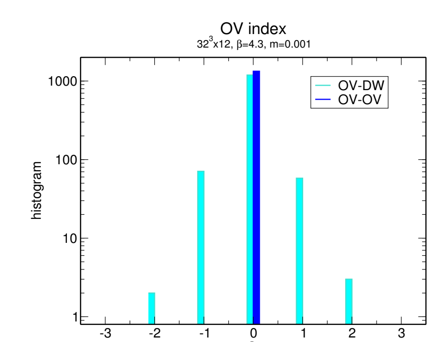
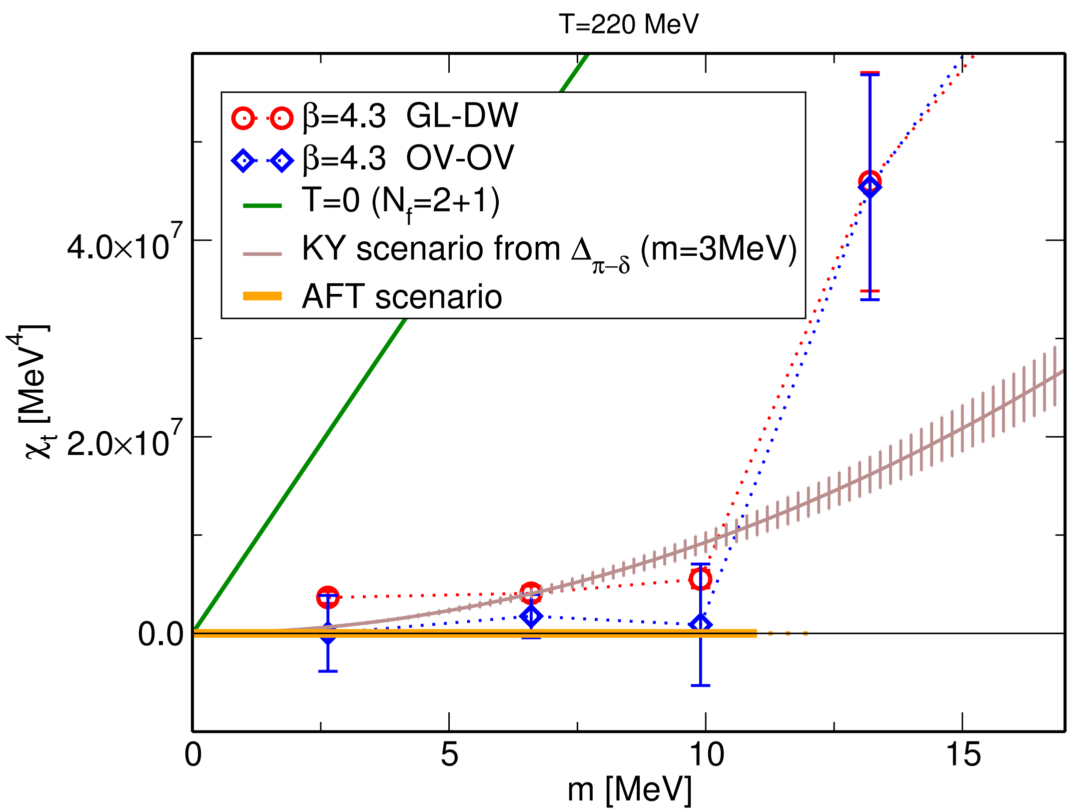
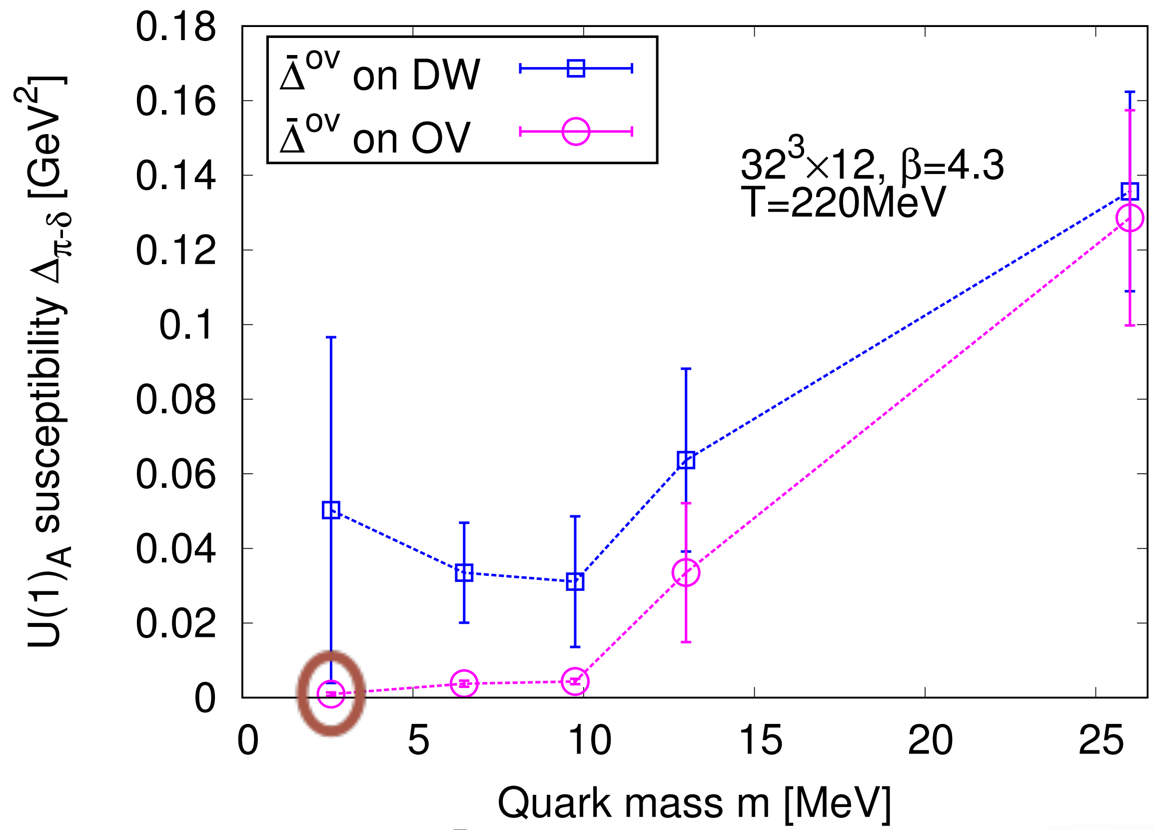
- inconsistent with m>10 MeV χ_t growth



competing scenarios for χ_t and $\Delta_{\pi\text{-}\delta}$ ($U_A(1)$ oder parameter) @ T=~220 MeV

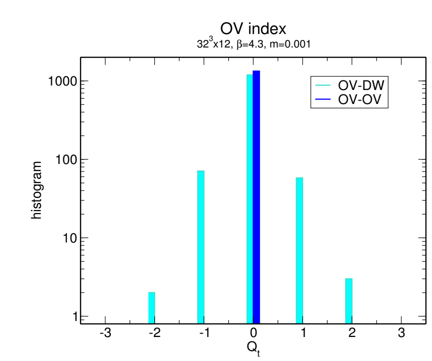




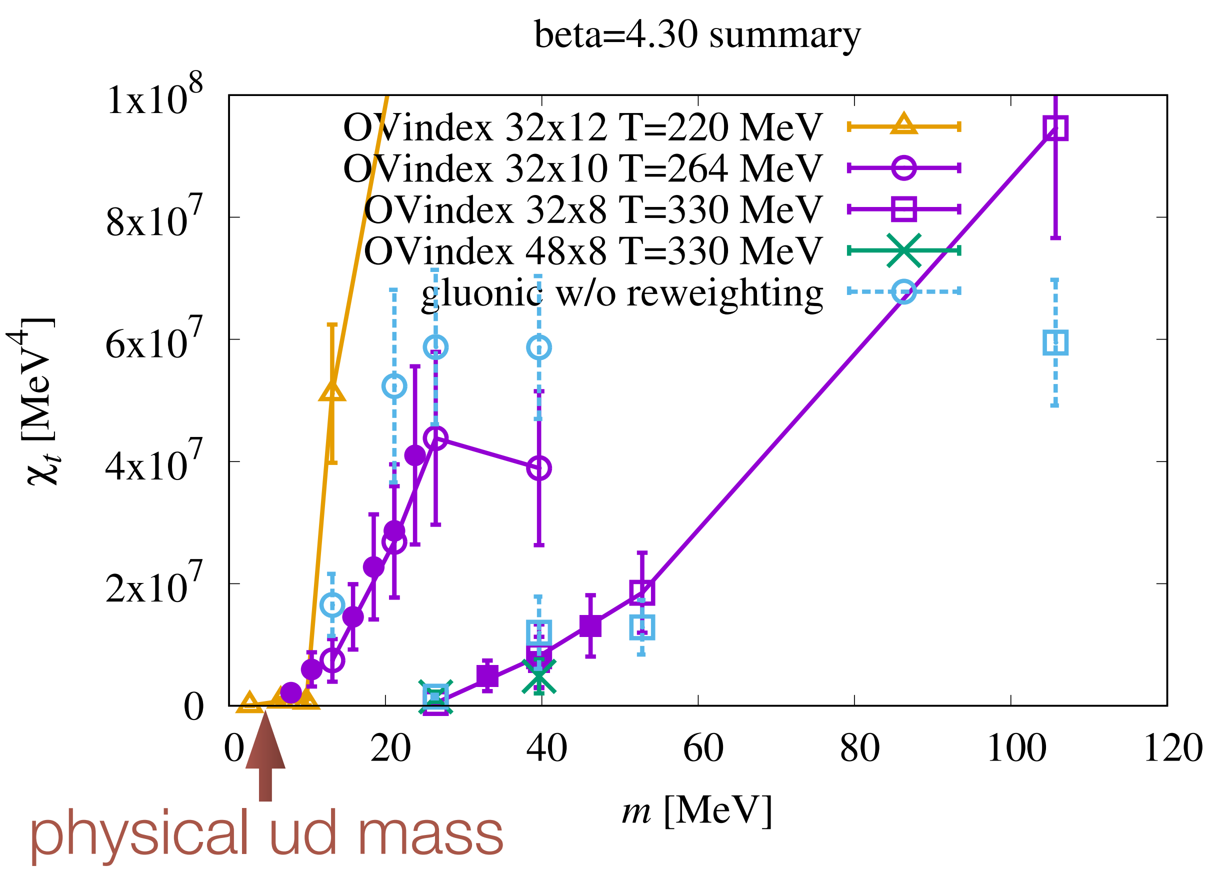
- KY scenario

Volume study would be useful to check this

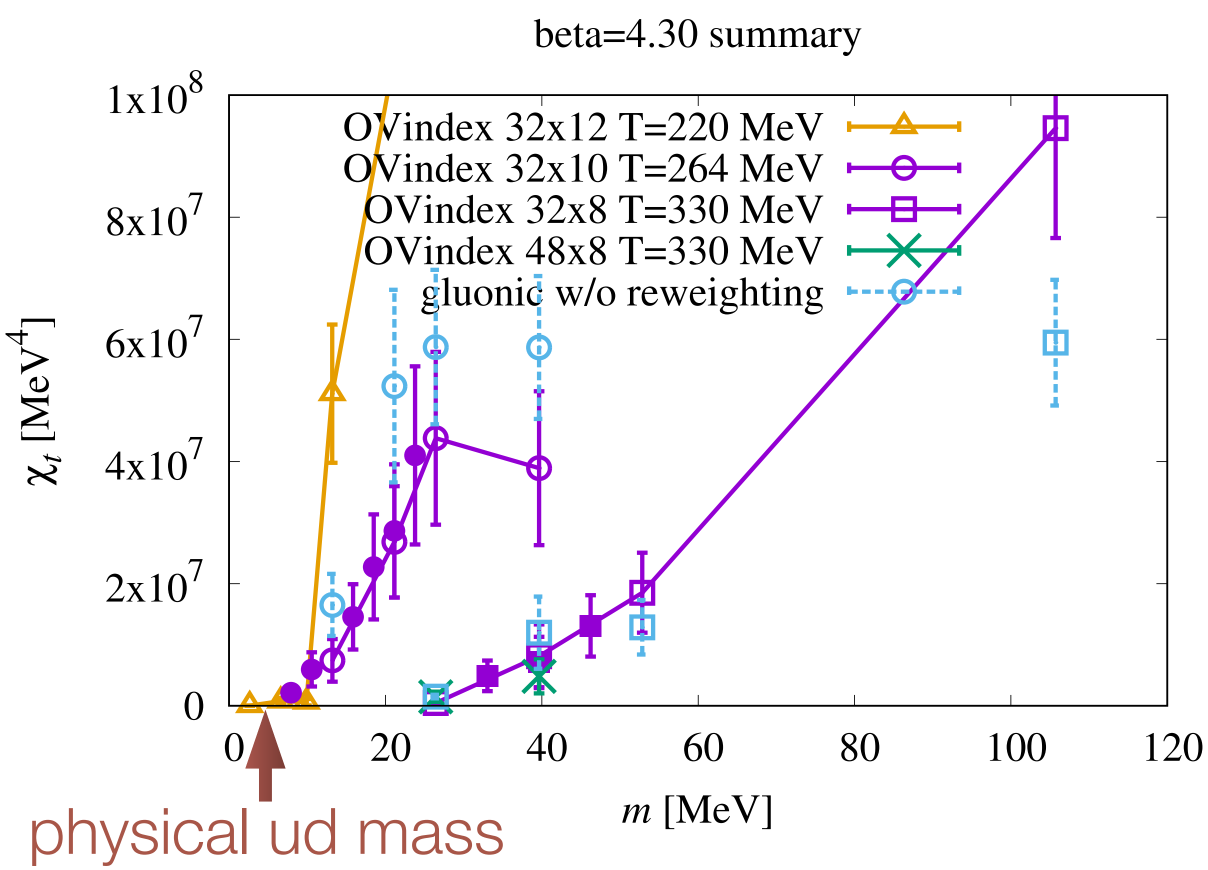
  - $\Delta_{\pi\text{-}\delta}$: incl
  - $\Delta_{\pi\text{-}\delta}$ = const >0
  - $\Delta_{\pi\text{-}\delta} \simeq 8\, V\, f_A^2\, m^2$ for Q=0 sector (for $2V f_A m^2 < 1$)
- $\Delta_{\pi\text{-}\delta}$ @ lightest point only from Q=0
- $\chi_t = 2\, f_A\, m^2$
- inconsistent with m>10 MeV χ_t growth



T=220, 264 & 330 MeV ; 1/a=2.6 GeV fixed

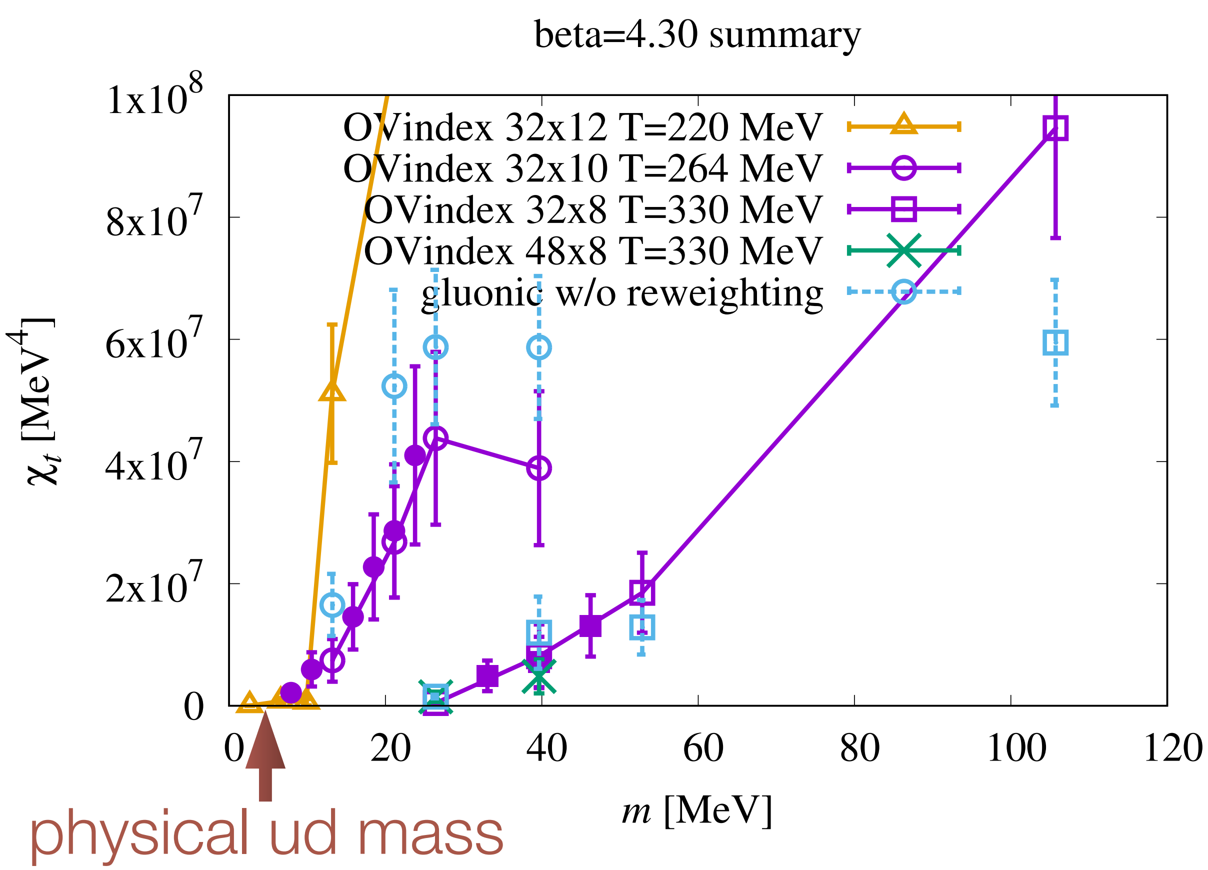


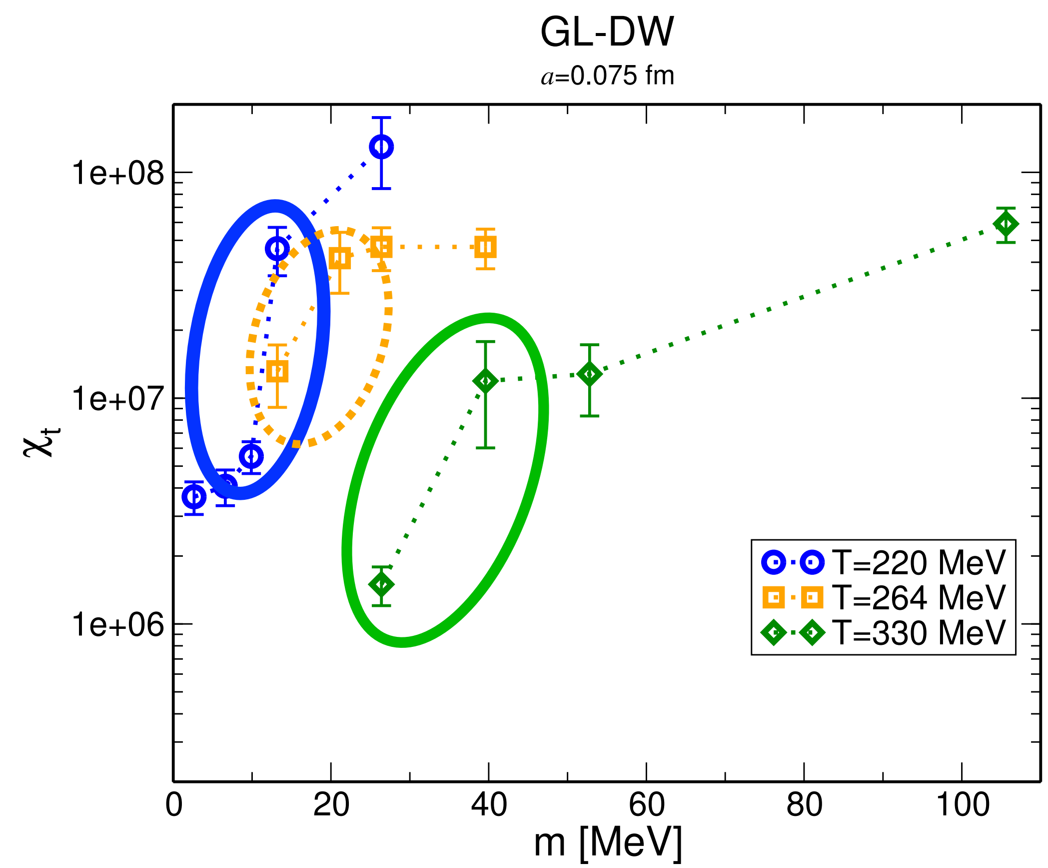
T=220, 264 & 330 MeV ; 1/a=2.6 GeV fixed



- filled symbol: by mass reweighting
- green: L x 1.5 (T=330 MeV) finite size effect small
- getting tiny value for larger mass with higher temperature
  - then lose the resolution
- similar drop as T=220 might have stated to be seen…

T=220, 264 & 330 MeV ; 1/a=2.6 GeV fixed





- filled symbol: by mass reweighting
- green: L x 1.5 (T=330 MeV) finite size effect small
- getting tiny value for larger mass with higher temperature
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Summary and Discussion

- N_f=2 QCD under investigation for topological susceptibility χ_t for high T
- fully utilizing the exact chiral symmetry Mobius DW (fine latt)→ OV reweighting
- Extensive study for T=220 MeV with a= 0.07 fm (& 0.11 fm)
- χ_t: sudden grow for m>10 MeV observed
- naive continuum scaling from two different definition of chit suggest it vanishes in the continuum limit for m<~10 MeV (eg: physical mud=~4 MeV)

➡ consistent with the prediction of AFT χ_t=0 for 0<=m<m_c (small on-zero mc)

- caveat of this scaling analysis is the difference in physical volume

  ➡ needs further investigation with different volume

- The numerical data shown here does not rule out KY scenario → vol. study
- any case, sudden change of the χ_t may suggest an existence of phase transition → volume study is useful to check this, too.