Plasmon mass scale and quantum fluctuations of classical fields on a real time lattice

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Based on:
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Standard model of ultrarelativistic heavy-ion collisions

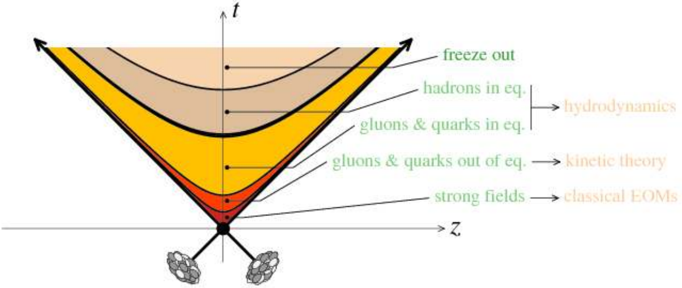


Figure : E. Iancu 1105.0751 [hep-ph]

- Hydro: thermalization in less than $1\frac{\mathrm{fm}}{\mathrm{c}}$, however $r_{\mathrm{pb}} \approx 7\mathrm{fm} \rightarrow$ Extremely fast!

Why Classical Yang-Mills theory (CYM) ?

CYM = pure glue classical QCD, equations of motion

$$\left[D_\mu, F^{\mu\nu}\right] = 0. \quad (1)$$

- Color Glass Condensate (CGC, effective theory of QCD at high energy (weak coupling) limit) → nonperturbatively high occupation numbers for gluons $f \sim 1/g^2$
- Highly occupied gluon states → classical fields
- Pauli principle $\rightarrow f_q(p) < 1 \rightarrow$ gluons are the dominant dof!

Simulating CYM on a real time lattice

- In "standard" LQCD, rotate to imaginary time, compute path integral (gives partition function)
- Real time approach: take $\mathscr{L}$ (or $\mathscr{H}$), derive classical EOMS.

Two dynamical variables, the gauge links, and chromoelectric fields E. EOMS:

$$\dot{U}_i(\mathbf{x}) = iE^i(\mathbf{x})U_i(\mathbf{x}) \quad (2)$$

$$a^2\dot{E}^i(\mathbf{x}) = -\sum_{j\neq i}\left[\Box_{i,j}(\mathbf{x}) + \Box_{i,-j}(\mathbf{x})\right]_{\mathrm{ah}}, \quad (3)$$

Plasmons & mass

Quasiparticle picture of classical fields?

- Gluon field modes of classical fields → plasmons

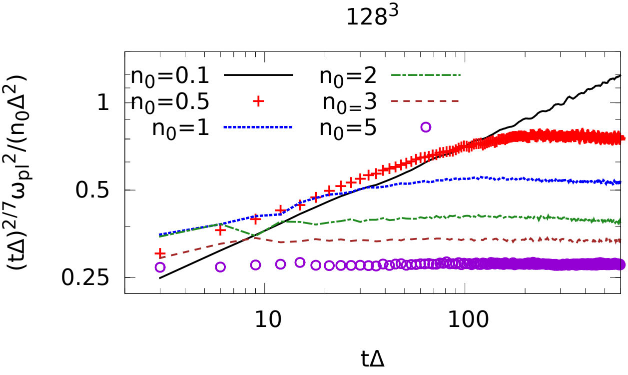
3 methods to determine their mass

- Effective dispersion relation (DR) at zero momentum $\frac{\left\langle \left|\dot{E}(k)\right|^2 \right\rangle}{\langle |E(k)| \rangle}$
- Add uniform chromoelectric field, measure oscillations (UE , Aleksi & G. D. Moore arXiv:1207.1663 [hep-ph]).
- Perturbation theory, Hard Thermal Loops (HTL):

$$\omega_{\mathrm{pl}}^2 = \frac{4}{3} g^2 N_c \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{f(k)}{|k|} \tag{4}$$

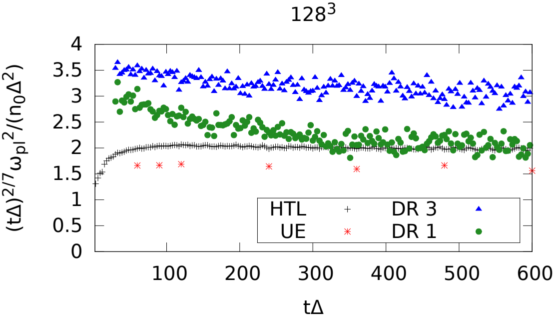
Time dependence, HTL

- Asymptotic late time behaviour consistent with $t^{-2/7}$ power law (arXiv:1207.1663 [hep-ph]).
- Systems with higher occupation numbers pass the initial transient behaviour faster.



Time dependence, comparison

- All three methods agree with the $t^{-2/7}$ powerlaw at late times.
- Significant fit cutoff and time dependence in the DR method. Needs also more statistics
- Main result: HTL and UE methods in rough agreement, the DR method is more cumbersome.



Linearized fluctuations

Why explicitly compute linearized fluctuations of CYM on top of classical background?

Two main applications in the future

- Plasma instabilities in CYM (possible contribution to thermalization!)
- Linear response of CYM, e.g. dispersion relation (ongoing)

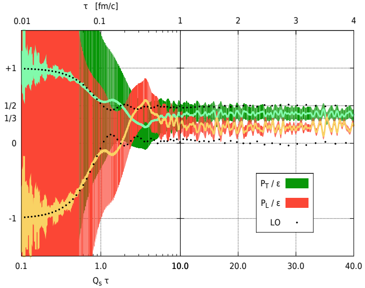


Figure : Gelis et. al. 1307.2214 [hep-ph]

EOM for the electric field fluctuation

$$
\begin{aligned}
a^2 e^i(t+\mathrm{d}t) = a^2 e^i(t) - \mathrm{d}t \sum_{j\neq i} \Big[& a_i(\mathbf{x})\Box_{i,j}(\mathbf{x}) + a_j(\mathbf{x}+\hat{\imath}\to\mathbf{x})\Box_{i,j}(\mathbf{x}) \\
& - \Box_{i,j}(\mathbf{x})a_i(\mathbf{x}+\hat{\jmath}\to\mathbf{x}) - \Box_{i,j}(\mathbf{x})a_j(\mathbf{x}) + a_i(\mathbf{x})\Box_{i,-j}(\mathbf{x}) \\
& - a_j(\mathbf{x}+\hat{\imath}-\hat{\jmath}\to\mathbf{x}+\hat{\imath}\to\mathbf{x})\Box_{i,-j}(\mathbf{x}) \\
& - \Box_{i,-j}(\mathbf{x})a_i(\mathbf{x}-\hat{\jmath}\to\mathbf{x}) + \Box_{i,-j}(\mathbf{x})a_j(\mathbf{x}-\hat{\jmath}\to\mathbf{x}) \Big]_{\mathrm{ah}} \qquad (5)
\end{aligned}
$$

Here

$$
a_j(\mathbf{x}+\hat{\imath}\to\mathbf{x}) \equiv U_i(\mathbf{x})a_j(\mathbf{x}+\hat{\imath})U_i^\dagger(\mathbf{x}), \qquad (6)
$$

and

$$
\begin{aligned}
& a_j(\mathbf{x}+\hat{\imath}-\hat{\jmath}\to\mathbf{x}+\hat{\imath}\to\mathbf{x}) \qquad (7) \\
& \quad \equiv U_i(\mathbf{x})a_j(\mathbf{x}+\hat{\imath}-\hat{\jmath}\to\mathbf{x}+\hat{\imath})U_i^\dagger(\mathbf{x}),
\end{aligned}
$$

Discretized Gauss' law for the fluctuations

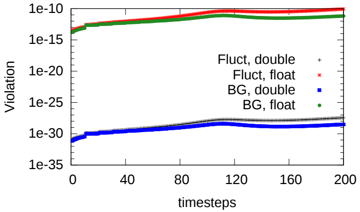
$$c(\mathbf{x},t) = \sum_i \frac{1}{a_s^2}\Big\{e^i(x) - U_i^\dagger(\mathbf{x}-\hat{\imath})e^i(\mathbf{x}-\hat{\imath})U_i(\mathbf{x}-\hat{\imath}) + iU_i^\dagger(\mathbf{x}-\hat{\imath})[a_i(\mathbf{x}-\hat{\imath}),E^i(\mathbf{x}-\hat{\imath})]U_i(\mathbf{x}-\hat{\imath})\Big\}. \qquad (8)$$

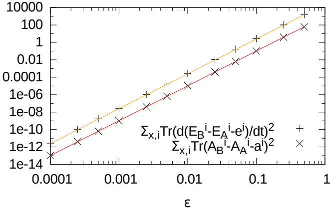
Problem Gauss' law is not conserved by the naive discretization of the continuum update of a_i! I.e. unphysical charges are created, breaks linearization (2nd order effect in dt)

Solution Reverse engineer the correct update for a_i from the Gauss's law.

$$a_i\,(t+\mathrm{d}t) = \frac{-i\left[E^i,\left(\Box_{0i}e^i_\perp\Box^\dagger_{0i}\right)\right]}{2\mathrm{Tr}\,(E^iE^i)} + \Box_{0i}a_i^\perp\Box^\dagger_{0i} + \mathrm{d}te^{i\parallel} + a_i^\parallel\,(t)\,. \qquad (9)$$

Numerical tests





- Violation of Gauss's law, square of sum vs. sum of squares with double and single precision numbers.

Use two fields, the background and the fluctuation (scale ε):

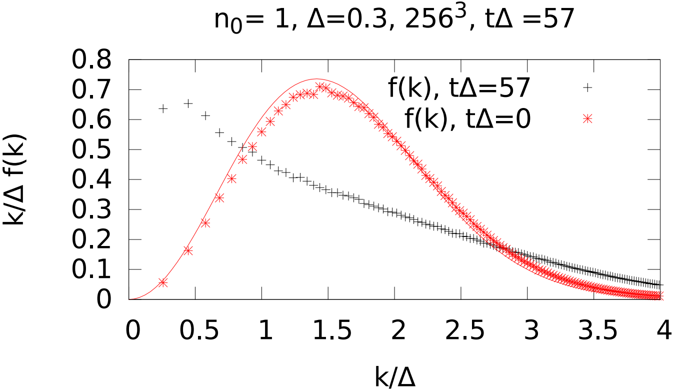
- E_B Fluctuation is absorbed in the background
- E_A Contains only the background
- e^i Is the fluctuation evolved with the linearized equations

Conclusions

- We have studied the plasmon mass in pure glue QCD using
  - HTL expression
  - dispersion relation
  - plasma oscillations
- DR method agrees with other methods within a factor of two. However fit cutoff effects have to be dealt with carefully.
- We have also derived, implemented and tested linearized equations for fluctuations in CYM, which conserve the Gauss' law exactly.
- Next steps: two dimensional systems, expanding coordinates, extract the dispersion relation using fluctuations

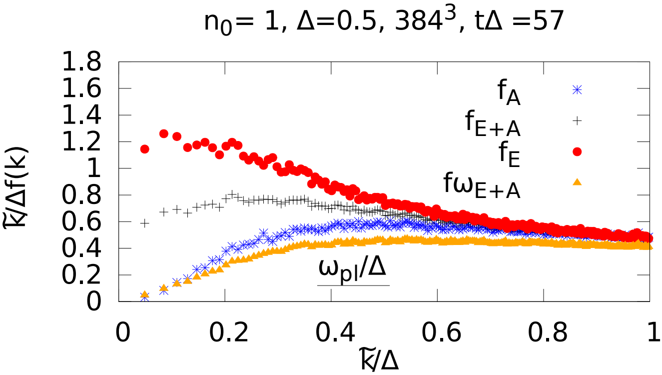
Quasiparticle spectrum

- Sample initial gauge fields s.t. $f = \frac{n_0}{g^2}\frac{k}{\Delta}\exp\left(-\frac{k^2}{2\Delta^2}\right)$.



Quasiparticle distribution

- Significant deviations between different definitions below the Debye-scale
- $f(k) = \frac{1}{2}\frac{1}{2\left(N_c^2-1\right)}\frac{1}{V}\left(\frac{|E_C(k)|^2}{|k|} + |k|\,|A_C(k)|^2\right).$



Small fluctuations in the continuum

Introduce fluctuations:

$$(E^i, A_i) \rightarrow (E^i + e^i, A_i + a_i), \tag{10}$$

This leads into equations of motion

$$\begin{aligned} \dot{a}_i &= e^i & (11) \\ \dot{e}^i &= \left[D_j, \left[D_j, a_i\right]\right] - \left[D_i, \left[D_j, a_j\right]\right] + 2ig\left[a_j, F_{ji}\right] & (12) \end{aligned}$$

The small fluctuations have their own Gauss' law

$$c(\mathbf{x}, t) = \left[D_i, e^i\right] + ig\left[a_i, E^i\right] = 0. \tag{13}$$

We have derived these equations of motion on the lattice and tested an implementation of a solver.

EOMS & Gauss' law

$$\dot{U}_i(\mathbf{x}) = iE^i(\mathbf{x})U_i(\mathbf{x}) \tag{14}$$

$$a^2\dot{E}^i(\mathbf{x}) = -\sum_{j\neq i}\left[\Box_{i,j}(\mathbf{x}) + \Box_{i,-j}(\mathbf{x})\right]_{\mathrm{ah}}, \tag{15}$$

where $\Box_{i,-j}(\mathbf{x}) = U_i(\mathbf{x})U_j^\dagger(\mathbf{x}+\hat{\imath}-\hat{\jmath})U_i^\dagger(\mathbf{x}-\hat{\jmath})U_j(\mathbf{x}-\hat{\jmath})$.
The discretized Gauss' law is given by

$$C(\mathbf{x},t) = \sum_i \frac{1}{a^2}\left\{E^i(\mathbf{x}) - U_i^\dagger(\mathbf{x}-\hat{\imath})E^i(\mathbf{x}-\hat{\imath})U_i(\mathbf{x}-\hat{\imath})\right\}. \tag{16}$$

This is separately conserved for both, timestep of E and timestep of U!

Time-evolution equation for a_i

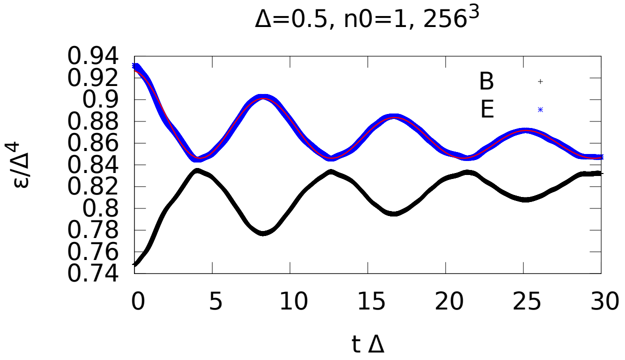
To satisfy Gauss' law, a_i has to satisfy the following equation

$$\left[E^i, a_i(t+\mathrm{d}t)\right] = -i\left(\Box_{0i} e^i \Box_{0i}^\dagger - e^i\right) + \left[E^i, \Box_{0i} a_i(t) \Box_{0i}^\dagger\right], \qquad (17)$$

with $\Box_{0i} = e^{iE^i \mathrm{d}t}$.
Solve this for the perpendicular component (to E) of a_i. Use the naive discretization for the parallel component. Solution (for SU(2)) is

$$a_i(t+\mathrm{d}t) = -\frac{i}{2\,\mathrm{Tr}\,[E^i E^i]}\left[E^i, \left(\Box_{0i} e^{i\perp} \Box_{0i}^\dagger - e^{i\perp}\right)\right] + \Box_{0i} a_i \Box_{0i}^\dagger + \mathrm{d}t e^{i\|} + a_i^{\|}(t) \qquad (18)$$

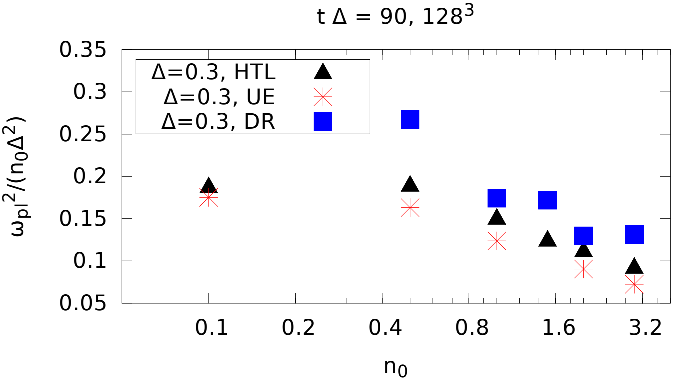
Uniform electric field



- The extracted plasmon mass relatively independent of the amount of energy introduced

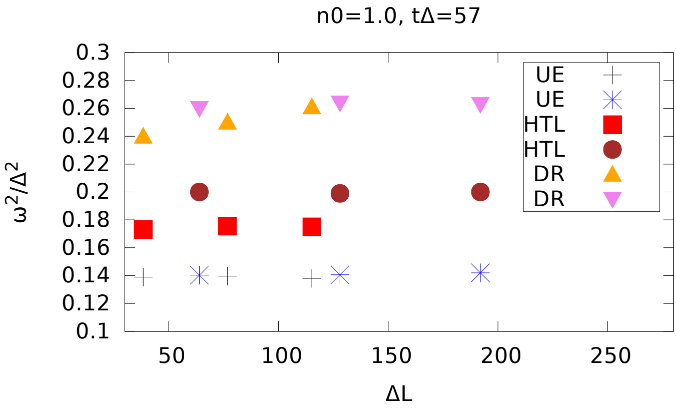
Occupation number dependence

- Observe that the plasmon mass decreases faster in more densely occupied systems.



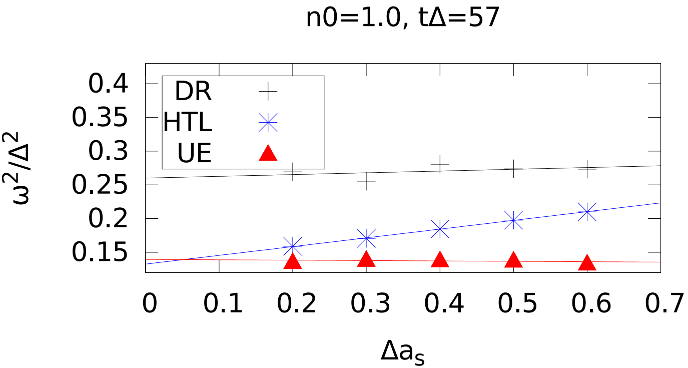
IR cut-off

- UV cutoff $a_s\Delta$ held fixed here.
- No significant IR effects



UV cut-off

- IR cutoff $L\Delta$ held fixed here.
- The HTL formula seems to be sensitive to the UV cut-off.
- The HTL and UE methods seem to converge to same value in the continuum limit.



UV cutoff dependence for f