

# Heavy Domain Wall Fermions: The RBC and UKQCD charm physics program

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RBC-UKQCD Collaborations

Granada

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THE UNIVERSITY *of* EDINBURGH

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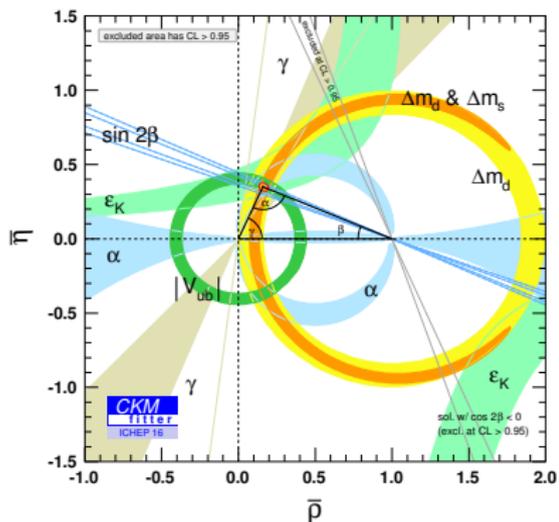
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- 1 Motivation
- 2 Set-Up 1 - charm
  - $f_D$ ,  $f_{D_s}$  and  $f_{D_s}/f_D$
  - $\xi$
  - $m_c$
- 3 Set-Up 2 - pushing further
  - First results
- 4 Aside: Combined Analysis of JLQCD + RBC/UKQCD data
- 5 Summary and Outlook



CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005) [hep-ph/0406184], updated results and plots available at: <http://ckmfitter.in2p3.fr>

## Experiment

- Belle, BaBar, CLEO-c
- LHCb, Belle II

## Theory

- $K$ ,  $D$  and  $B$  physics to test unitarity of the CKM matrix.

⇒ Place tight bounds on SM predictions

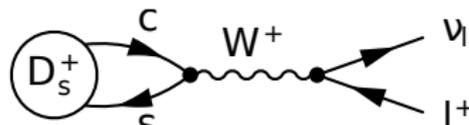
## Experimental decay rate

$$\Gamma_{\text{exp.}} = V_{CKM} \times (\text{weak} + \text{em}) \times (\text{strong}) \quad [+ \mathcal{O}(\alpha_{EM})]$$

# Motivation - Charm (and $b$ ) Observables

Large number of processes of interest:

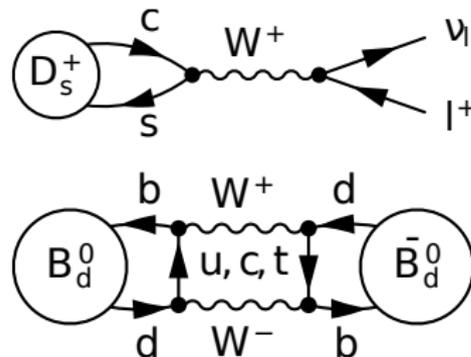
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  - ⇒ Access  $|V_{cd}|, |V_{cs}|$
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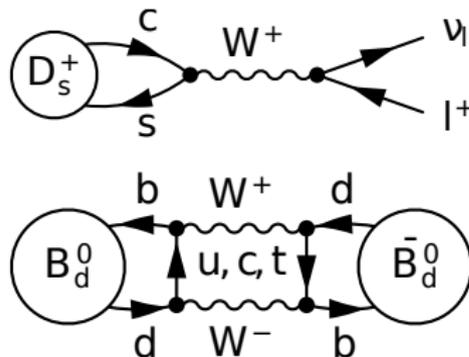
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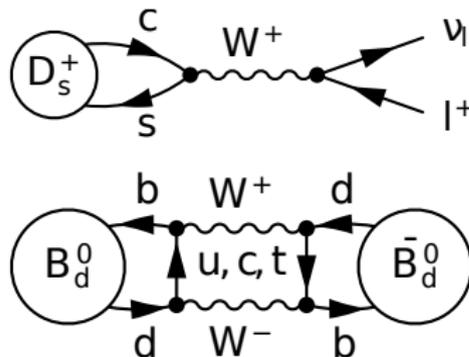
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  - ⇒ Input for further simulations



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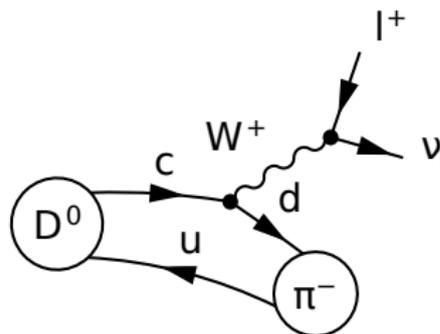
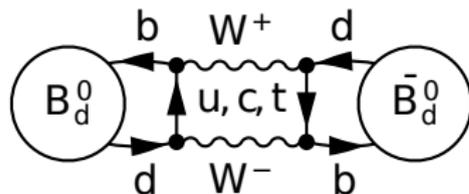
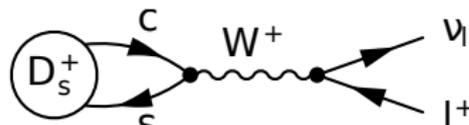
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- charm contribution to HVP
- Semi-leptonic decays
  - ⇒ Access  $|V_{cd}|, |V_{cs}|$



**DWFs provide a method to simulate (approximately) chiral fermions on the lattice at the expense of a fifth dimension**

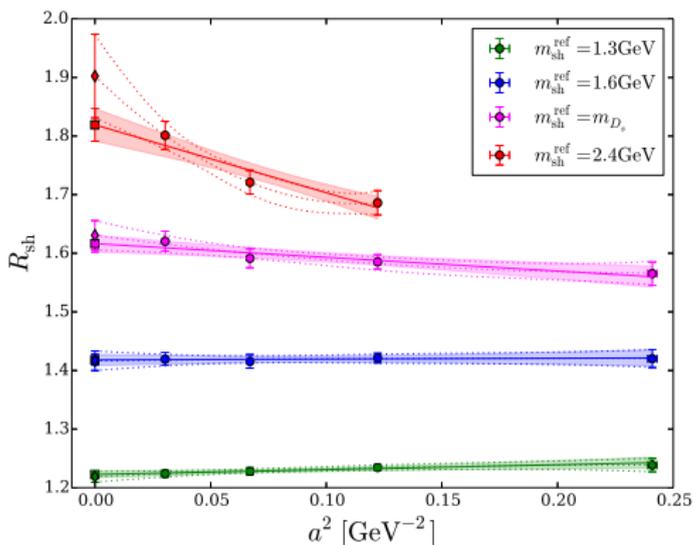
- ⇒ Automatic  $\mathcal{O}(a)$  improvement
- ⇒ No operator mixing:
  - ⇒ easier renormalisation
- ⇒ Additional parameters  $L_5$  and  $M_5$ :
  - $M_5$  only affects UV behaviour:
  - ⇒ use to reduce discretisation effects?

**BUT:**

- More expensive due to fifth dimension

# Heavy Domain Wall Fermion Set-up

- Scan parameter space for good action for heavy quarks.
- Keep light and strange unitary ( $M_5 = 1.8$ )



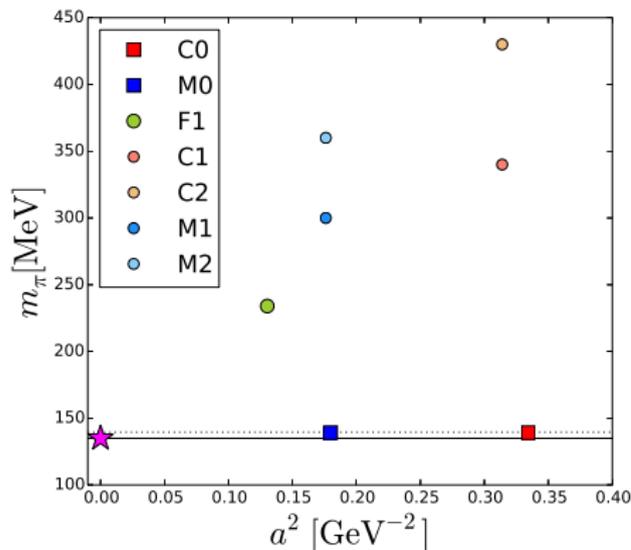
⇒ **Mixed action**

## Quenched Pilot Study:

JHEP **05** (2015) 072

JHEP **04** (2016) 037

- Good chiral properties and  $O(a)$ -improvement
- Find flat CL for modification of DW param's for charm:  
 $M_5 = 1.6$  and  $am_h \lesssim 0.4$



	$L^3 \times T/a^4$	$a^{-1}/\text{GeV}$	$m_\pi/\text{MeV}$
<b>C0</b>	<b><math>48^3 \times 96</math></b>	<b>1.73</b>	<b>139</b>
C1	$24^3 \times 64$	1.78	340
C2	$24^3 \times 64$	1.78	430
<b>M0</b>	<b><math>64^3 \times 128</math></b>	<b>2.36</b>	<b>139</b>
M1	$32^3 \times 64$	2.38	300
M2	$32^3 \times 64$	2.38	360
<b>F1</b>	<b><math>48^3 \times 96</math></b>	<b>2.77</b>	<b>230</b>

(**C0** + **M0**: arXiv:1411.7017)

(**F1**: arXiv:1701.02644)

- $N_f = 2 + 1$  Domain Wall Fermions
- **2 ensembles with physical pion masses**
- 3 Lattice spacings

- $\mathbb{Z}_2$ -Wall sources
- placed on many source planes
- binned into one effective measurements per config

Name	hits/conf	confs	total
C0	48	88	4224
C1	32	100	3200
C2	32	101	3232
M0	32	80	2560
M1	32	83	2656
M2	16	77	1232
F1	48	82	3936

- $\mathbb{Z}_2$ -Wall sources
- placed on many source planes
- binned into one effective measurements per config
- strange quark mass slightly mistuned on some ensembles

Name	$am_s^{\text{phys}}$	$am_s^{\text{sim}}$
C0	0.03580(16)	0.0362
C1	0.03224(18)	
C2	0.03224(18)	
M0	0.02539(17)	0.02661
M1	0.02477(18)	
M2	0.02477(18)	
F1	0.02132(17)	0.02144

## Experimental decay rate

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## Leptonic decays

$$\mathcal{B}(D_{(s)} \rightarrow l\bar{\nu}_l) = |V_{cq}|^2 \mathcal{K}(m_l, m_{D_q}) f_{D_q}^2, \quad q = d, s$$

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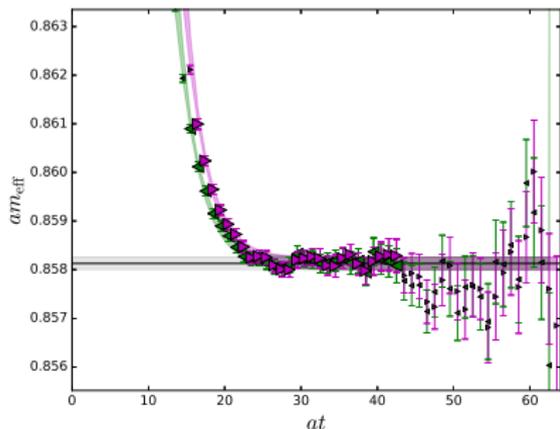
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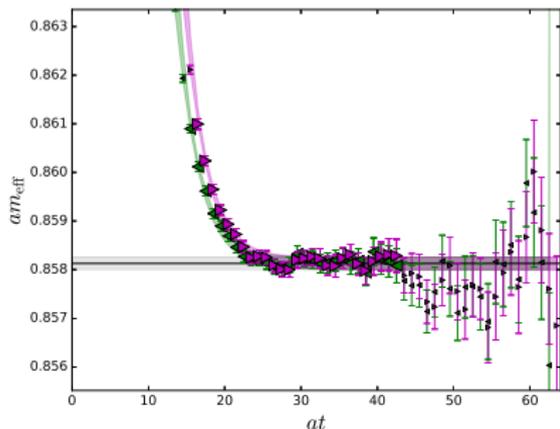
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## Lattice calculation

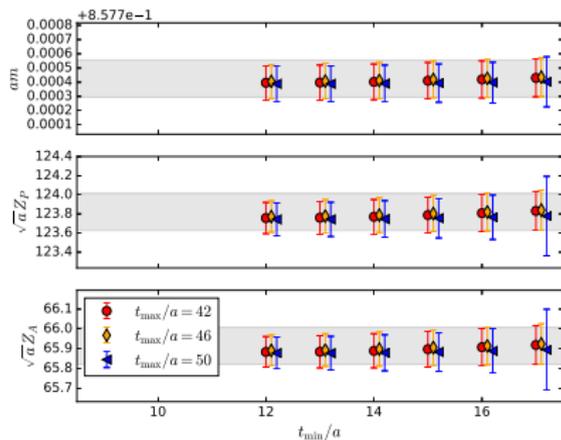
$$\mathcal{Z}_A \langle 0 | \bar{c} \gamma_4 \gamma_5 q | D_q(0) \rangle = f_{D_q} m_{D_q}, \quad q = d, s$$



Uncorrelated simultaneous  
two-exponential fit to  $\langle PP \rangle$  and  
 $\langle AP \rangle$  channels for  $D_S$  at  
 $am_h = 0.34$  on M0.

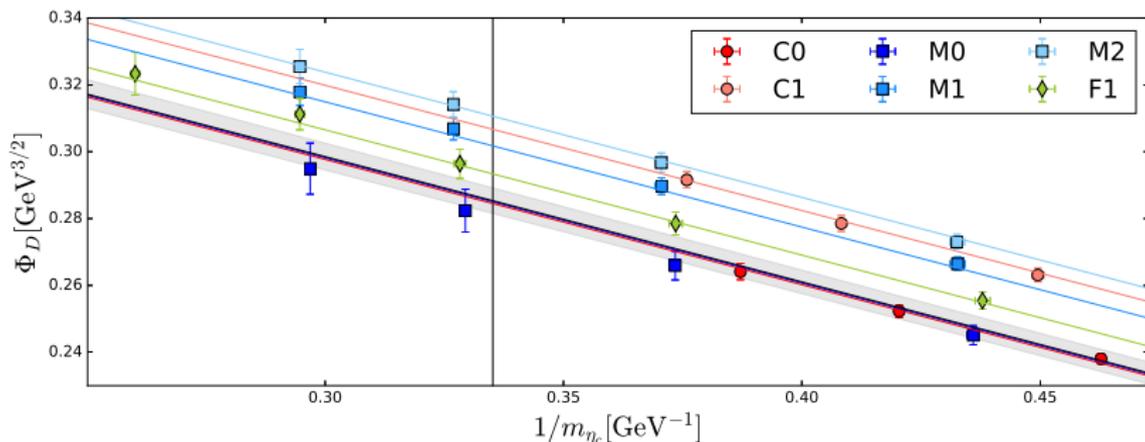


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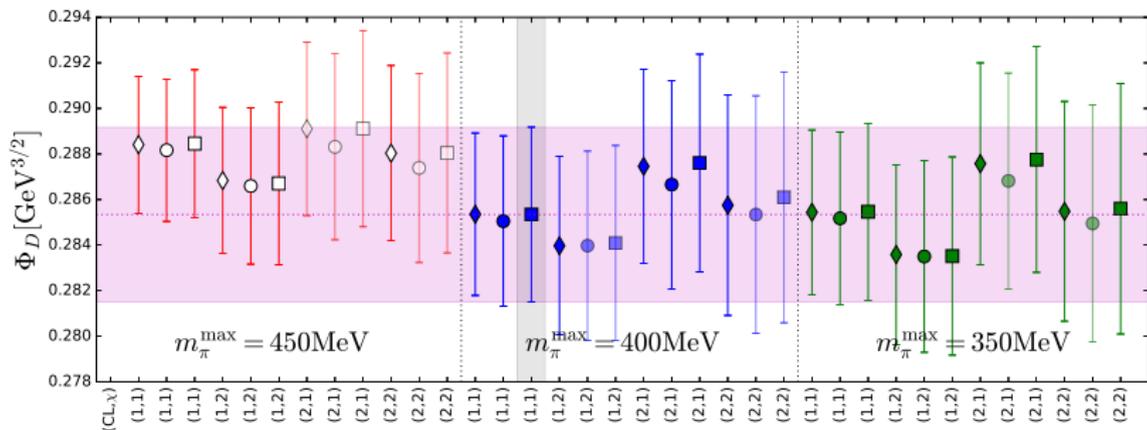


Variation of  $t_{\min}$  and  $t_{\max}$ .

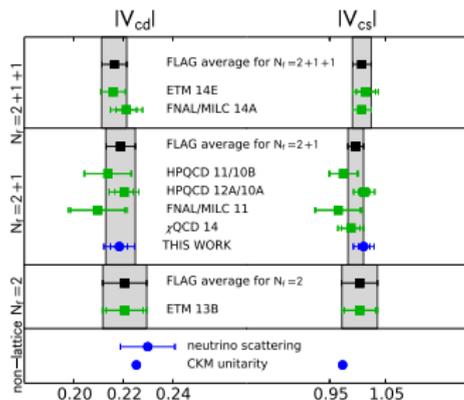
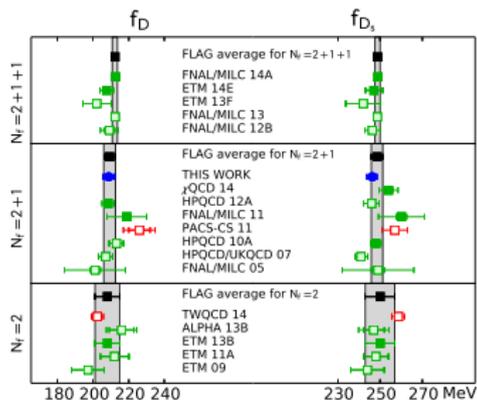
$$\mathcal{O}(a, m_l, m_h) = \mathcal{O}(0, m_l^{\text{phys}}, m_h^{\text{phys}}) + \left( C_{CL}^0 + C_{CL}^1 \Delta m_{P_h}^{-1} \right) a^2 \\ + C_{\chi}^0 \Delta m_{\pi}^2 + C_{P_h}^0 \Delta m_{P_h}^{-1}$$



$$\Phi_P = f_P \sqrt{m_P}$$



- Mixed action renormalisation:  $\left| 1 - \frac{\mathcal{Z}_A(M_5=1.8)}{\mathcal{Z}_A(M_5=1.6)} \right| < 0.5\%$
- Variations of fit ansatz and pion cuts to determine systematics
- Different ways to fix charm ( $D(\diamond)$ ,  $D_s(\circ)$ , and  $\eta_c(\square)$ )



Plots inspired by FLAG III [arXiv:1607.00299]

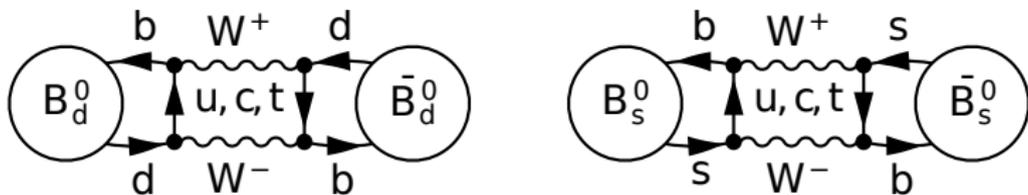
$$f_D = 208.7(2.8)_{\text{stat}} \left( {}_{-1.8}^{+2.1} \right)_{\text{sys}} \text{ MeV}$$

$$|V_{cd}| = 0.2185(50)_{\text{exp}} \left( {}_{-37}^{+35} \right)_{\text{lat}}$$

$$f_{D_s} = 246.4(1.3)_{\text{stat}} \left( {}_{-1.9}^{+1.3} \right)_{\text{sys}} \text{ MeV}$$

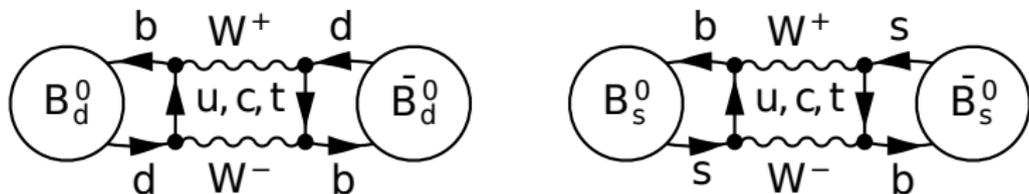
$$|V_{cs}| = 1.011(16)_{\text{exp}} \left( {}_{-9}^{+4} \right)_{\text{lat}}$$

# Neutral Meson Mixing



$$\Delta m_q = |V_{tq}^* V_{tb}|^2 \times \mathcal{K} \times \mathcal{M}_q$$

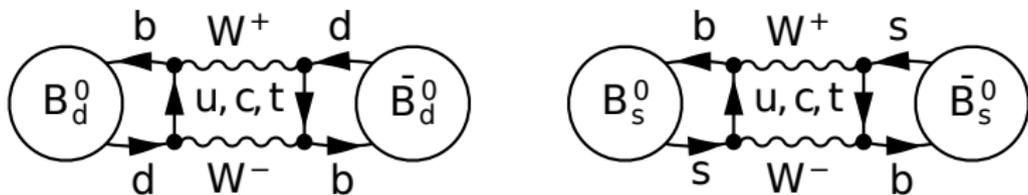
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$$\mathcal{M}_q = \langle \bar{B}_q^0 | [\bar{b}\gamma^\mu(1 - \gamma_5)q] [\bar{b}\gamma^\mu(1 - \gamma_5)q] | B_q^0 \rangle$$

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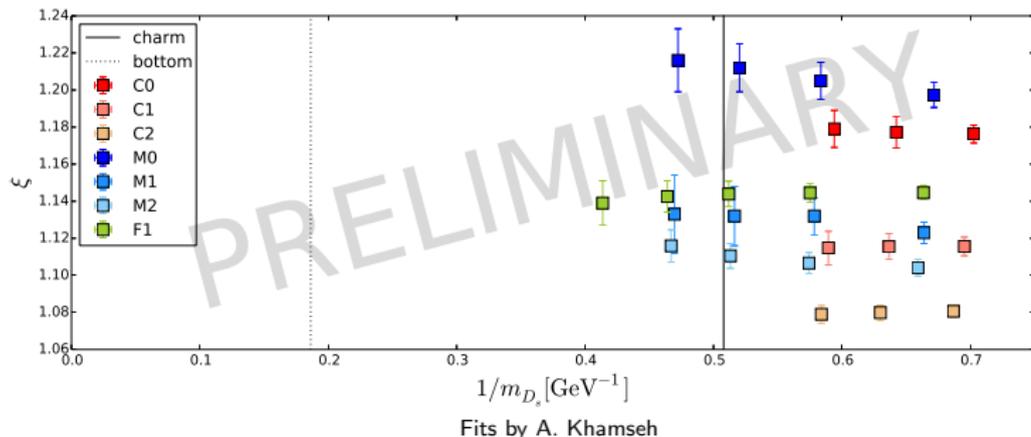


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# Ratio of Bag parameters $\xi$ (1511.09328)

$$B_P = \frac{\langle P^0 | O_{VV+AA} | \bar{P}^0 \rangle}{\frac{8}{3} f_P^2 m_P^2}, \quad \xi = \frac{f_{hs} \sqrt{B_{hs}}}{f_{hl} \sqrt{B_{hl}}} \propto \left| \frac{V_{td}}{V_{ts}} \right|$$

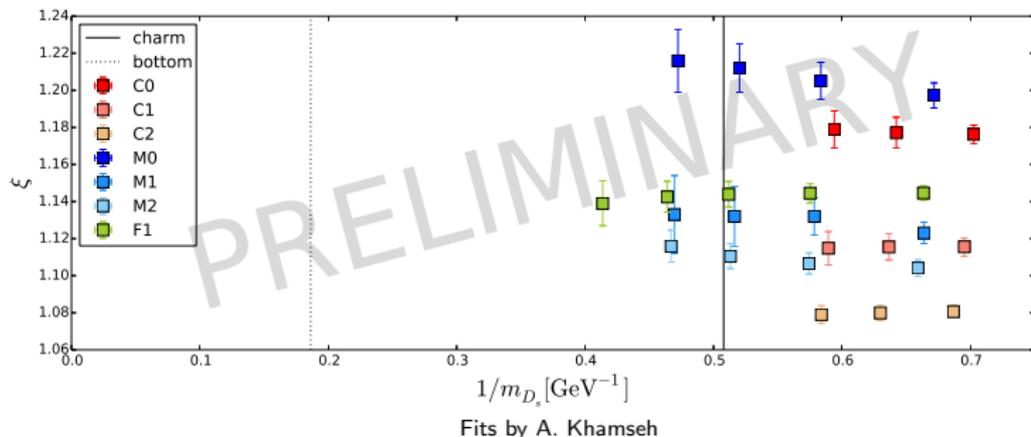


## STATUS:

- Data at charm: percent level precision
- Mild heavy mass dependence
- More data on disk + many src/snk separations

# Ratio of Bag parameters $\xi$ (1511.09328)

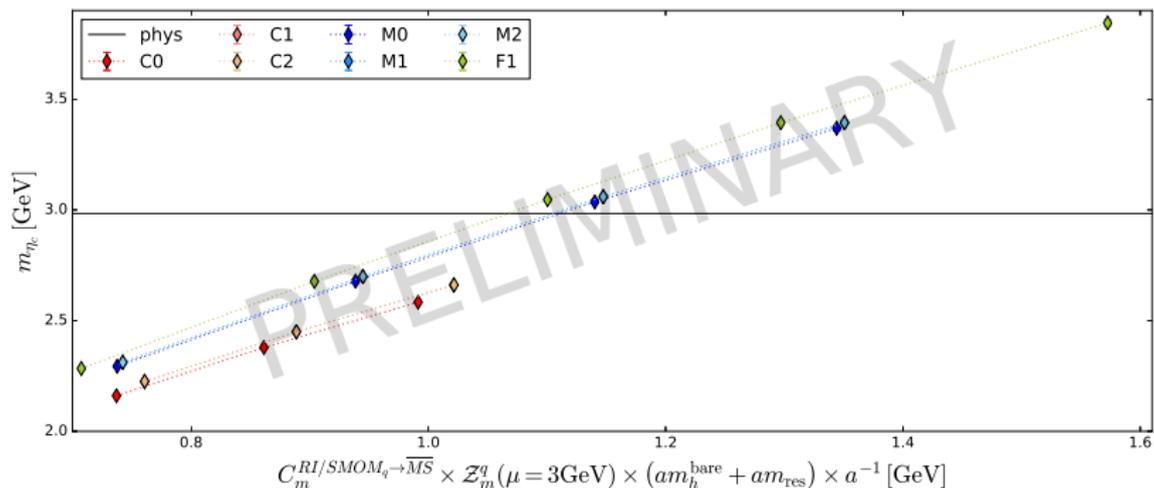
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## PLAN:

- Renormalisation of mixed action for Bag parameters.
- Analyse further data and add to global fit
- Extrapolate to  $B$

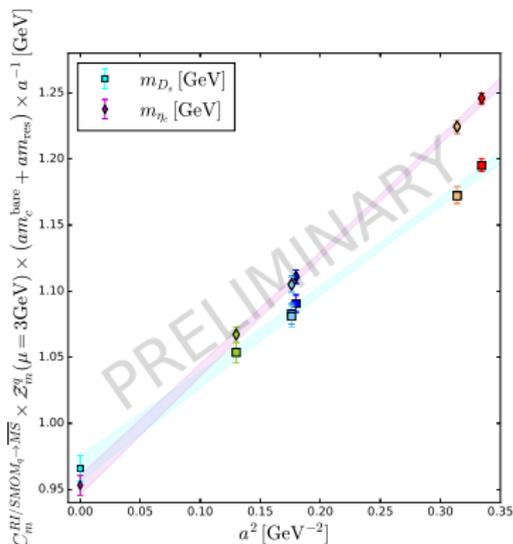
# Charm quark mass



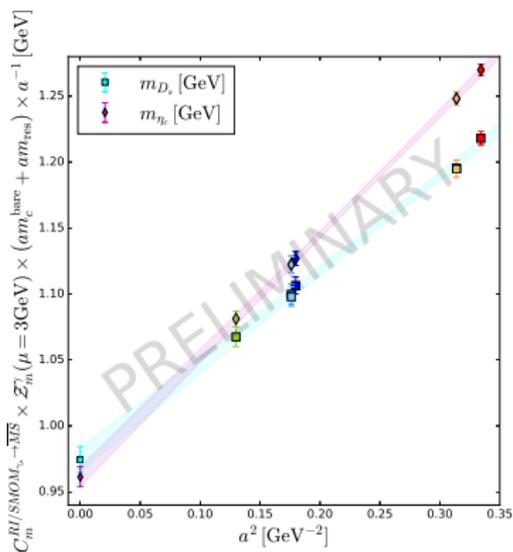
Definitions and coefficients from 1208.4412 and 1411.7017

- ⇒ Determine  $m_c(a)$  from  $m_{\eta_c}$  (or  $m_{D_s}$ )
- ⇒ Renormalisation of mixed action still ongoing
- ⇒ Currently use  $Z_m(M_5 = 1.8)$

$\not{d}$  – scheme



$\gamma\mu$  – scheme



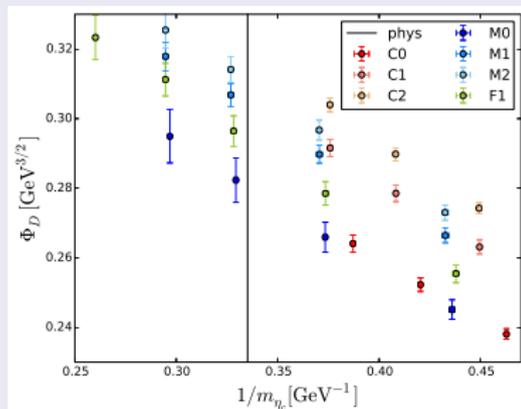
- ⇒ Renormalisation of mixed action still ongoing
- ⇒ No chiral extrapolation done yet
- ⇒ Good agreement between two schemes

# Current Limitations

## Problems

- $am_h^{\max} < am_c^{\text{phys}}$  on Coarse

## heavy quark reach

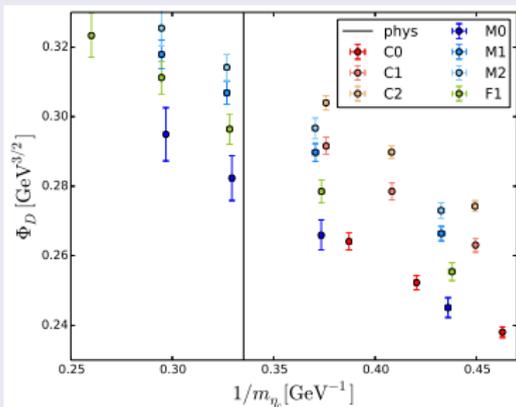


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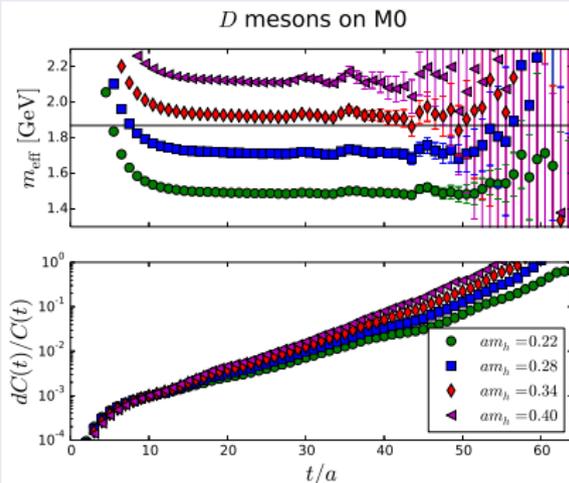
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## Noise growth

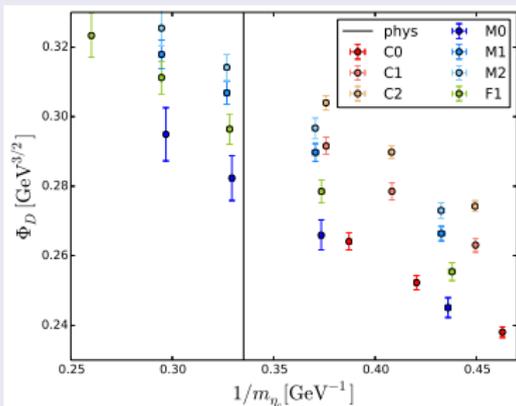


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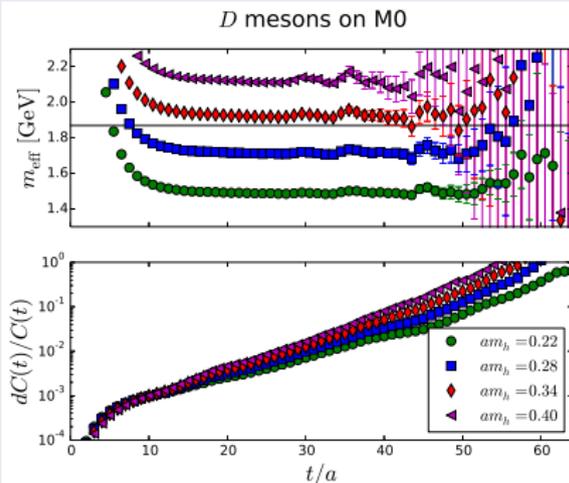
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## Noise growth



## Solutions?

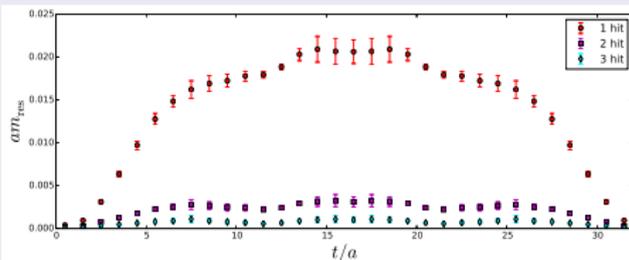
- Stout Smearred charm
- Gaussian Smearing for source + sink

## Stout Smearing

Found sweet-spot for

- $M_5 = 1.0$
- 3 hits of stout smearing
- Standard Stout parameter  $\rho = 0.1$

## hit comparison

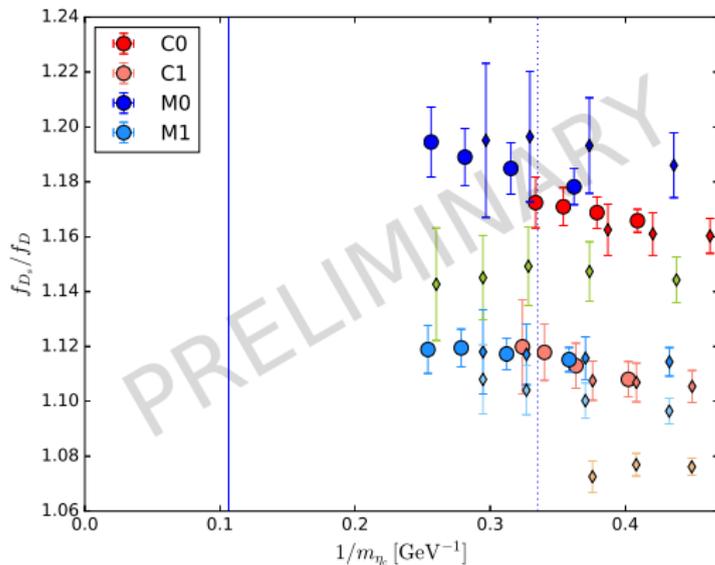


Fits by A. Khamseh

## Conclusion

- ⇒ Gaussian smearing of light and strange
- ⇒ Stout smearing of heavy quarks
- ⇒  $am_h \lesssim 0.7$

## First Data (Limited statistic)

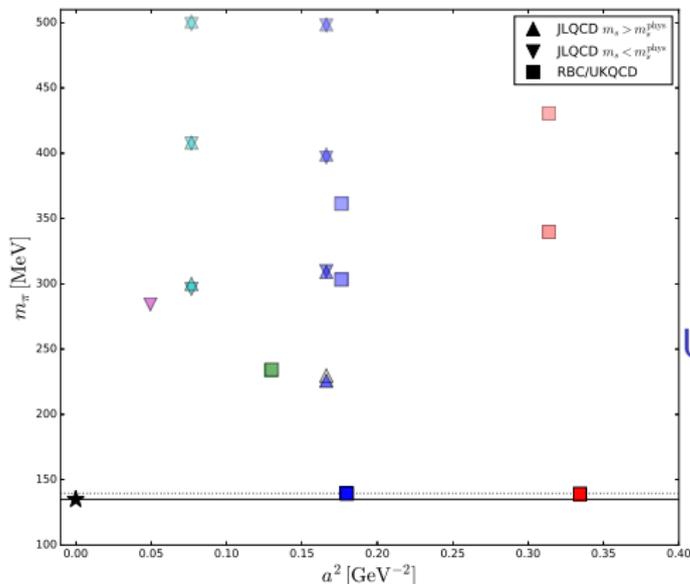


## Comments

- **Reach physical charm quark mass on all ensembles**
- More data soon
- Renormalisation in progress
- **Physics program:** decay constants, semi-leptonics, bag parameters,  $\xi$ , HVP
- Push towards  $b$ .

# Combined Data from JLQCD and RBC/UKQCD

in collaboration with Guido Cossu, Brendan Fahy, Shoji Hashimoto



Both:  $N_f = 2 + 1$  DWF

3+3 Lattice Spacings

JLQCD: (triangles)

Fine lattices:

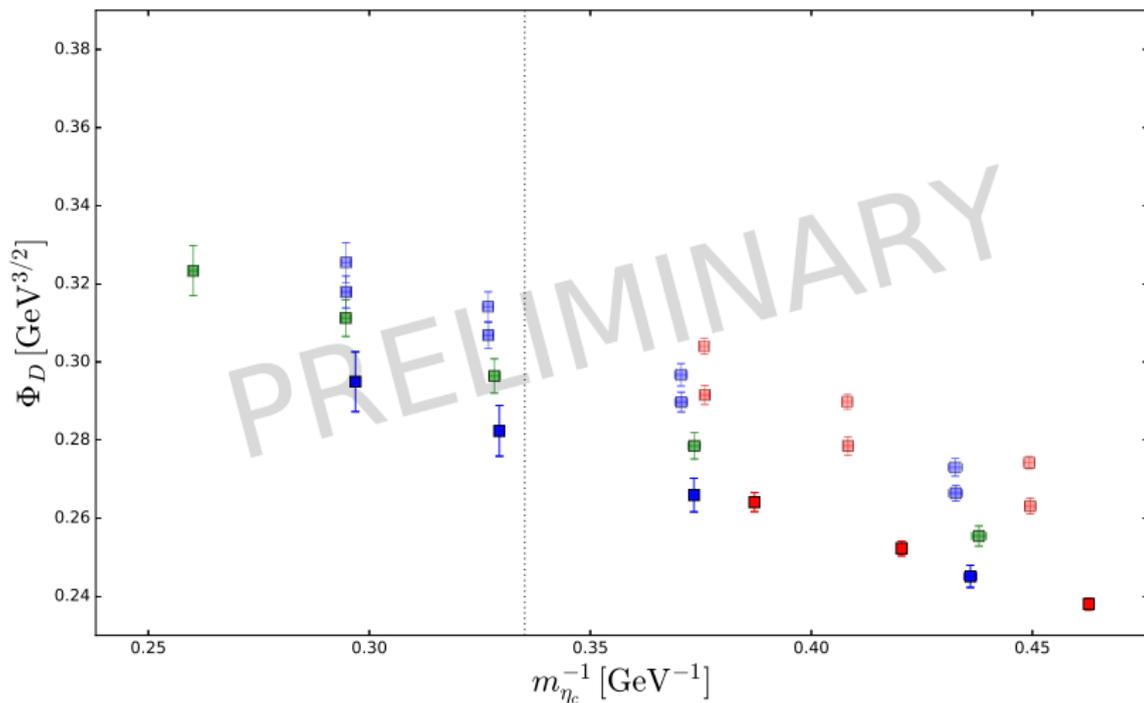
$a^{-1} = 2.4 - 4.5$  GeV

UKQCD: (squares)

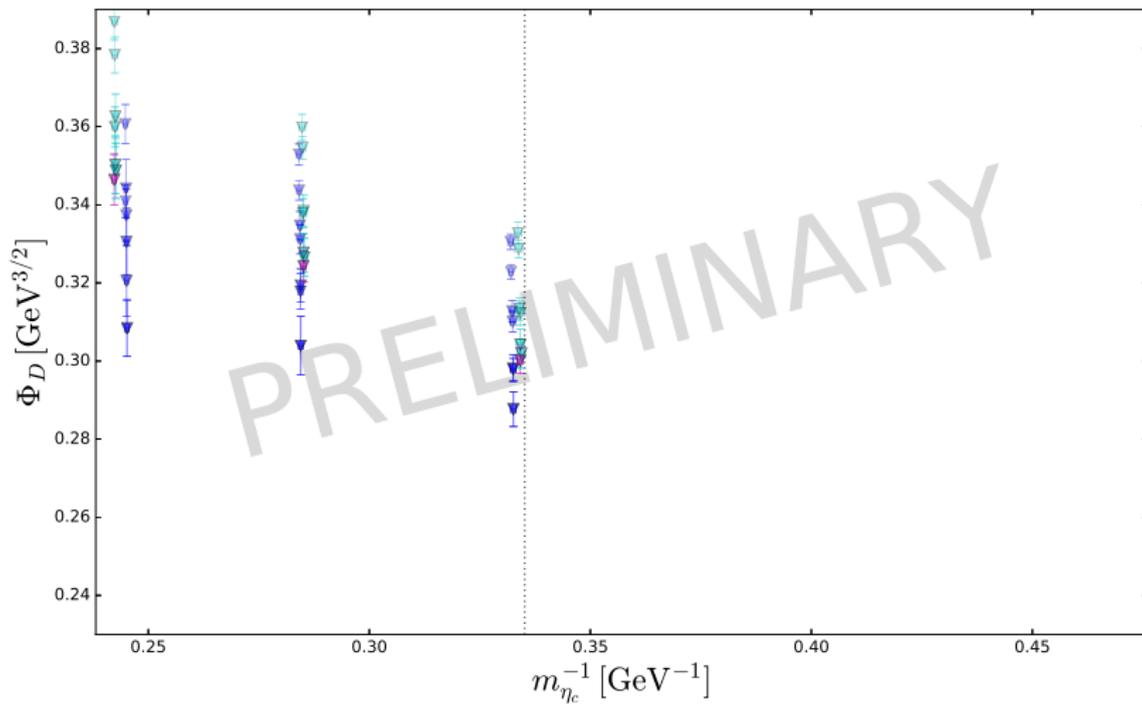
Physical Pion masses

Combine data sets for well controlled chiral and continuum limit.

## RBC/UKQCD data set



## JLQCD data set





- Data on both sides of charm
- Global fit with universality constraint is ongoing.  
First try:

$$\begin{aligned}\mathcal{O}_i(a, m_l, m_h) &= \mathcal{O}(0, m_l^{\text{phys}}, m_h^{\text{phys}}) \\ &\quad + C_X \Delta m_\pi^2 + C_H \Delta m_H^{-1} \\ &\quad + \left( C_{CL}^{UK} \delta_{UK,i} + C_{CL}^{KEK} \delta_{KEK,i} \right) a^2 \\ &\quad + C_K \Delta(2 m_K^2 - m_\pi^2)\end{aligned}$$

- Can include data with  $am_s \neq am_s^{\text{phys}}$  without corrections
- Include new smeared RBC/UKQCD data

## DONE

### Established HDWF

- DWF as heavy quark discretisation:  
 $M_5 = 1.6$ ,  $am_h \lesssim 0.4$
- JHEP **05** (2015) 072  
JHEP **04** (2016) 037
- Good Continuum Scaling
- $N_f = 2 + 1$ :  $f_D$ ,  $f_{D_s}$  with competitive precision (arXiv:1701.02644)
- 3 lattice spacings, 2 physical pion mass ensembles.

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- DWF as heavy quark discretisation:  
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### Data in Production

- beyond  $c$
- semi-leptonics
- NPR for smeared kernel

# ADDITIONAL SLIDES

obs	val	stat	sys	fit	FV	h.o. CL	$m_u \neq m_d$	ren	$m_s$
		$\times 10^4$							
$\Phi_D [\text{GeV}^{3/2}]$	0.2853	38	$^{+29}_{-24}$	$^{+24}_{-18}$	10	-	4.7	11	-
$\Phi_{D_s} [\text{GeV}^{3/2}]$	0.3457	26	$^{+18}_{-26}$	$^{+3}_{-19}$	6	7	4.4	14	0.9
$f_{D_s}/f_D$	1.1667	77	$^{+57}_{-43}$	$^{+44}_{-23}$	35	-	8	-	3

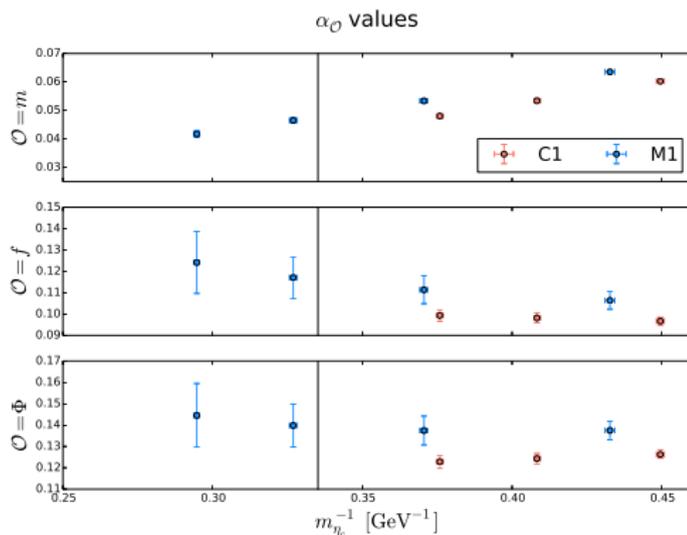
Systematic error budget for decay constants (arXiv:1701.02644)

# Strange Quark Mass Correction

⇒ Parameterise mistuning in terms of dimensionless  $\alpha$ :

$$\mathcal{O}^{\text{phys}} = \mathcal{O}^{\text{uni}} \left( 1 + \alpha_{\mathcal{O}} \frac{m_s^{\text{phys}} - m_s^{\text{uni}}}{m_s^{\text{phys}}} \right)$$

- Calculate 2 values of  $m_s$  on C1 and M1

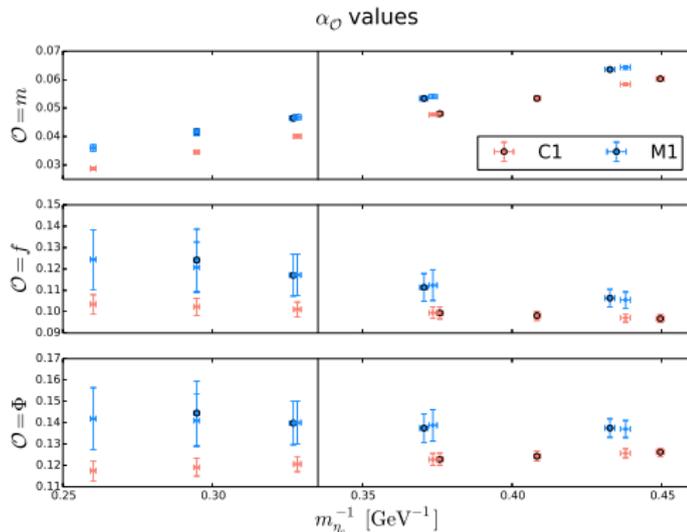


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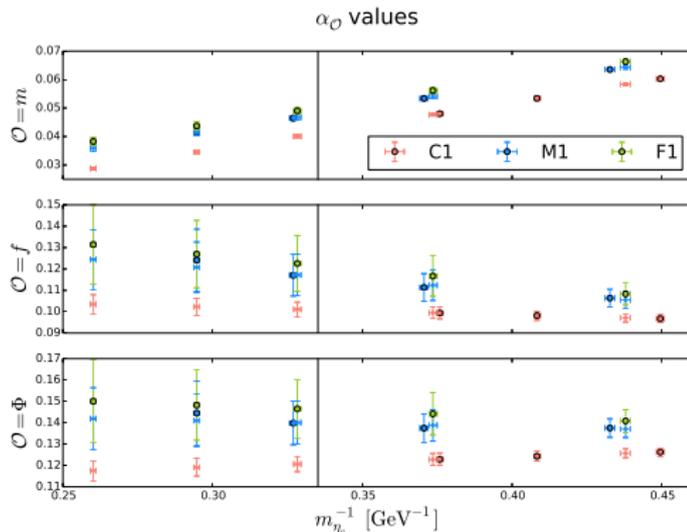


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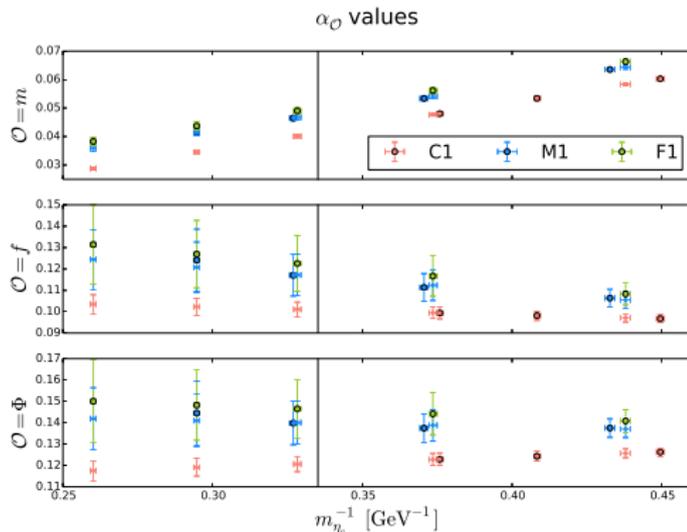


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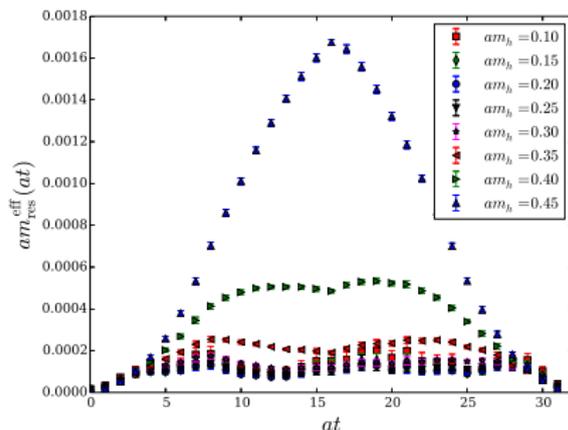
- Calculate 2 values of  $m_s$  on C1 and M1
  - Extrapolate to F1 masses
  - Extrapolate to F1 lattice spacing
- ⇒ Small correction



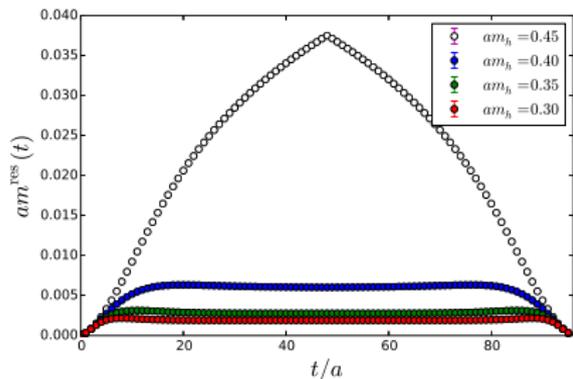
What happens at large input quark masses? Check residual mass

$$C_{m_{\text{res}}}(t) = \frac{\left\langle \sum_{\mathbf{x}} J_{5q}^a(\mathbf{x}) P(0) \right\rangle}{\left\langle \sum_{\mathbf{x}} P(\mathbf{x}) P(0) \right\rangle}$$

- Expect plateau
- For  $M_5 = 1.6$  mechanism seems to break down for  $am_h \gtrsim 0.4$
- No longer simulating (approx.) chiral QCD

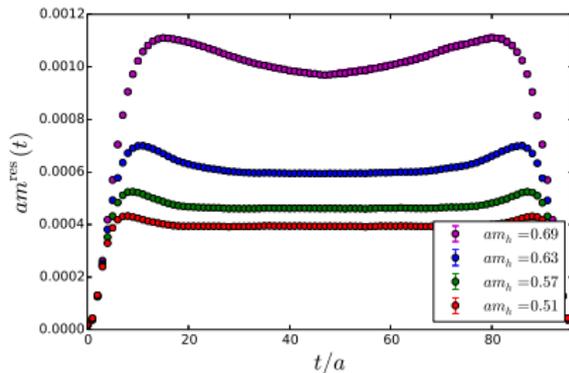


No smearing



$$am_h^{\text{bare}} \lesssim 0.4$$

Stout smeared



$$am_h^{\text{bare}} \lesssim 0.7$$