Universality in conformal and near-conformal systems

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Lattice 2017, June 23, 2017

In collaboration with
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Universality

The concept of Universality is the driving principle of LQCD:
It ensures that lattice simulations study continuum physics even with different lattice actions.

We expect universality, i.e. universal critical behavior, in systems

- *at criticality (basin of attraction of a FP)*
- *with identical field content & dimension*
- *identical symmetries*

Staggered fermions

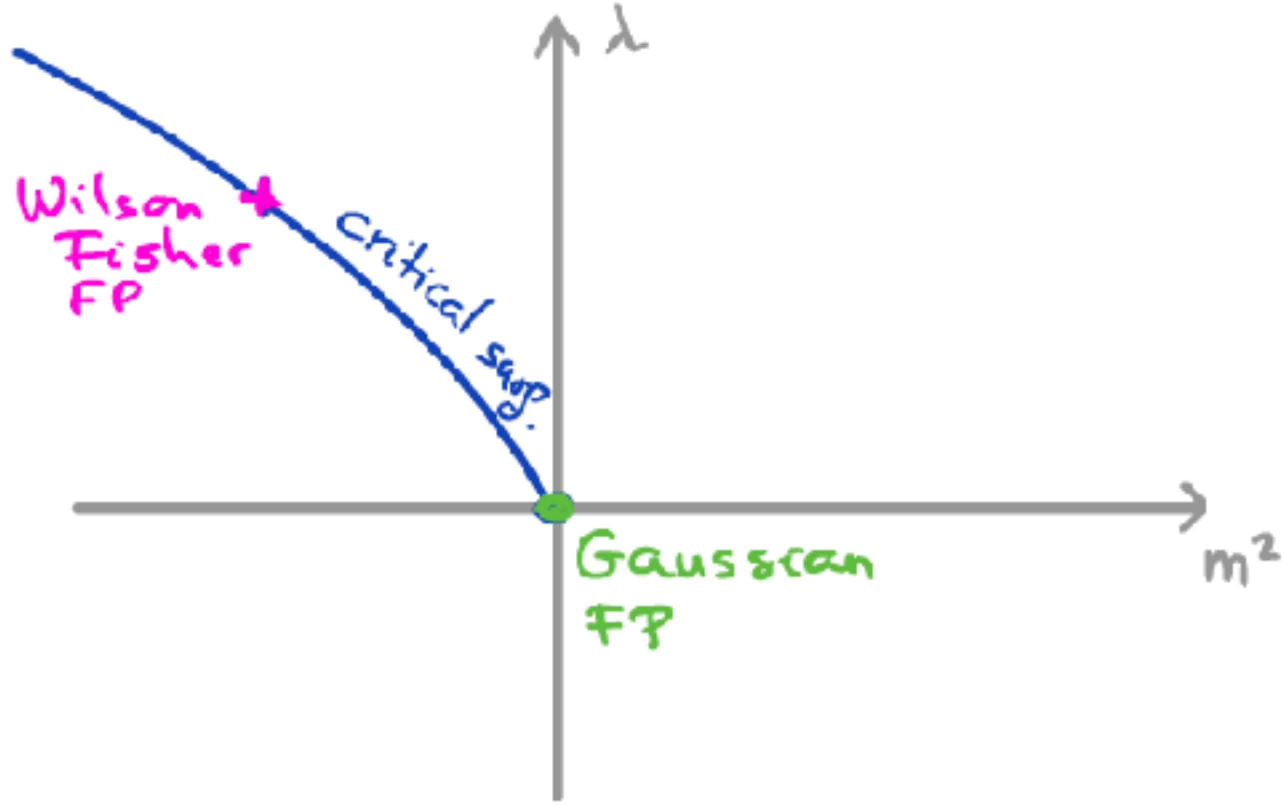
Staggered fermions do not have continuum
$SU(N_f)xSU(N_f)$ flavor symmetry

- Not a problem in QCD-like systems
  The taste breaking /chiral breaking terms vanish as $g^2 \rightarrow 0$
  At $g^2 = 0$ fixed point continuum chiral symmetry is restored ✔
  (This is quite non-trivial - S. Sharpe in "Good, bad, or ugly?")

- Conformal systems are different
  At *the conformal IRFP $g^2 \neq 0$ and universality could be violated*
  —> *different physics ?*

Universality in 3D spin models

Simple example: 3D O(n) model $$V = \frac{1}{2} m^2 \phi^2 + \lambda (\phi^2)^2$$



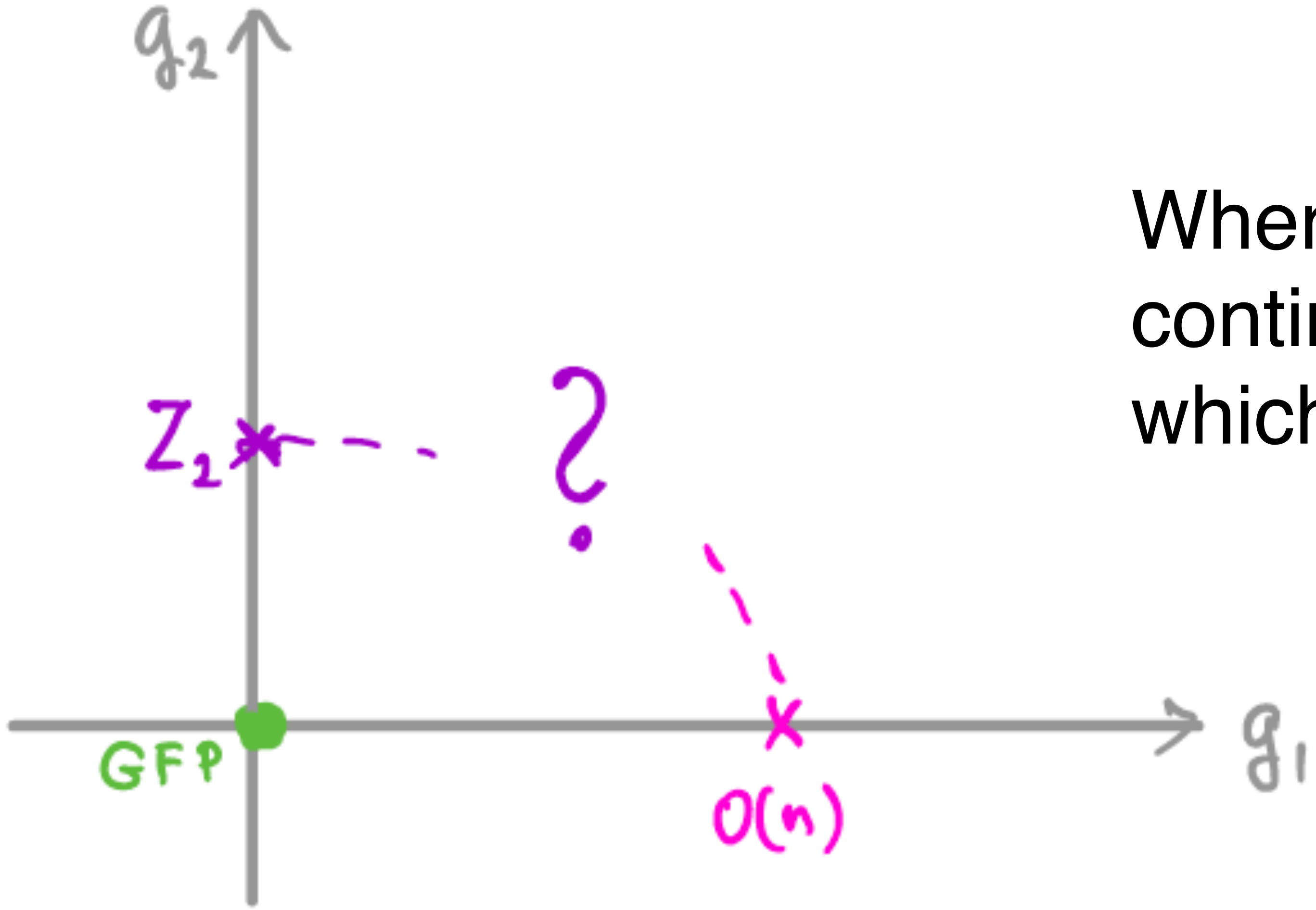
Perturbative FP is mean-field with 2 relevant directions

Wilson-Fisher FP has only one relevant direction (mass)

Example: 3dim O(N) scalar model

Break the symmetry : O(n)→Z_2 $$V = \frac{1}{2}m^2\phi^2 + g_1(\phi^2)^2 + g_2(\sum_\alpha \phi_\alpha^4)$$

On critical surface:



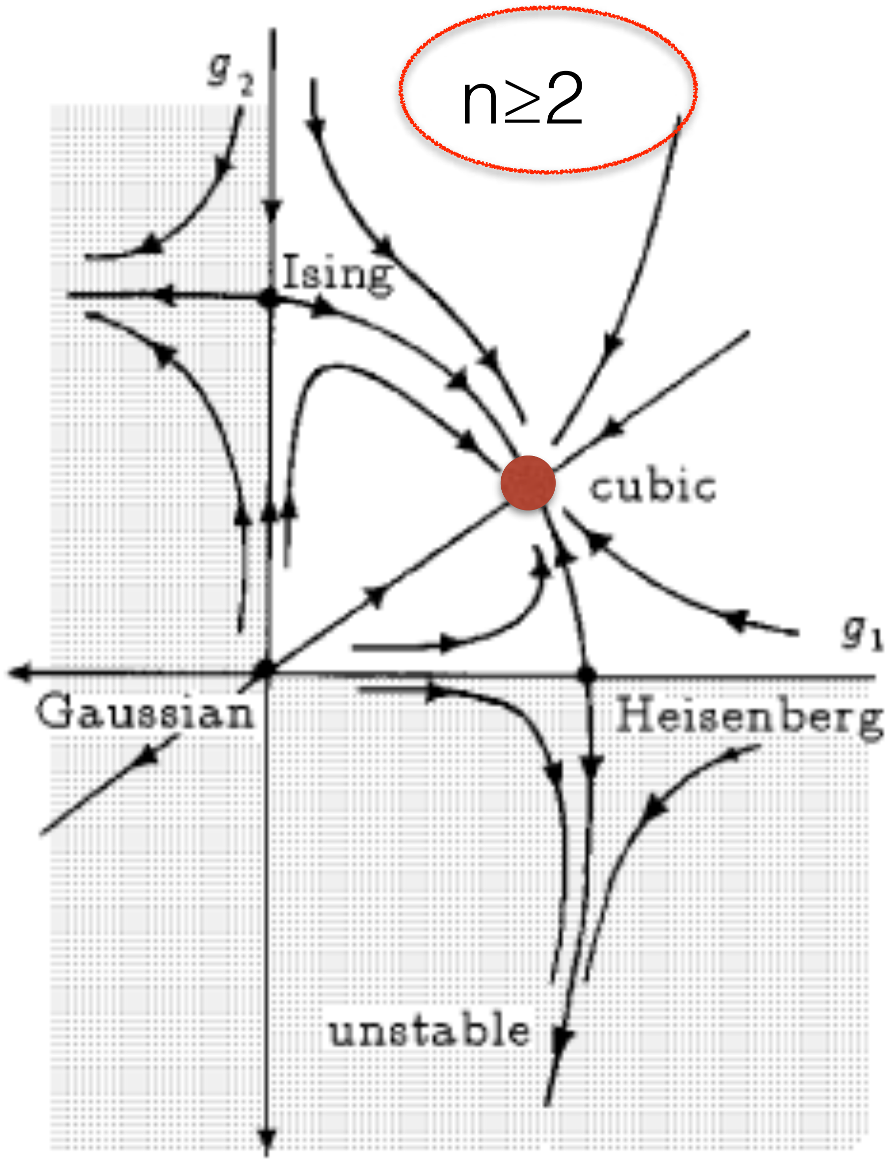
Where is the conformal continuum limit, i.e.
which fixed point is IR stable?

3dim O(n)→Z_2 scalar model

Kleinert et al studied a system with O(n)→Z_2 symmetry

$$V = \frac{1}{2} m^2 \phi^2 + g_1 (\phi^2)^2 + g_2 (\sum_\alpha \phi_\alpha^4)$$

Kleinert, Schulte-Frohlinde
cond-mat/9503038



The stable fixed point in neither the O(n), nor the Ising one, but a new FP!

Based on 5th order ε expansion

Testing universality in many-flavor systems

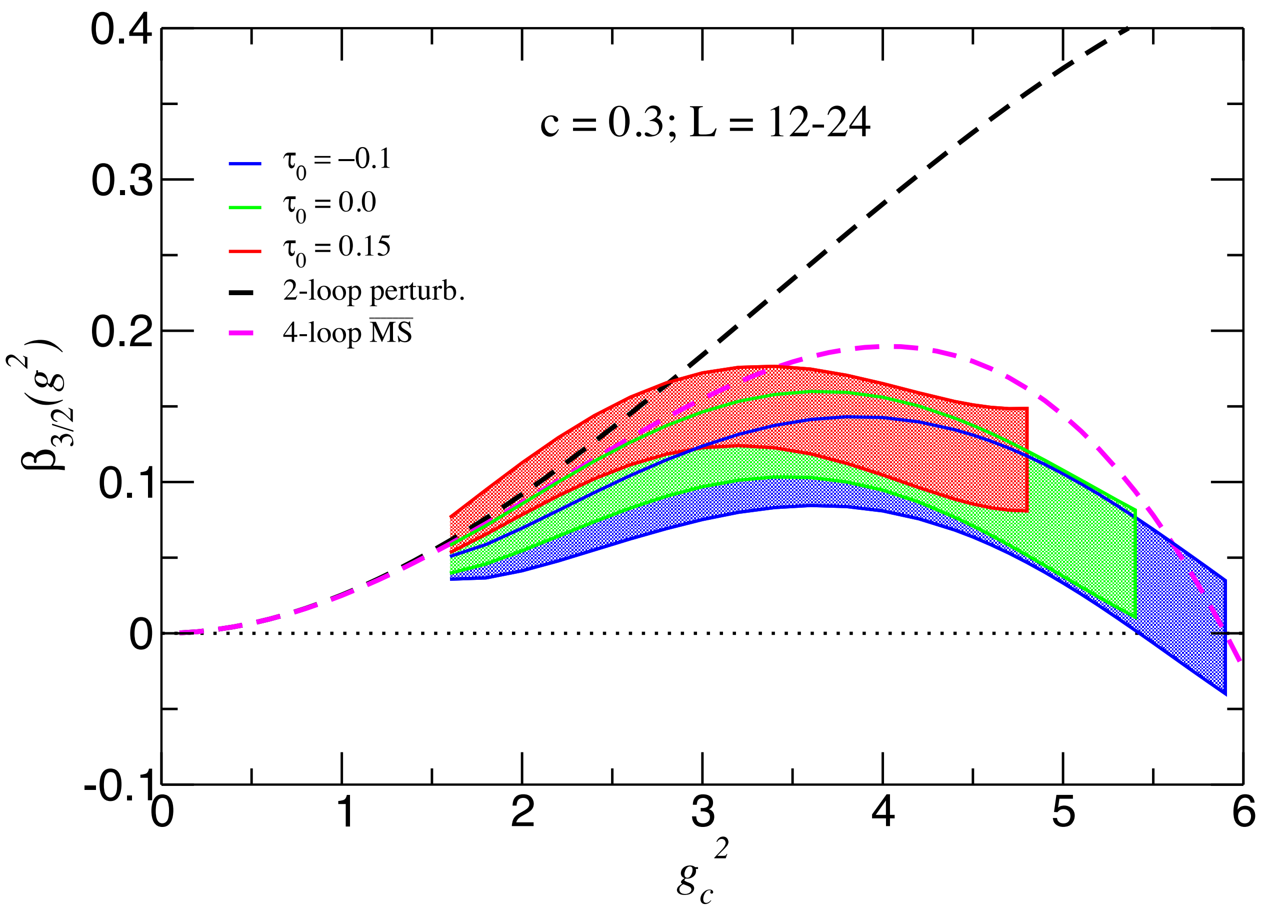
Compare scheme-independent quantities

- critical exponents
- spectrum
- renormalized step scaling functions (Wflow)
  (compare the $\beta(g^2)$ functions in the same scheme)

Motivation-I: SU(3) N_f=2 sextet (circa Lattice 2015):

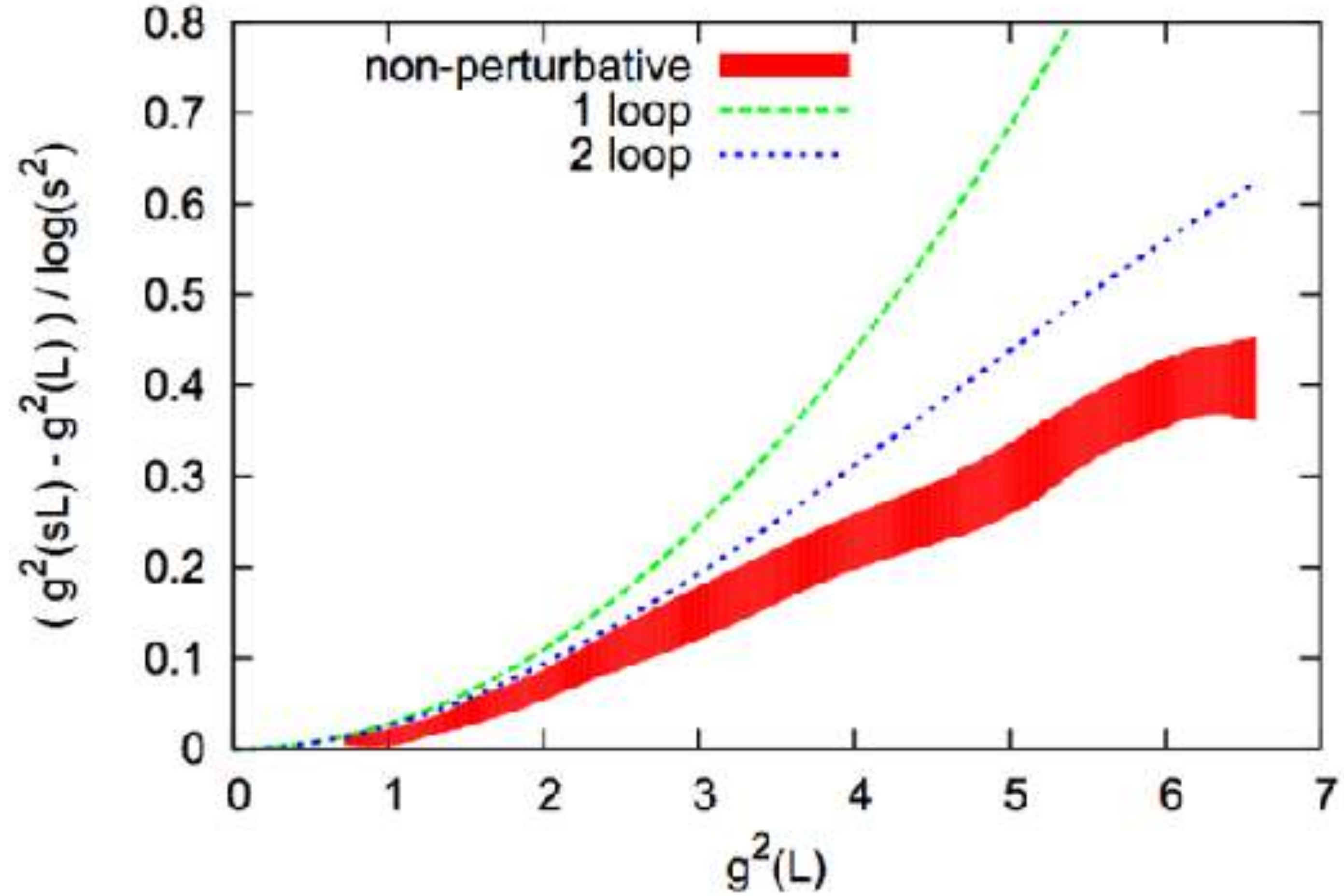
Wilson

(A.H., Y. Liu)



Staggered

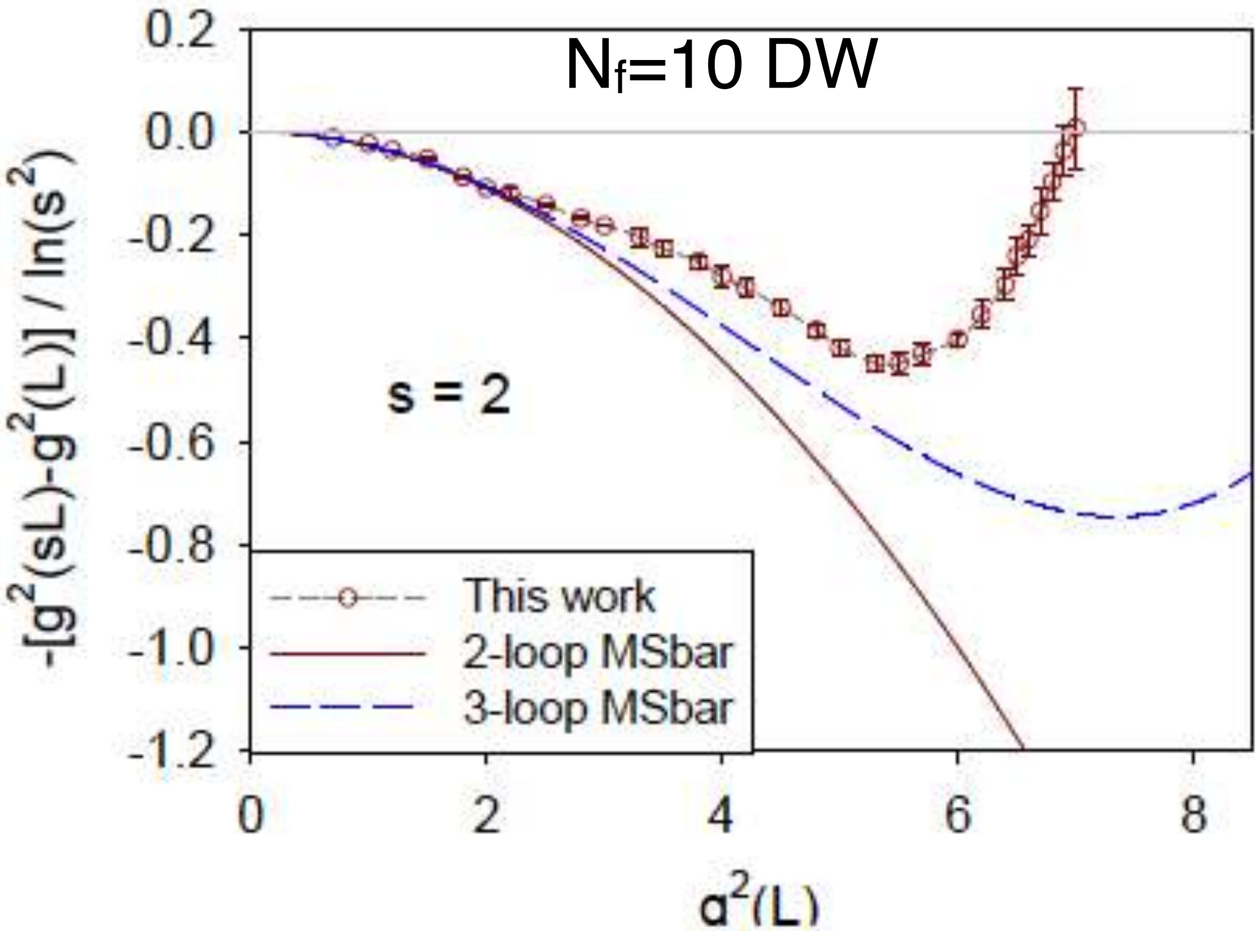
(LatHiggsColl)



- Wilson and staggered results are not consistent at large g^2

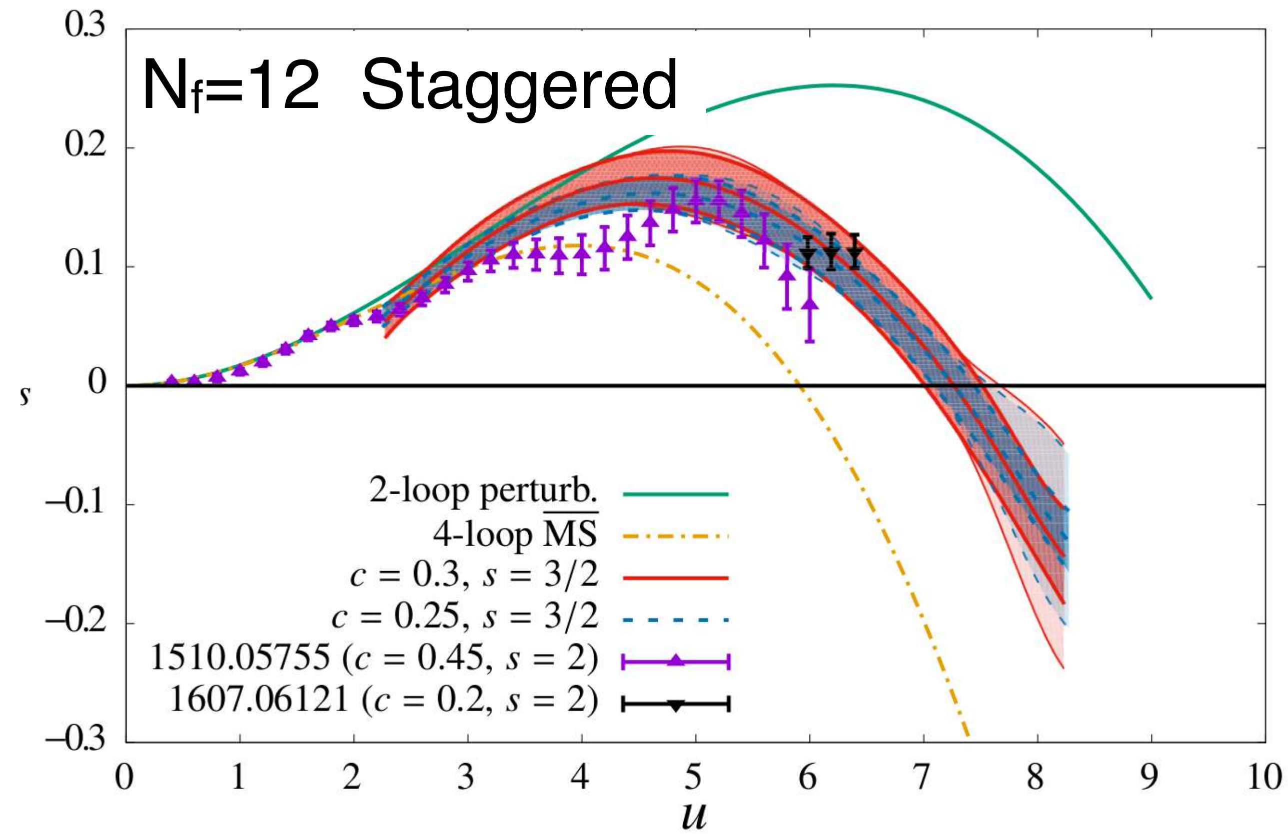
Motivation-II (circa Lattice 2016):

ArXiv:1603.08854 and PoS(Latice2016) 228



β function is below 4-loop prediction

A.H., D. Schaich, ArXiv:1610.10004



β function is between 2- and 4-loop prediction

10 < 12 : if $N_f=10$ is conformal it must have an IRFP that is at stronger coupling than $N_f=12$

Yet : $g^2(N_f=10) \approx 7.0 \le g^2(N_f=12) \approx 7.3(3)$

Lattice study of step scaling function with DWF

(AH, C. Rebbi, O. Witzel + GRID developers)

Simulations:

- *3-stout MobiusDW fermions*
- *Symanzik gauge action*
- *Wilson flow, clover and plaquette operators*
- *Volumes 8^4 - 24^4*
- *L_5 = 12 in most cases; 16 in some, 24 and 32 in others : needed to control residual mass* 😧

Could it work?

- *DWF have smaller lattice artifacts —> smaller volumes are OK*
- *Stout smearing is more efficient as there is no shift in the gauge coupling ~ smoother gauge fields*
- *Consider larger c values*

Lattice study of step scaling function with DWF

(AH, C. Rebbi, O. Witzel + GRID developers)

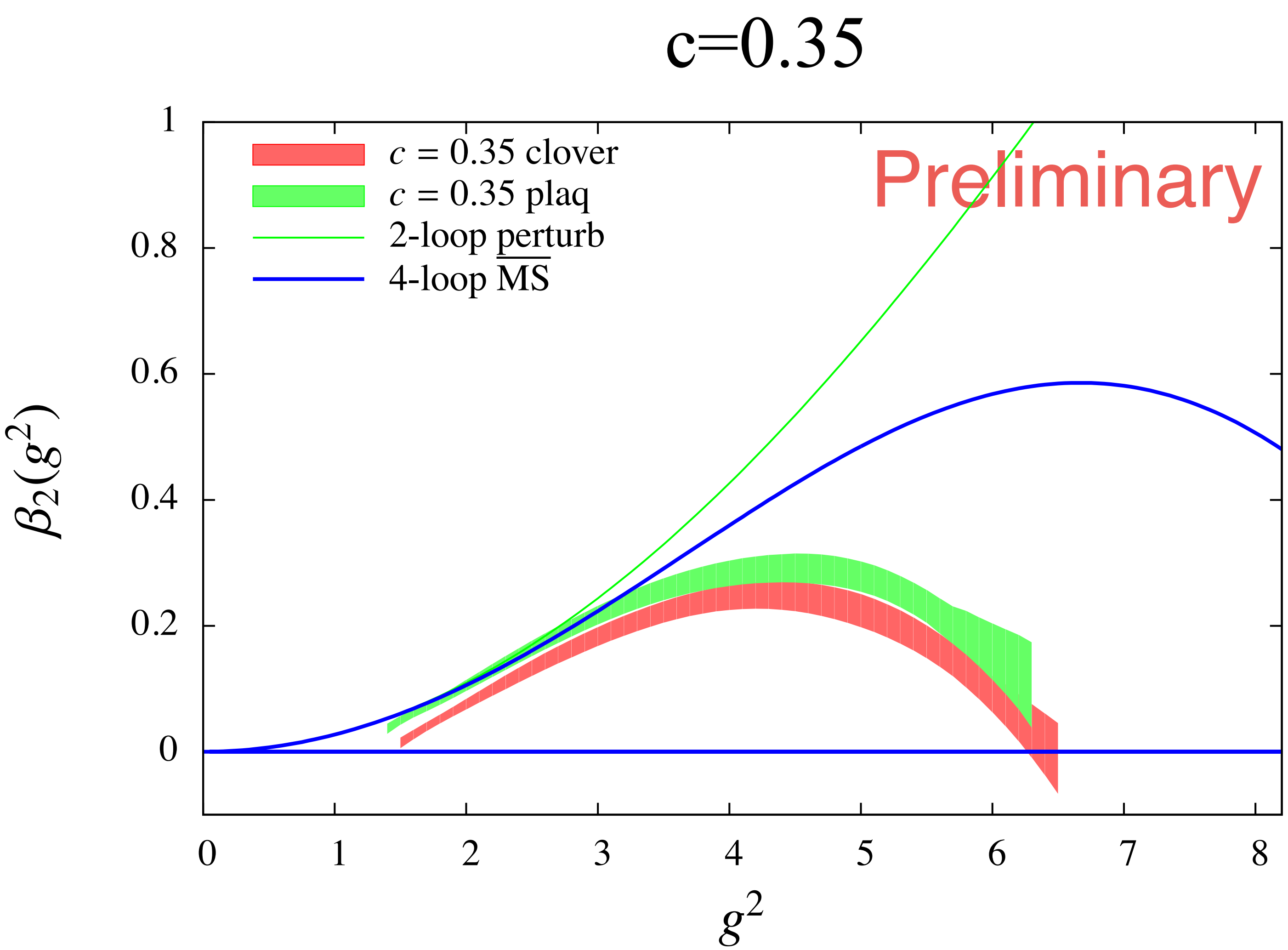
There is a new code base, GRID*, that is efficient and can do almost any action. We are testing it with

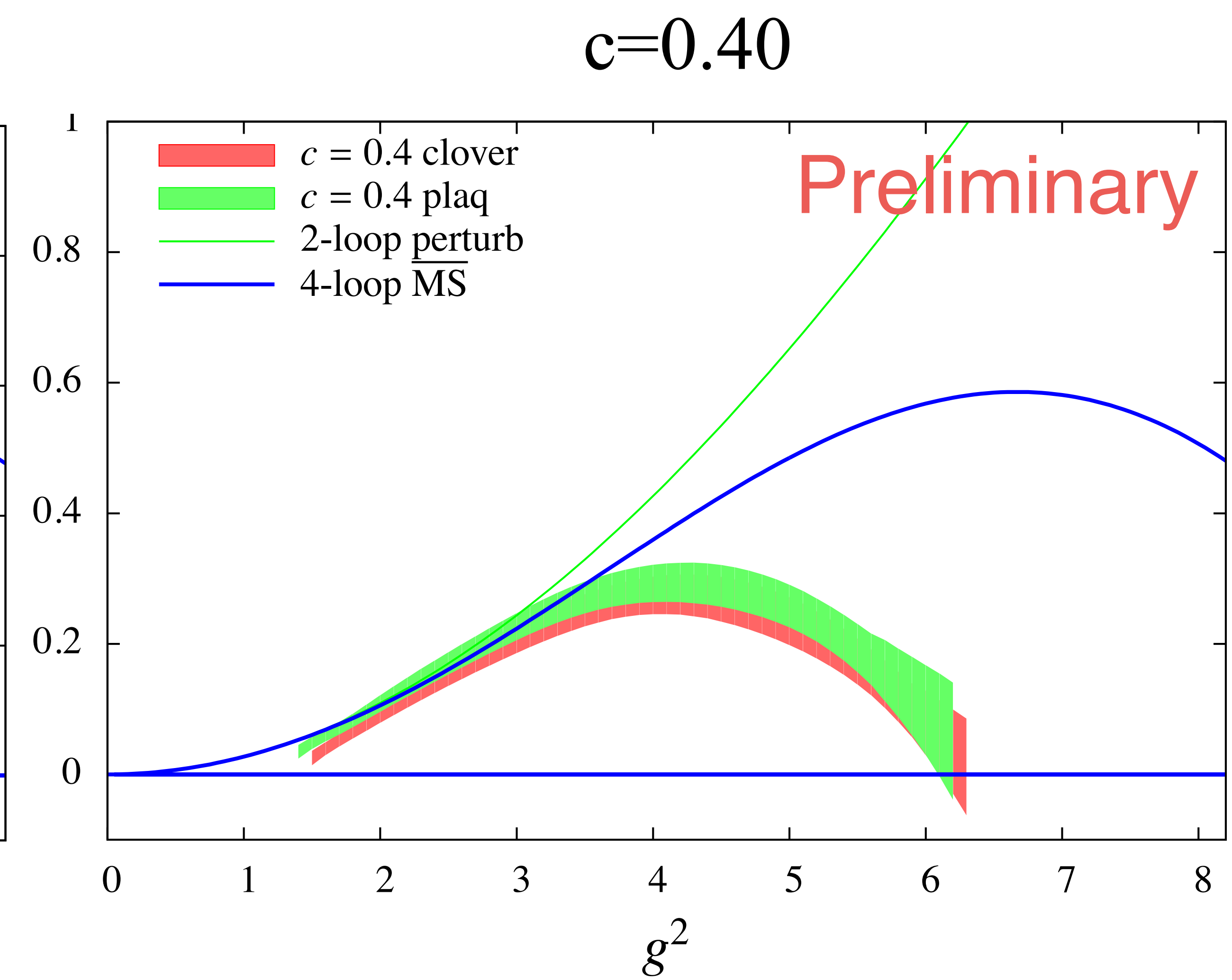
- $N_f = 10$ (to compare with T-W Chiu)
- $N_f = 12$ to compare it with staggered

*We greatly appreciate the opportunity to test the GRID code while still in development
*Also acknowledge the support of the Summit computing system at CU Boulder

N_f=10 step scaling function

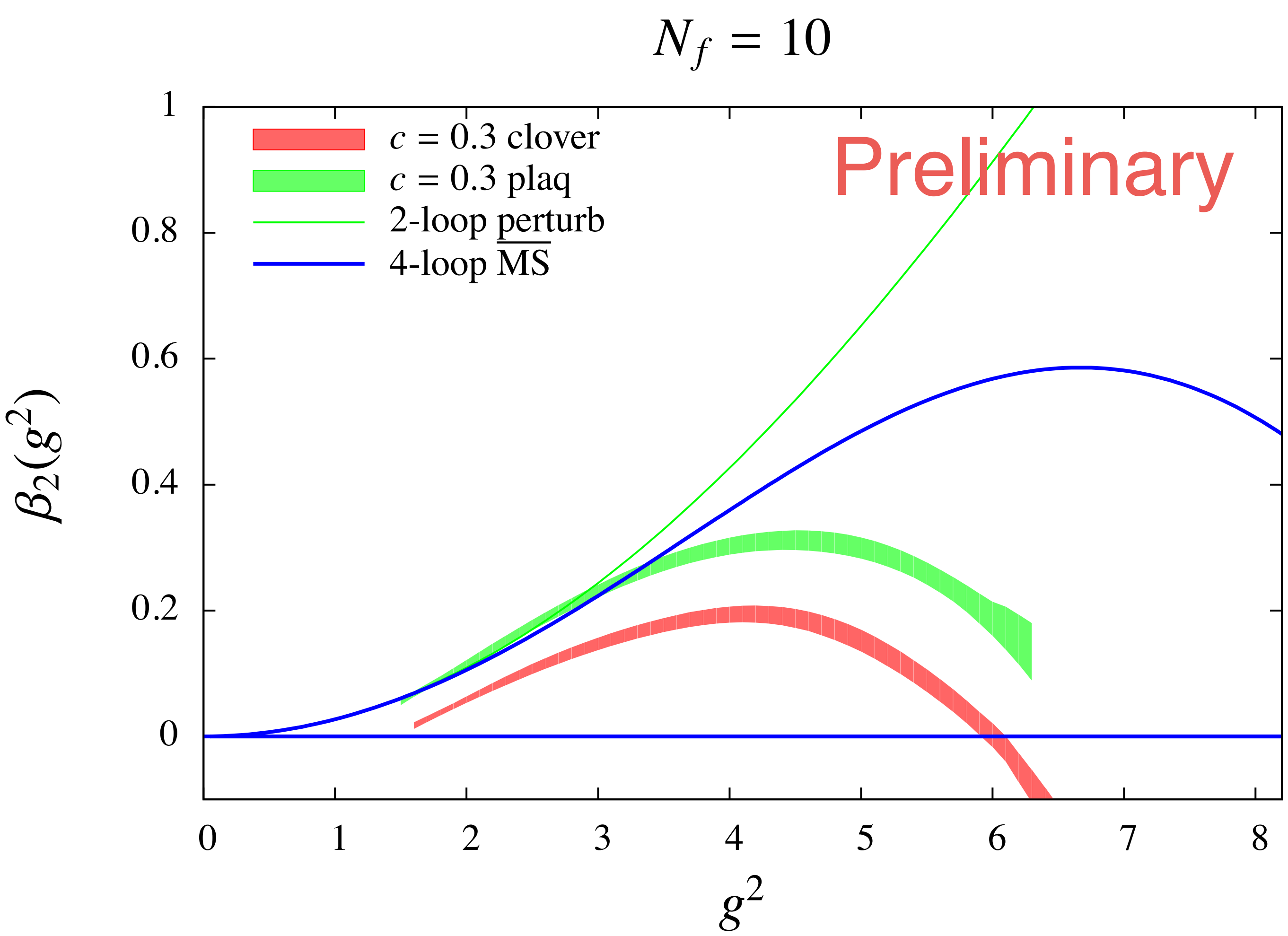
Plaquette and Clover operators are consistent if c ≥ 0.35



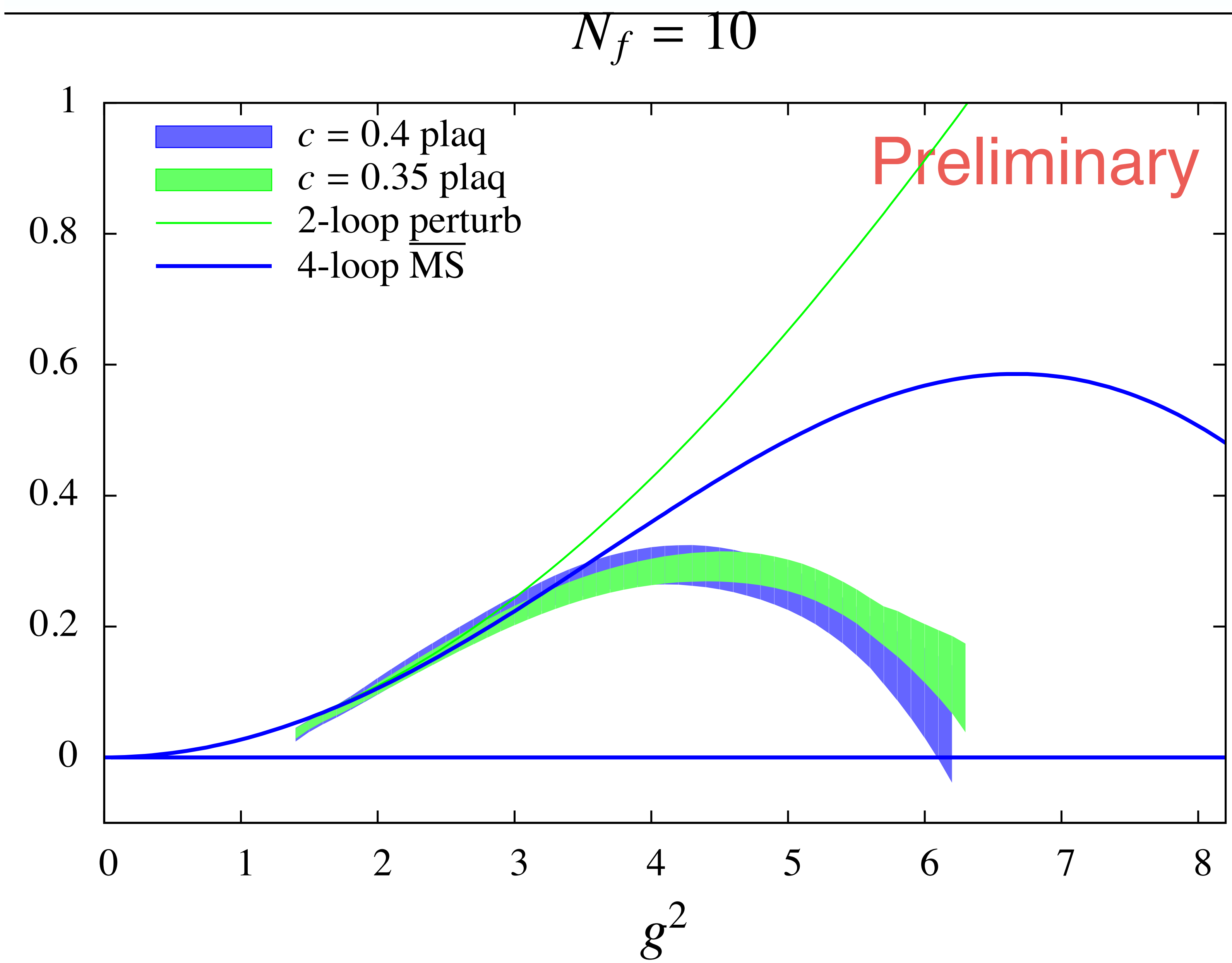


N_f=10 step scaling function

c ≤ 0.3 is not large enough



N_f=10 step scaling function



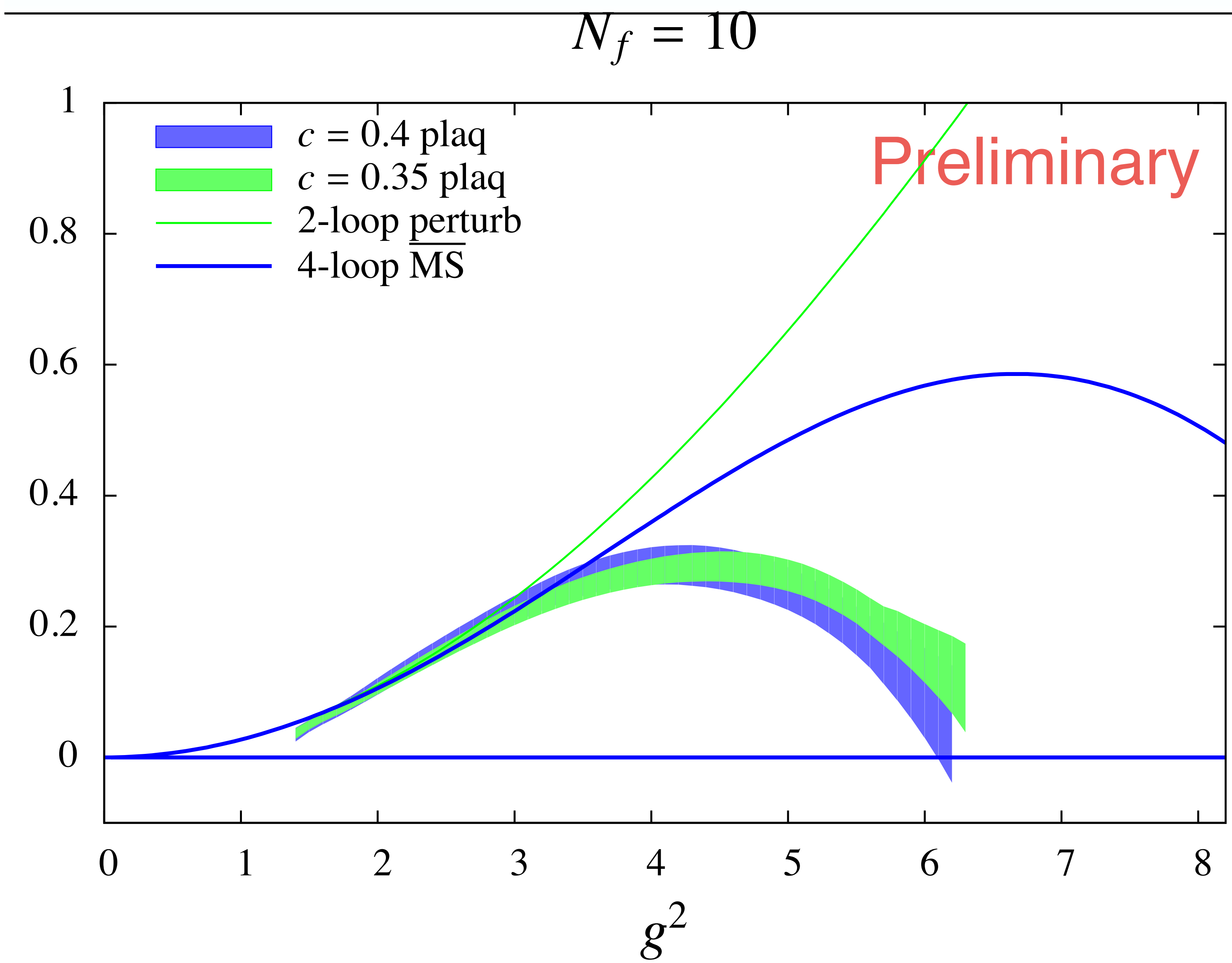
(AH, C. Rebbi, O. Witzel + GRID developers)

- The results could be consistent with Chiu
- **Much below 4-loop**
- suggestive IRFP at $g^2 \approx 7$ (we cannot yet tell)

The tension between N_f=10 and 12 is still there ….

Shown: Plaquette operator, c=0.35 and 0.4
They are consistent (do not have to be!)

N_f=10 step scaling function



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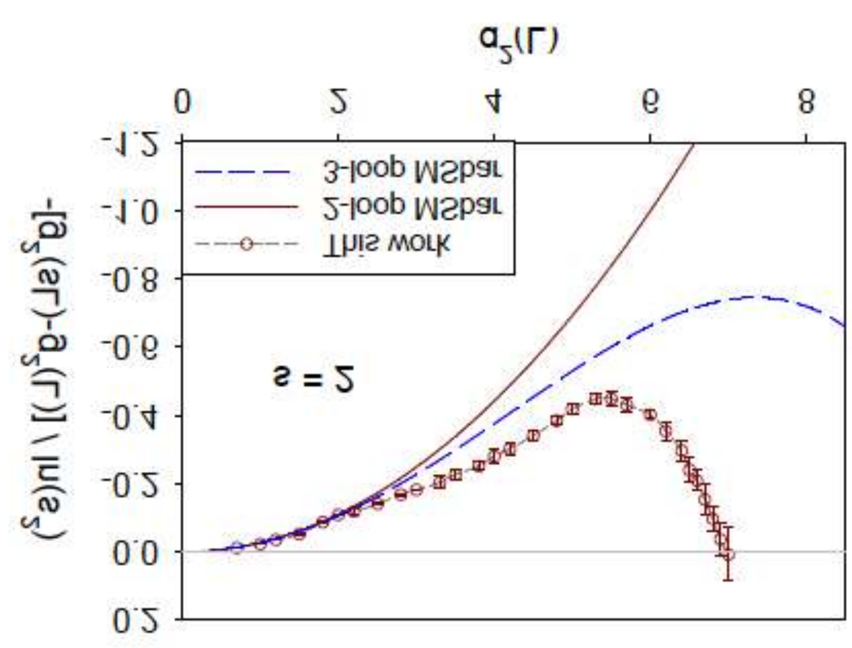
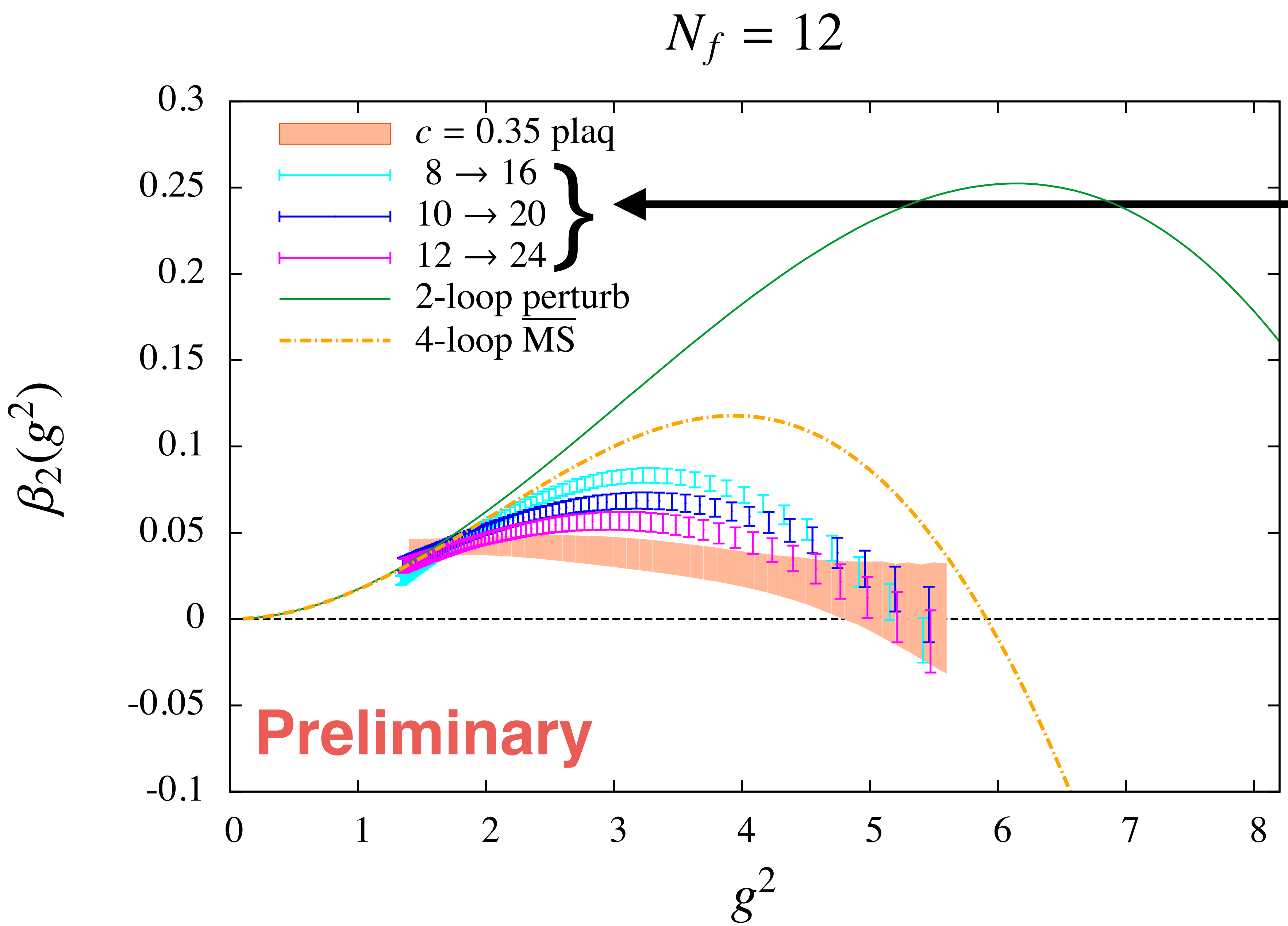


N_f=12 step scaling function

N_f=12 is hard … with DWF even harder; With ~5000 MDTU per configuration set even c=0.35 has large errors

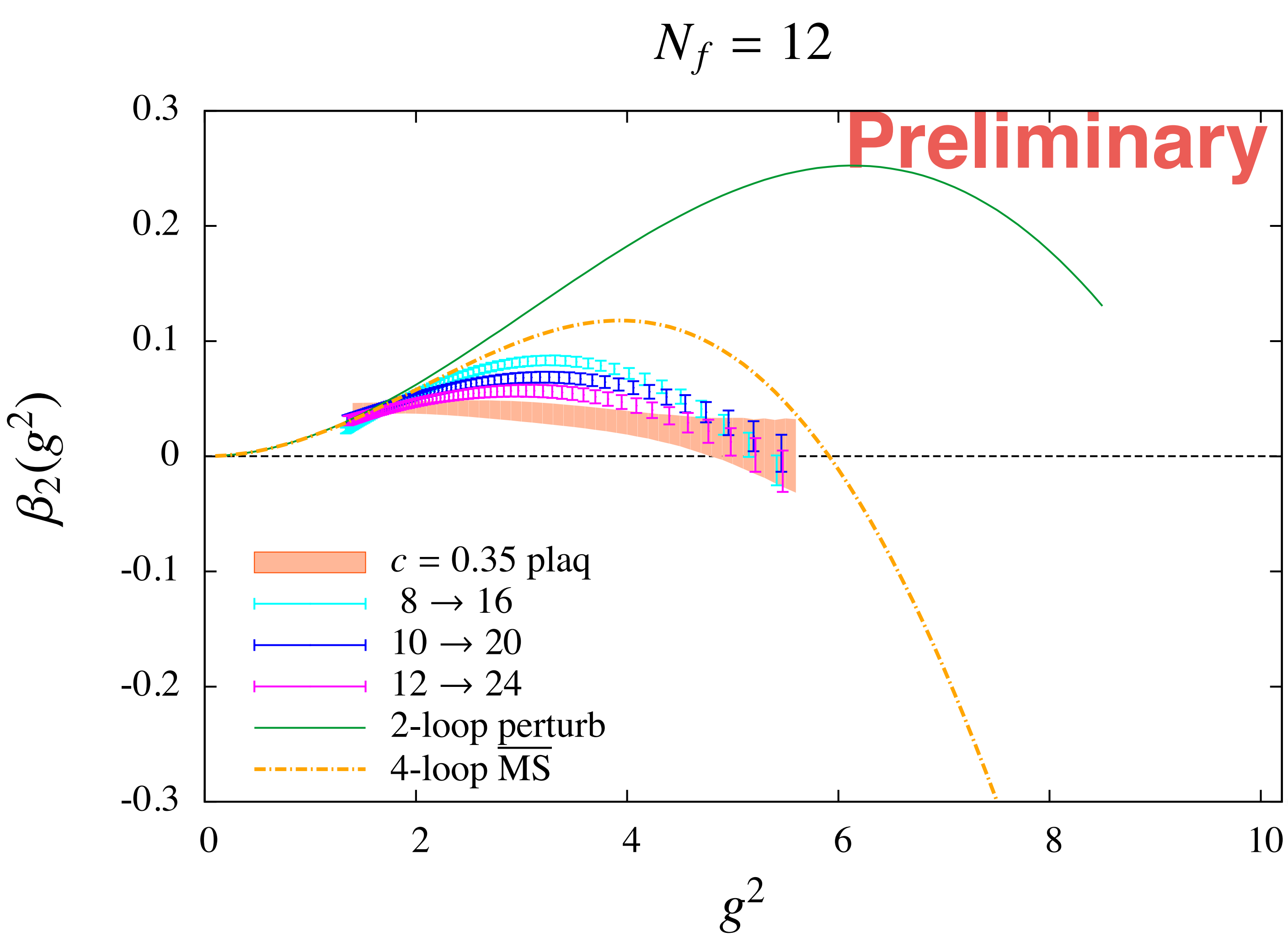
Compare finite volume data with L→∞ :



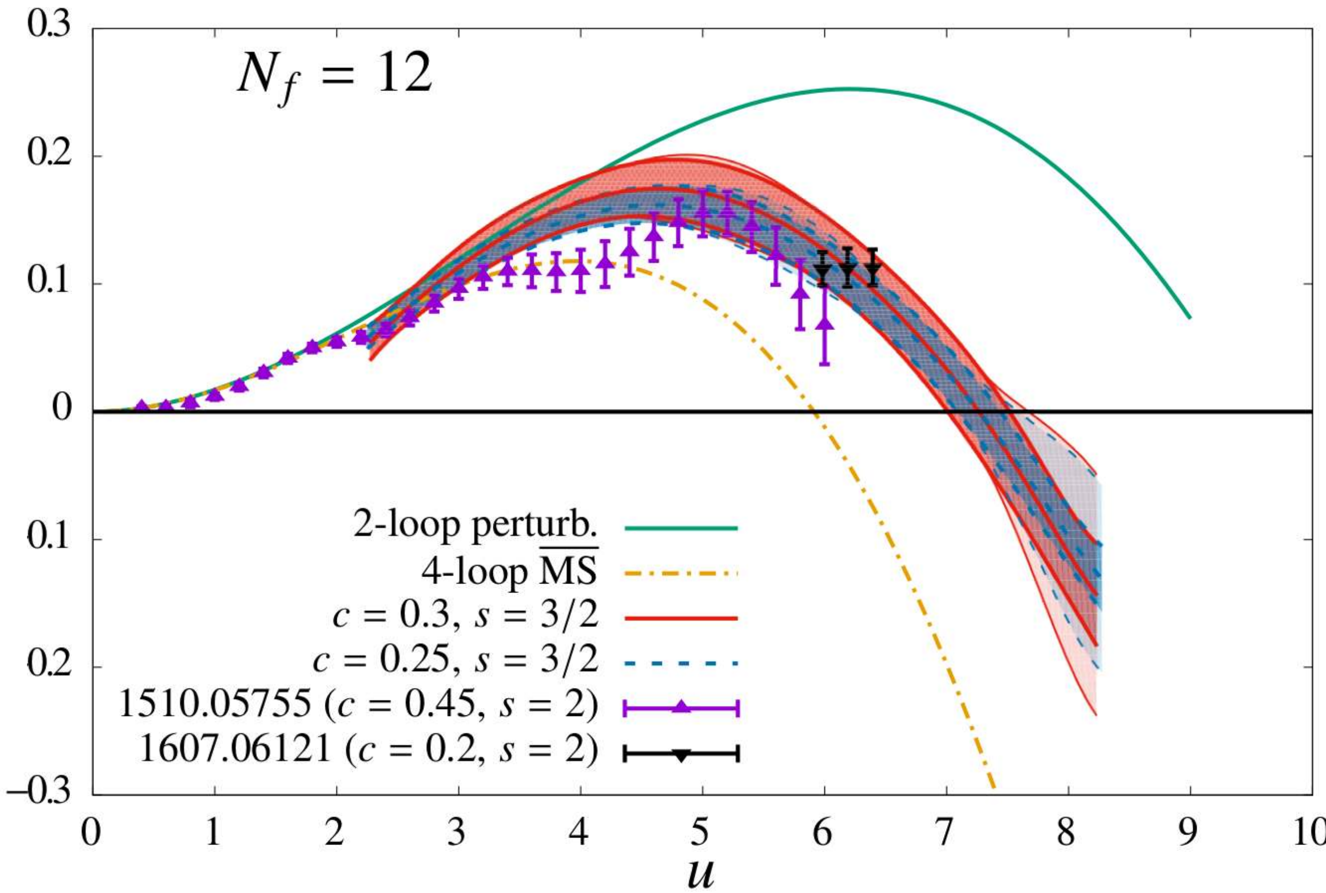
no $1/L^2$ extrapolation; Predicted difference on matched volumes

N_f=12, DW vs staggered

DW



Staggered



- DW result is well below 4-loop, suggesting an IRFP smaller than 4-loop; similar to N_f=10 but very different from staggered
- The difference is significant, *independent of the existence of the IRFP*

Conclusion & Summary

It is (perhaps) not surprising that lattice models with fermions with different chiral symmetries have different conformal fixed points.
This has direct consequences for lattice simulations.

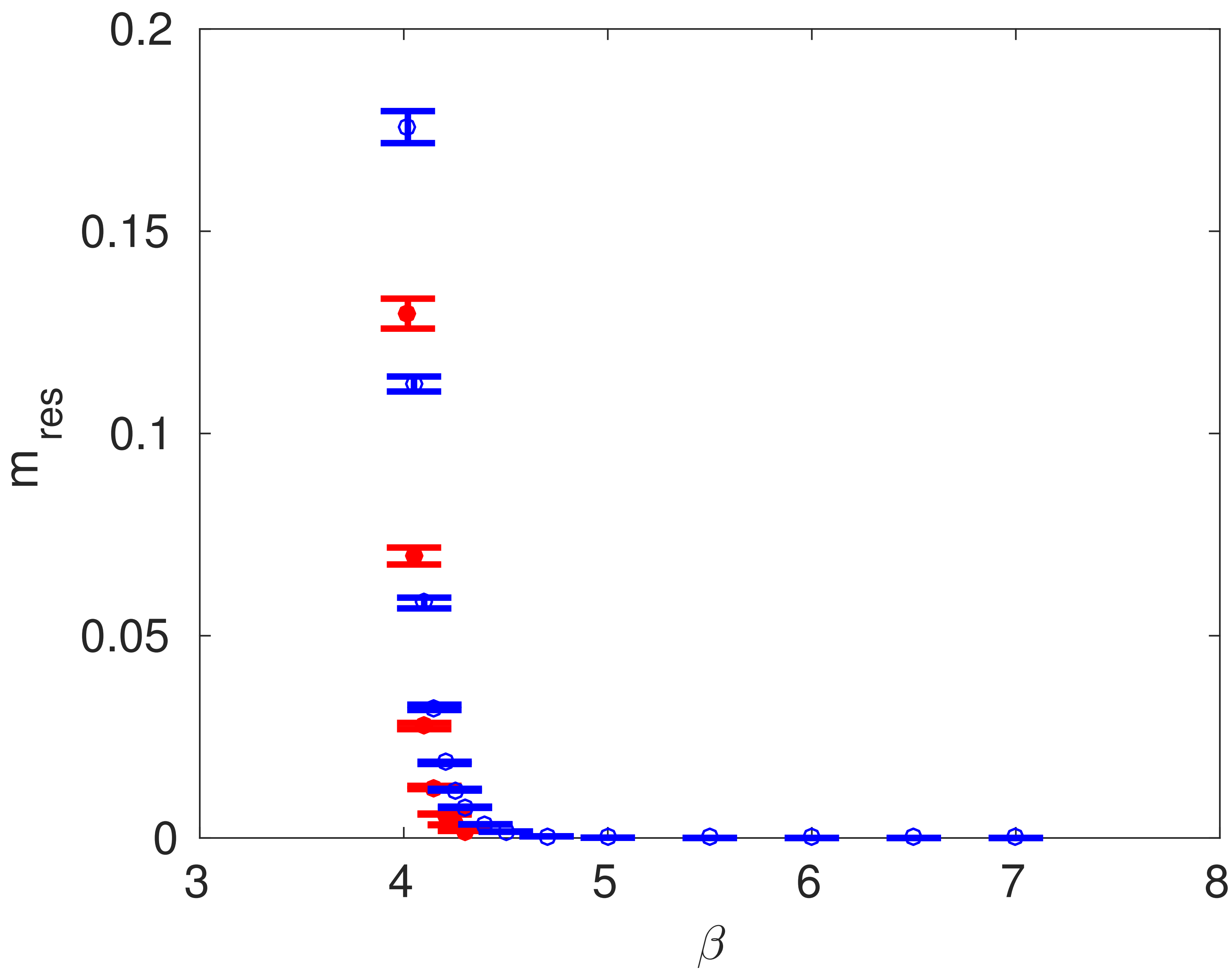
- Models that rely on a conformal IRFP should be simulated using DWF (or Wilson) fermions
  - Expensive, but necessary; Improved chiral properties help

- Models that are chirally broken but close to a conformal IRFP might show universality violations as well - just compare the RG β functions!

EXTRA SLIDES

Step scaling function with N_f=10 - II

L_5 = 12 in most cases; 16 in some, 24 in others : needed to control residual mass 😧



There is a first order bulk transition around β=4.0

The residual mass explodes near the bulk transition

(we use tanh approximation; T-W Chiu uses Zolotarev)