Structure of the Nucleon and its Excitations

Derek Leinweber

In collaboration with: Waseem Kamleh, Zhan-Wei Liu, Ben Owen, Anthony Thomas, Finn Stokes, Sam Thomas, Jia-Jun Wu

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

THE UNIVERSITY of ADELAIDE

Outline

Relativistic Components of the Nucleon Wave Function
- Techniques
- Ground state and first radial excitation results
- Non-trivial gluonic components in the ground state

Parity-Expanded Variational Analysis (PEVA)
- Essence of the PEVA Method
- Ground-state electromagnetic form factors
- Odd-parity excited-state electromagnetic form factors
- Magnetic moments of the $N^*(1535)$ and $N^*(1650)$

Hamiltonian Effective Field Theory
- Meson-baryon excited state contaminations in correlators

PACS-CS $(2+1)$-Flavour Ensembles

- We consider the PACS-CS $(2+1)$-flavour ensembles, available through the ILDG.
  - S. Aoki *et al.* (PACS-CS Collaboration), Phys. Rev. D **79**, 034503 (2009)
- Lattice size of $32^3 \times 64$ with $\beta = 1.90$. $L \simeq 3$ fm.
- Five pion masses: 702, 570, 411, 296 and 156 MeV.
- Current results are based on the order of 700 sources per configuration.

Hamiltonian Effective Field Theory (HEFT)

- An extension of chiral effective field theory incorporating the Lüscher relation
  - Linking the energy levels observed in finite volume to the Q^2 dependence of the scattering phase shift.

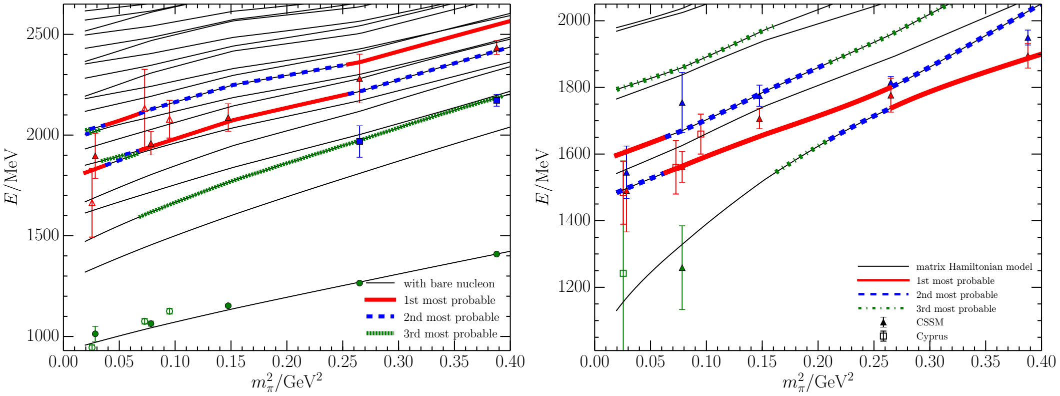
Hamiltonian Effective Field Theory (HEFT)

- An extension of chiral effective field theory incorporating the Lüscher relation
  - Linking the energy levels observed in finite volume to the Q^2 dependence of the scattering phase shift.
- Fitting resonance phase-shift data and inelasticities
  - Predictions of the finite-volume spectrum are made.

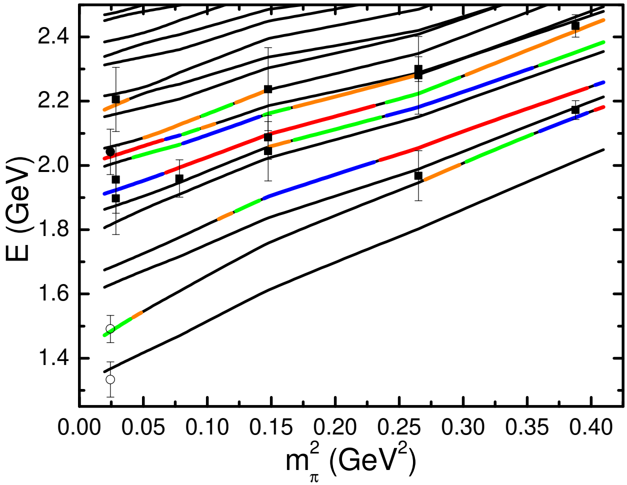
Hamiltonian Effective Field Theory (HEFT)

- An extension of chiral effective field theory incorporating the Lüscher relation
  - Linking the energy levels observed in finite volume to the Q^2 dependence of the scattering phase shift.
- Fitting resonance phase-shift data and inelasticities
  - Predictions of the finite-volume spectrum are made.
- The eigenvectors of the Hamiltonian provide insight into the energy eigenstates most likely to be excited with localised 3-quark operators.
  - Indicated in colour in the following plots.
  - Probability is ordered red, blue, green.

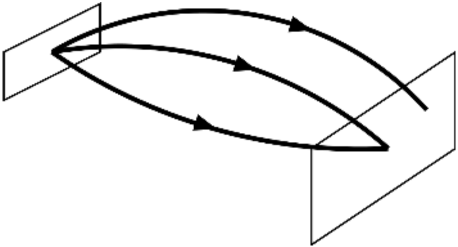
Even- and Odd-Parity Nucleon Spectrum on PACS-CS Lattices



Even-Parity Nucleon Spectrum: J. Wu, *et al.* (CSSM), arXiv:1703.10715 [nucl-th]



Landau-Gauge Wave functions from the Lattice



- Measure the *overlap* of the annihilation operator with the state as a function of the quark positions.

Landau-Gauge Wave functions from the Lattice

- Generalize the baryon annihilation operator to

$$\epsilon^{abc}\left(u^{Ta}(\vec{x}+\vec{s},t)\,C\gamma_5\,d^{b}(\vec{x}+\vec{y},t)\right)\,u^{c}(\vec{x}-\vec{s},t)+$$
$$\epsilon^{abc}\left(u^{Ta}(\vec{x}-\vec{s},t)\,C\gamma_5\,d^{b}(\vec{x}+\vec{y},t)\right)\,u^{c}(\vec{x}+\vec{s},t)$$

- Measure the *overlap* of this operator with the state as a function of $\vec{y}$ for fixed $\vec{s}$.

Landau-Gauge Wave functions from the Lattice

- Generalize the baryon annihilation operator to

$$\epsilon^{abc}\left(u^{Ta}(\vec{x}+\vec{s},t)\,C\gamma_5\,d^b(\vec{x}+\vec{y},t)\right)\,u^c(\vec{x}-\vec{s},t)+$$
$$\epsilon^{abc}\left(u^{Ta}(\vec{x}-\vec{s},t)\,C\gamma_5\,d^b(\vec{x}+\vec{y},t)\right)\,u^c(\vec{x}+\vec{s},t)$$

- Measure the *overlap* of this operator with the state as a function of $\vec{y}$ for fixed $\vec{s}$.
- In this case, one obtains the wave function of the d quark.

Landau-Gauge Wave functions from the Lattice

- Generalize the baryon annihilation operator to

$$\epsilon^{abc}\left(u^{Ta}(\vec{x}+\vec{s},t)\,C\gamma_5\,d^b(\vec{x}+\vec{y},t)\right)\,u^c(\vec{x}-\vec{s},t)+$$
$$\epsilon^{abc}\left(u^{Ta}(\vec{x}-\vec{s},t)\,C\gamma_5\,d^b(\vec{x}+\vec{y},t)\right)\,u^c(\vec{x}+\vec{s},t)$$

- Measure the *overlap* of this operator with the state as a function of $\vec{y}$ for fixed $\vec{s}$.
- In this case, one obtains the wave function of the d quark.
- At the source, use an optimised $\bar{\phi}^\alpha = \sum_i u_i^\alpha\,\bar{\chi}_i$ to create the state α of interest.

Landau-Gauge Wave functions from the Lattice

- Generalize the baryon annihilation operator to

$$\epsilon^{abc}\left(u^{Ta}(\vec{x}+\vec{s},t)\,C\gamma_5\,d^b(\vec{x}+\vec{y},t)\right)\,u^c(\vec{x}-\vec{s},t)+$$
$$\epsilon^{abc}\left(u^{Ta}(\vec{x}-\vec{s},t)\,C\gamma_5\,d^b(\vec{x}+\vec{y},t)\right)\,u^c(\vec{x}+\vec{s},t)$$

- Measure the *overlap* of this operator with the state as a function of $\vec{y}$ for fixed $\vec{s}$.
- In this case, one obtains the wave function of the d quark.
- At the source, use an optimised $\bar{\phi}^\alpha = \sum_i u_i^\alpha\,\bar{\chi}_i$ to create the state α of interest.
- Consider the u-quark offset $\vec{s} = 0$;
  - *i.e.* explore the distribution of the d quark about two u quarks at the origin.

Orbital Angular Momentum Selection

- The two-point function for the nucleon is a 4×4 matrix in Dirac space, $G_{ba}(t)$.

Orbital Angular Momentum Selection

- The two-point function for the nucleon is a 4×4 matrix in Dirac space, $G_{ba}(t)$.
- Dirac indices at the source, a, select the spin and parity of the state, α, under investigation.
  - Positive parity: $a = 1$ is spin up, $|\uparrow\rangle$.
  - Positive parity: $a = 2$ is spin down, $|\downarrow\rangle$.

Orbital Angular Momentum Selection

- The two-point function for the nucleon is a 4×4 matrix in Dirac space, $G_{ba}(t)$.
- Dirac indices at the source, a, select the spin and parity of the state, α, under investigation.
  - Positive parity: $a = 1$ is spin up, $|\uparrow\rangle$.
  - Positive parity: $a = 2$ is spin down, $|\downarrow\rangle$.
- Dirac indices at the sink select one of the four components of the Dirac spinor wave function.
  - $b = 1$ is $|\uparrow\rangle_{\mathrm{upper}}$ $b = 2$ is $|\downarrow\rangle_{\mathrm{upper}}$
  - $b = 3$ is $|\uparrow\rangle_{\mathrm{lower}}$ $b = 4$ is $|\downarrow\rangle_{\mathrm{lower}}$

Orbital Angular Momentum Selection

- The two-point function for the nucleon is a 4×4 matrix in Dirac space, $G_{ba}(t)$.
- Dirac indices at the source, a, select the spin and parity of the state, α, under investigation.
  - Positive parity: $a = 1$ is spin up, $|\uparrow\rangle$.
  - Positive parity: $a = 2$ is spin down, $|\downarrow\rangle$.
- Dirac indices at the sink select one of the four components of the Dirac spinor wave function.
  - $b = 1$ is $|\uparrow\rangle_{\text{upper}}$ $b = 2$ is $|\downarrow\rangle_{\text{upper}}$
  - $b = 3$ is $|\uparrow\rangle_{\text{lower}}$ $b = 4$ is $|\downarrow\rangle_{\text{lower}}$
- The spatial wave function will compensate for spin and parity mismatches between the source and sink.

Upper Dirac Component Wave Functions

- Denote the spatial wave functions for state α by $\langle \vec{r} \,|\, \alpha, \ell, m \,\rangle$

Upper Dirac Component Wave Functions

- Denote the spatial wave functions for state α by $\langle \vec{r} \,|\, \alpha, \ell, m \,\rangle$
- Our smeared source is spherically symmetric $\langle \vec{r} \,|\, \alpha, \ell, m \,\rangle = \langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle$

Upper Dirac Component Wave Functions

- Denote the spatial wave functions for state α by $\langle \vec{r} \,|\, \alpha, \ell, m \,\rangle$
- Our smeared source is spherically symmetric $\langle \vec{r} \,|\, \alpha, \ell, m \,\rangle = \langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle$
- G_{11}: $\langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle \; |{\uparrow}\rangle_{\rm upper} \rightarrow |{\uparrow}\rangle_{\rm upper} \; \langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle$

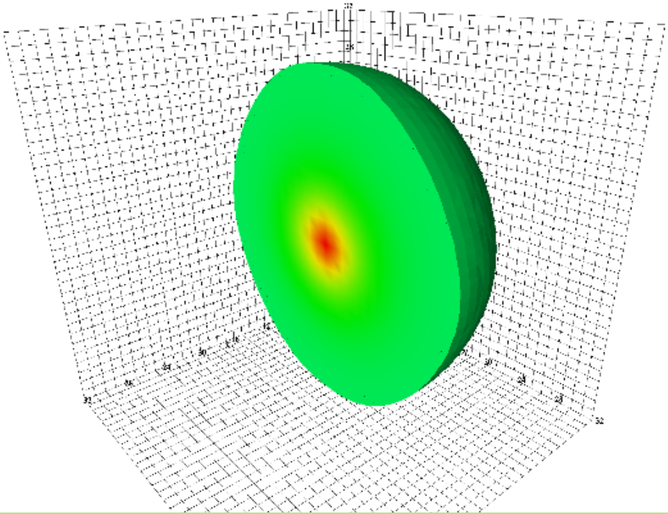
Upper Dirac Component Wave Functions

- Denote the spatial wave functions for state α by $\langle \vec{r} \,|\, \alpha, \ell, m \,\rangle$
- Our smeared source is spherically symmetric $\langle \vec{r} \,|\, \alpha, \ell, m \,\rangle = \langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle$
- G_{11}: $\langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle \; |{\uparrow}\rangle_{\rm upper} \to |{\uparrow}\rangle_{\rm upper} \; \langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle$
- G_{22}: $\langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle \; |{\downarrow}\rangle_{\rm upper} \to |{\downarrow}\rangle_{\rm upper} \; \langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle$

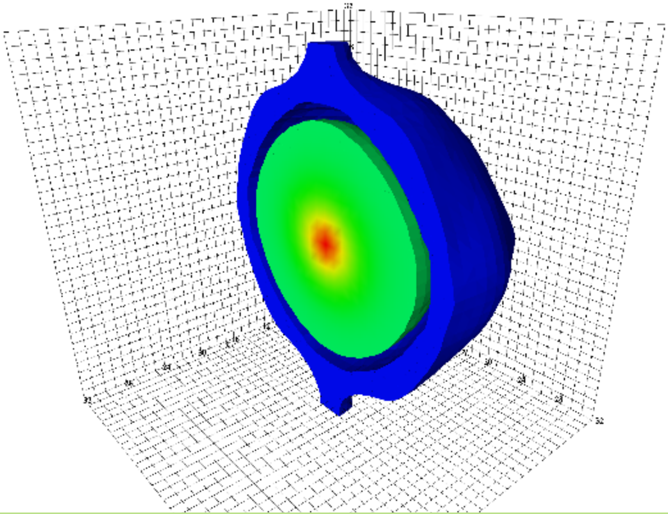
Upper Dirac Component Wave Functions

- Denote the spatial wave functions for state α by $\langle \vec{r} \,|\, \alpha, \ell, m \,\rangle$
- Our smeared source is spherically symmetric $\langle \vec{r} \,|\, \alpha, \ell, m \,\rangle = \langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle$
- G_{11}: $\langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle \ |{\uparrow}\rangle_{\rm upper} \to |{\uparrow}\rangle_{\rm upper} \ \langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle$
- G_{22}: $\langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle \ |{\downarrow}\rangle_{\rm upper} \to |{\downarrow}\rangle_{\rm upper} \ \langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle$
- Anticipate a spherically-symmetric wave function.

Ground-state upper-component wave function at lightest quark mass



1st-excited-state upper-component $\langle \vec{r} \,|\, 0, 0 \,\rangle$ wave function



Lower Dirac Component Wave Functions

- Lower Dirac components have opposite parity.

Lower Dirac Component Wave Functions

- Lower Dirac components have opposite parity.
- Orbital angular momentum $\ell = 1$ is required.

Lower Dirac Component Wave Functions

- Lower Dirac components have opposite parity.
- Orbital angular momentum $\ell = 1$ is required.
- G_{31}: $\langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle \; |{\uparrow}\rangle_{\rm upper} \to |{\uparrow}\rangle_{\rm lower} \; \langle \vec{r} \,|\, \alpha, 1, 0 \,\rangle$

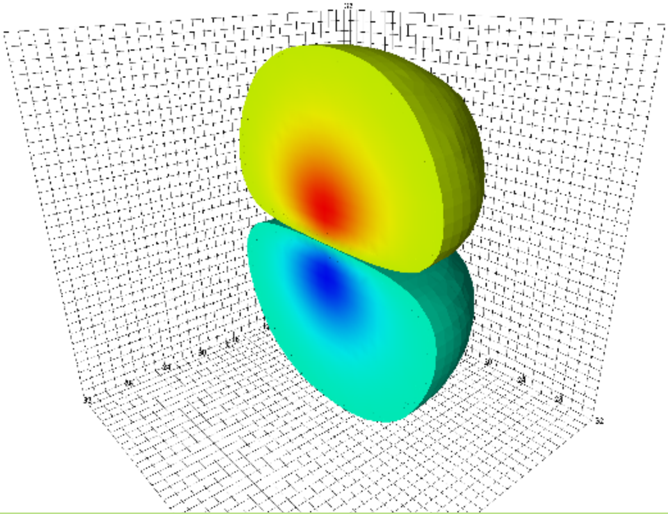
Lower Dirac Component Wave Functions

- Lower Dirac components have opposite parity.
- Orbital angular momentum $\ell = 1$ is required.
- G_{31}: $\langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle \; |{\uparrow}\rangle_{\rm upper} \to |{\uparrow}\rangle_{\rm lower} \; \langle \vec{r} \,|\, \alpha, 1, 0 \,\rangle$
- G_{42}: $\langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle \; |{\downarrow}\rangle_{\rm upper} \to |{\downarrow}\rangle_{\rm lower} \; \langle \vec{r} \,|\, \alpha, 1, 0 \,\rangle$

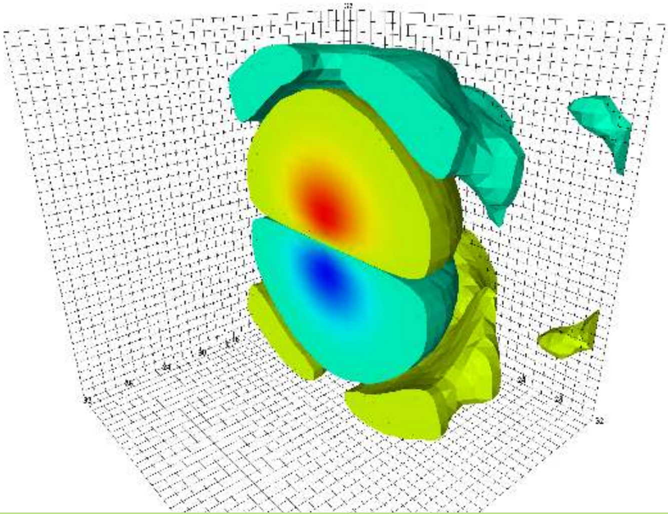
Lower Dirac Component Wave Functions

- Lower Dirac components have opposite parity.
- Orbital angular momentum $\ell = 1$ is required.
- G_{31}: $\langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle \; |{\uparrow}\rangle_{\mathrm{upper}} \to |{\uparrow}\rangle_{\mathrm{lower}} \; \langle \vec{r} \,|\, \alpha, 1, 0 \,\rangle$
- G_{42}: $\langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle \; |{\downarrow}\rangle_{\mathrm{upper}} \to |{\downarrow}\rangle_{\mathrm{lower}} \; \langle \vec{r} \,|\, \alpha, 1, 0 \,\rangle$
- The $\ell = 1$, $m_\ell = 0$ spatial wave function is resolved.

Ground-state $\langle \vec{r} \,|\, 1, 0 \,\rangle$ wave function at lightest quark mass



1st-excited-state $\langle \vec{r} \,|\, 1, 0 \,\rangle$ wave function



Lower Dirac Component Wave Functions: Spin flip

- Orbital angular momentum $\ell = 1$, $m_\ell = \pm 1$ is required.

Lower Dirac Component Wave Functions: Spin flip

- Orbital angular momentum $\ell = 1$, $m_\ell = \pm 1$ is required.
- G_{41}: $\langle \vec{r} \mid \alpha, 0, 0 \rangle \ |\uparrow\rangle_{\rm upper} \to |\downarrow\rangle_{\rm lower} \ \langle \vec{r} \mid \alpha, 1, 1 \rangle$

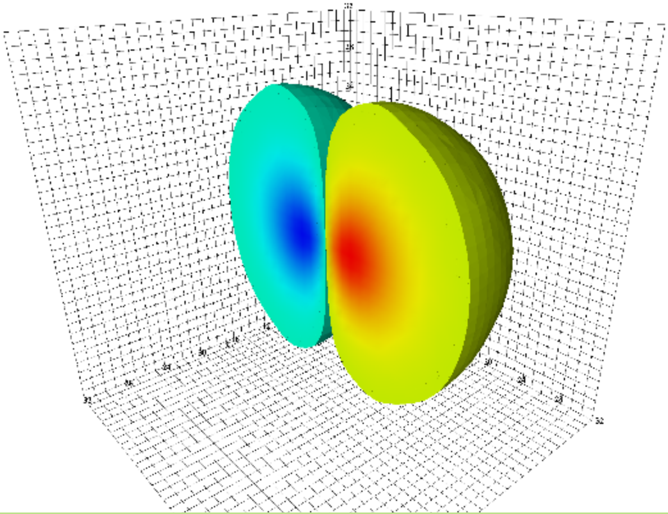
Lower Dirac Component Wave Functions: Spin flip

- Orbital angular momentum $\ell = 1$, $m_\ell = \pm 1$ is required.
- G_{41}: $\langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle \; |\!\uparrow\rangle_{\text{upper}} \to |\!\downarrow\rangle_{\text{lower}} \; \langle \vec{r} \,|\, \alpha, 1, 1 \,\rangle$
- G_{32}: $\langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle \; |\!\downarrow\rangle_{\text{upper}} \to |\!\uparrow\rangle_{\text{lower}} \; \langle \vec{r} \,|\, \alpha, 1, -1 \,\rangle$

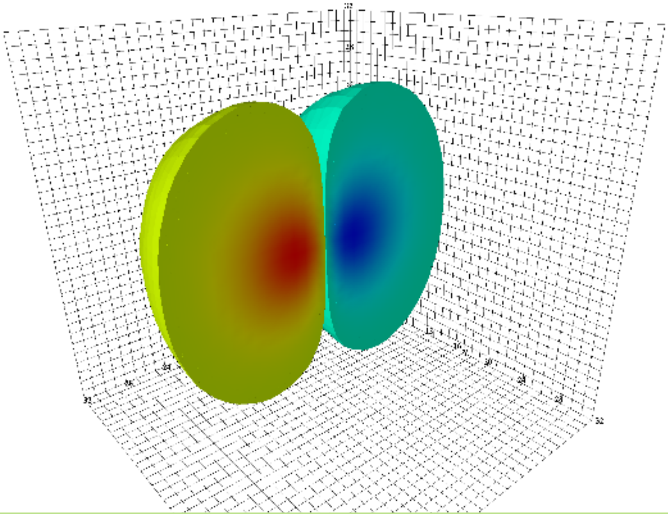
Lower Dirac Component Wave Functions: Spin flip

- Orbital angular momentum $\ell = 1$, $m_\ell = \pm 1$ is required.
- G_{41}: $\langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle \; |{\uparrow}\rangle_{\rm upper} \to |{\downarrow}\rangle_{\rm lower} \; \langle \vec{r} \,|\, \alpha, 1, 1 \,\rangle$
- G_{32}: $\langle \vec{r} \,|\, \alpha, 0, 0 \,\rangle \; |{\downarrow}\rangle_{\rm upper} \to |{\uparrow}\rangle_{\rm lower} \; \langle \vec{r} \,|\, \alpha, 1, -1 \,\rangle$
- The $\ell = 1$, $m_\ell = \pm 1$ spatial wave function is resolved.

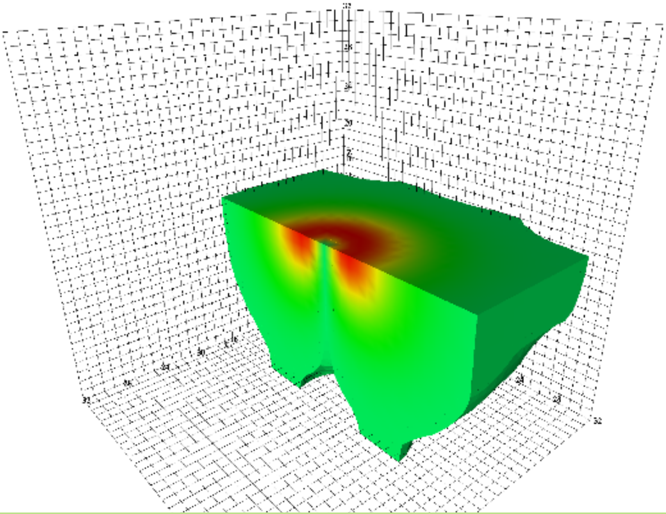
Ground-state $\mathrm{Re}\ \langle \vec{r} \,|\, 1, +1 \,\rangle$ wave function at lightest quark mass



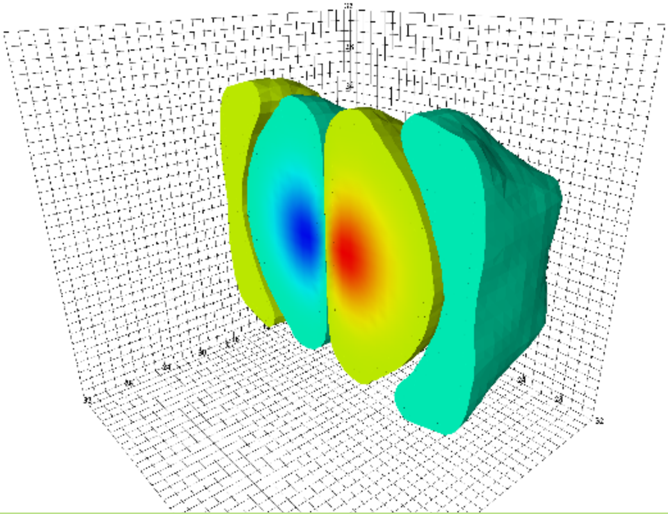
Ground-state $\mathrm{Im}\ \langle \vec{r} \,|\, 1, +1 \,\rangle$ wave function at lightest quark mass



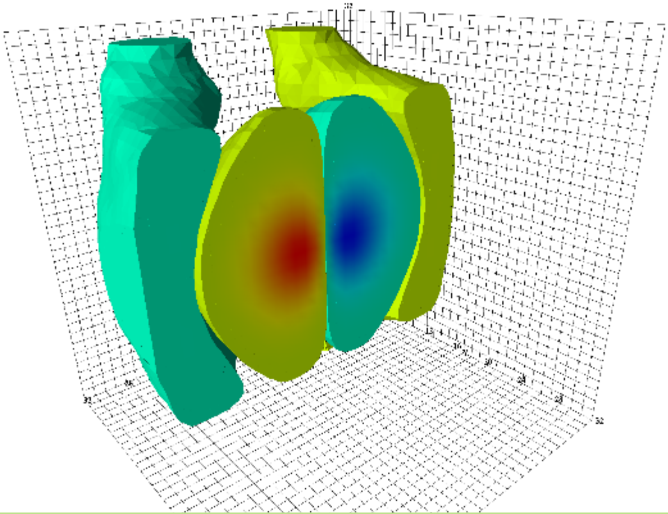
Ground-state $|\langle \vec{r} \,|\, 1, +1 \rangle|$ wave function at lightest quark mass



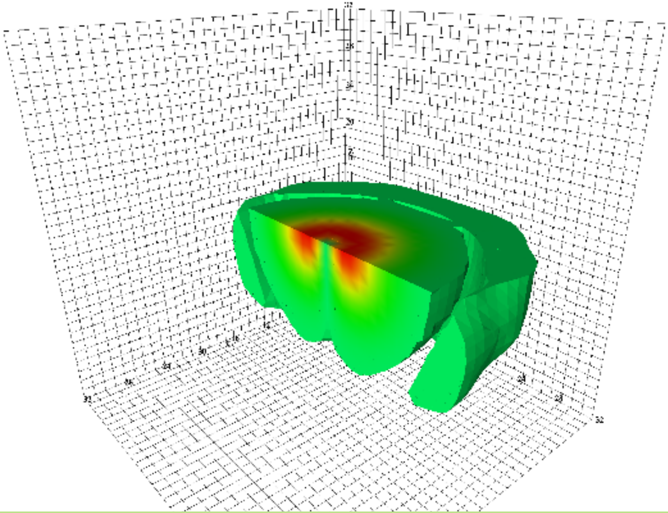
1st-excited-state $\mathrm{Re}\,\langle \vec{r} \,|\, 1, +1 \,\rangle$ wave function



1st-excited-state $\mathrm{Im}\ \langle \vec{r} \,|\, 1, +1 \,\rangle$ wave function



1st-excited-state $|\langle \vec{r} \,|\, 1, +1 \,\rangle|$ wave function



What about G_{21} or G_{12}?

- Spin flip requires nontrivial m_ℓ.

What about G_{21} or G_{12}?

- Spin flip requires nontrivial m_ℓ.
- Upper component to upper component requires even parity.

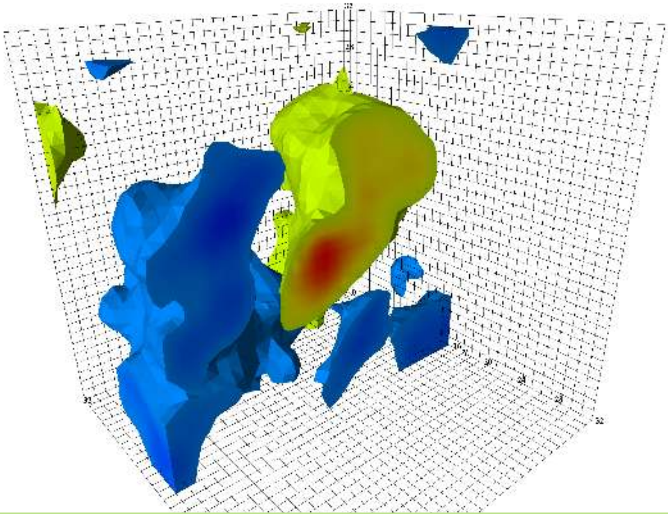
What about G_{21} or G_{12}?

- Spin flip requires nontrivial m_ℓ.
- Upper component to upper component requires even parity.
- $\ell + s = 2 + 1/2 \neq 1/2$

What about G_{21} or G_{12}?

- Spin flip requires nontrivial m_ℓ.
- Upper component to upper component requires even parity.
- $\ell + s = 2 + 1/2 \neq 1/2$
- Expect this matrix element to vanish.

Ground-state $\mathrm{Im}\, G_{21}$ wave function



Gluons play a non-trivial role

- G_{21}:

$$\langle \vec{r} \,|\, 0,0 \,\rangle \, |\uparrow\rangle_{\text{upper}} \rightarrow |\downarrow\rangle_{\text{upper}} \left(\langle \vec{r} \,|\, 1,1 \,\rangle_{\text{quark}} \, \langle \vec{r} \,|\, 1,0 \,\rangle_{\text{gluon}} + \langle \vec{r} \,|\, 1,0 \,\rangle_{\text{quark}} \, \langle \vec{r} \,|\, 1,1 \,\rangle_{\text{gluon}} \right)$$

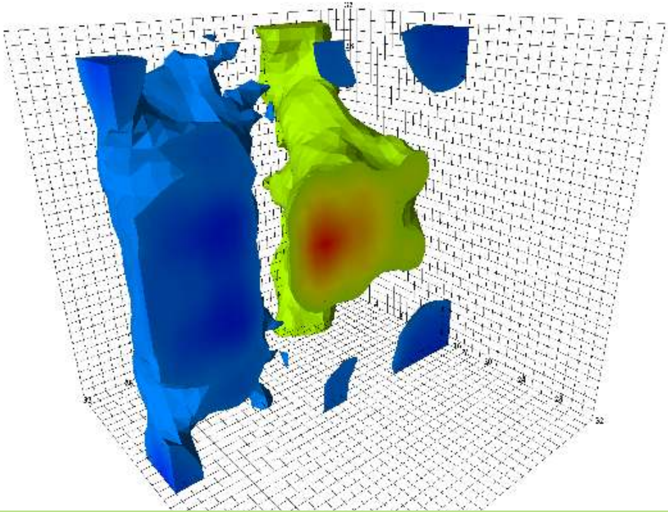
Gluons play a non-trivial role

- G_{21}:

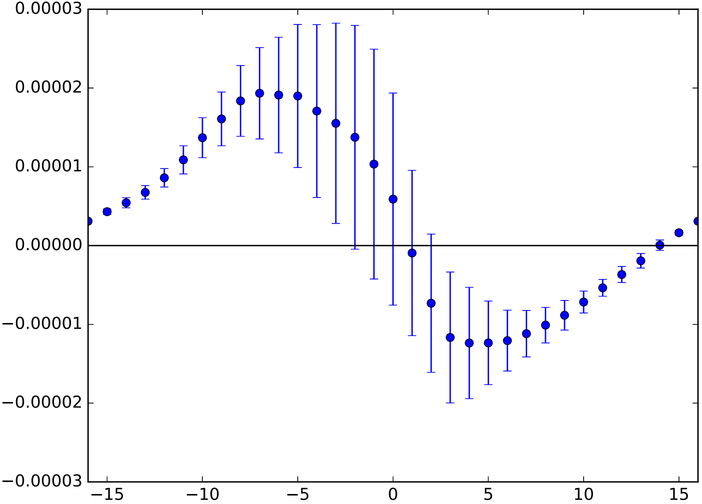
$$\langle \vec{r} \,|\, 0,0 \,\rangle \, |\uparrow\rangle_{\text{upper}} \to |\downarrow\rangle_{\text{upper}} \left(\langle \vec{r} \,|\, 1,1 \,\rangle_{\text{quark}} \, \langle \vec{r} \,|\, 1,0 \,\rangle_{\text{gluon}} + \langle \vec{r} \,|\, 1,0 \,\rangle_{\text{quark}} \, \langle \vec{r} \,|\, 1,1 \,\rangle_{\text{gluon}} \right)$$

- Symmetrize along the z axis to reveal $\langle \vec{r} \,|\, 1,1 \,\rangle_{\text{quark}}$

$\mathrm{Im}\ \langle \vec{r} \,|\, 1, 1 \rangle_{\mathrm{quark}}$ from the G_{21} matrix element



$\mathrm{Im}\ \langle \vec{r} \,|\, 1, 1 \,\rangle_{\mathrm{quark}}$ along y-axis from the G_{21} matrix element



Electromagnetic Form Factors: Conventional variational analysis

- Start with some basis of operators $\{\chi^i\}$

Electromagnetic Form Factors: Conventional variational analysis

- Start with some basis of operators $\{\chi^i\}$
- We use local three-quark spin-1/2 nucleon operators

$$\chi_1 = \epsilon^{abc} \left[u^{a\mathrm{T}} (C\gamma_5)\, d^b \right] u^c$$
$$\chi_2 = \epsilon^{abc} \left[u^{a\mathrm{T}} (C)\, d^b \right] \gamma_5\, u^c$$

Electromagnetic Form Factors: Conventional variational analysis

- Start with some basis of operators $\{\chi^i\}$
- We use local three-quark spin-$1/2$ nucleon operators

$$\chi_1 = \epsilon^{abc}\left[u^{a\,\mathrm{T}}(C\gamma_5)\, d^b\right] u^c$$
$$\chi_2 = \epsilon^{abc}\left[u^{a\,\mathrm{T}}(C)\, d^b\right] \gamma_5\, u^c$$

- Apply 16, 35, 100 and 200 sweeps of gauge invariant Gaussian smearing

Electromagnetic Form Factors: Conventional variational analysis

- Start with some basis of operators $\{\chi^i\}$
- We use local three-quark spin-1/2 nucleon operators

$$\chi_1 = \epsilon^{abc}\left[u^{a\mathrm{T}}(C\gamma_5)\, d^b\right] u^c$$
$$\chi_2 = \epsilon^{abc}\left[u^{a\mathrm{T}}(C)\, d^b\right] \gamma_5\, u^c$$

- Apply 16, 35, 100 and 200 sweeps of gauge invariant Gaussian smearing

8×8 Correlation Matrix

$$\mathcal{G}^{ij}(\mathbf{p};\, t) := \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}}\, \langle\Omega|\chi^i(x)\,\overline{\chi}^j(0)|\Omega\rangle$$

Electromagnetic Form Factors: Conventional variational analysis

- Start with some basis of operators $\{\chi^i\}$
- We use local three-quark spin-1/2 nucleon operators

$$\chi_1 = \epsilon^{abc} \left[u^{a\mathrm{T}}(C\gamma_5)\, d^b\right] u^c$$
$$\chi_2 = \epsilon^{abc} \left[u^{a\mathrm{T}}(C)\, d^b\right] \gamma_5\, u^c$$

- Apply 16, 35, 100 and 200 sweeps of gauge invariant Gaussian smearing

8×8 Correlation Matrix

$$\mathcal{G}^{ij}(\mathbf{p};\, t) := \sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}}\, \langle\Omega|\chi^i(x)\,\overline{\chi}^j(0)|\Omega\rangle$$

- Seek optimised operators that couple strongly to a single energy eigenstate

Parity-Expanded Variational Analysis (PEVA)

- Expand basis to simultaneously isolate finite momentum energy eigenstates of both parities.

Parity-Expanded Variational Analysis (PEVA)

- Expand basis to simultaneously isolate finite momentum energy eigenstates of both parities.
- Terms in unprojected correlation matrix have Dirac structure

$$\begin{pmatrix} E_{B^\pm}(\mathbf{p}) \pm m_{B^\pm} & -\sigma_k p_k \\ \sigma_k p_k & -\left(E_{B^\pm}(\mathbf{p}) \mp m_{B^\pm}\right) \end{pmatrix}$$

Parity-Expanded Variational Analysis (PEVA)

- Expand basis to simultaneously isolate finite momentum energy eigenstates of both parities.
- Terms in unprojected correlation matrix have Dirac structure

$$\begin{pmatrix} E_{B^\pm}(\mathbf{p}) \pm m_{B^\pm} & -\sigma_k p_k \\ \sigma_k p_k & -\left(E_{B^\pm}(\mathbf{p}) \mp m_{B^\pm}\right) \end{pmatrix}$$

Parity-Expanded Variational Analysis (PEVA)

- Expand basis to simultaneously isolate finite momentum energy eigenstates of both parities.
- Terms in unprojected correlation matrix have Dirac structure

$$\begin{pmatrix} E_{B^\pm}(\mathbf{p}) \pm m_{B^\pm} & -\sigma_k p_k \\ \sigma_k p_k & -\left(E_{B^\pm}(\mathbf{p}) \mp m_{B^\pm}\right) \end{pmatrix}$$

Parity-Expanded Variational Analysis (PEVA)

- Expand basis to simultaneously isolate finite momentum energy eigenstates of both parities.
- Terms in unprojected correlation matrix have Dirac structure

$$\begin{pmatrix} E_{B^\pm}(\mathbf{p}) \pm m_{B^\pm} & -\sigma_k p_k \\ \sigma_k p_k & -\left(E_{B^\pm}(\mathbf{p}) \mp m_{B^\pm}\right) \end{pmatrix}$$

Parity-Expanded Variational Analysis (PEVA)

- Expand basis to simultaneously isolate finite momentum energy eigenstates of both parities.
- Terms in unprojected correlation matrix have Dirac structure

$$\begin{pmatrix} E_{B^\pm}(\mathbf{p}) \pm m_{B^\pm} & -\sigma_k p_k \\ \sigma_k p_k & -(E_{B^\pm}(\mathbf{p}) \mp m_{B^\pm}) \end{pmatrix}$$

- Define PEVA projector

$$\Gamma_{\mathbf{p}} = \frac{1}{4}(\mathbb{I} + \gamma_4)(\mathbb{I} - i\gamma_5\gamma_k\hat{p}_k)$$

Parity-Expanded Variational Analysis (PEVA)

- Expand basis to simultaneously isolate finite momentum energy eigenstates of both parities.
- Terms in unprojected correlation matrix have Dirac structure

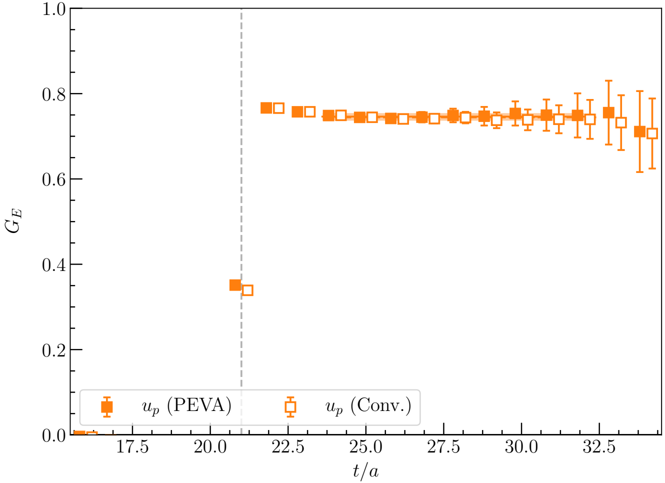
$$\begin{pmatrix} E_{B^\pm}(\mathbf{p}) \pm m_{B^\pm} & -\sigma_k p_k \\ \sigma_k p_k & -\left(E_{B^\pm}(\mathbf{p}) \mp m_{B^\pm}\right) \end{pmatrix}$$

- Define PEVA projector

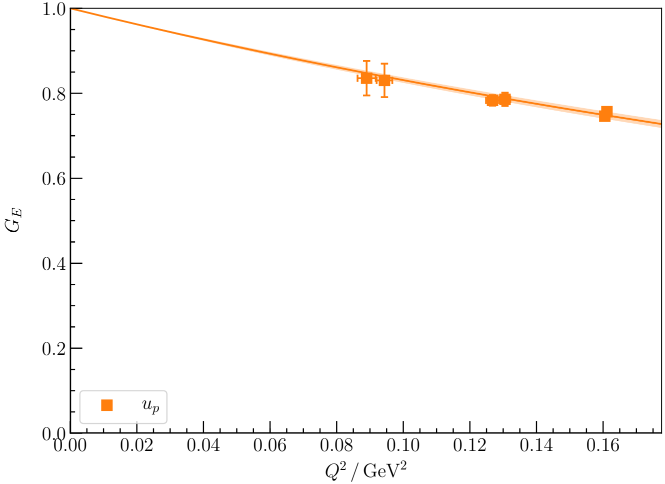
$$\Gamma_{\mathbf{p}} = \frac{1}{4}(\mathbb{I} + \gamma_4)(\mathbb{I} - i\gamma_5\gamma_k\hat{p}_k)$$

- Double the correlation matrix size:
  - $\chi^i_{\mathbf{p}} := \Gamma_{\mathbf{p}}\chi^i$ couples to positive parity states at zero momentum.
  - $\chi^{i'}_{\mathbf{p}} := \Gamma_{\mathbf{p}}\gamma_5\chi^i$ couples to negative parity states at zero momentum.

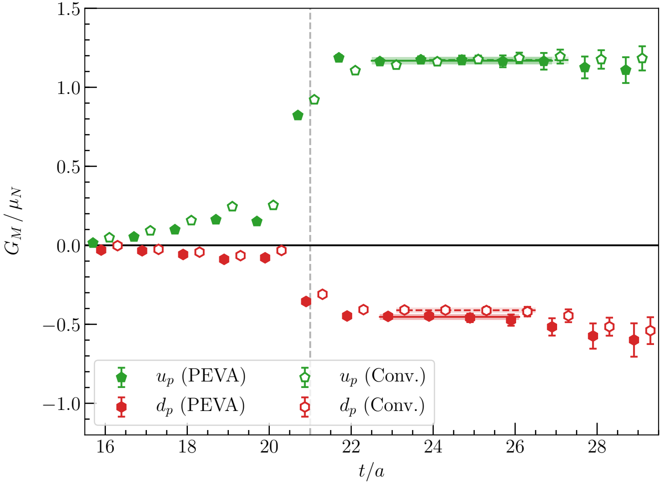
G_E for u_p for $p = (0,0,0) \to (1,0,0)$ at $m_\pi = 296$ MeV



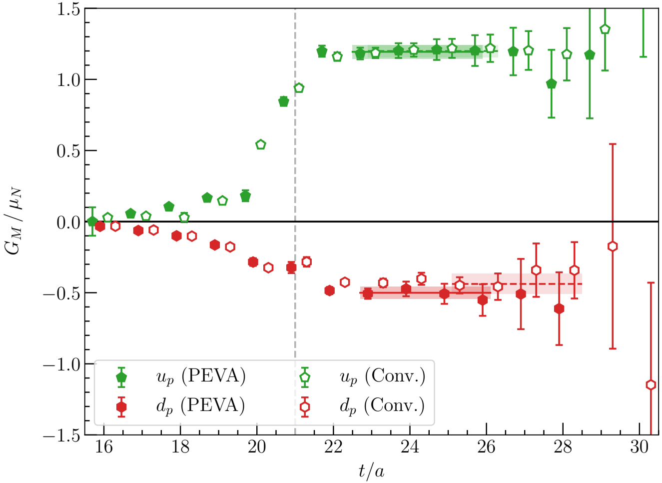
G_E for u_p for several $p' - p = (1, 0, 0)$ at $m_\pi = 296$ MeV



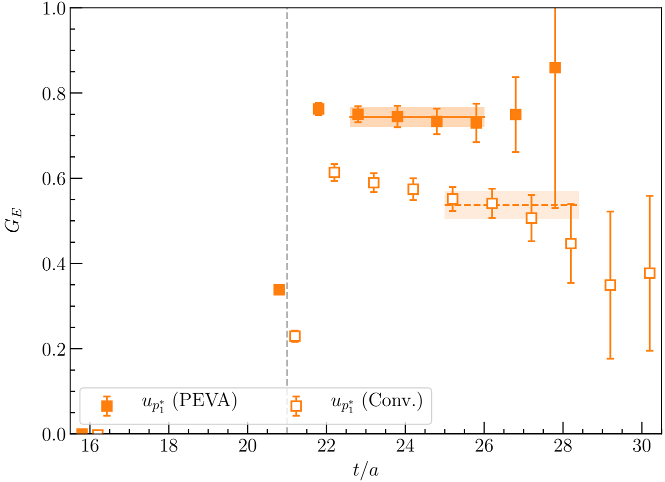
G_M for u_p & d_p for $p = (0,0,0) \to (1,0,0)$ at $m_\pi = 296$ MeV



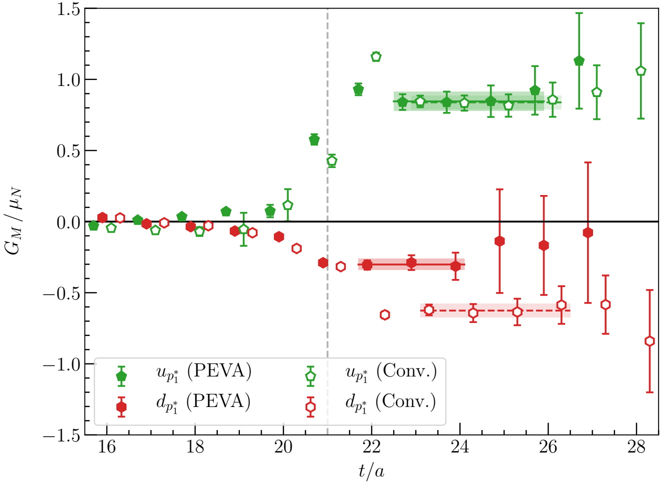
G_M for u_p & d_p for $p = (1,0,0) \to (2,0,0)$ at $m_\pi = 296$ MeV



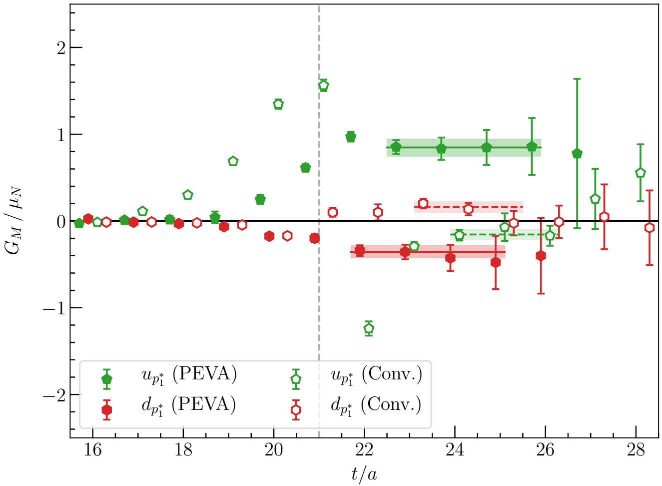
G_E for $u_{p_1^*}$ for $p = (0,0,0) \to (1,0,0)$ at $m_\pi = 411$ MeV



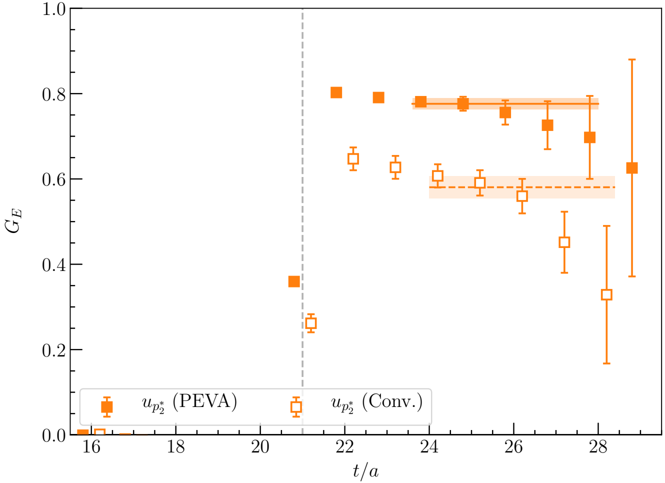
G_M for $u_{p_1^*}$ & $d_{p_1^*}$ for $p = (0,0,0) \to (1,0,0)$ at $m_\pi = 411$ MeV



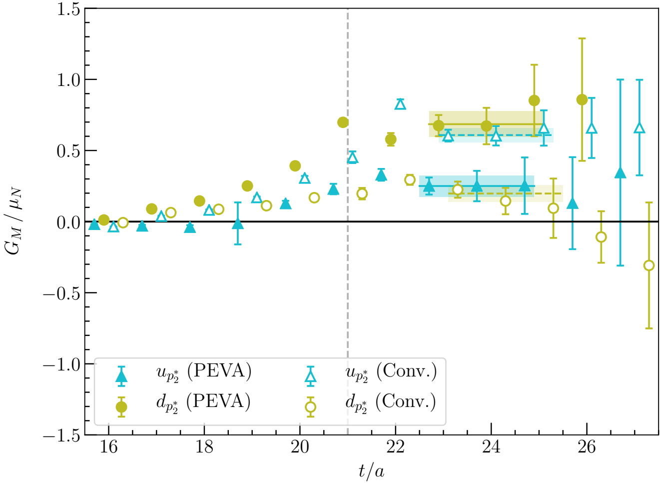
G_M for $u_{p_1^*}$ & $d_{p_1^*}$ for $p = (1,0,0) \to (2,0,0)$ at $m_\pi = 411$ MeV



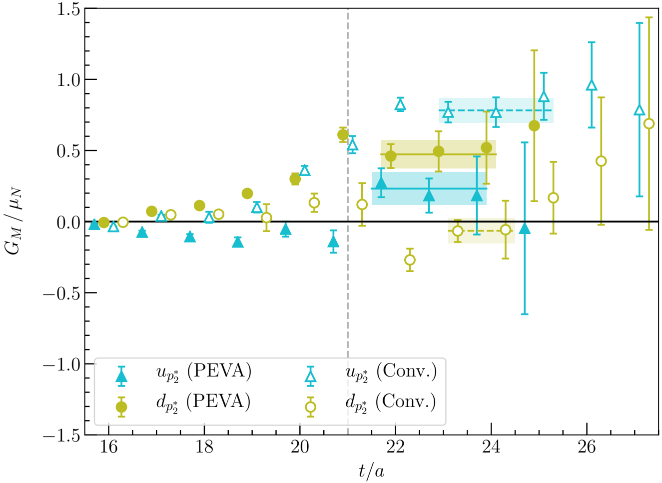
G_E for $u_{p_2^*}$ for $p = (0,0,0) \to (1,0,0)$ at $m_\pi = 411$ MeV



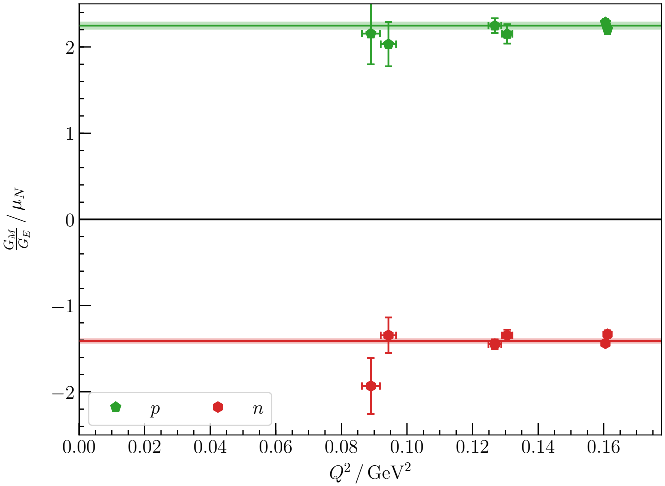
G_M for $u_{p_2^*}$ & $d_{p_2^*}$ for $p = (0,0,0) \to (1,0,0)$ at $m_\pi = 411$ MeV



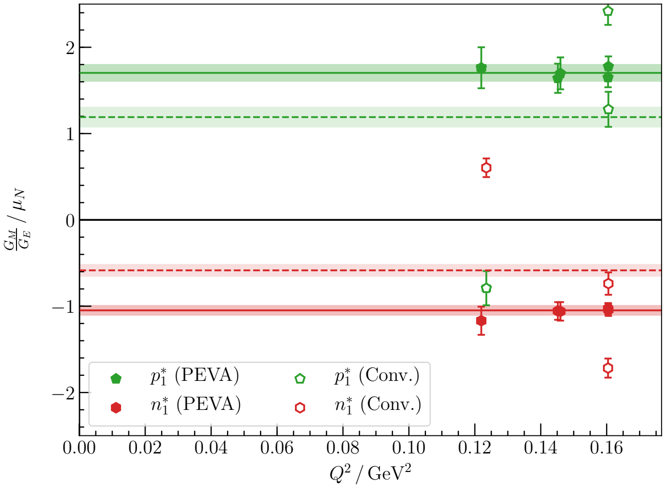
G_M for $u_{p_2^*}$ & $d_{p_2^*}$ for $p = (1,0,0) \rightarrow (2,0,0)$ at $m_\pi = 411$ MeV



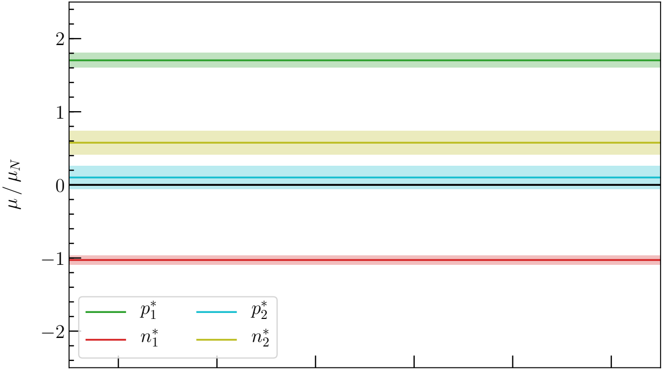
G_M/G_E ratio for p & n at $m_\pi = 296$ MeV



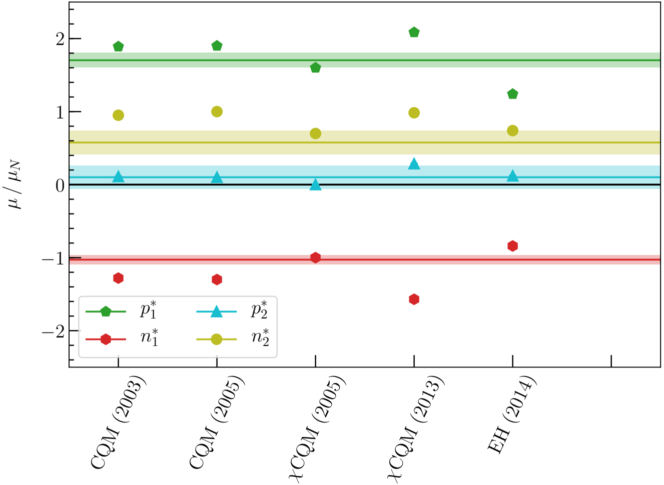
G_M / G_E ratio for p_1^* & n_1^* at $m_\pi = 411$ MeV



Magnetic moments of odd-parity nucleons at $m_\pi = 411$ MeV



Magnetic moments of odd-parity nucleons at $m_\pi = 411$ MeV



Model Calculation References

- CQM (2003)

  W.-T. Chiang, S. N. Yang, M. Vanderhaeghen, and D. Drechsel, Magnetic dipole moment of the S 11 (1535) from the $\gamma p \to \gamma \eta p$ reaction, Nucl. Phys. **A723**, 205 (2003), nucl-th/0211061

- χCQM (2005)

  J. Liu, J. He, and Y. Dong, Magnetic moments of negative-parity low-lying nucleon resonances in quark models, Phys. Rev. **D71**, 094004 (2005).

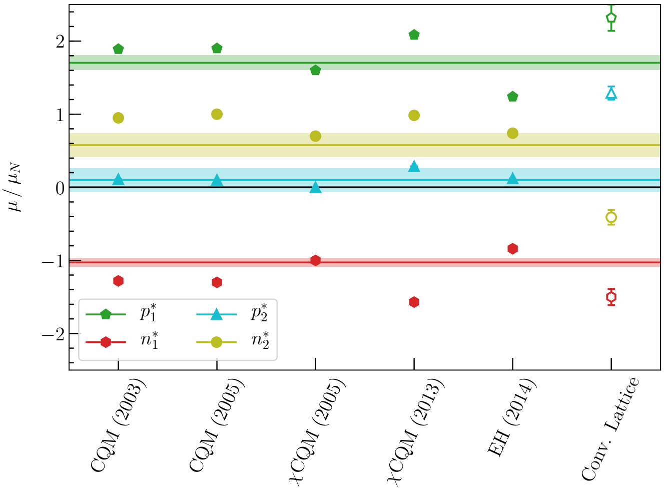
- χCQM (2013)

  N. Sharma, A. Martinez Torres, K. Khemchandani, and H. Dahiya, Magnetic moments of the low-lying $1/2^-$ octet baryon resonances, Eur. Phys. J. **A49**, 11 (2013), arXiv:1207.3311

- EH (2014)

  I. Narodetskii and M. Trusov, Magnetic moments of negative parity baryons from the effective Hamiltonian approach to QCD, JETP Lett. **99**, 57 (2014), arXiv:1311.2407

Magnetic moments of odd-parity nucleons at $m_\pi = 411$ MeV



What about scattering state contaminations?

- Consider the zero-momentum two-point function of localised 3-quark interpolators

$$G(t) = \sum_{\vec{x}} \sum_{i} \langle \Omega | \, \chi(x) \, | E_i \rangle \, \langle E_i | \, \overline{\chi}(0) \, | \Omega \rangle \ .$$

What about scattering state contaminations?

- Consider the zero-momentum two-point function of localised 3-quark interpolators

$$G(t) = \sum_{\vec{x}} \sum_{i} \langle\Omega|\, \chi(x)\, |E_i\rangle \langle E_i|\, \overline{\chi}(0)\, |\Omega\rangle \ .$$

- In Hamiltonian EFT, the only localised basis state is the bare state

$$\overline{\chi}(0)\, |\Omega\rangle = |m_0\rangle \ ,$$

What about scattering state contaminations?

- Consider the zero-momentum two-point function of localised 3-quark interpolators

$$G(t) = \sum_{\vec{x}} \sum_i \langle\Omega|\, \chi(x)\, |E_i\rangle \, \langle E_i|\, \overline{\chi}(0)\, |\Omega\rangle \; .$$

- In Hamiltonian EFT, the only localised basis state is the bare state

$$\overline{\chi}(0)\, |\Omega\rangle = |m_0\rangle \; ,$$

- Overlap with two-particle basis states is volume suppressed. Thus,

$$G(t) = \sum_i |\, \langle m_0 \,|\, E_i\rangle\, |^2 e^{-E_i\, t} \, .$$

What about scattering state contaminations?

- Consider the zero-momentum two-point function of localised 3-quark interpolators

$$G(t) = \sum_{\vec{x}} \sum_i \langle \Omega | \chi(x) | E_i \rangle \langle E_i | \overline{\chi}(0) | \Omega \rangle \ .$$

- In Hamiltonian EFT, the only localised basis state is the bare state

$$\overline{\chi}(0) | \Omega \rangle = | m_0 \rangle \ ,$$

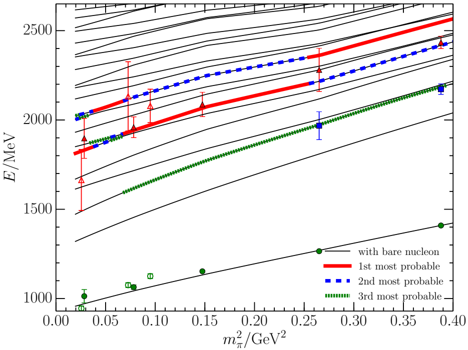
- Overlap with two-particle basis states is volume suppressed. Thus,

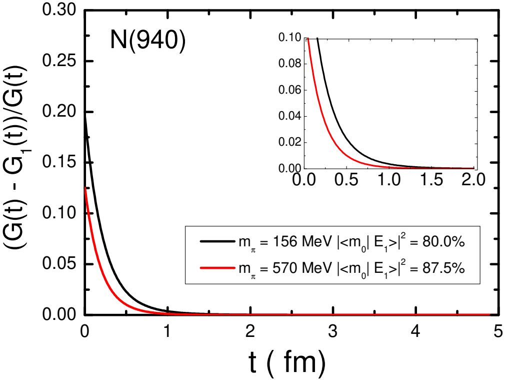
$$G(t) = \sum_i | \langle m_0 \, | \, E_i \rangle |^2 e^{-E_i \, t} \ .$$

- Hamiltonian EFT provides the eigenstate energies E_i and the overlaps $\langle m_0 \, | \, E_i \rangle$.

Even-Parity Nucleon Spectrum on 3 fm lattices

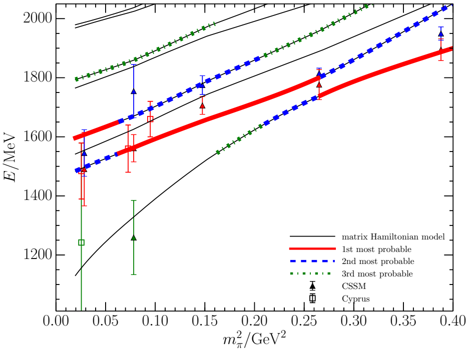
- The relative contamination is $(G(t) -$ states accessed in LQCD $)/G(t)$

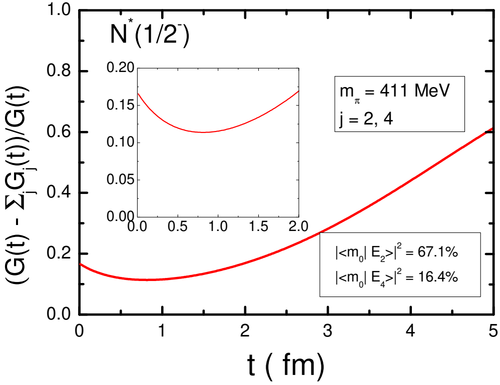




Odd-Parity Nucleon Spectrum on 3 fm lattices at $m_\pi = 411$ MeV

- The relative contamination is $\left(G(t) - \text{states accessed in LQCD}\right)/G(t)$





Conclusions

- The Parity Expanded Variational Analysis (PEVA) Technique is
  - Vital to the determination of excited state form factors.
  - Important in precision matrix elements of the nucleon.

Conclusions

- The Parity Expanded Variational Analysis (PEVA) Technique is
  - Vital to the determination of excited state form factors.
  - Important in precision matrix elements of the nucleon.
- The two low-lying odd-parity states of the nucleon spectrum
  - Are unlikely to be finite volume manifestations of a single resonance.
  - They have magnetic moments consistent with quark model expectations
    - For the $N^*(1535)$ and $N^*(1650)$.

Conclusions

- The Parity Expanded Variational Analysis (PEVA) Technique is
  - Vital to the determination of excited state form factors.
  - Important in precision matrix elements of the nucleon.
- The two low-lying odd-parity states of the nucleon spectrum
  - Are unlikely to be finite volume manifestations of a single resonance.
  - They have magnetic moments consistent with quark model expectations
    - For the $N^*(1535)$ and $N^*(1650)$.
- Non-trivial gluonic degrees of freedom are manifest in quark wave functions.

Conclusions

- The Parity Expanded Variational Analysis (PEVA) Technique is
  - Vital to the determination of excited state form factors.
  - Important in precision matrix elements of the nucleon.
- The two low-lying odd-parity states of the nucleon spectrum
  - Are unlikely to be finite volume manifestations of a single resonance.
  - They have magnetic moments consistent with quark model expectations
    - For the $N^*(1535)$ and $N^*(1650)$.
- Non-trivial gluonic degrees of freedom are manifest in quark wave functions.
- Future studies will need to address multi-particle scattering states in the spectrum.
  - Induce $\sim 10\%$ contaminations in correlation functions otherwise.
  - These form factors are required to connect lattice QCD to resonance structure.

References

- Parity Expanded Variational Analysis (PEVA) technique

  - F. M. Stokes, *et al.* (CSSM), Phys. Rev. D **92**, 114506 (2015) arXiv:1302.4152 [hep-lat]
  - F. M. Stokes, *et al.* (CSSM), PoS LATTICE **2016**, 161 (2016) arXiv:1701.07177 [hep-lat]

- Hamiltonian Effective Field Theory

  - J. j. Wu, *et al.* (CSSM), arXiv:1703.10715 [nucl-th]
  - Z. W. Liu, *et al.* (CSSM), Phys. Rev. D **95**, 034034 (2017) arXiv:1607.04536 [nucl-th]
  - Z. W. Liu, *et al.* (CSSM), Phys. Rev. Lett. **116**, 082004 (2016) arXiv:1512.00140 [hep-lat]
  - J. M. M. Hall, *et al.* (CSSM), Phys. Rev. D **87**, 094510 (2013) arXiv:1303.4157 [hep-lat]
  - Z. W. Liu, *et al.* (CSSM), Phys. Rev. D **95**, 014506 (2017), arXiv:1607.05856 [nucl-th]

- Wave Functions of Nucleon Excitations

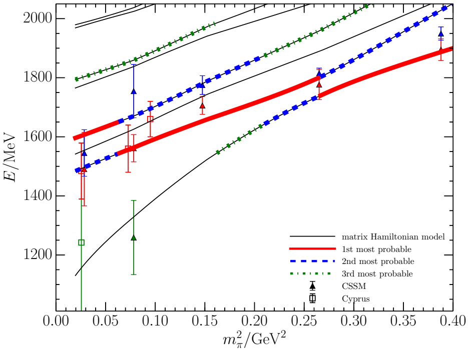
  - D. S. Roberts, *et al.* (CSSM), Phys. Rev. D **89**, 074501 (2014) arXiv:1311.6626 [hep-lat]

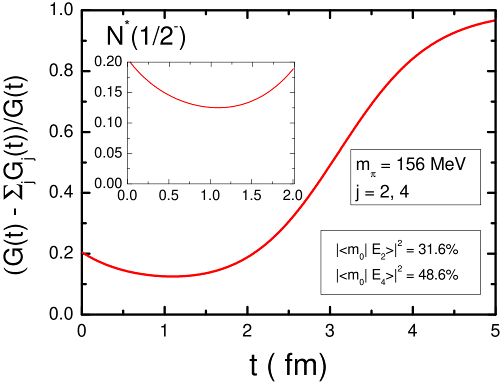
Supplementary Information

The following slides provide additional information which may be of interest.

Odd-Parity Nucleon Spectrum on 3 fm lattices at $m_\pi = 156$ MeV

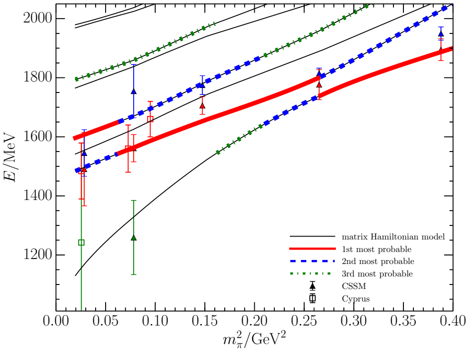
- The relative contamination is $(G(t) -$ states accessed in LQCD $)/G(t)$

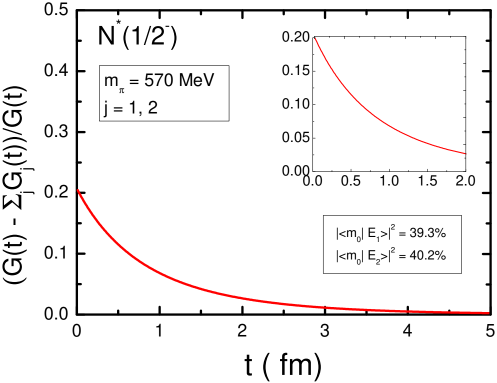




Odd-Parity Nucleon Spectrum on 3 fm lattices at $m_\pi = 570$ MeV

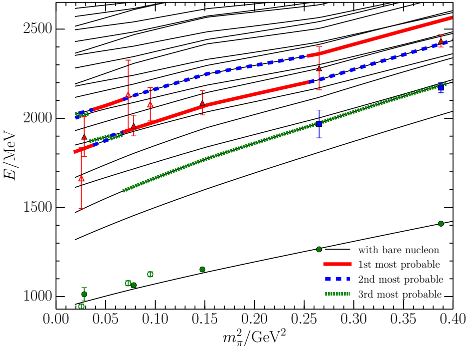
- The relative contamination is $(G(t) -$ states accessed in LQCD $)/G(t)$

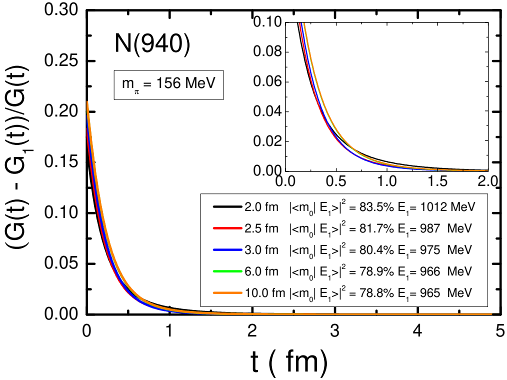




Even-Parity Nucleon Spectrum: Volume Dependence

- The relative contamination is $(G(t) - \text{states accessed in LQCD })/G(t)$





Even-Parity Nucleon Spectrum: Volume Dependence

- The relative contamination is $(G(t) -$ states accessed in LQCD $)/G(t)$