Lattice simulations of gravitational waves from non-abelian gauge fields at a tachyonic transition

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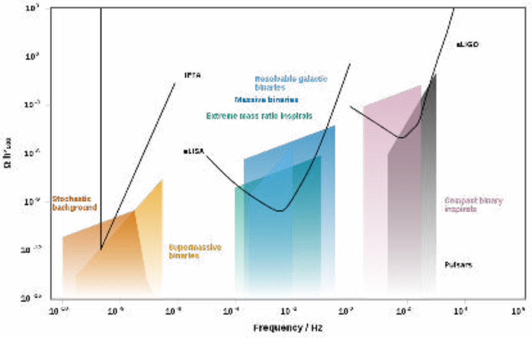
Gravitational waves

- Briefly: predicted by GR, detected by aLIGO
- Variety of origins: supernovae, neutron star, black hole coalescence
  - very different spectral shapes and amplitudes

  → observations reveal origin of the GW waves?



Ref. [Moore,Cole,Berry 2014]

- GW detectors work at frequencies of 1-100Hz, planned observatories from 10^{-3} Hz (LISA) to 10^{3} Hz (Advanced-LIGO)

Tachyonic Transition

- Reheating:
  - energy density converted into radiation and matter
  - the first stage of conversion: preheating
- Tachyonic preheating:
  - conversion into radiation almost instantaneously
  - production of dark matter particles, topological defects, ..?
  - But the most important: could produce gravitational waves within the frequency range of LISA
- Modelled by: self-interacting scalar fields that may be coupled to gauge fields
  - Previous simulations: scalar (+ abelian gauge fields)
- Our study: scalar + SU(2) non-abelian gauge fields
  - Non-abelian fields self-interact strongly $\rightarrow$ a significant contribution?

Model of tachyonic transition

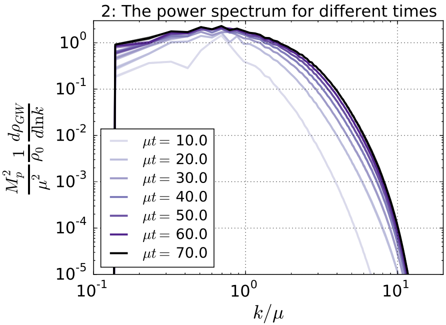
- GWs: perturbations of the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- After some algebra: $\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{TT}$
  - source of GWs: T_{ij}^{TT} = transverse traceless part of energy momentum tensor
- Potential in our model: $V(\phi) = V_0 + \mu_{\mathrm{eff}}^2(t)\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$, where $\mu_{\mathrm{eff}}^2(t) = \mu^2\left(1 - \frac{2t}{\tau_q}\right)$
  - τ_q = quench time, how fast is the transition

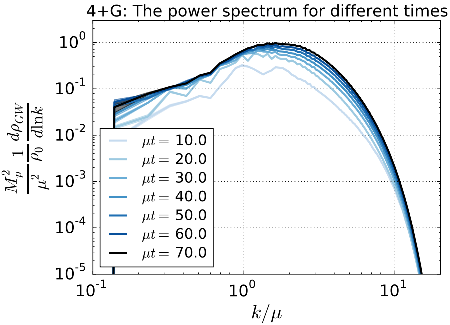
- Standard steps of numerical GW simulations:
  1. use real-time simulations to model nonperturbative field dynamics
  2. compute the spectrum and strength of the GW signal
  3. compare to the range of observation

On our present work

- Aim: to study the effect of including a non-abelian gauge field
- Notation:
  - "4+G": non-abelian gauge field + complex doublet
  - "2" and "4": complex singlet and complex doublet with no gauge fields
- We compute the spectrum of gravitational waves:
  - using different quench times
  - comparing "4+G" to reduced models "2" and "4"
  - varying λ
- We assume Standard Model-like theory: $g = 0.65$ and $\lambda = 0.13$
- We use real time, non-equilibrium, classical lattice simulations with leapfrog updates
- Lattice simulation parameters:
  - Lattice size: $N^3 = 384^3$
  - Lattice spacing: $a\mu = 0.17$ with $m_H = \sqrt{2}\mu$

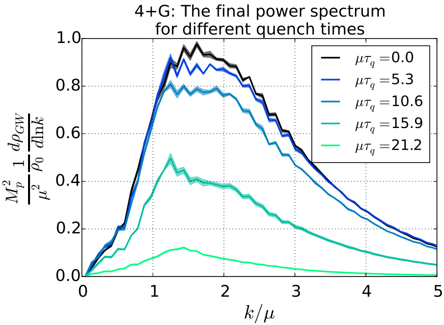
Results: "2" and "4+G" power spectra for different times

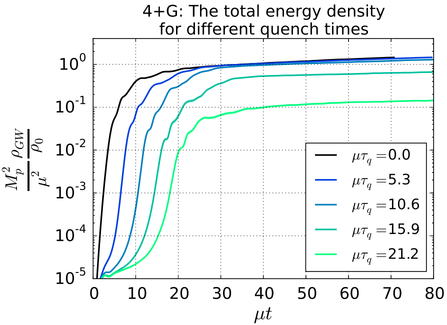




- Evolve until spectrum doesn't change
- The energy density:
  - normalised by vacuum energy $\rho_0 = \lambda v^4/4$, where $v = \mu/\sqrt{\lambda}$
  - scaled by the prefactor M_p^2/μ^2, where M_p is the Planck mass

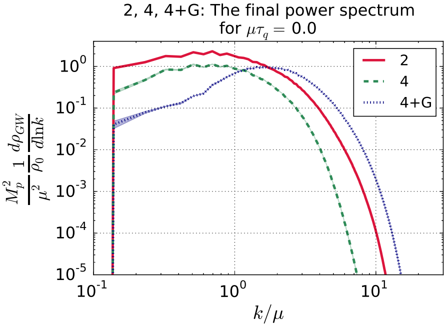
Results: Dependence of the quench time, "4+G"

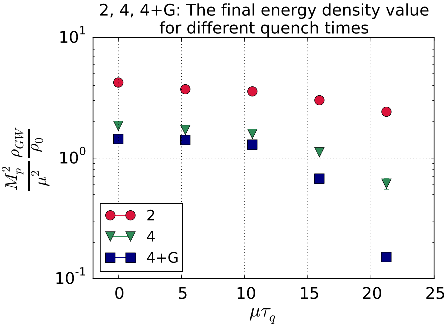




- Five different quench times, $\mu t_{final} = \mu\tau_q + 70$
- Peak value of the final power spectrum decreases with longer quench times
- The violent transition causes the gravitational energy to grow exponentially until a time $\mu t \simeq 10$ after the quench

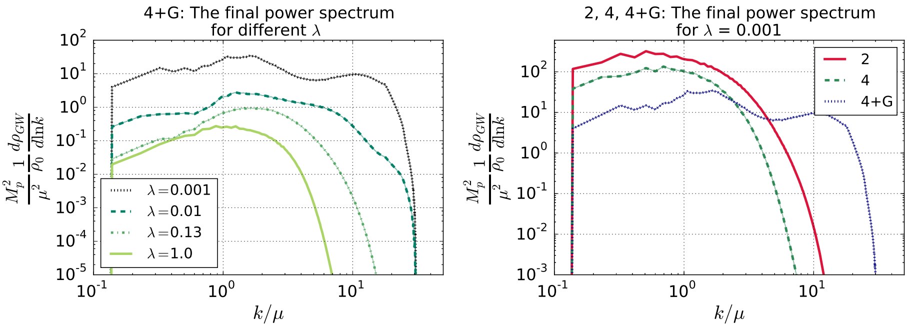
Results: Dependence to quench time, all models





- Maximum at $k/\mu \simeq 0.7$ for "2" and "4", and $k/\mu \simeq 1.5$ for "4+G"
- "2" has about two times higher peak amplitude
- Final energy density: first only little dependence on quench time, then decreases with quench time
  $\rightarrow$ if quench is below a certain cut-off: the time-scale of the dynamics is the spinodal roll-off itself

Results: varying λ



- In SM: $\frac{m_H}{m_W} = \sqrt{\frac{8\lambda}{g^2}} \simeq 1.57$, but changes if varying λ
  $\Rightarrow$ a way to examine the two mass-scales
- When λ decreases (m_W increases): the amplitude increases and the peak resolves into two distinct peaks
  - the second peak due to the gauge field; compare to "2" and "4" with $\lambda = 0.001$

Results: converted to physical units

- The frequency f today (using $\lambda = 0.13$):

$$f = 4 \times 10^{10}\ \mathrm{Hz} \left(\frac{k}{\mu}\right) (4\lambda)^{1/4} = 3.4 \times 10^{10}\ \mathrm{Hz} \times \frac{k}{\mu}$$

- The amplitude of the spectrum:

$$\Omega_{\mathrm{gw}} h^2 = 9.3 \times 10^{-6} \times \frac{1}{\rho_0} \frac{d\rho_{\mathrm{GW}}}{d \ln k}$$

- If assuming SM: $\mu \simeq$88 GeV $\Rightarrow \frac{\mu^2}{M_{\mathrm{p}}^2} = 1.3 \times 10^{-33}$

- Our maximum signal at $k/\mu \simeq 1.5$ corresponds to 5×10^{10} Hz
- Our peak amplitude at $\Omega_{\mathrm{gw}} h^2 = 9.3 \times 10^{-6} \left(\frac{\mu}{M_{\mathrm{p}}}\right)^2$

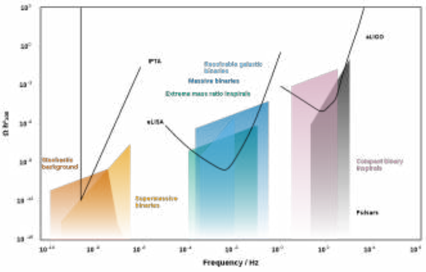
$\Rightarrow$ Far off from the future detectors' ranges :(

Conclusions

- Idea: to study the effect of including a non-abelian gauge field
- Use: real-time, classical simulations
- Compare "4+G" to reduced models, vary: quench time, λ
- Results:
  - max signal at 5×10^{10} Hz
  - the peak amplitude at

    $$\Omega_{\rm gw} h^2 = 9.3 \times 10^{-6} \left(\frac{\mu}{M_{\rm p}}\right)^2$$

    $\rightarrow$ refers to 10^{-38} for a electroweak scale and 10^{-12} for a GUT-scale



Ref. [Moore,Cole,Berry 2014]

- Final conclusion: results far off from the future detectors
  $\Rightarrow$ tachyonic preheating will not be observable at LISA