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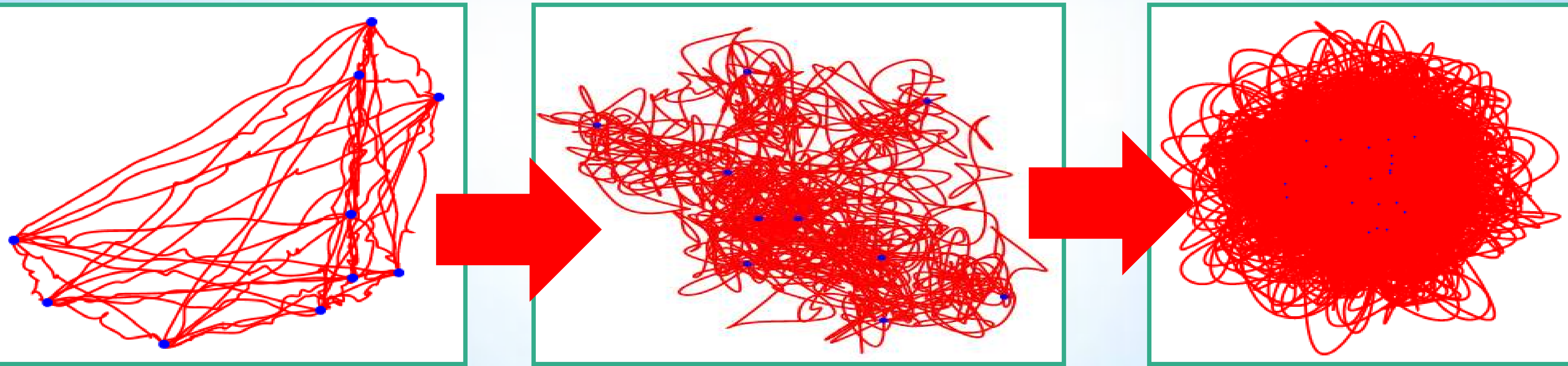
Black hole evolution from real-time simulations of BFSS matrix model

(in collaboration with M. Hanada, A. Schäfer)



Motivation

Dimensionally reduced (9+1) dimensional N=1 Super-Yang-Mills in (d+1) dimensions

$$S_{(d+1)} = \int d^{d+1}x \,\mathrm{Tr}\left(\frac{1}{4}F_{AB}^2 + \frac{i}{2}\bar{\psi}\not{D}\psi + \right.$$
$$\left. + \frac{1}{2}(D_A X_\mu)^2 - \frac{1}{4}[X_\mu, X_\nu]^2 + \frac{1}{2}\bar{\psi}\tilde{\gamma}^\mu[X_\mu, \psi]\right)$$

A,B = 0…d, μ,ν = d+1…9

N x N hermitian matrices

Majorana-Weyl fermions, N x N hermitian matrices

"Holographic" duality [Witten'96]:

- X^{ii}_μ = Dp brane positions
- X^{ij}_μ = open string excitations

Motivation

N=1 Supersymmetric Yang-Mills in D=1+9:

gauge bosons+adjoint Majorana-Weyl fermions

Reduce to a single point = BFSS matrix model

[Banks, Fischler, Shenker, Susskind'1997]

$$L = \frac{1}{2g}\left[\mathrm{tr}\dot{X}^i\dot{X}^i + 2\theta^T\dot{\theta} - \frac{1}{2}\mathrm{tr}[X^i, X^j]^2 - 2\theta^T\gamma_i[\theta, X^i]\right]$$

N x N hermitian matrices

Majorana-Weyl fermions, N x N hermitian

System of *N* D0 branes joined by open strings

Motivation

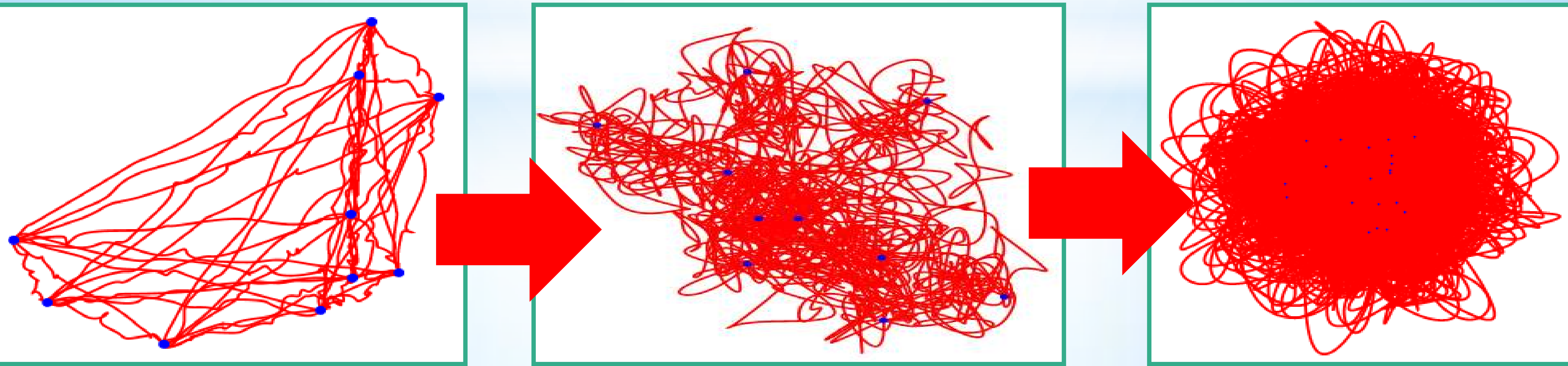
Classical equations of motion are chaotic

[Saviddy; Berenstein;Susskind;Hanada;…]

- **Positive Lyapunov exponents**
- **X_μ matrices forget initial conditions**
- **Kolmogorov entropy produced**

Stringy interpretation:

black hole formation



Motivation

So far, mostly classical simulations …

Quantum effects?

$$\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$$

- Thermalization and scrambling?
- Fast thermalization in QCD plasma?
- Evaporation of black holes [Hanada'15]
- Quantum entanglement?

In this talk:

Numerical methods to look at these problems (not "solve exactly")

- **Beyond classical simulations: quantum fluctuations of "gauge" fields, thermalization and scrambling**
- **Classical-statistical approximation: effect of fermions → black hole "evaporation"**

Quantum equations of motion

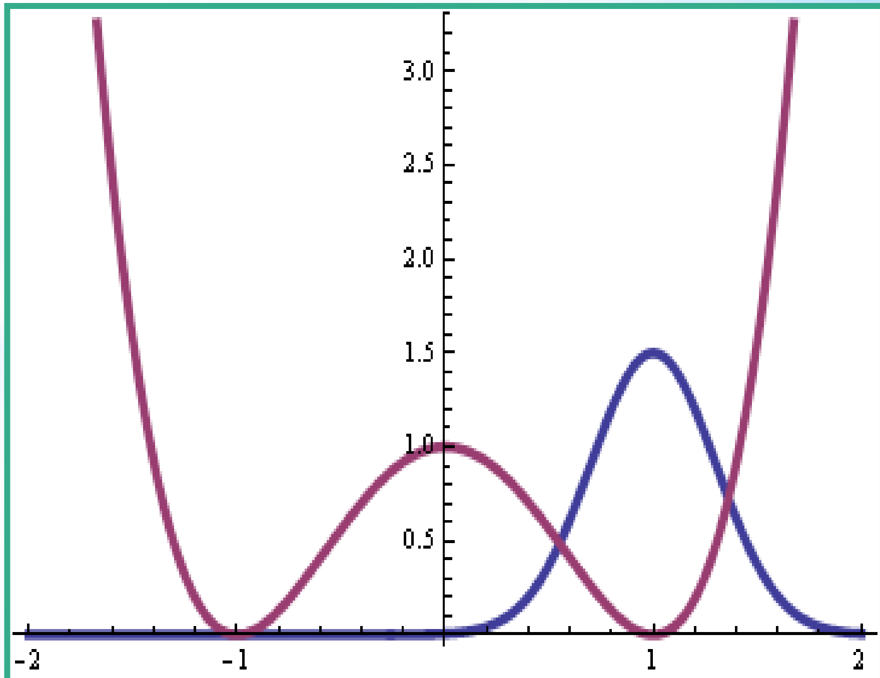
- Start with Heisenberg eqs of motion
- VEV with more non-trivial correlators

First, test this idea on the simplest example

$$\hat{H} = \frac{\hat{p}^2}{2} + \frac{a\hat{x}^2}{2} + \frac{b\hat{x}^3}{3} + \frac{c\hat{x}^4}{4}$$

$$\partial_t \hat{x} = \hat{p},$$
$$\partial_t \hat{p} = -a\hat{x} - b\hat{x}^2 - c\hat{x}^3$$

Tunnelling between potential wells?



Next step: Gaussian Wigner function

Assume Gaussian wave function <u>at any t</u>

Translates to <u>Gaussian Wigner function</u>

$$\langle \hat{x}^2 \rangle = x^2 + \sigma_{xx},$$
$$\langle \hat{p}^2 \rangle = p^2 + \sigma_{pp},$$
$$\langle \frac{\hat{x}\hat{p}+\hat{p}\hat{x}}{2} \rangle = xp + \sigma_{xp}$$

For other correlators: use Wick theorem!

$$\langle \hat{x}^4 \rangle = x^4 + 6x^2\sigma_{xx} + 3\sigma_x x^2,$$
$$\langle \hat{x}^2\hat{p} \rangle = x^2 p + 2x\sigma_{xp} + p\sigma_{xx}$$

Derive closed equations for

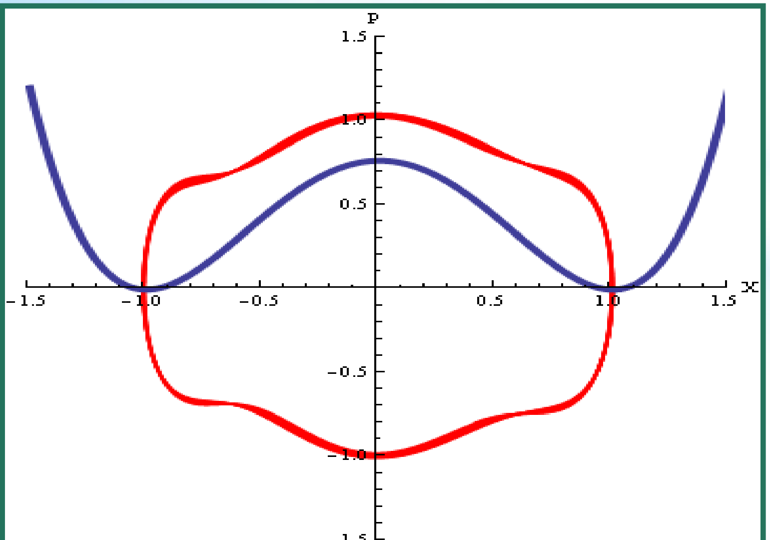
x, p, σ_{xx} , σ_{xp} , σ_{pp}

Quantum "force" and tunnelling

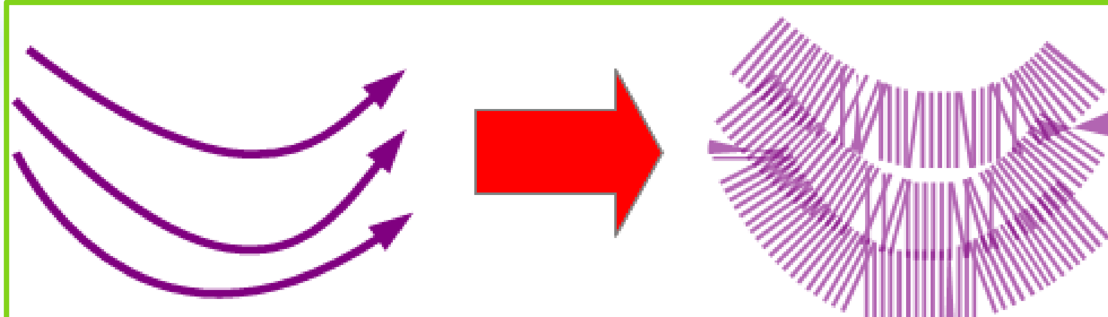
$$\partial_t p = -ax - bx^2 - cx^3 - b\sigma_{xx} - 3cx\sigma_{xx},$$
$$\partial_t x = p$$

Positive force even at *x=0* (classical minimum)

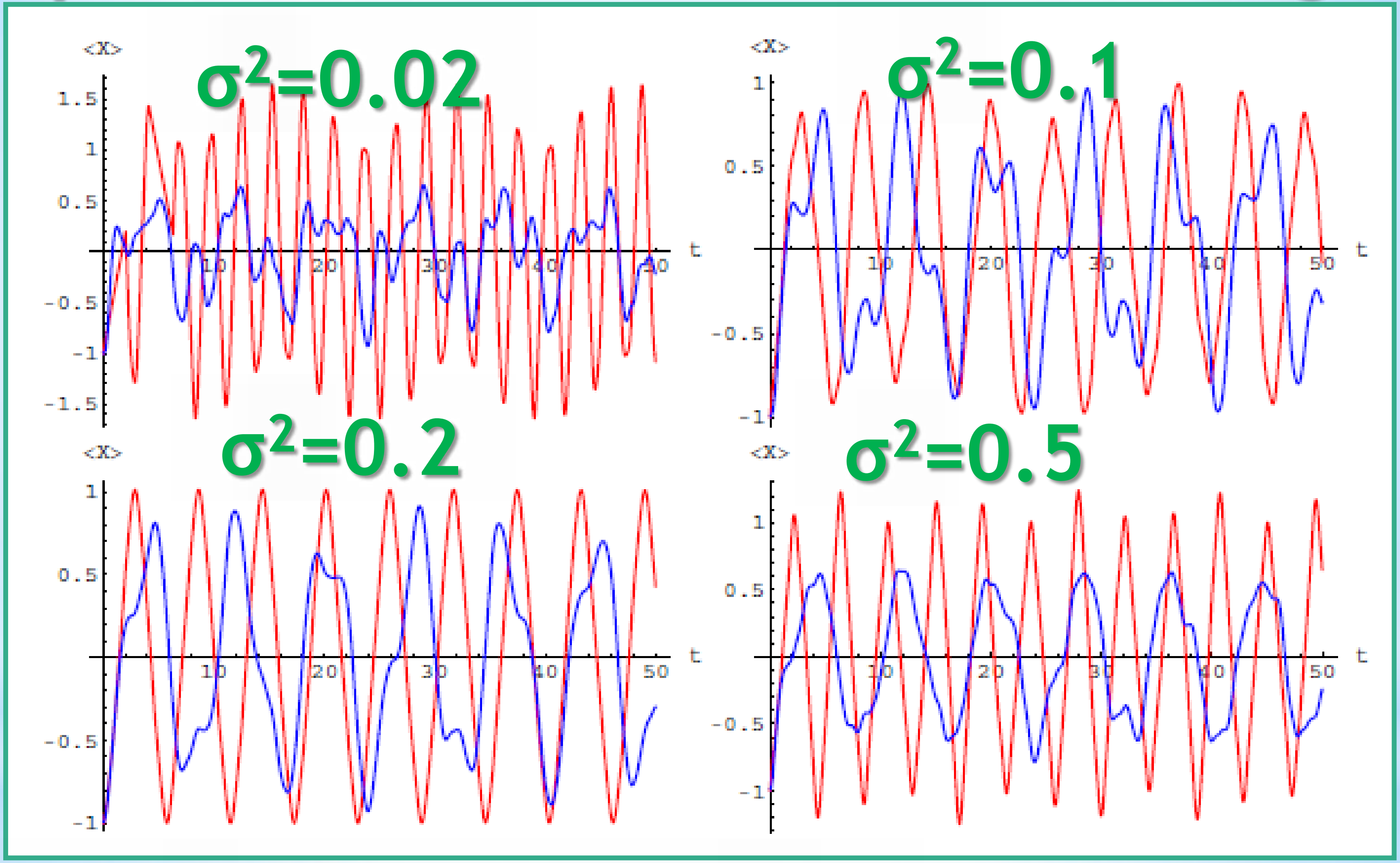
$$\partial_t \sigma_{xx} = 2\sigma_{xp},$$
$$\partial_t \sigma_{xp} = \sigma_{pp} - a\sigma_{xx} - 2bx\sigma_{xx} - 3cx^2\sigma_{xx} - 3c\sigma_{xx}^2,$$
$$\partial_t \sigma_{pp} = -2\left(a\sigma_{xp} + 2bx\sigma_{xp} + 3cx^2\sigma_{xp} + 3c\sigma_{xx}\sigma_{xp}\right)$$



Quantum force pushes <x> away from minimum



Improved CSFT vs exact Schrödinger



- **Early-time evolution OK**
- **Tunnelling period qualitatively OK**

Gaussian approx. for BFSS model

$$\partial_t P^a_i = -C_{abc} C_{cde} X^b_j X^d_i X^e_j - \frac{i}{2} C_{bac} \sigma^i_{\alpha\beta} \langle \psi^b_\alpha \psi^c_\beta \rangle -$$
$$- C_{abc} C_{cde} X^b_j [XX]^{de}_{ij} - C_{abc} C_{cde} [XX]^{be}_{jj} X^d_i - C_{abc} C_{cde} [XX]^{bd}_{ji} X^e_j$$

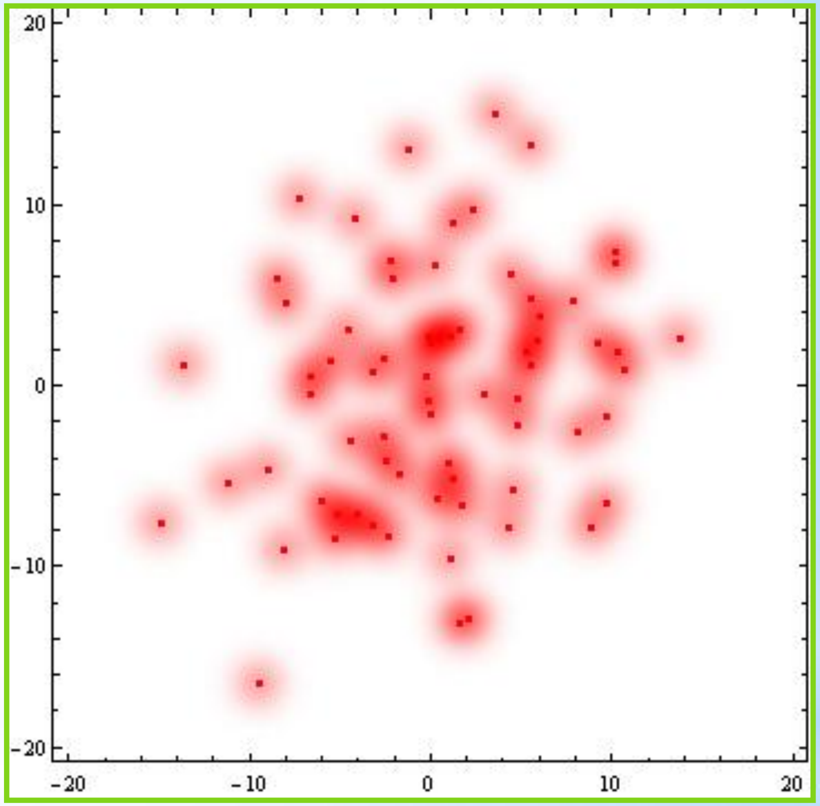
$$\partial_t [XX]^{ab}_{ij} = [XP]^{ab}_{ij} + [XP]^{ba}_{ji},$$
$$\partial_t [XP]^{af}_{ik} = [PP]^{af}_{ik} - C_{abc} C_{cde} \left(X^d_i X^e_j + [XX]^{de}_{ij} \right) [XX]^{bf}_{jk} -$$
$$- C_{abc} C_{cde} \left(X^b_j X^e_j + [XX]^{be}_{jj} \right) [XX]^{df}_{ik} -$$
$$- C_{abc} C_{cde} \left(X^b_j X^d_i + [XX]^{bd}_{ji} \right) [XX]^{ef}_{jk},$$
$$\partial_t [PP]^{af}_{ik} = -C_{abc} C_{cde} \left(X^d_i X^e_j + [XX]^{de}_{ij} \right) [XP]^{bf}_{jk} -$$
$$- C_{abc} C_{cde} \left(X^b_j X^e_j + [XX]^{be}_{jj} \right) [XP]^{df}_{ik} -$$
$$- C_{abc} C_{cde} \left(X^b_j X^d_i + [XX]^{bd}_{ji} \right) [XP]^{ef}_{jk} + (\{a, i\} \leftrightarrow \{f, k\})$$

- **CPU time ~ *N^5* (double commutators)**
- **RAM memory ~ *N^4***
- ***Conserves pure states & entropy***

Initial conditions

$\langle X^a_i \rangle$ are random Gaussian

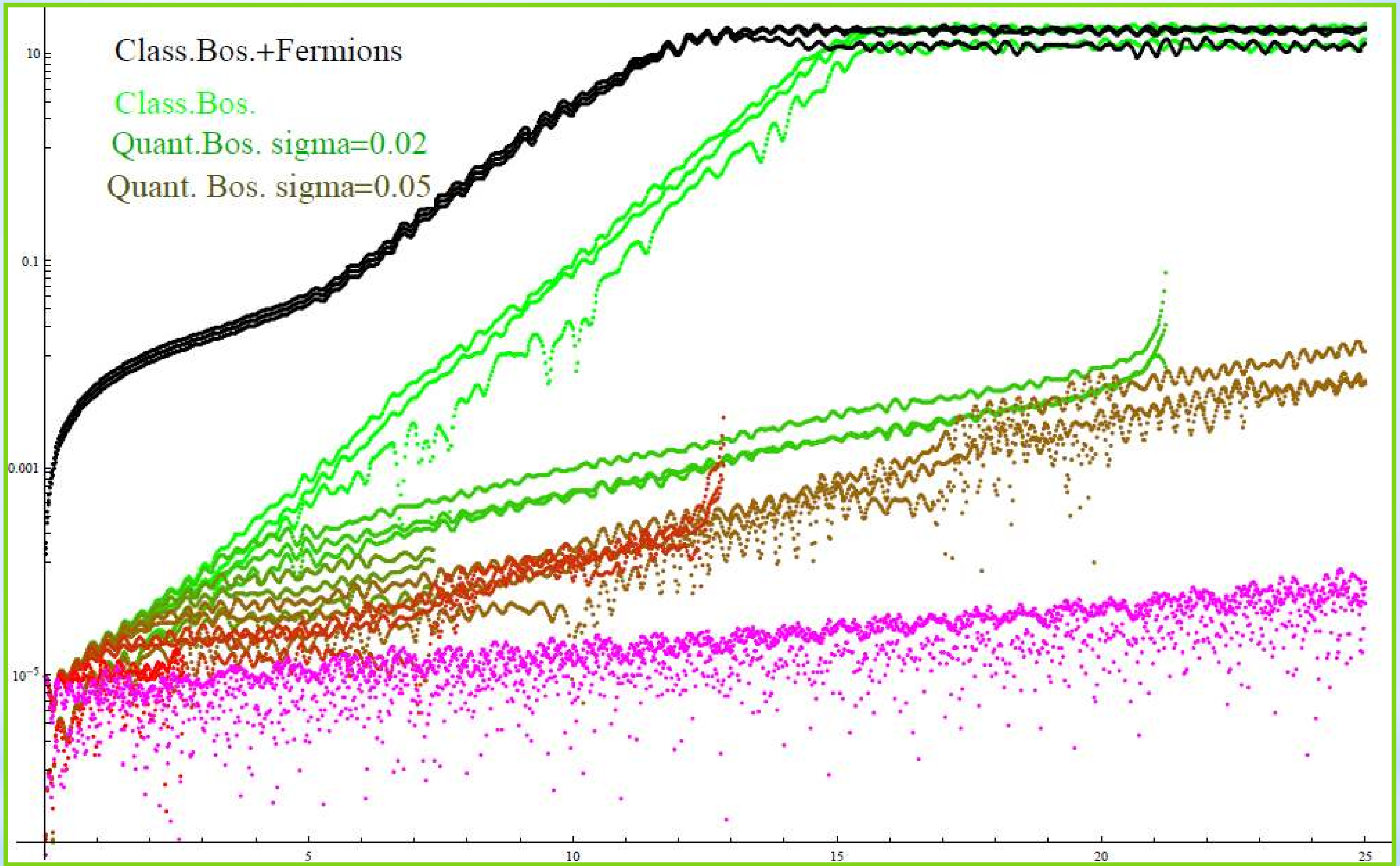
$$[XX]^{ab}_{ij} = \sigma^2\,\delta^{ab}\delta_{ij},$$
$$[PP]^{ab}_{ij} = \sigma^2\,\delta^{ab}\delta_{ij},$$
$$[XP]^{ab}_{ij} = 0$$



Minimal quantum dispersion (Uncertainty principle)

(Classical) dispersion of $\langle X^a_i \rangle$ roughly corresponds to temperature

Quantum effects decrease instability

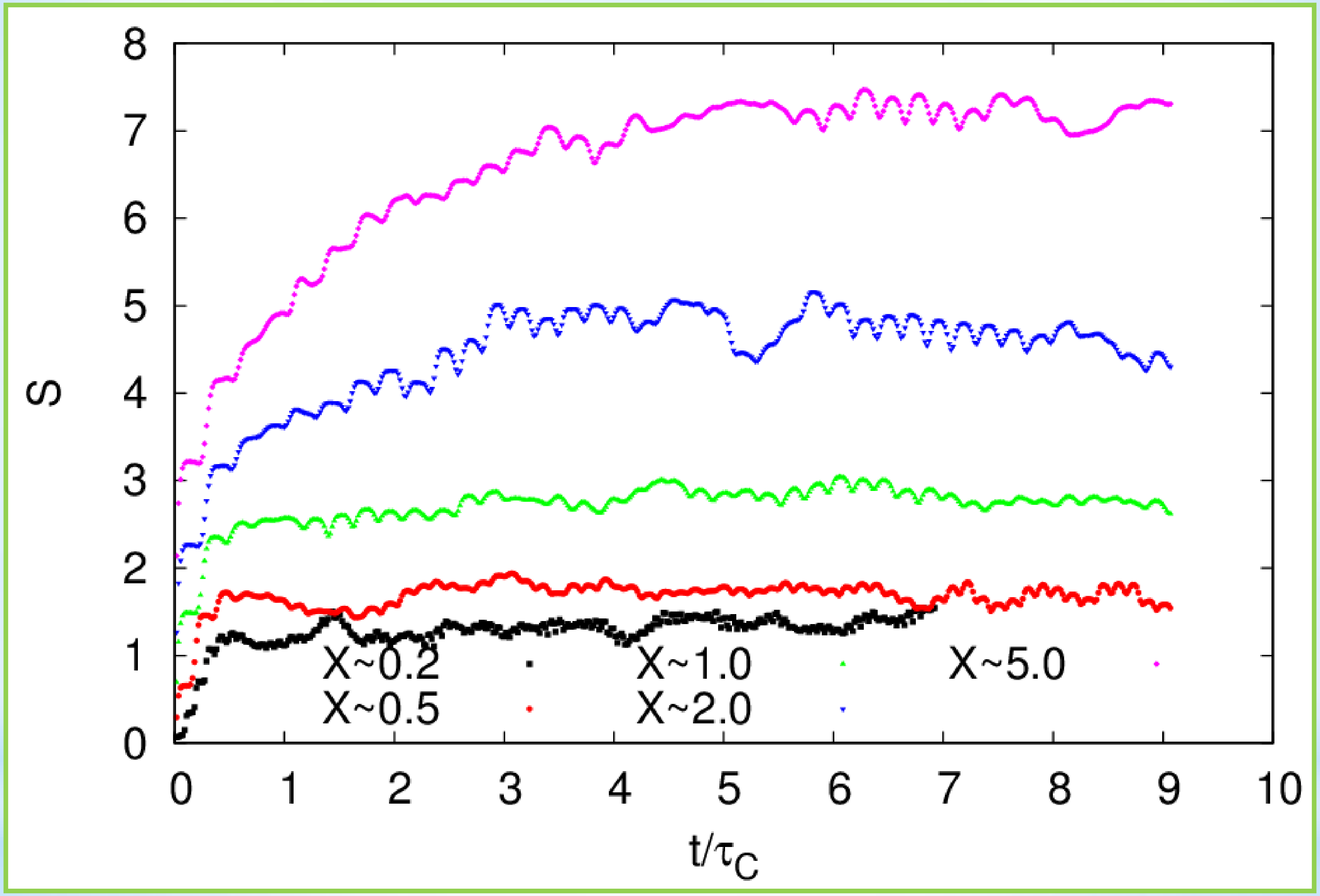


$$\langle in|\, e^{i\hat{p}\delta x}\hat{x}(t)\, e^{-i\hat{p}\delta x}\,|in\rangle - \langle in|\,\hat{x}(t)\,|in\rangle =$$

$$= i\langle in|\,[\hat{p}(0), \hat{x}(t)]\,|in\rangle$$

Lyapunov exponents ~ OTO correlators !!!

Entanglement entropy production



Single matrix entry entangled with others

Initially, *dS/dt* scales as classical Lyapunov t

Effect of fermions

Fermions introduce repulsive force

Evolution of fermionic 2pt correlators!

Closed system of equations: $\partial_t X_i^a = P_i^a$

$$\partial_t P_i^a = -C_{abc} C_{cde} X_j^b X_i^d X_j^e - \frac{i}{2} C_{bac} \sigma^i_{\alpha\beta} \langle \hat{\psi}^b_\alpha \hat{\psi}^c_\beta \rangle$$

$$\partial_t \langle \hat{\psi}^a_\alpha \hat{\psi}^b_\beta \rangle = C_{ade} X_i^d \sigma^i_{\alpha\gamma} \langle \hat{\psi}^e_\gamma \hat{\psi}^b_\beta \rangle + C_{bde} X_i^d \sigma^i_{\beta\gamma} \langle \hat{\psi}^a_\alpha \hat{\psi}^e_\gamma \rangle$$

CPU time + RAM memory scaling ~ N^4

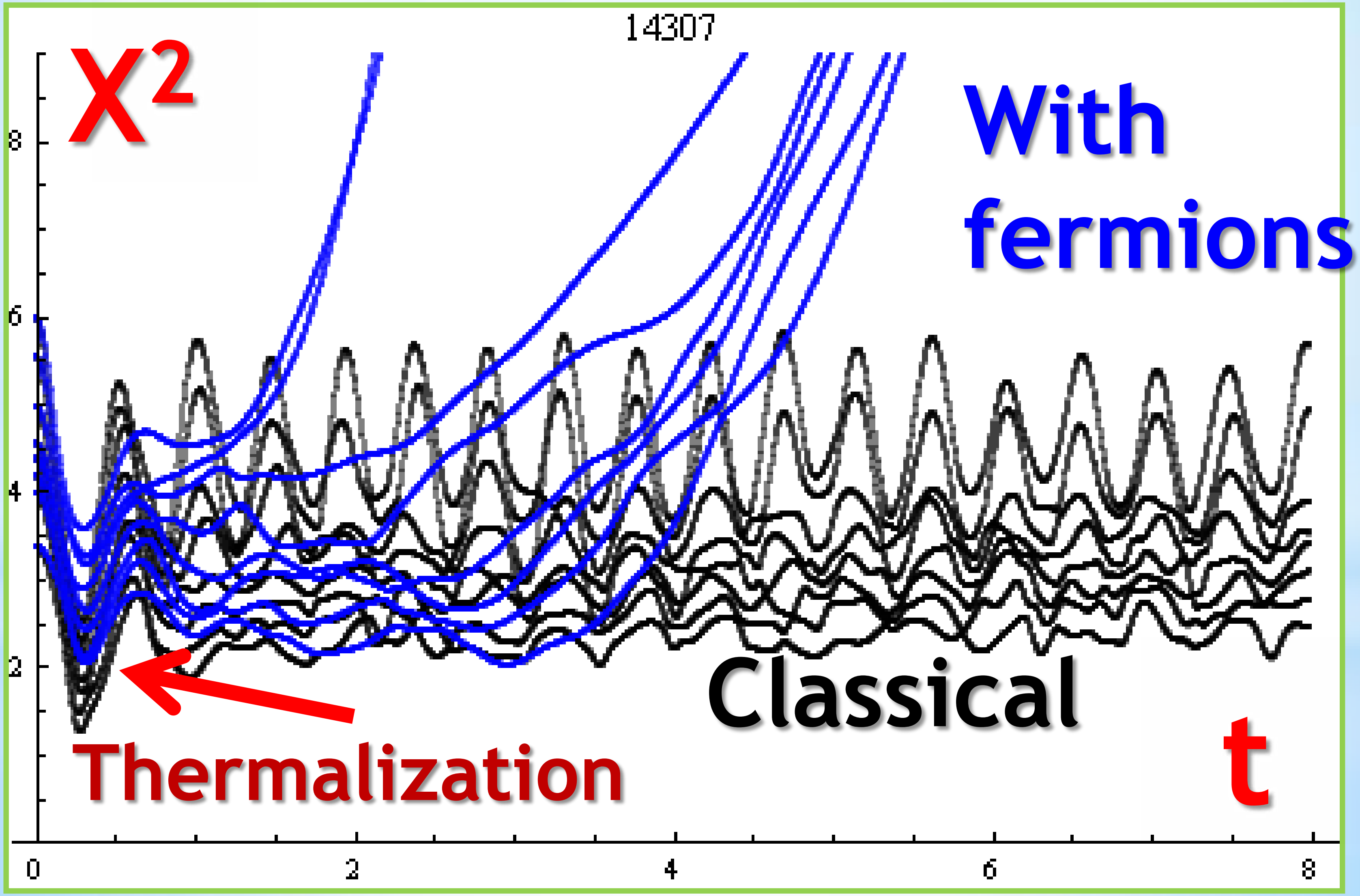
In fact, CSFT approximation

[Son, Aarts, Smit, Berges, Tanji, Gelis,…]

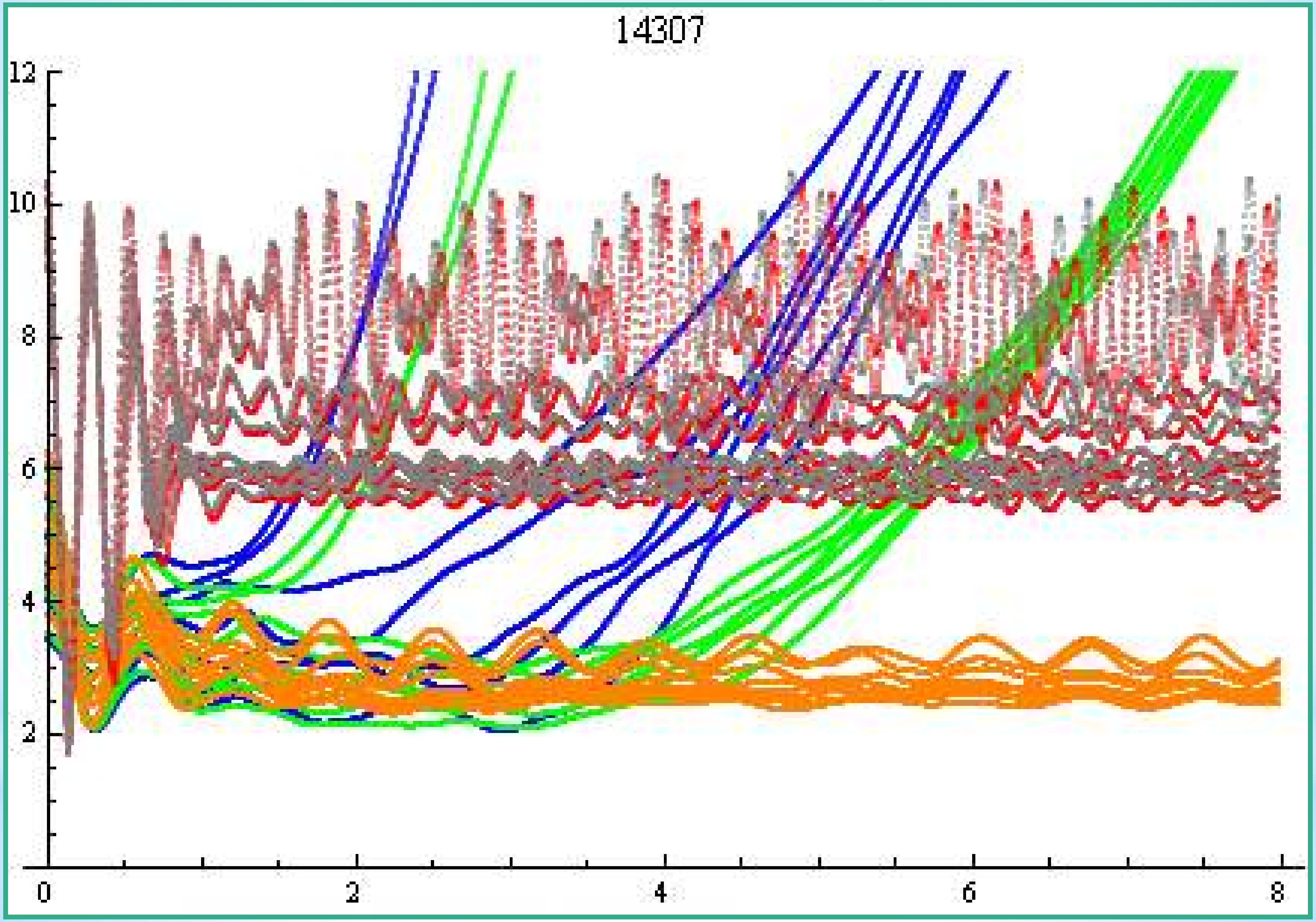
E.g. Schwinger pair production, Axial charge creation in glasma, …

Some results N = 8, f = 0.48

Hawking radiation of D0 branes

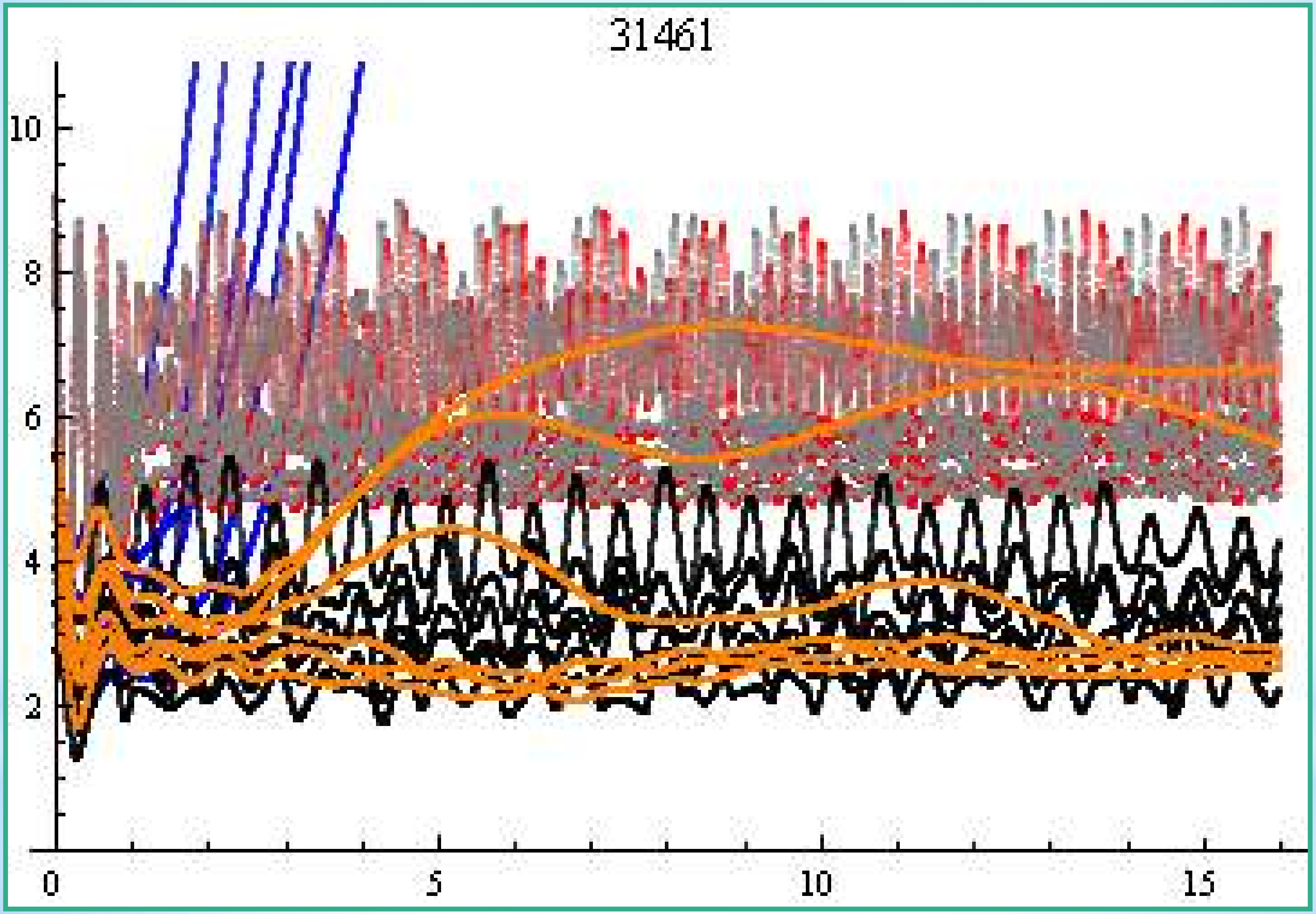


Some results N = 8, f = 0.48



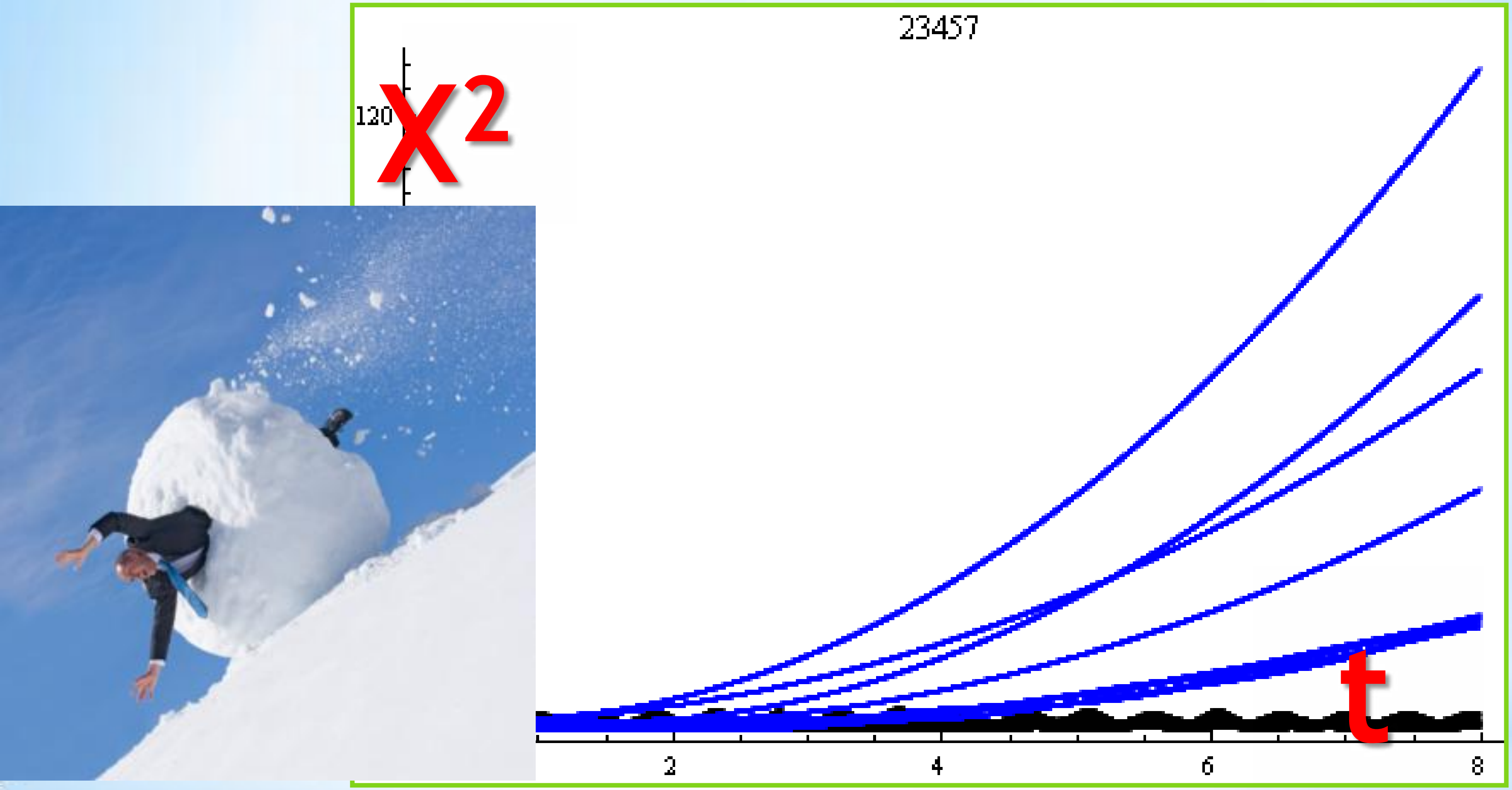
σ2 = 0.0, 0.1, 0.2, 1.0 + no fermions

Evaporation threshold: N = 6, f = 0.50



$\sigma 2$ = 0.0, 0.2, 1.0 + no fermions

Constant acceleration at late times



- Boson kinetic energy grows without bound
- Fermion energy falls down
- Origin of const force – SUSY violation?

Summary

- **Quantum fluctuations suppress classical Lyapunov exponents**
- **Scrambling time still proportional to classical Lyapunov exponents**
- **Quantum fermions in BFSS model trigger real-time instability, “black hole evaporation”?**

Backup slides

CSFT and SUSY

16 supercharges in BFSS model:

$$\hat{Q}_\alpha = \hat{P}_i^a \left[\sigma_i\right]_{\alpha\beta} \hat{\psi}_\beta^a - \frac{1}{4} C_{abc} \hat{X}_i^b \hat{X}_j^c \left[\sigma_{ij}\right]_{\alpha\beta} \hat{\psi}_\beta^a$$

$$\sigma_{ij} \equiv \sigma_i \sigma_j - \sigma_j \sigma_i$$

$$\left\{\hat{Q}_\alpha, \hat{Q}_\beta\right\} = 2\delta_{\alpha\beta} \hat{H} - 2\left(\sigma_i\right)_{\alpha\beta} \hat{X}_i^a \hat{J}^a$$

$$\left[\hat{H}, \hat{Q}_\gamma\right] = -i \hat{\psi}_\gamma^a \hat{J}^a$$

Gauge transformations

$$\hat{J}^a = C_{abc} \hat{X}_i^b \hat{P}_i^c - \frac{i}{2} C_{abc} \hat{\psi}_\alpha^b \hat{\psi}_\alpha^c$$

CSFT and SUSY

In full quantum theory

$$\partial_t \hat{Q}_\delta = \frac{i}{2} C_{abc} \hat{\psi}^a_\alpha \hat{\psi}^b_\beta \hat{\psi}^c_\gamma \left(\sigma^i_{\alpha\beta} \sigma^i_{\gamma\delta} - \delta_{\alpha\beta}\delta_{\gamma\delta} \right) = 0$$

Fierz identity (cyclic shift of indices):

$$[\sigma_i]_{\alpha\beta} [\sigma_i]_{\gamma\delta} + [\sigma_i]_{\alpha\gamma} [\sigma_i]_{\beta\delta} + [\sigma_i]_{\alpha\delta} [\sigma_i]_{\gamma\beta} = \\ = \delta_{\alpha\beta}\delta_{\gamma\delta} + \delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\gamma\beta}$$

In CSFT approximation

$$\partial_t \hat{Q}_\delta = \frac{i}{2} C_{abc} \langle \hat{\psi}^a_\alpha \hat{\psi}^b_\beta \rangle \hat{\psi}^c_\gamma \left(\sigma^i_{\alpha\beta} \sigma^i_{\gamma\delta} - \delta_{\alpha\beta}\delta_{\gamma\delta} \right) \neq 0$$

Fermionic 3pt function seems necessary!