Real-time CSFT simulations of chiral plasma with overlap fermions

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Outline

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Why studying chiral plasma ?

- Important in lot of different fields:
  - High energy physics:
    - Quark-Gluon Plasma
    - Hadronic matter
    - Cosmology of the early universe
  - Condensed matter phenomena
    - Weyl semi-metals
    - Topological insulators
    - Liquid Helium
- We understand very little about it beyond hydrodynamics and kinetic approaches

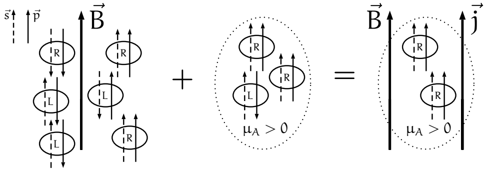
Anomalous transport phenomena

- Chiral imbalance (axial charge $\neq$0)

  $\Rightarrow$Anomalous transport phenomena!
- Chiral Separation Effect, Chiral Vortex effect, ...
- Chiral Magnetic Effect (CME):

$$\vec{j}_V = \sigma_{CME} \vec{B}$$

with $\sigma_{CME} = qN_c\mu_A/2\pi^2$: the CM conductivity



See: axXiv:1511.04050

Anomalous Maxwell Equation

Mawell eq. + ohmic conductivity + CME

- $\partial_t \vec{B} = -\vec{\nabla} \times \vec{E}$
- $\partial_t \vec{E} = -\vec{\nabla} \times \vec{B} - \sigma \vec{E} - \sigma_{CME} \vec{B}$

Applied to plane waves:

Mawell eq. + ohmic conductivity + CME

- $iw\vec{B} = -i\vec{k} \times \vec{E}$
- $iw\vec{E} = -\vec{k} \times \vec{B} - \sigma \vec{E} - \sigma_{CME} \vec{B}$

Dispersion relation:

$$w = \frac{i\sigma}{2} \pm \sqrt{k^2 \pm \sigma_{CME} k - \frac{\sigma^2}{4}}$$

Chiral Plasma instability

$$w = \frac{i\sigma}{2} \pm \sqrt{k^2 \pm \sigma_{CME} k - \frac{\sigma^2}{4}}$$

Instable Solutions

- $E_x = f e^{\gamma t} cos(kz)$ $\quad E_y = -f e^{\gamma t} sin(kz)$ $\quad E_z = 0$
- $B_x = -f \frac{k}{\kappa} e^{\gamma t} cos(kz)$ $\quad B_y = f \frac{k}{\kappa} e^{\gamma t} sin(kz)$ $\quad B_z = 0$
- $\gamma = -\frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \sigma_{CME} k - k^2}$

- For $\sigma_{CME} > |k|$: $\gamma > 0$ and the amplitude of solution grows exponentially.
- What can stop the instability ? The decay of the axial charge

$$\partial_t Q_A = \frac{g^2}{2\pi^2} \int d^3x \, \vec{E}.\vec{B} < 0 \Rightarrow \partial_t \sigma_{CME} < 0$$

Different Approach

- Previous approach is far too simple as the k and w dependences of σ and σ_{CME} have been neglected
- Other approaches used so far:
  - Chiral kinetic theory (linear response, relaxation time, long - wavelength...)
  - Chiral Hydrodynamics (long - wavelength)
  - ...
- What has not been taken into account:
  - Non-trivial dispersion of conductivities
  - Developing (axial) charge inhomogeneities
  - Non-linear responses

$\Rightarrow$ We can used simulations

Outline

CSFT simulations

- CSFT: Classical Statistical Field Theory: Out of equilibrium simulation.
- Idea: Semi-classical approximation along Keldysh contour

more details in [Phys. Rev. D 94, 025009]

- In practice: solving the following set of equations:

$$\begin{aligned}
\partial_t^2 A_{x,i}(t) &= -J_{ext,x,i}(t) - \langle j_{x,i}(t)\rangle - \partial_j F_{x,ij}(t) \\
\langle j_{x,i}(t)\rangle &= \sum_n n_f(\varepsilon_n)\psi^\dagger_{n,x}(t) j_{x,i}(t)\psi_{n,x}(t) \\
\partial_t \psi_{n,x}(t) &= -i h_{x,y}\psi_{n,y}(t) \\
j_{x,i} &= \partial h_{x,y}/\partial A_{x,i}
\end{aligned}$$

with $h_{x,y}$ the one particle Hamiltonian.

- Solving equation using Leapfrog algorithm (symplectic integrator: energy conservation under control)

Wilson Dirac and Overlap Hamiltonian

Simulations with Wilson-DIrac and Overlap Hamiltonian.

- Wilson-Dirac Hamiltonian:

$$h^{WD}_{x,y}(m) = (3v_f\beta + m)\delta_{x,y} + \frac{iv_f}{2}\sum_{i=1}^{3}(i\beta + \alpha_i)e^{igA_{x,i}}\delta_{y,x+e_i} + \frac{iv_f}{2}\sum_{i=1}^{3}(i\beta - \alpha_i)e^{-igA_{x-ei,i}}\delta_{y,x-e_i}$$

- Overlap Hamiltonian:

$$h^{OV}_{x,y}(m) = \left(1 + \frac{m}{2\rho}\right)\beta + \left(1 - \frac{m}{2\rho}\right)\text{sign}\left(h^{WD}_{x,y}(\rho)\right)$$

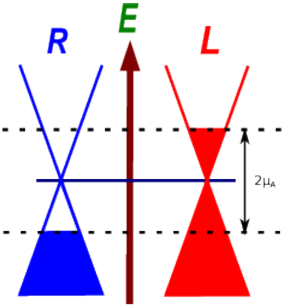
  - Important as chiral related problems.
  - Approximation for the sign function: Polynomial and Zolotarev
  - Comparison of computation time:

$$\text{Exact} \gg \text{Polynomial} > \text{Zolotarev}$$

Introducing chiral imbalance and excitations

Insertion of the chiral imbalance:

- left-handed eigenstates > right-handed eigenstates



Insertions of seeds of instabilities:

- n modes

$$A_{x,i}(t=0) = \sum_{m=1}^{n} f \frac{\cos(k_m x_3 + \phi_m)}{\sqrt{4\sin^2(k_m/2)}} \vec{n}_m$$

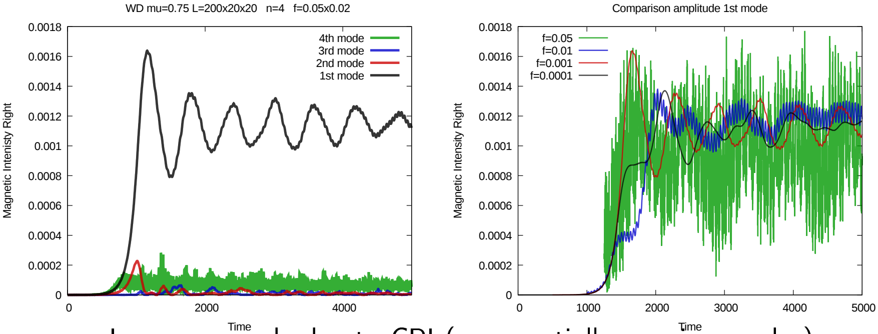
- Random polarization ($\vec{n}_m \in \{\vec{x}, \vec{y}\}$), same energy (same f)
- To have instable mode:

$$\sigma_{CME} > k \Rightarrow L > (4\pi^3)/\mu_A \quad (L > 125 \text{ for } \mu_A = 1)$$

Outline

1. Introduction
   - Motivation
   - Chiral Plasma Instability
2. Program
3. Numerical results
   - Helical modes
   - Comparison Wilson-Dirac and Overlap results
   - Axial charge

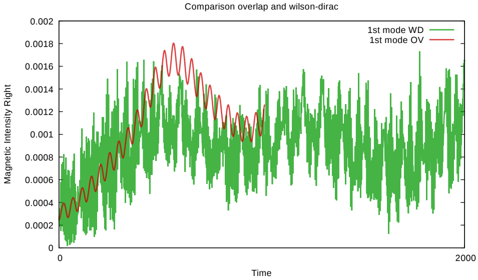
Helical modes



- Inverse cascade due to CPI (exponentially growing modes) with saturation!
- Saturation does not depend on amplitude of pertubations
- Reason for saturation?
  - WD artifacts? Q_A decays? Non-linearity?

Comparison Wilson-Dirac/Overlap

- Both behaved similarly (not a WD artifact)
  - Same inverse cascade (CPI)
  - Same saturation



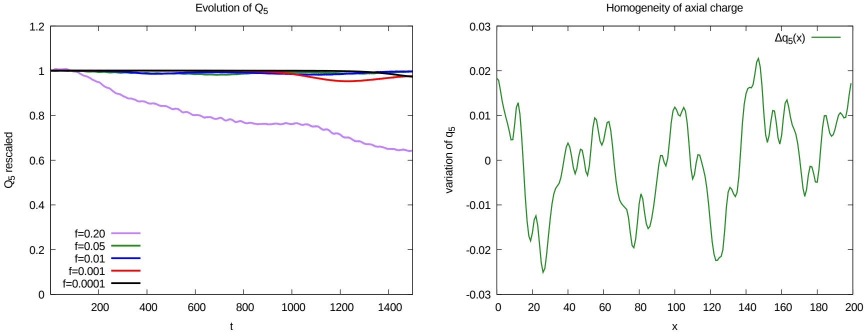
- Possibility to study with WD (cheaper) with a check of the results with OV.

Axial charge

- Decay only appears for large energy

$\Rightarrow$ Cannot explain the Saturation !

$\Rightarrow$ Strong non-linear effect !



- Homogeneity: Variation $< 5\%$

Conclusion

- Observation of saturation of the instabilities!
- No Decay of the axial charge

$\Rightarrow$ Important non-linear phenomena: Simulation necessary!

Outlook:

- Explanation of the saturation.
- Computing occupation number (production of particles).
- Understanding the behavior with analytical model.