Non-leptonic kaon decays at large N_c

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Open problems in kaon physics…

1) Direct vs Indirect CP violation

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$= \begin{cases} 1.38(5.15)(4.43)\times 10^{-4} & \text{theory} \\ 16.6(2.3)\times 10^{-4} & \text{experiment} \end{cases}$$

[Bai et al., PRL 115 (2015) 212001]

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2) The ΔI = ½ "rule" (by now 60 years old)

$$\left|\frac{A_0}{A_2}\right| = 22.35$$

[Gell-Mann, Pais, PR 97 (1955)]

Anatomy of ΔI = ½ "rule"

- a PT enhancement of A_0 with respect to A_2?

[Gaillard, Lee; Altarelli, Maiani 1974]

$$\left|\frac{A_0}{A_2}\right| = \frac{k_1^-(M_W)}{k_1^+(M_W)} \frac{\langle(\pi\pi)_{I=0}|Q_1^-|K\rangle}{\langle(\pi\pi)_{I=2}|Q_1^+|K\rangle}$$

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- a consequence of the charm decoupling?

[Shifman, Vainshtein, Zakharov 1975-77]

checking requires a reliable NP QCD computation

[Cabibbo, Martinelli, Petronzio; Brower, Maturana, Gavela, Gupta 1984]

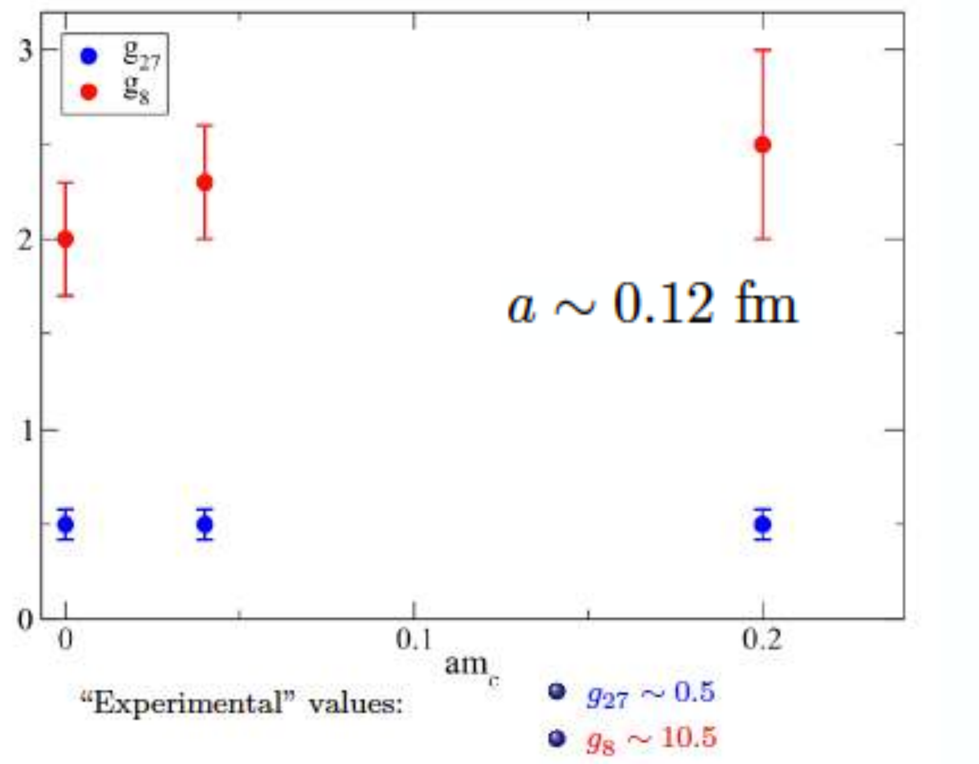
Numerical charm decoupling

$$m_c = m_u = m_d = m_s \quad \longrightarrow \quad m_c \gg m_u = m_d \leq m_s$$

No apparent charm decoupling enhancement

However:

- Quenched
- Far from physical charm mass



[Giusti, Hernández, Laine, Pena, Wennekers, Wittig 2007]

[Endress, Pena 2014]

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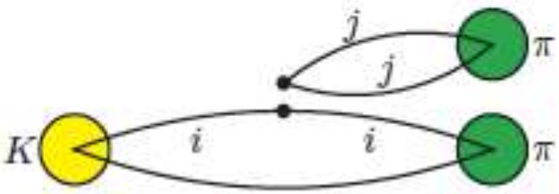
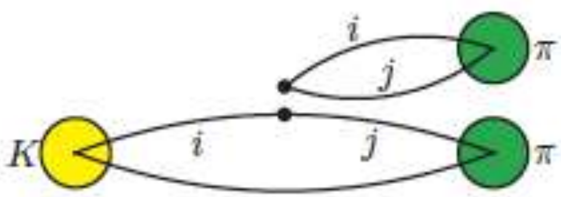
- other effects....

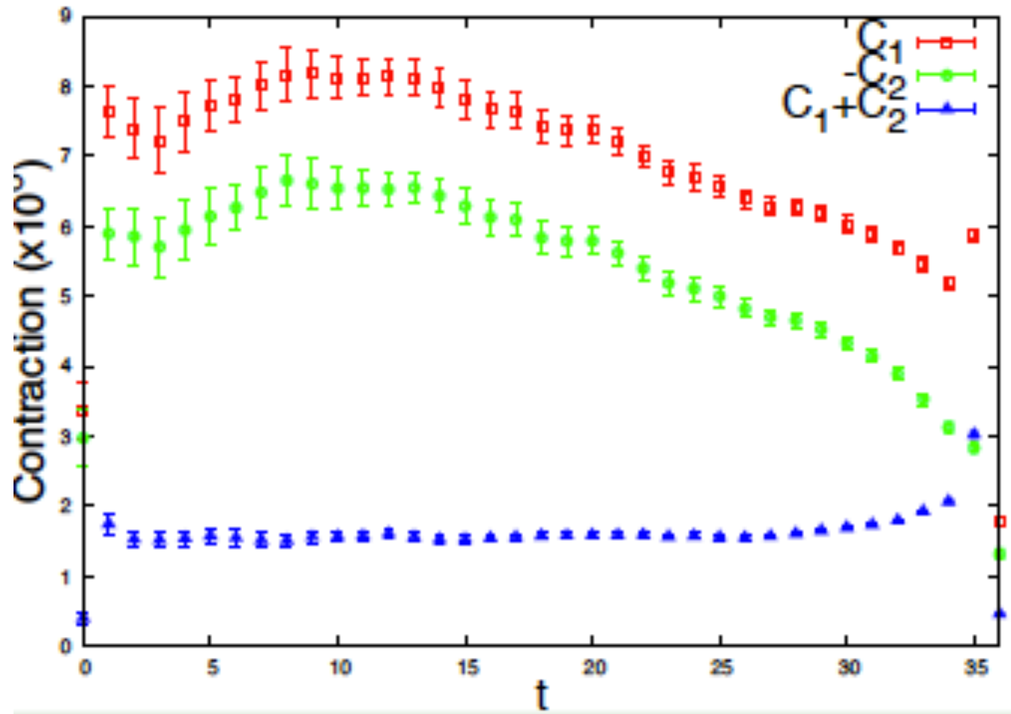
On the other hand: RBC/UKQCD



C_1: color diagonal



C_2: color mixed



[RBC-UKQCD, PRD91 (2015) 074502]

Color counting in LO PT $\Rightarrow$ $C_2 = C_1/3$; Non-PT effects $\Rightarrow$ $C_2 \approx -0.8 C_1$

CLAIM: Puzzle of $\Delta I = 1/2$ rule is resolved from first principles

Emerging "conspiracy" picture

- The $\Delta I = \frac{1}{2}$ "rule" is the combination of several effects, all of them enhancing $\Delta I = 1/2$ with respect to $\Delta I = 3/2$ kaon decays
  - PT enhancement of Wilson coefficients ($k^-_1 > k^+_1$)
  - Charm decoupling (maybe larger near m_c)
  - Final state interactions? still to study…
  - Bulk long-distance QCD effect (connected diagram is larger than expected at $N_c = 3$): let's study the N_c dependence of physical amplitudes $A^\pm$

Simulation strategy for N_c dependence

- work in the SU(4) limit: $m_c = m_u = m_d = m_s$
  - a simpler effective hamiltonian:

$$H_{\rm w}^{\Delta S=1} = \int d^4x \, \frac{g_{\rm w}^2}{4M_W^2} V_{us}^* V_{ud} \sum_{\sigma=\pm} k^\sigma(\mu)\, \bar{Q}^\sigma(x,\mu)$$

- use twisted mass QCD:
  - affordable computational time, multiplicative renormalization, $O(a^2)$ corrections [Frezzotti, Rossi 2004]
- quenched
- correlators and confs computed using code from

[Del Debbio, Patella and Pica 2010]
[Pica private communications]

RGI amplitudes

To work with scale-independent quantities:

$$\hat{Q}^\sigma \equiv \hat{c}^\sigma(\mu)\bar{Q}^\sigma(\mu)$$

Good N_c scaling

$$\hat{c}^\sigma(\mu) \equiv \left(\frac{N_c}{3}\frac{g^2(\mu)}{4\pi}\right)^{-\frac{\gamma_0^\sigma}{2b_0}} \exp\left\{-\int_0^{g(\mu)} \mathrm{d}g \left[\frac{\gamma^\sigma(g)}{\beta(g)} - \frac{\gamma_0^\sigma}{b_0\, g}\right]\right\}$$

from the Callan-Symanzik equation

The physical observable (two-loop PT running in RI):

$$\hat{k}^\sigma\,\hat{Q}^\sigma = \left[\frac{k^\sigma(M_W)}{\hat{c}^\sigma(M_W)}\right]\left[\hat{c}^\sigma(\mu)\,\bar{Q}^\sigma(\mu)\right] = k^\sigma(M_W)\,U^\sigma(\mu, M_W)\,\bar{Q}^\sigma(\mu)$$

[Ciuchini et al. 1998; Buras et al. 2000]

The observables

We compute ratios of (renormalized) three- to two-point functions

$$\hat{R}^{\pm} \equiv \frac{\langle \pi | \hat{Q}^{\pm} | K \rangle}{f_K f_\pi m_K m_\pi} = \hat{c}^{\pm}(\mu) Z_R^{\pm}(\mu) R^{\pm}$$

16^3 lattices

Renormalization constants (RI scheme) at ~ 2 GeV in one-loop PT from Constantinou et al., 2011; Alexandrou et al, 2012

$$A^{\pm} \equiv \hat{k}^{\pm} \hat{R}^{\pm}$$

$$\frac{A_0}{A_2} = \frac{1}{\sqrt{2}} \left(\frac{1}{2} + \frac{3}{2} \frac{A^-}{A^+} \right) \xrightarrow{\text{large } N_c} \sqrt{2}$$

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A by-product in the SU(3) limit: $\hat{B}_K = \frac{3}{4} \hat{R}^+$

Simulation parameters

N_c	T/a	β	$am_{\rm PCAC}$	$am_{\rm PS}$	$R^+_{\rm bare}$	$R^-_{\rm bare}$
3	48	6.0175	-0.002(14)	0.2718(61)	0.774(21)	1.218(31)
4	48	11.028	-0.0015(11)	0.2637(39)	0.783(15)	1.198(19)
5	48	17.535	0.0028(9)	0.2655(31)	0.839(8)	1.145(12)
6	32	25.452	0.0013(7)	0.2676(28)	0.871(6)	1.125(7)
7	32	34.8343	-0.0034(6)	0.2819(19)	0.880(5)	1.122(5)

β and k_c from detailed studies of large N_c spectrum

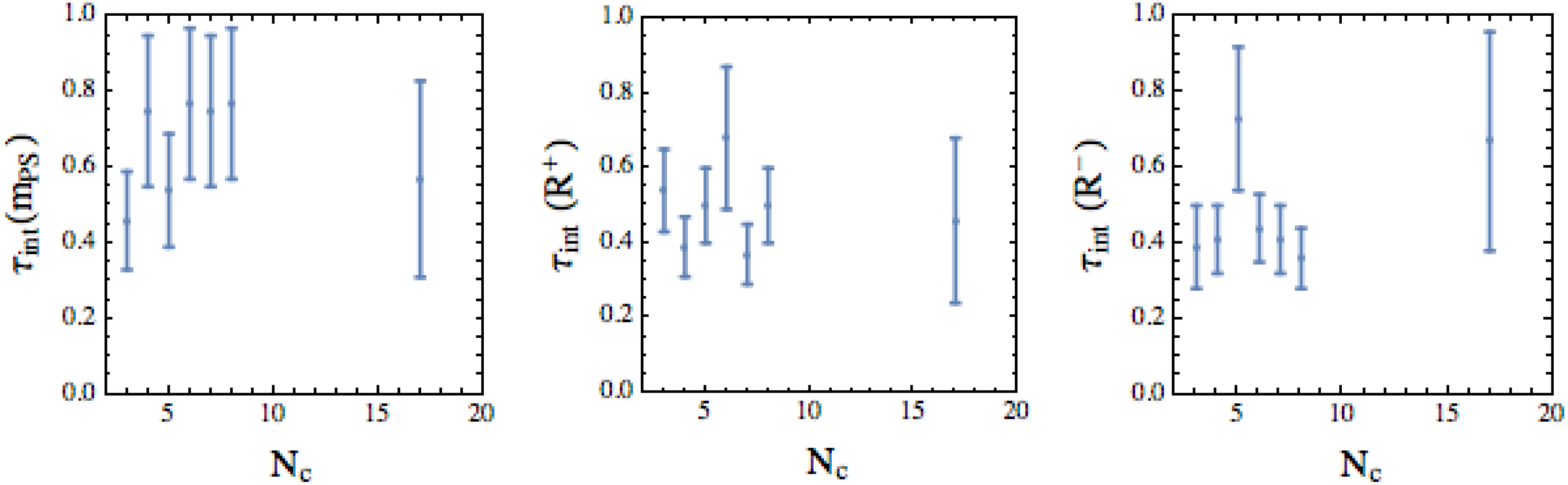
fixed $a\sqrt{\sigma} \simeq 0.2093 \Rightarrow a \sim 0.09$ fm

[Bali et al, 2013]

work at fixed quark mass at maximal twist with twisted bare mass $a\mu = 0.02$

roughly constant pseudoscalar mass $m_{PS} \sim m_K$

Observables autocorrelations



We checked that ALL measurements are (UWerr-)decorrelated

Chiral corrections

<π|Q_i|K> matrix elements can be related to <ππ|Q_i|K> in ChiPT

[Donoghue, Golowich, Holstein 1982; Bijnens, Sonoda, Wise 1984]
[Bernard et al. 1985]

the relation between B_K and A^+ holds beyond the chiral limit

as long as $m_u = m_d = m_s$

$$\left.\frac{\langle \pi^+\pi^0|H_W|K\rangle}{m_K^2 - m_\pi^2}\right|_{m_s=m_d} = \frac{iF}{\sqrt{2}} A^+ G_F V_{ud} V_{us}^*$$

$$\langle \pi^+\pi^0|H_W|K^+\rangle_{m_\pi\to 0} = m_K^2 \left.\frac{\langle \pi^+\pi^0|H_W|K^+\rangle}{m_K^2 - m_\pi^2}\right|_{m_s=m_d} \left(1 + \frac{9}{4}\frac{m_K^2}{(4\pi F)^2}\log\frac{m_K^2}{(4\pi F)^2}\right).$$

[Golterman, Leung 1997]

Chiral corrections: caveats

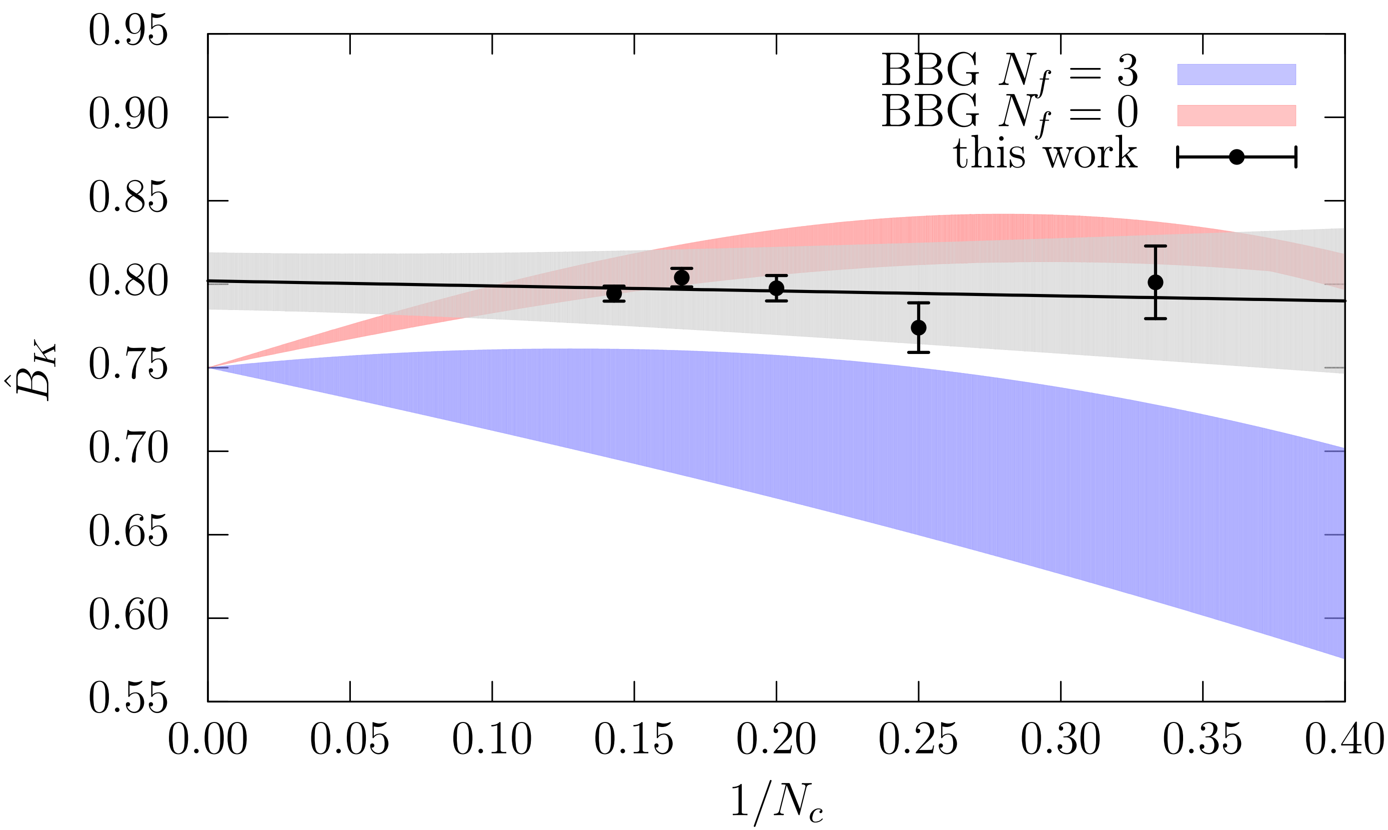
- out of SU(3) limit, chiral logs are much larger for kaon mixing than for kaon decay. B_K is no longer directly related to A^+

[Donoghue, Golowich, Holstein 1982; Bijnens, Sonoda, Wise 1984]

- higher-order ChiPT corrections argued to be large

[Truong 1988; Isgur, Maltman, Weinstein, Barnes 1990; Kambor, Missimer, Wyler 1991; Pallante, Pich 1998]

Results: B_k



Very small N_c dependence, consistent with theoretical expectations

[Buras, Gérard, Bardeen 2014]

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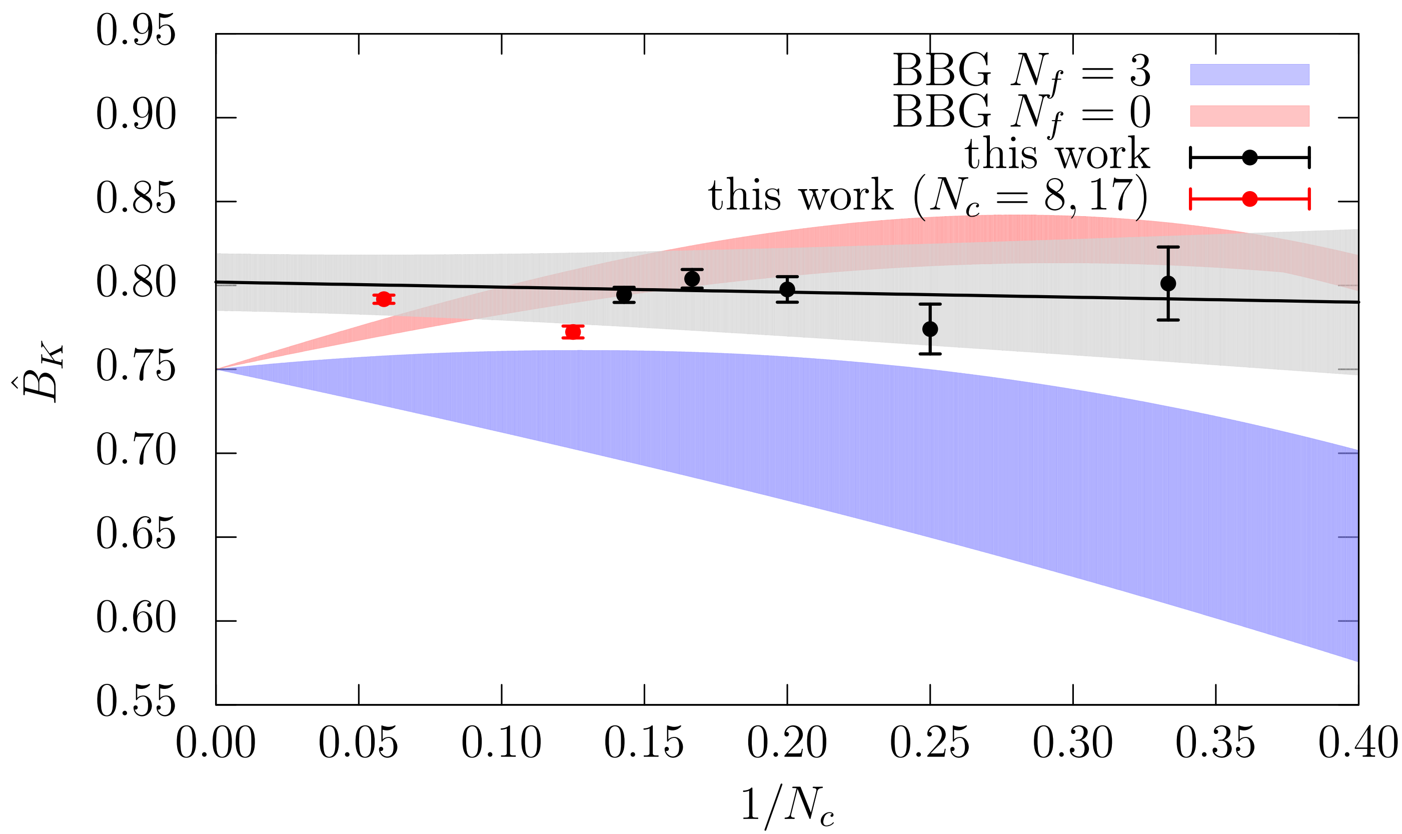

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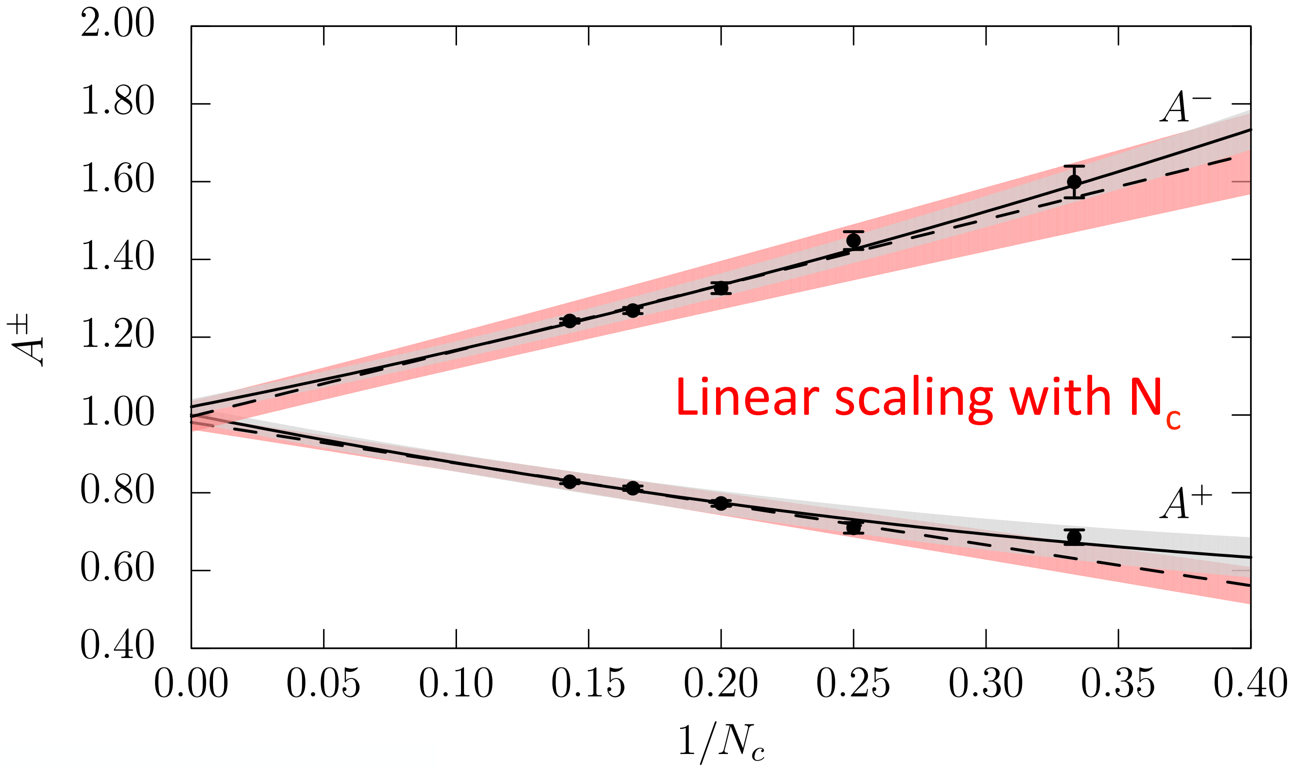

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Very large corrections in $1/N_c$!

Large N_c limit recovered

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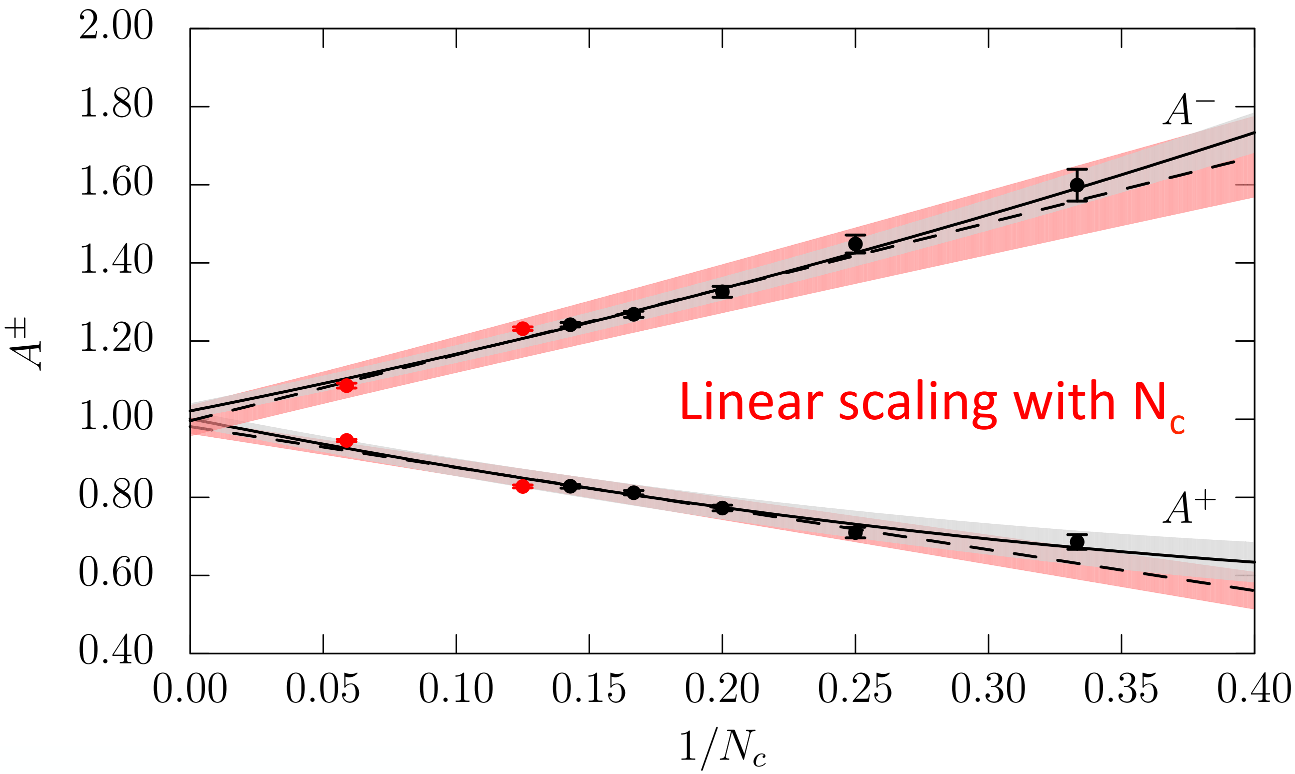

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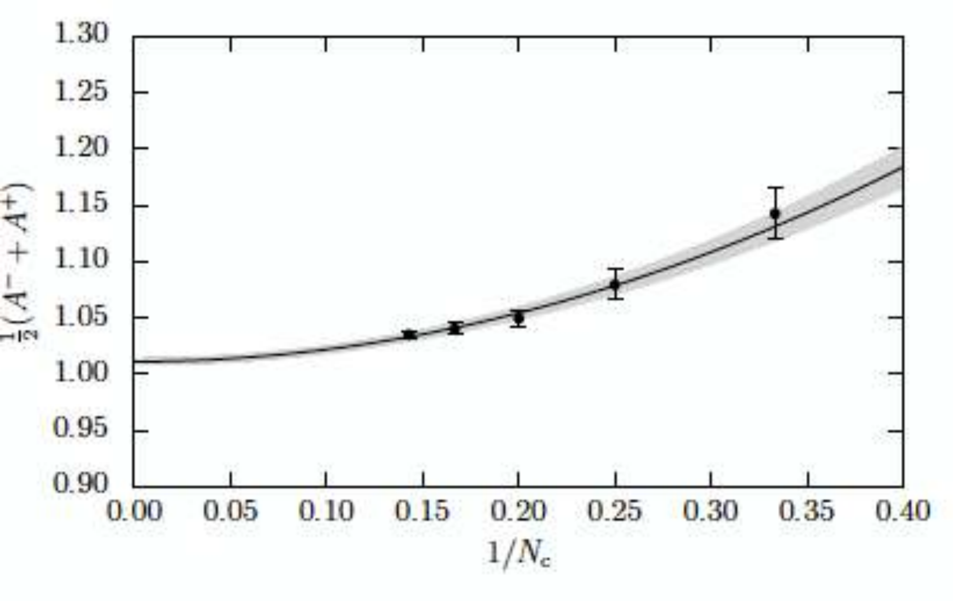
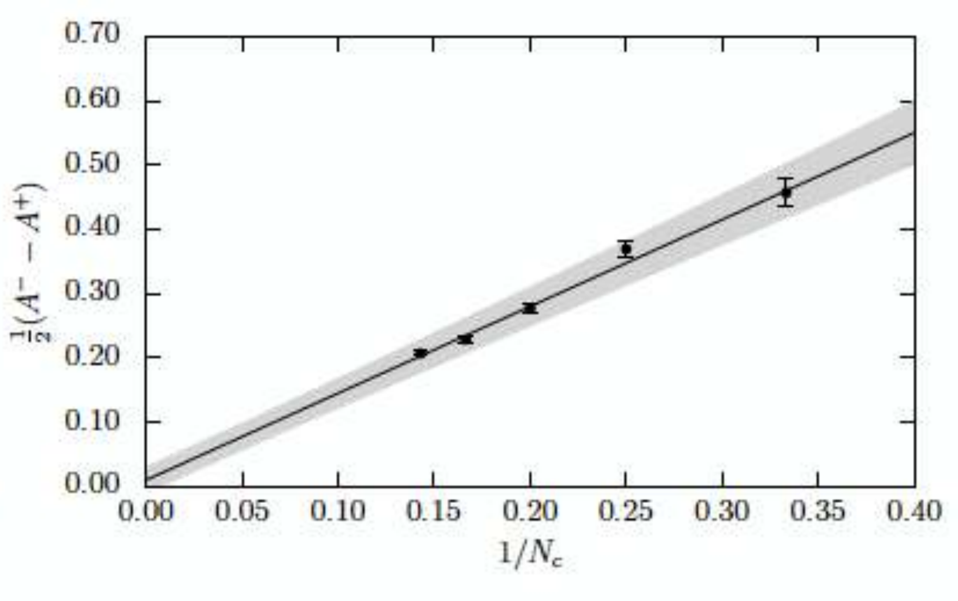
Results: from A^+ and A^- to A_1 and A_2



"Physical" disconnected diagram scales to ~1 quadratically in $1/N_c$



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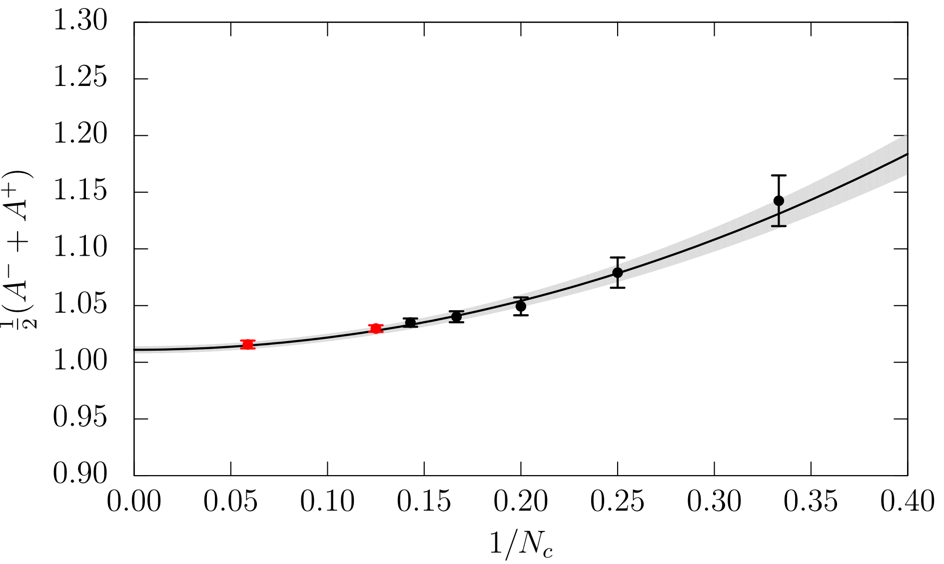

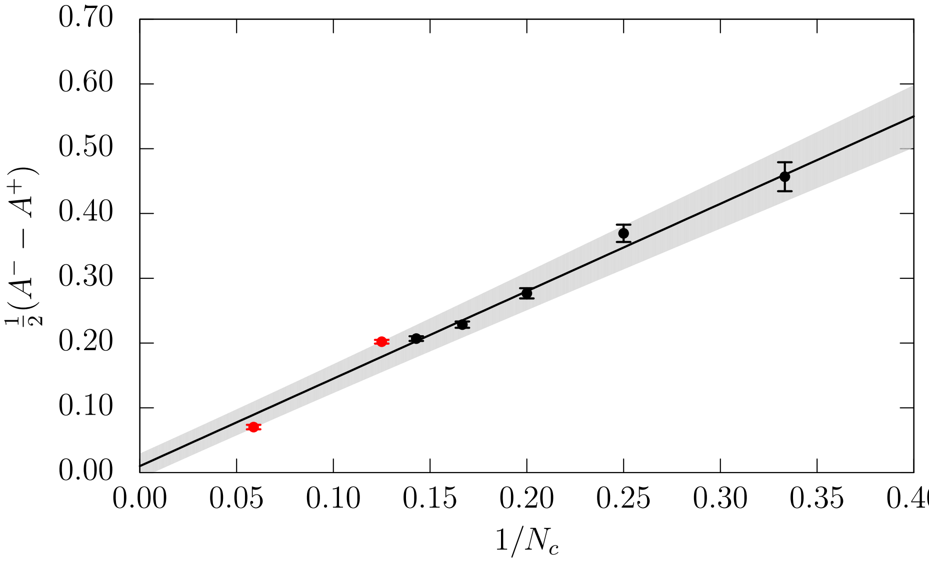

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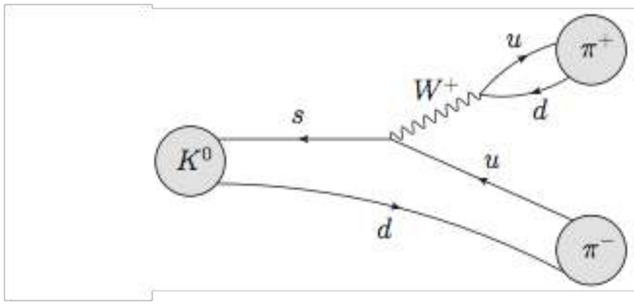
- Dynamical quarks are feasible [DeGrand, Liu 2016]
- Indeed large N_c corrections for $N_c = 3$, not enough to explain the $\Delta I = ½$ rule but in the good direction (compatible with naive exp)
- Large N_c-dependence for $m_s > m_u, m_d$?

Conclusions and Outlook

Is there an underlying reason for which ALL effects go in the same direction?

$K \rightarrow \pi\pi$ diagrammatics



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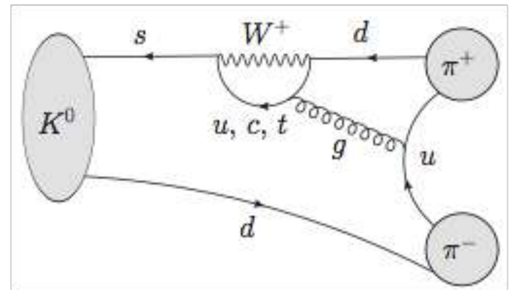

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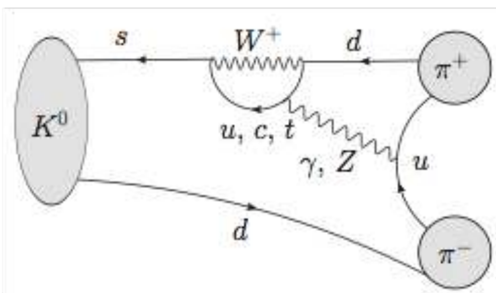

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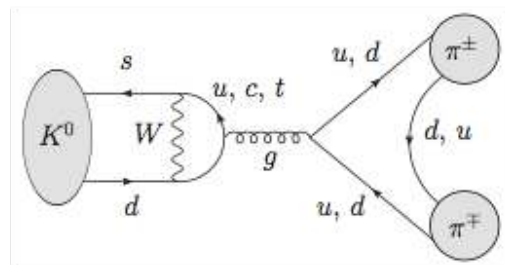

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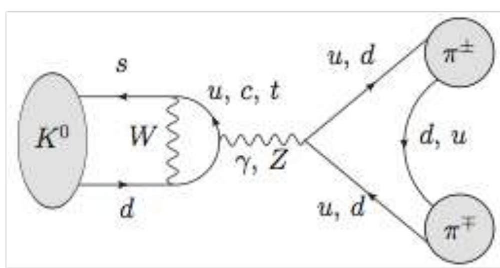

$+ \cdots$

Integrating out W, t and c
we get an effective lagrangian
with 10 four-fermion
operators

Plaquette autocorrelation