$B_s \to K\ell\nu$ form factors with 2+1 flavors

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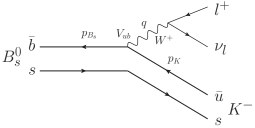
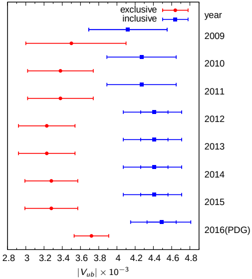
Motivation

- The Kabibbo-Kobayashi-Masakawa (CKM) matrix element $|V_{ub}|$ is one of the fundamental parameters in the Standard Model and is an important input to search for new physics.



- There is still tension between inclusive and exclusive $|V_{ub}|$.
- The charged current $B_s \to K^- \ell^+ \nu$ proceeds proportional to V_{ub}^*:
  - combining with further experimental measurements at LHCb or Belle II to determine $|V_{ub}|$.
- This is an alternative to $B \to \pi^- \ell^+ \nu$ in determining $|V_{ub}|$ from exclusive semileptonic decays, with smaller lattice errors.



Similar work by HPQCD(2014) and RBC&UKQCD(2015).

	sea	valence u,d,s	valence b
HPQCD	2+1 flavors MILC asqtad	HISQ	lattice NRQCD
RBC&UKQCD	2+1 flavors domain-wall	domain-wall	Columbia RHQ
Fermilab/MILC	2+1 flavors MILC asqtad	asqtad	Fermilab/clover

Matrix elements and form factors

- Amplitudes are decomposed into form factors:

$$\begin{aligned} \langle K(p_K)|\bar{u}\gamma^\mu b|B_s(p_{B_s})\rangle &= \left(p_K^\mu + p_{B_s}^\mu - q^\mu \frac{M_{B_s}^2 - M_K^2}{q^2}\right) f_+(q^2) + q^\mu \frac{M_{B_s}^2 - M_K^2}{q^2} f_0(q^2) \\ &= \sqrt{2M_{B_s}}\left[v^\mu f_\parallel(E_K) + p_\perp^\mu f_\perp(E_K)\right]. \end{aligned} \quad (1)$$

  f_+ and f_0 are vector and scalar form factors satisfying the kinematic constraint:

$$f_+(0) = f_0(0). \quad (2)$$

- $f_\parallel$ and $f_\perp$ are form factors related to the temporal and spatial components of the matrix element:

$$f_+(q^2) = \frac{1}{2M_{B_s}}[f_\parallel(E_K) + (M_{B_s} - E_K)f_\perp(E_K)], \quad (3a)$$

$$f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M_{B_s}^2 - M_K^2}[(M_{B_s} - E_K)f_\parallel(E_K) + (E_K^2 - M_K^2)f_\perp(E_K)]. \quad (3b)$$

Procedure to get the form factors

- For each ensemble:
  - Determine the lattice B_s meson masses, kaon masses and energies from the lattice 2-point correlation functions.
  - Determine the lattice form factors $f_{\parallel}^{\mathrm{lat}}$ and $f_{\perp}^{\mathrm{lat}}$ at several discrete kaon momentum $\boldsymbol{p}_K$ from two- and three-point correlation functions.
- Obtain the continuum $f_{\parallel}$ and $f_{\perp}$ at a finite $\boldsymbol{p}_K$ by extrapolating the lattice form factors to physical quark masses and continuum (zero lattice spacing) limits, and matching the corresponding currents.
- Construct the continuum form factors f_+ and f_0 from $f_{\parallel}$ and $f_{\perp}$ via Eqs.(3) and extrapolate to the whole kinematically allowed momentum transfer region, especially at $q^2 = 0$ point.
- Comprehensive error analysis at all the above steps.

Correlators

- On a Euclidean space-time lattice, the correlation functions with momentum $\boldsymbol{p}_K$ are defined as:

$$C_2^{B_s}(t;0) = \sum_{\boldsymbol{x}} \langle \mathcal{O}_{B_s}(t,\boldsymbol{x}) \mathcal{O}^\dagger_{B_s}(0,\boldsymbol{0}) \rangle, \tag{4a}$$

$$C_2^{K}(t;\boldsymbol{p}_K) = \sum_{\boldsymbol{x}} \langle \mathcal{O}_{K}(t,\boldsymbol{x}) \mathcal{O}^\dagger_{K}(0,\boldsymbol{0}) \rangle e^{-i\boldsymbol{p}_K\cdot\boldsymbol{x}}, \tag{4b}$$

$$C_{3,\mu}^{B_s\to K}(t,T;\boldsymbol{p}_K) = \sum_{\boldsymbol{x},\boldsymbol{y}} \langle \mathcal{O}_{K}(0,\boldsymbol{0}) V^\mu(t,\boldsymbol{y}) \mathcal{O}^\dagger_{B_s}(T,\boldsymbol{x}) \rangle e^{i\boldsymbol{p}_K\cdot\boldsymbol{y}}, \tag{4c}$$

with $\mathcal{V}^\mu \equiv \bar{u}\gamma^\mu b = Z_{V_\mu} V^\mu$.

- Two-point correlation functions are constructed to extract lattice meson masses and to verify dispersion relations.

- Three-point correlation functions are constructed to extract the lattice form factors:

$$f_\parallel(E_K) = \frac{\langle K|\bar{u}\gamma^0 b|B_s\rangle}{\sqrt{2M_{B_s}}}, \tag{5a}$$

$$f_\perp(E_K) = \frac{\langle K|\bar{u}\gamma^i b|B_s\rangle}{\sqrt{2M_{B_s}}} \frac{1}{p_K^i}. \tag{5b}$$

Lattice simulation details

- Sea quarks: MILC asqtad ensembles (2+1 flavors with rooted staggered fermions).
  - lattice spacing: $\approx$ 0.12, 0.09 and 0.06 fm; $m_l/m_s \approx$ 0.05, 0.1, 0.15 and 0.2.
- Valence quarks: asqtad for u, d and s; Fermilab/clover for b.
  - Valence light quarks are degenerate with the sea light quarks: $m'_l = m_l$.
  - Valence strange quarks are better tuned than the sea strange quarks $m'_s \neq m_s$.
- This is a subset of the MILC asqtad ensembles; chosen based on experience from $B \to \pi$ and $B \to K$ analysis.

$\approx$a(fm)	$N_s^3 \times N_t$	am_l/am_s	am'_l/am'_s	N_{config}	N_{source}	$M_\pi N_s$
0.12	$24^3 \times 64$	0.0050/0.0500	0.0050/0.0336	2099	4	3.8
0.09	$28^3 \times 96$	0.0062/0.031	0.0062/0.0247	1931	4	4.1
0.09	$32^3 \times 96$	0.00465/0.031	0.00465/0.0247	1015	8	4.1
0.09	$40^3 \times 96$	0.0031/0.031	0.0031/0.0247	1015	8	4.2
0.09	$64^3 \times 96$	0.00155/0.031	0.00155/0.0247	791	4	4.8
0.06	$64^3 \times 144$	0.0018/0.018	0.0018/0.0177	827	4	4.3

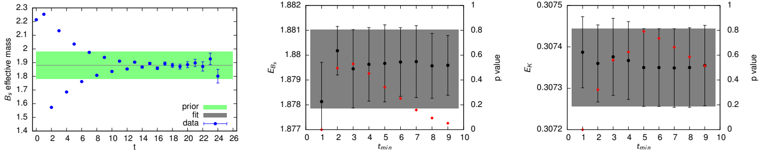
Two-point correlator fits

Two-point correlators are fit to the following functional forms

$$C_2^{B_s}(t;0) = \sum_{n=0}^{N-1} (-1)^{n(t+1)} \frac{|\langle 0|\mathcal{O}_{B_s}|B_s^{(n)}\rangle|^2}{2M_{B_s}^{(n)}} \left(e^{-M_{B_s}^{(n)} t} + e^{-M_{B_s}^{(n)}(N_t - t)} \right), \qquad \text{(6a)}$$

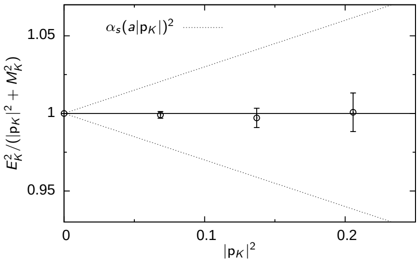
$$C_2^{K}(t;\boldsymbol{p}_K) = \sum_{n=0}^{N-1} (-1)^{n(t+1)} \frac{|\langle 0|\mathcal{O}_{K}|K^{(n)}\rangle|^2}{2E_{K}^{(n)}} \left(e^{-E_{K}^{(n)} t} + e^{-E_{K}^{(n)}(N_t - t)} \right). \qquad \text{(6b)}$$

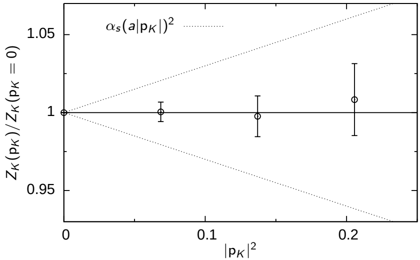
- N = 3 is used for the fits.
- Prior central values are set according to the lattice effective masses.
- Prior widths are wide enough to avoid bias in the fitting procedure.
- t_{max} is chosen such that the fractional error in the correlators are small (< 3%).
- t_{min} is set to have a good p-value and stable fit result.



$a \approx 0.12$ fm

- Lattice energies and amplitudes agree with continuum dispersion relation, $E_K^2 = M_K^2 + \boldsymbol{p}_K^2$ and $Z_K(p_K) \equiv |\langle 0|\mathcal{O}_K|K^{(n)}\rangle| = Z_K(p_K = 0)$, for all ensembles.
- The energy errors are larger for large lattice momentum and therefore only data with lattice momentum up to “(1,1,1)” are used.
- After the dispersion relation is verified, continuum dispersion relation is used to get the energies.





$a \approx 0.12$ fm

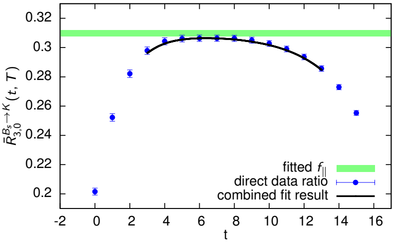
Three-point correlator fits

Three-point correlators have the following functional form

$$C_{3,\mu}^{B_s\to K}(t,T;\boldsymbol{p}_K) = \sum_{m,n=0}^{N-1}(-1)^{m(t+1)}(-1)^{n(T-t-1)}A_{mn}^{\mu}e^{-E_K^{(m)}t}e^{-M_{B_s}^{(n)}(T-t)}. \qquad (7)$$

- They share common ingredients, i.e., energies and amplitudes, with the two-point correlators.

- A simultaneous fit of two- and three-point correlators is possible and the data is accurate enough to take the excited states into account.
  - **Blue points** are the ratio of two- and three-point correlators that suppress opposite parity states.
  - **Black curve** is the simultaneous fit result; agrees well with the ratio data (**blue points**).
  - **Green band** is the resulting amplitude without excited states.



$a \approx 0.12$ fm

Chiral continuum extrapolation

- SU(2) HMrSχPT:
$$f_{P,\mathrm{NLO}} = f_P^{(0)}[c_P^0(1+\delta f_{P,\mathrm{logs}}) + c_P^1\chi_\mathrm{l} + c_P^2\chi_\mathrm{h} + c_P^3\chi_E + c_P^4\chi_E^2 + c_P^5\chi_a^2], \quad (8)$$
where the leading order term is
$$\frac{1}{f}\frac{g_\pi}{E_K + \Delta_P^*}. \quad (9)$$

- The Δ_P^* takes the pole contribution into account:
$$\Delta_P^* = \frac{M_{B^*}^2 - M_{B_s}^2 - M_K^2}{2M_{B_s}}. \quad (10)$$
They are determined by requiring $f_\|$ and $f_\perp$ to have the same pole as f_0 and f_+.

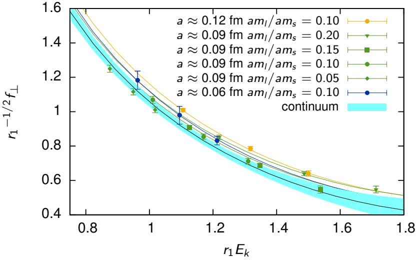
- The vector meson (with $J^P = 1^-$) has been experimentally measured [PDG(2016)] as $M_{B^*} = 5324.65(25)\mathrm{MeV}$.

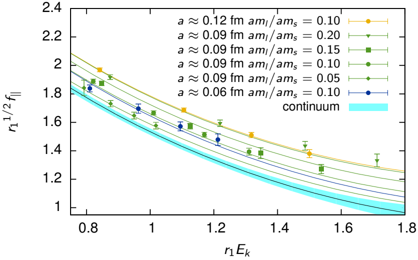
- The scalar B^* meson (with $J^P = 0^+$) has not been observed experimentally, but lattice calculation [HPQCD(2011)] suggests the mass difference between 0^+ and 0^- states to be around $400\mathrm{MeV}$:
$$M_{B^*}(0^+) - M_B \approx 400\mathrm{MeV}. \quad (11)$$

- The $J^P = 1^-$ pole is below the $B\pi$ production threshold, while the 0^+ one is slightly above it.

Chiral continuum extrapolation

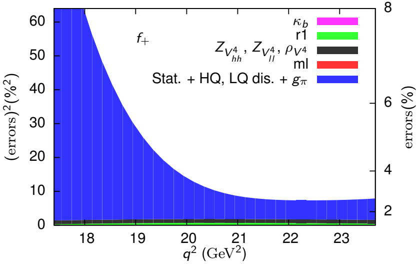
- Fitting the $f_{\parallel}$ and $f_{\perp}$ simultaneously.
- The next-to-next leading order (NNLO) HMrSχPT is used as the central fit.
- The statistical errors dominate over various systematic ones.
- $\chi^2/dof[dof] = 0.89[42]$; p-value is 0.68.

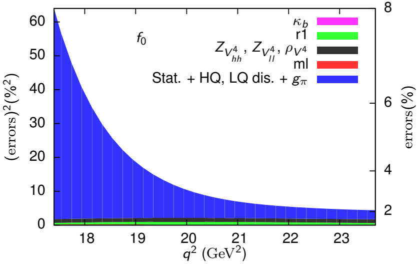




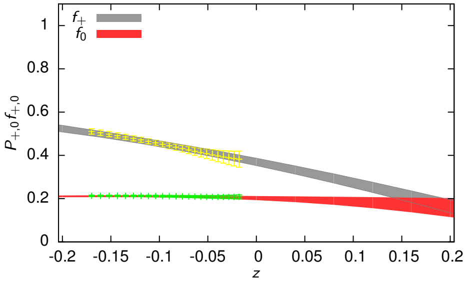
Error budgets (preliminary)

- The statistical errors, especially the ones coming from the chiral-continuum extrapolation and discretization, are large in the small q^2 region.
- A reliable method is needed to extrapolate the form factors to $q^2 = 0$.



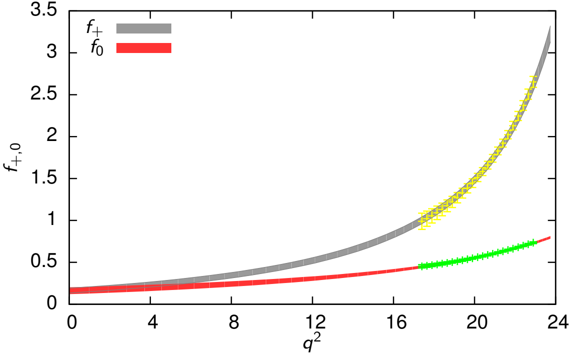


- Functional method (PRD92, 2015) with BCL z-expansion is used.
- Fit f_+ and f_0 simultaneously by keeping up to z^3 terms without any constraint.
- The kinematic constraint $f_+(q^2 = 0) = f_0(q^2 = 0)$ is satisfied automatically.
- The unitarity condition $\sum_{m,n=0}^{K} B_{mn} b_m b_n \leq 1$ is satisfied automatically.
- Adding the heavy quark and/or kinematic constraints decreases the form factor errors at $q^2 = 0$ slightly.



Form factors (preliminary)

- Fitting the f_+ and f_0 simultaneously.
- K = 4 (up to z^3) without constraint is used as the central fit.
- $\chi^2/dof[dof] = 0.82[5]$; p-value is 0.54.
- Z factors are still "blinded".



Summary

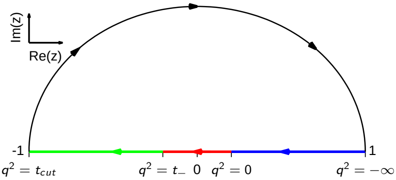
- Analysis is almost done for $B_s \to K\ell\nu$ form factors on 2+1 flavors asqtad ensembles.
- Need to:
  - Complete the error estimates.
  - Unblind the Z factors and get the final form factors for phenomenology.
  - Write-up a paper.
  - Wait for experimental input and determine $|V_{ub}|$.

BACKUP

z-parametrization

- Map the whole complex q^2 plane onto the unit disk in the z plane.

$$
\begin{aligned}
z(q^2, t_0) &= \frac{\sqrt{t_{\rm cut} - q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - q^2} + \sqrt{t_{\rm cut} - t_0}}, \\
q^2 &= t_{\rm cut} - (\frac{1+z}{1-z})^2 (t_{\rm cut} - t0), \\
t_{\rm cut} &= (M_B + M_\pi)^2, \\
t_- &= (M_{B_s} - M_K)^2, \\
t_0 &= t_{\rm cut}(1 - \sqrt{1 - t_-/t_{\rm cut}}).
\end{aligned}
$$



- $t_{\rm cut}$ is the $B\pi$ pair-production threshold.
- t_- is the maximum momentum-transfer squared allowed in the $B_s \to K^- \ell^+ \nu$ decay.
- t_0 is chosen such that the full kinematic range for $B_s \to K^- \ell^+ \nu$ decay is centered around the origin $z = 0$, i.e., by solving $z(q^2 = 0, t_0) = -z(q^2 = t_-, t_0)$.
- Lattice data range: $0.87 < r_1 E_k < 1.71$; $17.4 < q^2 < 23.1$; $-0.178 < z < -0.018$.
- Kinematically allowed range:
  $-0.204 = z(q^2 = t_-, t_0) \le z \le z(q^2 = 0, t_0) = 0.204$.

BCL parametrization of the z-expansion

By analyticity and positivity properties of vacuum polarization functions, the form factors can be expanded as (BGL)

$$f_+(q^2) = \frac{1}{B(q^2)\phi(q^2, t_0)} \sum_{n=0}^{\infty} a_n(t_0) z^n, \tag{12}$$

where $B(q^2) = z(q^2, M_{B*}^2)$ is the Blaschke factor,which takes the pole(s) into account; $\phi(q^2, t_0)$ is a complicated outer function, computable via perturbative QCD and the operator product expansion.
From unitarity and crossing symmetry, one gets (unitarity condition):

$$\sum_{n=0}^{\infty} a_n^2(t_0) \le 1. \tag{13}$$

An alternative simplier parametrization is

$$f_+(q^2) = \frac{1}{1 - q^2/m_{B*}^2} \sum_{k=0}^{K} b_k(t_0)\, z^k. \tag{14}$$

From angular momentum conservation and analycity, one can get $\frac{\partial f_+}{\partial z}|_{z=-1} = 0$, which means $b_K = \sum_{k=0}^{K-1} (-1)^{k-K-1} \frac{k}{K} b_k$.

Therefore, Eq. (14) can be written as (BCL)

$$f_+(q^2) = \frac{1}{1 - q^2/m_{B*}^2} \sum_{k=0}^{K-1} b_k \left[z^k - (-1)^{k-K} \frac{k}{K} z^K \right], \tag{15}$$

f_0 can be expanded as $\sum_{k=0}^{K} b_k z^k$ or as in Eq. (14) depending on the importance of the scalar pole.
The unitarity condition in BGL Eq. (13) becomes

$$\sum_{j,k=0}^{K} B_{jk}(t_0) b_j(t_0) b_k(t_0) \le 1, \tag{16}$$

where the B_{jk} is calculatable via the outer function $\phi(q^2, t_0)$.