Perturbative Renormalization of Wilson-line Operators

Martha Constantinou (Temple University)
Haris Panagopoulos (University of Cyprus)

Together with members of the ETM Collaboration

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Present work: `arXiv:1705.11193`

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Introduction

- **PDFs : Important information on distribution of momentum/spin among hadronic constituents**

  [Plenary talk by L. Del Debbio]

- **PDFs are light-cone correlation functions $\Rightarrow$**
  Not amenable to investigations on a Euclidean lattice !?!

- **Approach by Xiangdong Ji : Study quasi-PDFs in Euclidean space, then match them to physical PDFs**

- **Determination of quasi-PDFs in lattice simulations $\Rightarrow$**
  Study of hadronic matrix elements of nonlocal Wilson-line operators, e.g.:

$$\mathcal{O}_\Gamma \equiv \overline{\psi}(x)\,\Gamma\,\mathcal{P}\,e^{i\,g\int_0^z A_\mu(x+\zeta\hat{\mu})d\zeta}\,\psi(x+z\hat{\mu})$$

Introduction

- **Long history of nonlocal composite operators**
  Seminal work – different viewpoints – by:
  Mandelstam, Polyakov, Migdal, Makeenko, Witten, ...

- **Renormalization of (continuum) Wilson loops**
  Dotsenko+Vergeles, Brandt+Neri+Sato :
  - **Finite functions of renormalized coupling (smooth loops)**
  - **Logarithmic singularities at cusps and self-intersections**

  - **Regularizations other than DR: Linear divergence**

$$Z \sim e^{-cL/a}$$

- **Old results on renormalization of (continuum) Wilson lines:**
  Anomalous dimension in $\mathrm{DR} - \overline{\mathrm{MS}}$ at:
  - **1 loop (Dorn)**
  - **3 loops (Chetyrkin+Grozin)**

Introduction

- Lattice: Many nonlocal operators studied in simulations
  - Wilson loops
  - Polyakov lines
  - Field correlators: Input to phenomenological models (Simonov, Dosch, ...)

- Their renormalization has not been sufficiently explored
- Perturbatively: To one loop in $\overline{\mathrm{MS}}$ (This work)
- Non-perturbatively: Most naturally in an RI$'$-like scheme
  $\Rightarrow$ Need also conversion factor from RI$'$ to $\overline{\mathrm{MS}}$ (This work)
- Presently, several aspects of Ji's approach are investigated:
  - Matching of quasi-PDFs to PDFs (Xiong, Ma, Chen, Li)
  - Relation with TMDs (Engelhardt, Negele, Radyushkin)
  - Linear divergence through static quark potential (Musch, Ishikawa, Chen)
  - Logarithmic divergences (Ishikawa, Monahan, Carlson)

Operators – Setup

$$\mathcal{O}_\Gamma \equiv \overline{\psi}(x)\,\Gamma\,\mathcal{P}\,e^{i\,g\,\int_0^z A_\mu(x+\zeta\hat{\mu})d\zeta}\,\psi(x+z\hat{\mu})$$

- Choose a straight path of length z in μ-direction ($\mu = 1$)
- $z \to 0$: Local operators. Discontinuity, due to contact term.
- 16 choices of Γ. Mixing on lattice within each of the pairs

$$(\hat{1},\,\gamma_1) \quad (\gamma_5\gamma_2, \gamma_3\gamma_4) \quad (\gamma_5\gamma_3, \gamma_4\gamma_2) \quad (\gamma_5\gamma_4, \gamma_2\gamma_3)$$

- Renormalized operators: $\begin{pmatrix} \mathcal{O}^R_{\Gamma_1} \\ \mathcal{O}^R_{\Gamma_2} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}^{-1} \begin{pmatrix} \mathcal{O}_{\Gamma_1} \\ \mathcal{O}_{\Gamma_2} \end{pmatrix}$

$$[Z^{X,Y}_{ij} : \quad X = \text{regulator}, \quad Y = \text{renormalization scheme}]$$

- Green's functions (2-pt, momentum q):

$$\langle \psi^R\, \mathcal{O}^R_{\Gamma_i}\, \bar{\psi}^R \rangle_{\text{amp}} = Z_\psi \sum_{j=1}^{2} (Z^{-1})_{ij}\, \langle \psi\, \mathcal{O}_{\Gamma_j}\, \bar{\psi} \rangle_{\text{amp}}\,, \quad \psi = Z_\psi^{1/2} \psi^R$$

$$[Z_{ij} = \delta_{ij} + g^2 z_{ij} + \mathcal{O}(g^4)\,, \qquad Z_\psi = 1 + g^2 z_\psi + \mathcal{O}(g^4)]$$

Operators – Setup

$$\langle\psi^R\,\mathcal{O}^R_{\Gamma_i}\,\bar\psi^R\rangle_{\rm amp} = Z_\psi\,\sum_{j=1}^{2}(Z^{-1})_{ij}\,\langle\psi\,\mathcal{O}_{\Gamma_j}\,\bar\psi\rangle_{\rm amp}\,,$$

- In DR ($D=4-2\epsilon$): $\mathcal{O}(1/\epsilon)$ terms in $\langle\psi\,\mathcal{O}_{\Gamma_i}\,\bar\psi\rangle^{DR}_{\rm amp}$ are (necessarily!) z-independent $\Rightarrow$ $Z^{DR,\,\overline{\rm MS}}$ is z-independent
  - $\langle\psi\,\mathcal{O}_{\Gamma_i}\,\bar\psi\rangle^{DR}_{\rm amp}\Big|_{1/\epsilon} \propto \langle\psi\,\mathcal{O}_{\Gamma_i}\,\bar\psi\rangle^{\rm tree} \Rightarrow Z^{DR,\,\overline{\rm MS}}$ is diagonal
- On the lattice (LR): Extract $Z^{LR,\,\overline{\rm MS}}_{ij}$ by requiring:

$$\langle\psi^R\,\mathcal{O}^R_{\Gamma_i}\,\bar\psi^R\rangle^{DR,\,\overline{\rm MS}} = \langle\psi^R\,\mathcal{O}^R_{\Gamma_i}\,\bar\psi^R\rangle^{LR,\,\overline{\rm MS}}\Big|_{a\to 0} \quad\Longrightarrow$$

$$\langle\psi^R\,\mathcal{O}^R_{\Gamma_1}\,\bar\psi^R\rangle^{DR,\,\overline{\rm MS}} - \langle\psi\,\mathcal{O}_{\Gamma_1}\,\bar\psi\rangle^{LR} = g^2\left(z^{LR,\,\overline{\rm MS}}_\psi - z^{LR,\,\overline{\rm MS}}_{11}\right)\langle\psi\,\mathcal{O}_{\Gamma_1}\,\bar\psi\rangle^{\rm tree}$$
$$-g^2\,z^{LR,\,\overline{\rm MS}}_{12}\langle\psi\,\mathcal{O}_{\Gamma_2}\,\bar\psi\rangle^{\rm tree} + \mathcal{O}(g^4)$$

  - The lhs must be a q-independent linear combination of tree-level quantities: Highly nontrivial check!

RI′ scheme for $\mathcal{O}_\Gamma$

- Non-perturbative evaluations of Z-factors: Need RI′-type scheme
  - Define RI′ for $\mathcal{O}_\Gamma$: $\mathrm{Tr}\left[\Lambda^{\mathrm{RI}'}_{\Gamma_i}(\Lambda^{\mathrm{tree}}_{\Gamma_j})^\dagger\right]_{q_\nu=\bar{q}_\nu} = \mathrm{Tr}\left[\Lambda^{\mathrm{tree}}_{\Gamma_i}(\Lambda^{\mathrm{tree}}_{\Gamma_j})^\dagger\right] = 12\delta_{ij}$

$$\left[\Lambda_\Gamma \equiv \langle\psi\,\mathcal{O}_\Gamma\,\bar{\psi}\rangle,\ \Lambda^{\mathrm{RI}'}_\Gamma \equiv \langle\psi^{\mathrm{RI}'}\,\mathcal{O}^{\mathrm{RI}'}_\Gamma\,\bar{\psi}^{\mathrm{RI}'}\rangle,\ \Lambda^{\mathrm{tree}}_\Gamma = \Gamma\,e^{i\,q_\mu z}\right]$$

- $\bar{q}_\nu$: RI′ renormalization scale. Direction-dependent!
  - Different choices of $\bar{q}_\nu$ $\equiv$ Different schemes, related by finite renormalization
- In terms of bare Green's functions:

$$\frac{1}{12}\,Z^{LR,\mathrm{RI}'}_\psi \sum_{k=1}^{2} (Z^{LR,\mathrm{RI}'})^{-1}{}_{ik}\,\mathrm{Tr}\left[\Lambda_{\Gamma_k}\,(\Lambda^{\mathrm{tree}}_{\Gamma_j})^\dagger\right]_{q_\nu=\bar{q}_\nu} = \delta_{ij}$$

  - 4 conditions for $Z^{LR,\mathrm{RI}'}_{ij}$. Values of Λ_{Γ_k} from simulations.

Conversion from RI′ to $\overline{\mathrm{MS}}$

- Relies necessarily on perturbation theory

$$\begin{pmatrix} \mathcal{O}_{\Gamma_1}^{\overline{\mathrm{MS}}} \\ \mathcal{O}_{\Gamma_2}^{\overline{\mathrm{MS}}} \end{pmatrix} = (Z^{LR,\overline{\mathrm{MS}}})^{-1} \cdot (Z^{LR,\mathrm{RI}'}) \cdot \begin{pmatrix} \mathcal{O}_{\Gamma_1}^{\mathrm{RI}'} \\ \mathcal{O}_{\Gamma_2}^{\mathrm{RI}'} \end{pmatrix} \equiv (\mathcal{C}^{\overline{\mathrm{MS}},\mathrm{RI}'}) \cdot \begin{pmatrix} \mathcal{O}_{\Gamma_1}^{\mathrm{RI}'} \\ \mathcal{O}_{\Gamma_2}^{\mathrm{RI}'} \end{pmatrix}$$

  - $\mathcal{C}^{\overline{\mathrm{MS}},\mathrm{RI}'}$ is finite, regularization independent

$$\mathcal{C}^{\overline{\mathrm{MS}},\mathrm{RI}'} = (Z^{LR,\overline{\mathrm{MS}}})^{-1} \cdot (Z^{LR,\mathrm{RI}'}) = (Z^{DR,\overline{\mathrm{MS}}})^{-1} \cdot (Z^{DR,\mathrm{RI}'})$$

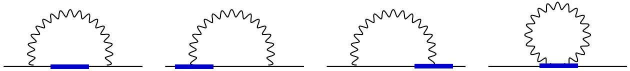
- We have calculated $\mathcal{C}^{\overline{\mathrm{MS}},\mathrm{RI}'}$ in DR to 1 loop
  $\longrightarrow$ Currently extending to 2 loops

- Actually $\mathcal{C}^{\overline{\mathrm{MS}},\mathrm{RI}'}$ is diagonal (unlike $Z^{LR,\overline{\mathrm{MS}}}$, $Z^{LR,\mathrm{RI}'}$)

- $\mathcal{C}^{\overline{\mathrm{MS}},\mathrm{RI}'}$ is q-independent (as it should)
  Predictable $(\bar{q}^2/\bar{\mu}^2)$ dependence + Complicated $z\bar{q}_\nu$ dependence!

Calculation - Dimensional Regularization

Feynman Diagrams



- Typical contribution:

$$\int_0^z d\zeta \int \frac{dp^D}{(2\pi)^D} \frac{e^{-i\,\zeta\, p_\mu}\, p_\mu}{p^2\,(p+q)^2} \quad = \quad \frac{i}{16\pi^2}\Big(\frac{1}{\epsilon} + 2 - \gamma_E + \log(4\pi/q^2)$$

$$-2\int_0^1 dx\, e^{-ixzq_\mu}\, K_0(|z|\sqrt{q^2x(1-x)}\,)\Big)$$

  - x: Feynman parameter, q_ν : external momentum
- Constant pole coefficient $\Rightarrow$ Constant $Z^{DR,\overline{\mathrm{MS}}}$
- Standard logarithmic dependence on renormalization scale $\bar{\mu}$
- Complicated dependence on zq, zq_ν
- Complex Green's functions $\Rightarrow$ Complex conversion factors

Results - Dimensional Regularization

- $Z_\Gamma^{DR,\,\overline{\rm MS}} = 1 + \frac{3}{\epsilon}\,\frac{g^2\,C_f}{16\,\pi^2}$
  - Known result (Dorn, Chetyrkin+Grozin)
  - Independent of Γ — No mixing
  - Independent of z, α (gauge parameter) : to all loops
  - Independent of D-dimensional prescription for γ_5

- Conversion Factors: Same expressions for:
  $(S,\,P),\ (V_\mu\,,\,A_\mu),\ (V_\nu\,,\,A_\nu)\ [\nu \neq \mu],\ T_{\nu\rho}$ [all 6 components]
  - For example, $\mathcal{C}_{V_\nu}^{\overline{\rm MS},{\rm RI}'}$ $\Longrightarrow$

    (Vector operator perpendicular to Wilson line)

Results - Dimensional Regularization

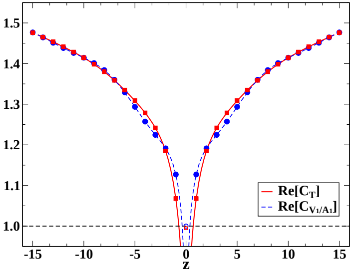
$$
\begin{aligned}
\mathcal{C}^{\overline{\rm MS},{\rm RI}'}_{V_\nu} \quad = \quad & 1 - \frac{g^2\,C_f}{16\,\pi^2}\Bigg(-7-4\gamma_E+\log(16)+4F_2+\frac{4\bar{q}_\nu^2|z|}{\bar{q}}F_5 \\
& +(\beta-4)\log\left(\frac{\bar{\mu}^2}{\bar{q}^2}\right)-(\beta+2)\log(\bar{q}^2 z^2) \\
& +\beta\Big[3-2\gamma_E+\log(4)-2\left(\frac{\bar{q}_\nu^2|z|}{\bar{q}}F_4+\left(\bar{q}^2+\bar{q}_\mu^2\right)G_3\right)-2F_1 \\
& +z^2\left(\bar{q}^2\left(F_3-\frac{F_1-F_2}{2}\right)+\bar{q}_\nu^2(F_1-F_2-F_3)\right)\Big] \\
& +i\left\{4\bar{q}_\mu G_1+\beta\bar{q}_\mu\left[\bar{q}(z|z|F_5+2(G_4-2G_5))-2G_1+2G_2\right]\right\}\Bigg)
\end{aligned}
$$

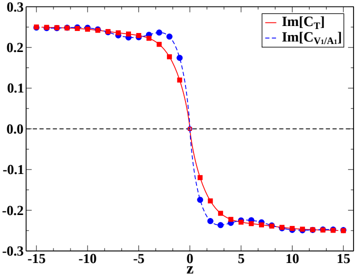
- $\beta = 1-\alpha$, and F_i , G_i are integrals over Bessel functions, e.g.:

$$G_1(q,z) = \int_0^1 dx \int_0^z d\zeta\, e^{-iq_\mu x\zeta}\, K_0\left(q\,|\zeta|\,\sqrt{(1-x)x}\right)$$

Results - Dimensional Regularization

Numerical Value of Conversion Factors vs. z/a (ETMC ensembles)





- $g^2 = 3.077$, $a = 0.082\,$fm, Lattice size: $32^3 \times 64$
  - $\overline{\text{MS}}$ scale: $\bar{\mu} = 2\,$GeV
  - RI′ scale: $a\bar{q} = \frac{2\pi}{32}(\frac{n_t}{2}+\frac{1}{4}, 0, 0, n_z)$, for $n_t = 8$ and $n_z = 4$
  - Landau gauge
- Dependence on operator weakens for large z

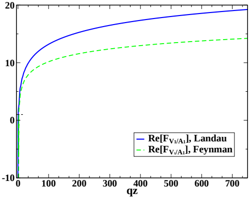
$\overline{\text{MS}}$-Renormalized Green's Functions

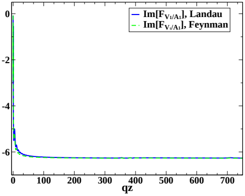
- Illustration: Select $q_\mu = (q,\,0,\,0,\,0) \Rightarrow \Lambda_\Gamma^{1-\text{loop}} \propto \Lambda_\Gamma^{\text{tree}}$

$$\Lambda_\Gamma^{1-\text{loop}}\Big|_{(q,0,0,0)} = \Lambda_\Gamma^{\text{tree}} \left(\frac{\bar{\mu}^2}{q^2}\right)^{(4-\beta)g^2\,C_f/(16\,\pi^2)} \left[1 + \frac{g^2\,C_f}{16\,\pi^2} F_\Gamma(qz)\right]$$

  - F_Γ : complex functions of qz, satisfying $F_\Gamma(-x) = F_\Gamma^\dagger(x)$
  - $\bar{\mu}$-dependence: exponentiated, shows anomalous dimension

- Example, $F_{V_1} = F_{A_1}$:





- Singular $q \to 0$ limit: Expected, due to contact terms as $z \to 0$

$$\lim_{q\to 0} F_{V_1(A_1)}(qz) = 3\left(1 + \gamma_E + \log(\tfrac{qz}{2})\right)$$

Lattice Actions

Clover Fermions

$$S_F = -\frac{a^3}{2}\sum_{x,\,f,\,\mu}\left[\bar\psi_f(x)\,(r-\gamma_\mu)\,U_{x,\,x+a\,\mu}\psi_f(x+a\,\mu)\right.$$
$$\left. +\bar\psi_f(x+a\,\mu)\,(r+\gamma_\mu)\,U_{x+a\,\mu,\,x}\psi_f(x)\right]$$
$$+ \quad a^4\sum_{x,\,f}(\frac{4r}{a}+m_0^f)\bar\psi_f(x)\psi_f(x)$$
$$- \quad \frac{a^5}{4}\sum_{x,\,f,\,\mu,\,\nu} c_{\mathrm{SW}}\,\bar\psi_f(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi_f(x)$$

- c_{SW} : free parameter
- Mass-independent schemes $\Rightarrow m_0^f = 0$ at one loop

  $\Rightarrow$ Twisted mass action is accommodated

Lattice Actions

Symanzik Gluons

$$S_G = \frac{2}{g_0^2}\Big[c_0 \sum_{\text{plaq.}} \mathrm{Re\,Tr}\{1 - U_{\text{plaq.}}\} + c_1 \sum_{\text{rect.}} \mathrm{Re\,Tr}\{1 - U_{\text{rect.}}\}$$
$$+ c_2 \sum_{\text{chair}} \mathrm{Re\,Tr}\{1 - U_{\text{chair}}\} + c_3 \sum_{\text{paral.}} \mathrm{Re\,Tr}\{1 - U_{\text{paral.}}\}\Big]$$

- c_i : Symanzik coefficients
  - Our calculations employ several choices of c_i
  - We will present results for values of c_i corresponding to: Plaquette, Tree-level Symanzik, Iwasaki gluons

Results - Lattice

- Same Feynman diagrams, more complicated vertices
- First diagram is finite $\Rightarrow$ Same as in DR
- Lattice Green's functions: As in DR, with additional q-independent (z-dependent!) terms
- Typical example: $I_{lat} \equiv \int \frac{dp^4}{(2\pi)^4} \frac{e^{-i\,z\,p_\mu/\mathbf{a}} - 1}{\sin\left(\frac{p_\mu}{2}\right)} \frac{\sin(p_\rho + a\,q_\rho)}{\widehat{p}^2\,\widehat{(p + a\,q)}^2}$
- By iterative additions/subtractions, brought into a form: (Factorizable integral) + (Terms with $a \to 0$) + (Continuum result)

$$I_{lat} = \int \frac{dp^4}{(2\pi)^4} \frac{e^{-i\,z\,p_\mu/\mathbf{a}} - 1}{\sin\left(\frac{p_\mu}{2}\right)} \frac{\sin(\overline{p}_\rho + a\,q_\rho)}{\widehat{\overline{p}}^2\,\widehat{(\overline{p} + a\,q)}^2} + (\text{terms with a} \to 0)$$

$$+ \int_{p \leq r/a} \frac{dp^4}{(2\pi)^4} \frac{e^{-i\,z\,p_\mu} - 1}{p_\mu/2} \left(\frac{(p_\rho + q_\rho)}{p^2\,(p+q)^2} - \frac{(\overline{p}_\rho + q_\rho)}{\overline{p}^2\,(\overline{p}+q)^2} \right)$$

- ($\overline{p} \equiv p - p_\mu \hat{\mu}$), r : arbitrary. Nontrivial check: r-independent result

Results - Lattice - Linear Divergence

- As with Wilson loops: $|z|/a$ term arises from "tadpole" diagram
  $\Rightarrow$ Independent of operator – Independent of fermion action
- Term leading to $|z|/a$:

$$\int_{-\pi}^{\pi} \frac{dp^4}{(2\pi)^4} S_g(p)_{\mu\mu} \frac{\sin^2\left(\frac{z}{a}\frac{p_\mu}{2}\right)}{\sin^2(p_\mu/2)}$$
$$= \int_{-\pi}^{\pi} \frac{dp^3}{(2\pi)^3} S_g(\overline{p})_{\mu\mu} \cdot \left(\frac{|z|}{a}\right) + \mathcal{O}(a^0, \log a)$$

  - $S_g(p)_{\mu\mu}$: gluon propagator in the direction of the Wilson line (μ), $\overline{p} \equiv p - p_\mu \hat{\mu}$
- Resummation to all orders in perturbation theory (shown by Dotsenko for Wilson loops): $\Lambda_\Gamma = e^{-c\,|z|/a}\,\tilde{\Lambda}_\Gamma$
  - One-loop expression for c : as above
  - $\tilde{\Lambda}_\Gamma$ related to $\Lambda_\Gamma^{\overline{\mathrm{MS}}}$ by a further renormalization factor which is at most logarithmically divergent with a.

Results - Lattice Renormalization

Green's functions: Bare (lattice) - Renormalized ($\overline{\mathrm{MS}}$)

$$\langle\psi\,\mathcal{O}_\Gamma\,\bar\psi\rangle^{DR,\overline{\mathrm{MS}}} - \langle\psi\,\mathcal{O}_\Gamma\,\bar\psi\rangle^{LR} = \frac{g^2\,C_f}{16\,\pi^2}\,e^{i\,q_\mu z}\Bigg[\Gamma\bigg(\alpha_1 + 5.792\,\beta + \alpha_2\,\frac{|z|}{a} + \log\left(a^2\bar\mu^2\right)(4-\beta)\bigg) + (\Gamma\cdot\gamma_\mu + \gamma_\mu\cdot\Gamma)\,\Big(\alpha_3 + \alpha_4\,c_{\mathrm{SW}}\Big)\Bigg]$$

- $\alpha_{\mathbf{i}}$: constants, depend on Symanzik coefficients, Γ-independent
- $(\Gamma\cdot\gamma_\mu + \gamma_\mu\cdot\Gamma)$: not a multiple of tree-level $\Rightarrow$ mixing! (finite)
  - Unpolarized quasi-PDF mixes with twist-3 "scalar" operator
  - Beyond 1 loop: coefficients change, mixing pattern identical
- Taking also into account $Z_\psi^{LR,\overline{\mathrm{MS}}}$:

$$Z_\psi^{LR,\overline{\mathrm{MS}}} = 1 + \frac{g^2\,C_f}{16\,\pi^2}\left[e_1^\psi + 5.792\,\beta + e_2^\psi\,c_{\mathrm{SW}} + e_3^\psi\,c_{\mathrm{SW}}^2 + (1-\beta)\log\left(a^2\bar\mu^2\right)\right]$$

Our results for renormalization and mixing coefficients are $\Longrightarrow$

Results - Lattice Renormalization

$$Z_{\mathcal{O}}^{LR,\overline{\rm MS}} = 1 + \frac{g^2\,C_f}{16\,\pi^2}\left(\mathbf{e_1} + \mathbf{e_2}\,\frac{|z|}{a} + \mathbf{e_3}\,c_{\rm SW} + \mathbf{e_4}\,c_{\rm SW}^2 - 3\log\left(a^2\bar{\mu}^2\right)\right)$$

$$Z_{mix}^{LR,\overline{\rm MS}} = 0 + \frac{g^2\,C_f}{16\,\pi^2}\left(\mathbf{e_5} + \mathbf{e_6}\,c_{\rm SW}\right)$$

Wherever mixing occurs

Results for selected actions

Action	e_1	e_2	e_3	e_4	e_5	e_6
Wilson	24.306	-19.955	-2.249	-1.397	14.450	-8.285
TL Symanzik	19.844	-17.293	-2.015	-1.242	12.756	-7.674
Iwasaki	12.558	-12.978	-1.601	-0.973	9.937	-6.528

- Gauge-invariant Z's
- Mixing coefficients are finite
- All Z's are operator independent
- Mixing vanishes at $c_{\rm SW} = -\,e_5/e_6$

$\Rightarrow$ Guidance for simulations: At $c_{\rm SW} \sim 1.5$ **mixing is suppressed**

Elimination of Linear Divergence

- Perturbative result: Important for investigating its properties
  However, higher order terms will dominate in the $a \to 0$ limit! $\Rightarrow$ Need a non-perturbative approach
- Approach by Ishikawa et al.: Estimate it through static potential
- Our approach [Alexandrou et al., arXiv:1610.03689], (see talk by K. Cichy): Relies on non-perturbative matrix elements of Wilson line operators: $q(P_3, z)$

◦ P_3 : momentum boost of nucleon (in the direction of the Wilson line)

◦ For simplicity, consider cases w/o mixing (helicity, transversity)

[Cases with mixing are similar: One must construct the 2×2 matrix of traces.]

Elimination of Linear Divergence

- Renormalized matrix element:

$$q^R(P_3 z, P_3/\bar{\mu}) = (P_3/\bar{\mu})^{2\gamma_\Gamma} \cdot \tilde{q}^R(P_3 z), \quad \gamma_\Gamma = -3g^2 C_f/(16\pi^2) + \mathcal{O}(g^4)$$

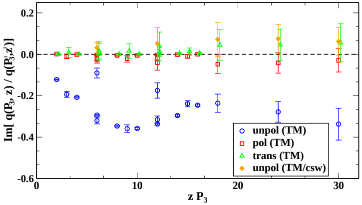
- Renormalization function: $Z_\Gamma^{LR,\overline{\text{MS}}}(a\bar{\mu}, z/a) = \tilde{Z}_\Gamma(a\bar{\mu}) \cdot \hat{Z}(z/a)$,

$$\text{where : } \quad \hat{Z}(z/a) = e^{-\delta m\,|z|/a}, \qquad \delta m = -\frac{g^2\,C_f}{16\,\pi^2}\,e_2 + \mathcal{O}(g^4)$$

- Form ratios: $\dfrac{q(P_3, z)}{q(P_3', z')} = \dfrac{e^{-\delta m\,|z|/a}\,(P_3)^{2\gamma_\Gamma}\,\tilde{q}^R(P_3 z)}{e^{-\delta m\,|z'|/a}\,(P_3')^{2\gamma_\Gamma}\,\tilde{q}^R(P_3' z')}$
- For $P_3 z = P_3' z'$: $\dfrac{q(P_3, z)}{q(P_3', z')} = e^{-\delta m\,(|z|-|z'|)/a}\left(\dfrac{P_3}{P_3'}\right)^{-6g^2 C_f/(16\pi^2)}$
- Choosing several values of $P_3 z = P_3' z'$, one extracts δm by a fit.
- Possible non-perturbative scale c: $\exp\{-(\delta m/a + c)|z|\}$ [R. Sommer, arXiv:1501.03060] One can still find $(\delta m/a + c)$ through a fit; δm by itself is more delicate.

Elimination of Linear Divergence

Checks of the ratio $q(P_3, z)/q(P_3', z')$



- Imaginary part of the ratio for data at $m_\pi = 375$ MeV using TM fermions [Alexandrou et al.] (open symbols), and preliminary data at $m_\pi = 130$ MeV using TM clover fermions (filled symbols)
- Real ratio for polarized and transversity, as expected
- Complex ratio for unpolarized (mixes with twist-3), as expected
- Extracted value for $(\delta m/a) + c$, using different combinations of $P_3 z$, is consistent within statistical accuracy
- Polarized and transversity: Very similar estimates for $(\delta m/a) + c$
- $c_{\rm SW} \sim 1.57$: Suppressed mixing $\Rightarrow$ Unpolarized ratio gets real

SUMMARY & TO-DO LIST

Progress in renormalization of quasi-PDFs

- **Techniques to understand and remove linear divergence**
- **Study of multiplicative renormalization**
  (perturbatively and non-perturbatively)
- **Eliminate mixing where present**

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Follow-up work

- **Finalize the renormalization of nucleon matrix elements**
- **Subtraction of lattice artifacts using perturbative results**
- **Non-perturbative elimination of mixing**
- **Conversion factor to 2 loops; inclusion of nonzero masses**
- **Study more complicated Wilson lines. Relevant for TMDs [Engelhardt et al.]**
- **Effect of smearing, perturbative and non-perturbatively**

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Gracias!