Westfälische Wilhelms-Universität Münster
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Ward identities in $\mathcal{N} = 1$ supersymmetric SU(3) Yang-Mills theory on the lattice

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Lattice2017

Noether Theorem in classical theory $\rightarrow$ WIs in quantum theory

$$\left\langle \left(\partial_\mu j^\mu(x)\right) Q(y) \right\rangle = -\left\langle \frac{\delta Q(y)}{\delta \bar{\varepsilon}(x)} \right\rangle$$

RHS of equation is contact term, which is zero if Q is localised at space-time points different from x.

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$$S = -\frac{\beta}{N_c}\sum_p \mathrm{Re}\,\mathrm{Tr}\; U_p$$

$$+\frac{1}{2}\sum_x \left\{ \overline{\lambda}^a_x \lambda^a_x - \kappa \sum_{\mu=1}^{4} \left[\overline{\lambda}^a_{x+\hat{\mu}} V_{ab,x\mu}(1+\gamma_\mu)\lambda^b_x + \overline{\lambda}^a_x V^t_{ab,x\mu}(1-\gamma_\mu)\lambda^b_{x+\hat{\mu}} \right] \right\}$$

$\beta = \dfrac{2N_c}{g^2}$, $\quad \kappa = \dfrac{1}{2m_0+8}$ (hopping parameter), $\quad m_0$: bare gluino mass

$V_{ab,x\mu} = 2\,\mathrm{Tr}\left(U^\dagger_{x\mu} T_a U_{x\mu} T_b\right)$, $\quad$ adjoint link variables

SUSY transformations on the lattice with P, T, Majorana nature and gauge invariance:

$$\delta U_\mu(x) = -\frac{iga}{2}\left(\bar{\varepsilon}(x)\gamma_\mu U_\mu(x)\lambda(x) + \bar{\varepsilon}(x+\hat{\mu})\gamma_\mu \lambda(x+\hat{\mu}) U_\mu(x)\right)$$

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Above transformations result in following Ward identities:

$$\big\langle\big(\nabla_\mu S_\mu(x)\big)Q(y)\big\rangle = m_0\big\langle D_S(x)Q(y)\big\rangle + \big\langle X(x)Q(y)\big\rangle - \left\langle \frac{\delta Q(y)}{\delta\bar{\varepsilon}(x)}\right\rangle$$

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$D_S(x)$: due to bare gluino mass (m_0)
$X(x)$: due to lattice regularization

} *Break SUSY*

After renormalization WI gets the following form:

$$\left\langle \left(\nabla_\mu S_\mu(x)\right) Q(y) \right\rangle + \frac{Z_T}{Z_S} \left\langle \left(\nabla_\mu T_\mu(x)\right) Q(y) \right\rangle = \frac{m_S}{Z_S} \left\langle D_S(x) Q(y) \right\rangle + O(a)$$

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Zero spatial momentum WI and expansion in a basis of 16 Dirac matrices; the surviving contributions form a set of two non-trivial independent equations:

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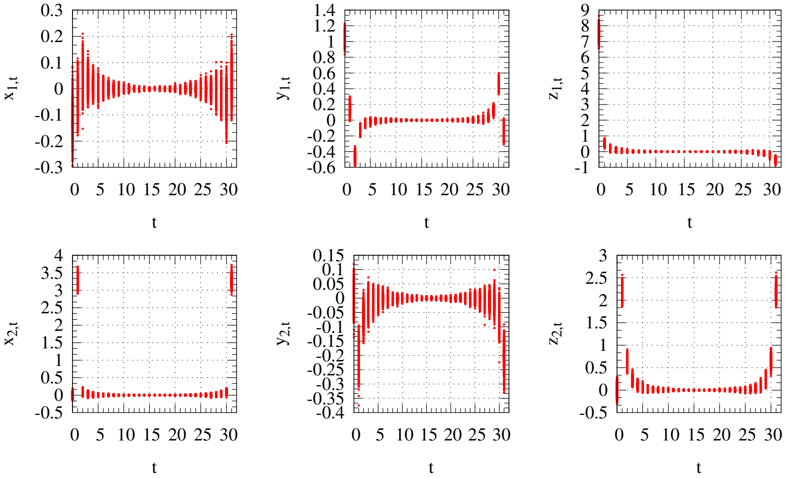
$\Longrightarrow$

$$\sum_\alpha A_\alpha x_{i\alpha} = 0, \quad i = (b,t), \alpha = 1,2,3$$

Figure: $V = 16^3 \cdot 32,\ \beta = 5.5,\ \kappa = 0.1673$



Global method

Calculate A and B by minimizing the quantity:

$$\sum_{i=1}^{2} \sum_{t=t_{min}}^{t_{max}} \frac{(x_{i,t} + A y_{i,t} - B z_{i,t})^2}{\sigma^2}$$

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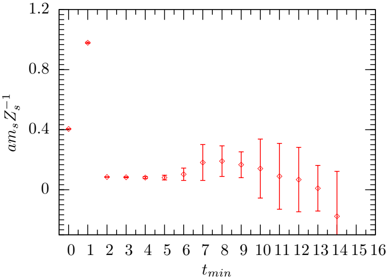

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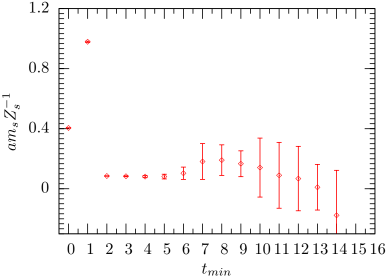
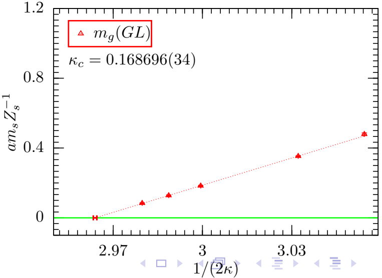


Extrapolation to the Chiral limit using m_g



Generalized χ^2 method

$$\sum_{\alpha} A_{\alpha} x_{i\alpha} = 0$$

We employ the method of
maximum likelihood

$$L = \frac{1}{2} \sum_{i,\alpha,j,\beta} (A_{\alpha} \bar{x}_{i\alpha})(D^{-1})_{ij}(A_{\beta} \bar{x}_{j\beta})$$

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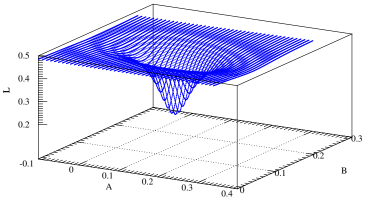

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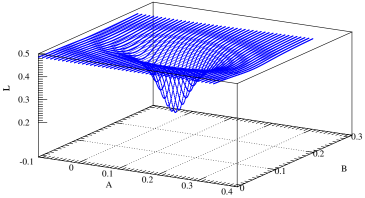

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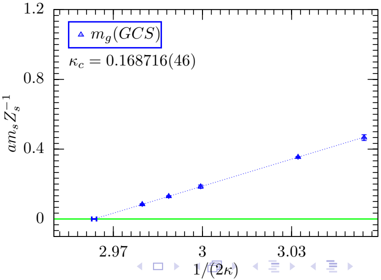

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Adjoint Pion

- The $m_{a-\pi}$ is obtained numerically in simulations of $\mathcal{N} = 1$ SYM theory
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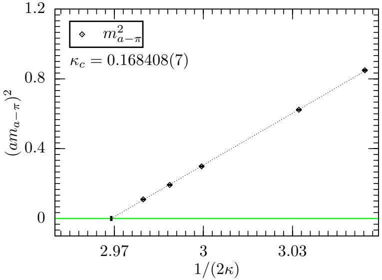

Figure: Extrapolation to the critical point /chiral limit using $m^2_{a-\pi}$

Adjoint Pion

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$$(am_{a-\pi})^2 \simeq A(\frac{1}{\kappa} - \frac{1}{\kappa_c})$$

$$am_S Z_S^{-1} = \frac{1}{2}(\frac{1}{\kappa} - \frac{1}{\kappa_c})$$

$$(am_{a-\pi})^2 \propto am_S Z_S^{-1}$$

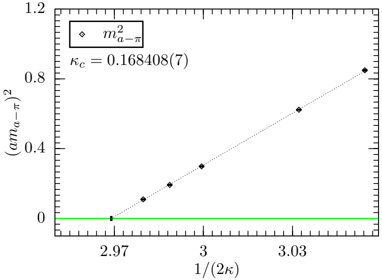

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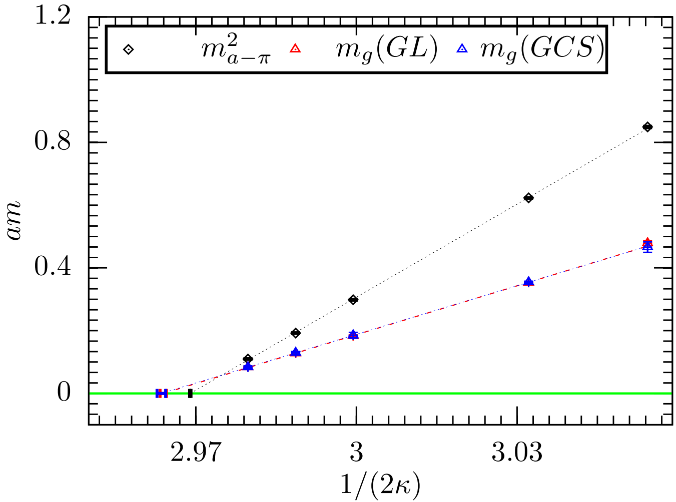
Figure: Extrapolation to Chiral limit using $m^2_{a-\pi}$ and m_g



Summary

- SUSY WIs on the lattice
- Determination of $am_S Z_S^{-1}$ using WIs by Global method
- Determination of $am_S Z_S^{-1}$ using WIs by GCS method
- Extrapolations towards vanishing gluino mass using WIs
- Extrapolations towards vanishing gluino mass using $m^2_{a-\pi}$
- Consistency between κ_c from WIs and from $m^2_{a-\pi}$
- Consistency with restoration of SUSY

Thank You!

Figure: Schlossgebäude: Westfälische Wilhelms-Universität Münster.