Complex Langevin simulation of QCD at finite density and low temperature using the deformation technique

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Collaborators

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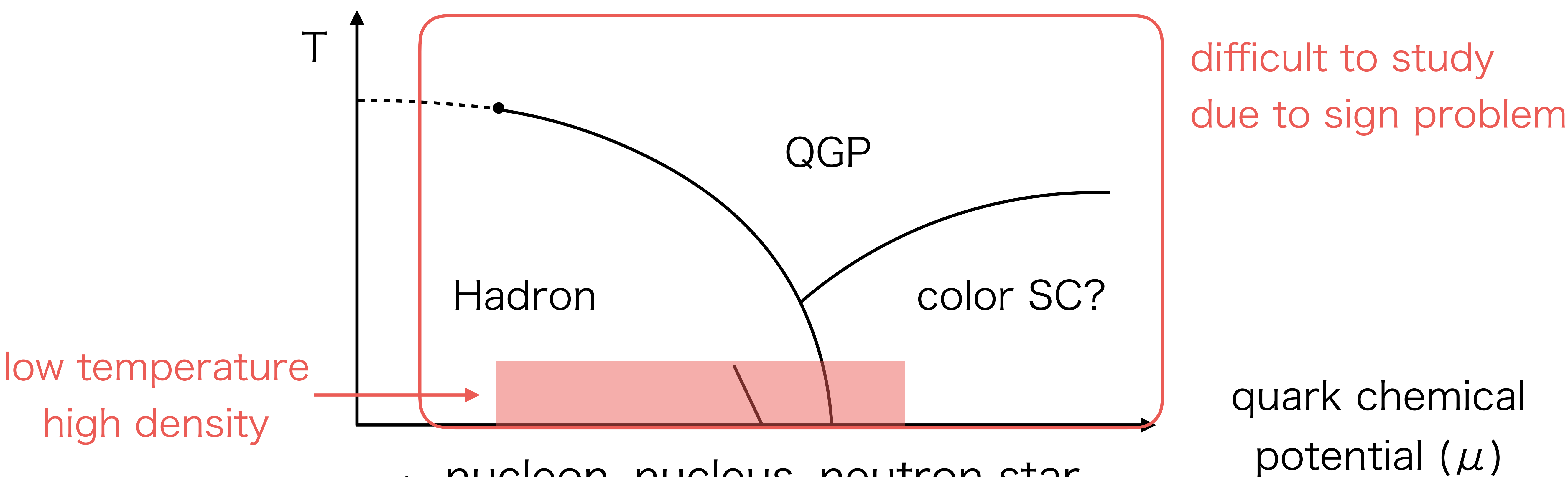
Sign problem in finite density QCD

- QCD partition function $Z = \int dU \det M[U, \mu] e^{-S_g[U]}$

complex ($\mu \neq 0$)

- QCD phase diagram



- nucleon, nucleus, neutron star
- Severe sign problem occurs

Our work

- We study QCD at finite μ and low T (N_f=4 staggered fermion) by using the complex Langevin method (CLM)

  [Parisi 83][Klauder 84]
  [Aarts, Seiler, Stamatescu 09][Aarts, James, Seiler, Stamatescu 11]…

- We successfully observe “the Silver Blaze phenomenon”

- Crucial techniques in our work :
  - ✓ Gauge cooling [Seiler, Sexty, Stamatescu 13]
  - ✓ Deformation of the Dirac operator [Ito, Nishimura 16]
  - ✓ Reliability check of the obtained results by the probability distribution of the drift term [Nagata, Nishimura, SS 16]

CLM for lattice QCD

- $Z = \int dU \det M[U, \mu] e^{-S_g[U]}$
- complexification $U_{x\mu} \in SU(3) \to \mathcal{U}_{x\mu} \in SL(3, C)$
  - action $S[U] \to S[\mathcal{U}]$ holomorphic function
  - gauge transformation $\mathcal{U}_{x\mu} \to g_x \, \mathcal{U}_{x\mu} \, g_{x+\hat{\mu}}^{-1}$ $g_x \in SL(3, C)$
- complex Langevin equation

$$\mathcal{U}_{x\mu}(t+\epsilon) = \exp\left\{ i \left(\epsilon v_{x\mu}(\mathcal{U}) + \sqrt{\epsilon}\eta_{x\mu} \right) \right\} \mathcal{U}_{x\mu}(t)$$

$$v_{x\mu}(\mathcal{U}) = -\mathcal{D}_{x\mu} S[\mathcal{U}]$$

Condition for the correct convergence

[Nagata, Nishimura, SS 16]

- Probability distribution of the magnitude of the drift term

$$p(u) = \frac{1}{4N_V}\left\langle \sum_{x\nu} \delta(u - u_{x\nu}) \right\rangle \qquad u_{x\nu} = \sqrt{\frac{1}{N_c^2 - 1}\mathrm{tr}(v_{x\nu} v_{x\nu}^\dagger)}$$

- Asymptotic behavior of $p(u)$ at large u

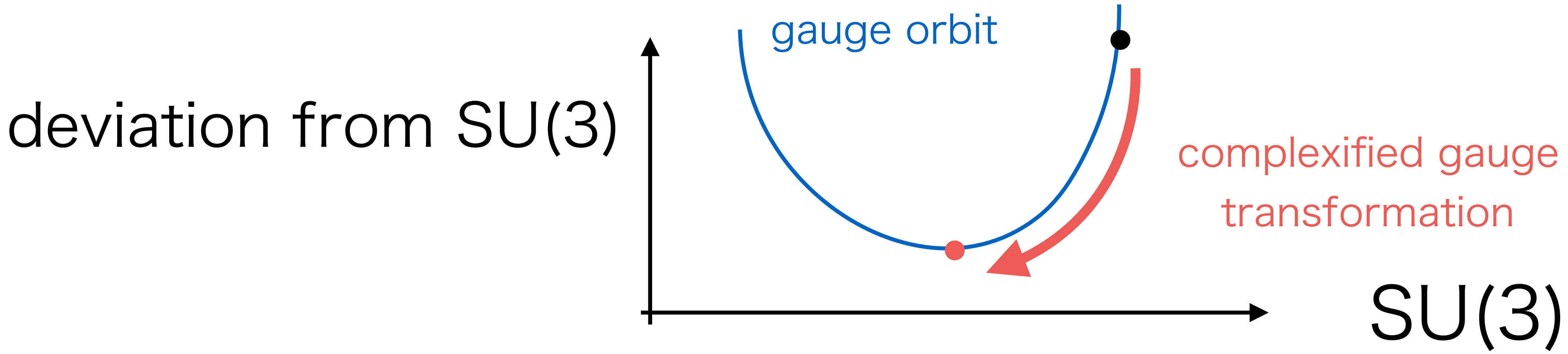
exponential fall-off or faster	$\rightarrow$	correct result
power-law fall-off	$\rightarrow$	wrong result

- The main causes of the power-law fall
  - large deviations of $\mathcal{U}_{x\mu}$ from SU(3) (excursion problem) $\leftarrow$ Gauge cooling
  - small eigenvalues of $M[\mathcal{U}, \mu]$ (singular drift problem) $\leftarrow$ Deformation of the Dirac operator

Gauge cooling

[Seiler, Sexty, Stamatescu 13]

- Perform the complexified gauge transformation after each Langevin step in such a way that the deviation of $\mathcal{U}_{x\mu}$ from SU(3) is minimized



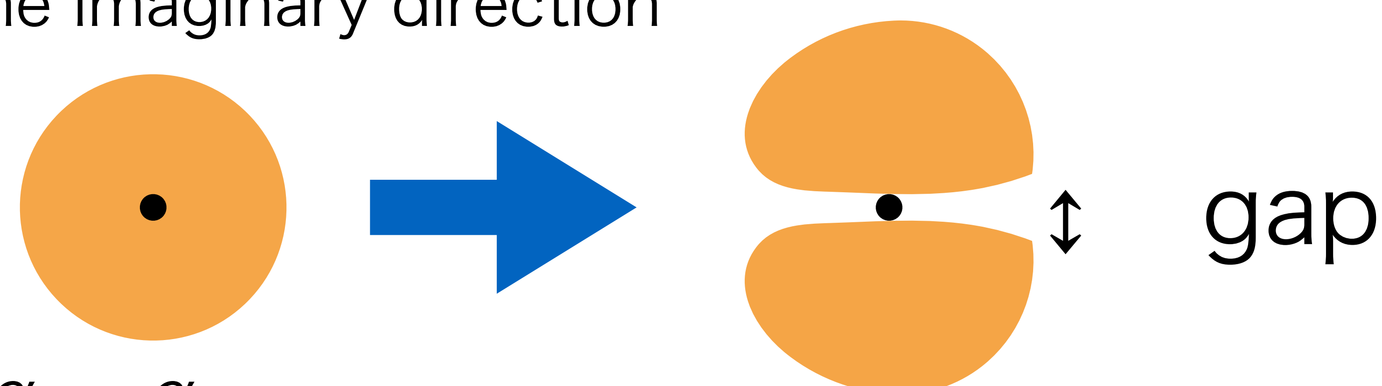
※ theoretical justification [Nagata, Nishimura, SS 16]

Deformation of the Dirac operator

[Ito, Nishimura 16]

Next talk by Y. Ito

- $$\det[M(\mathcal{U},\mu)] \to \det[M(\mathcal{U},\mu) + i\alpha\gamma_4]$$
- Formally, equivalent to adding the imaginary chemical potential in the continuum theory
  → Chiral sym and rotation sym kept intact
- $\alpha \neq 0$ → a gap in the imaginary direction



- symmetric under $\alpha \leftrightarrow -\alpha$
  ⇒ We fit the expectation values of observables to $c_0 + c_2\alpha^2 + \cdots$ by using only data points, which pass the reliability test, and extrapolate $\alpha \to 0$.

Simulation setup

cf) related talk by D. K. Sinclair

- Lattice setup :
  - * Staggered fermion (N_f=4)
  - * lattice size : N_x=N_y=N_z=4, N_t=8 [Nagata, Nishimura, SS, to appear]
  - * β=5.7
  - * quark mass : ma=0.05
  - * quark chemical potential : μa=0.4, 0.5, 0.6, 0.7

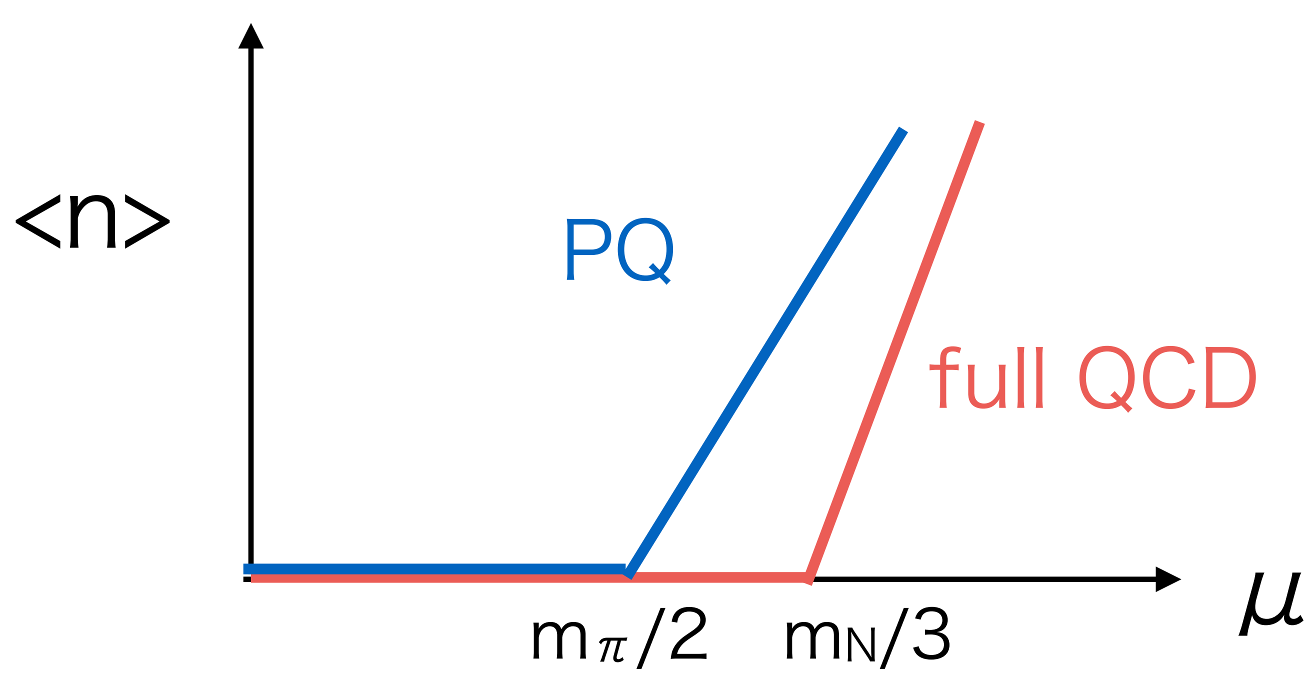
- CLM setup :
  - * $\varepsilon=10^{-4}$,
  - * total Langevin steps = 5×10^5 ~ 15×10^5
  - * gauge cooling (10~50 times)

- We also report some preliminary results for N_x=N_y=N_z=8, N_t=16

  [Ito, Matsufuru, Moritake, Nishimura, SS, Tsuchiya, Tsutsui]

Observables

- Baryon number density $$\langle n \rangle = \frac{1}{N_V N_c} \frac{\partial}{\partial(\mu a)} \log Z$$
- Chiral condensate $$\langle \Sigma \rangle = \frac{1}{N_V} \frac{\partial}{\partial(ma)} \log Z$$
- We compare the results with the RHMC results of the phase quenched model to confirm “the Silver Blaze phenomenon”.

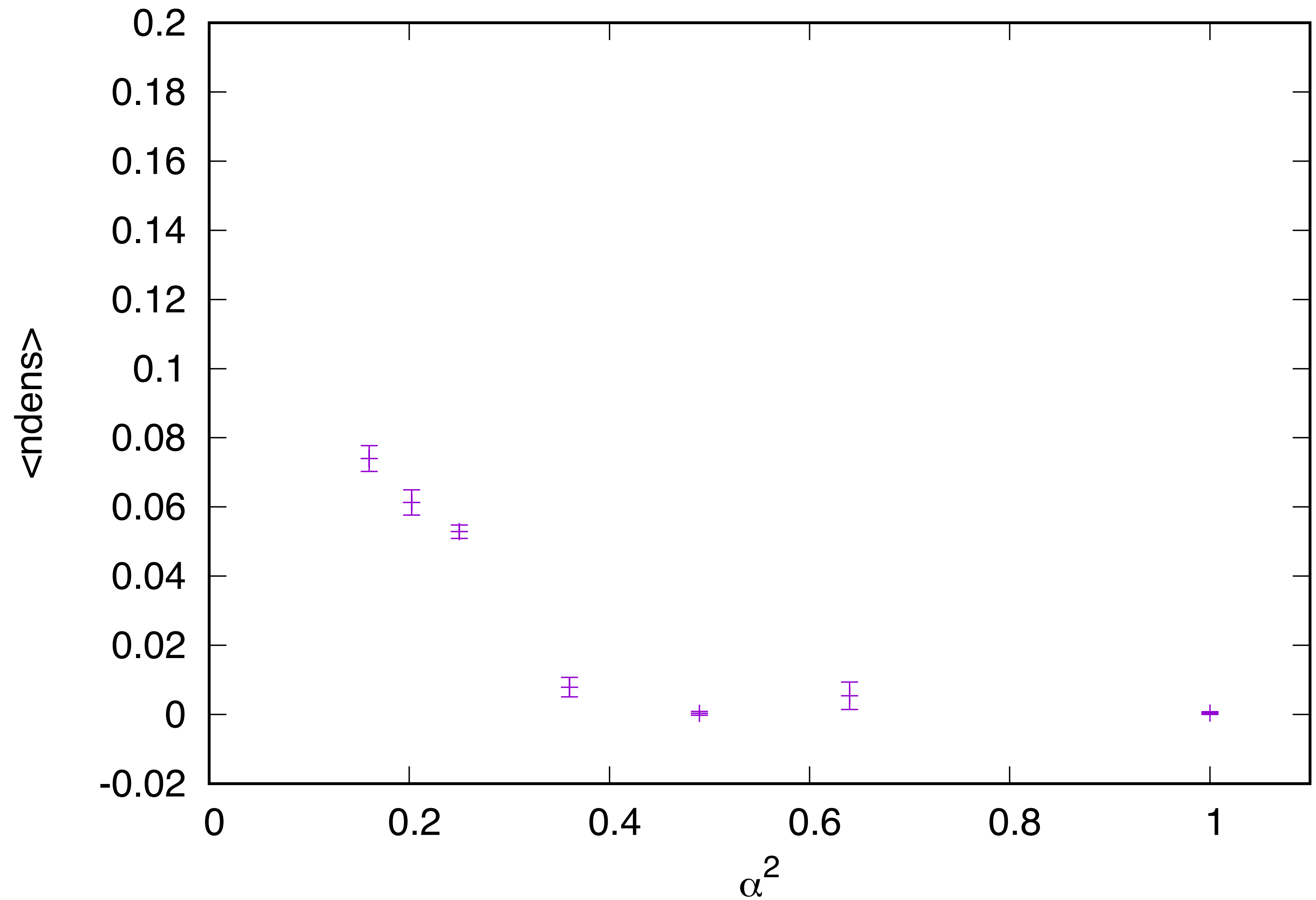


Results for $N_x=N_y=N_z=4$, $N_t=8$

[Nagata, Nishimura, SS, to appear]

" <n> vs α^2 " at μ=0.7

(4^3x8 lattice)



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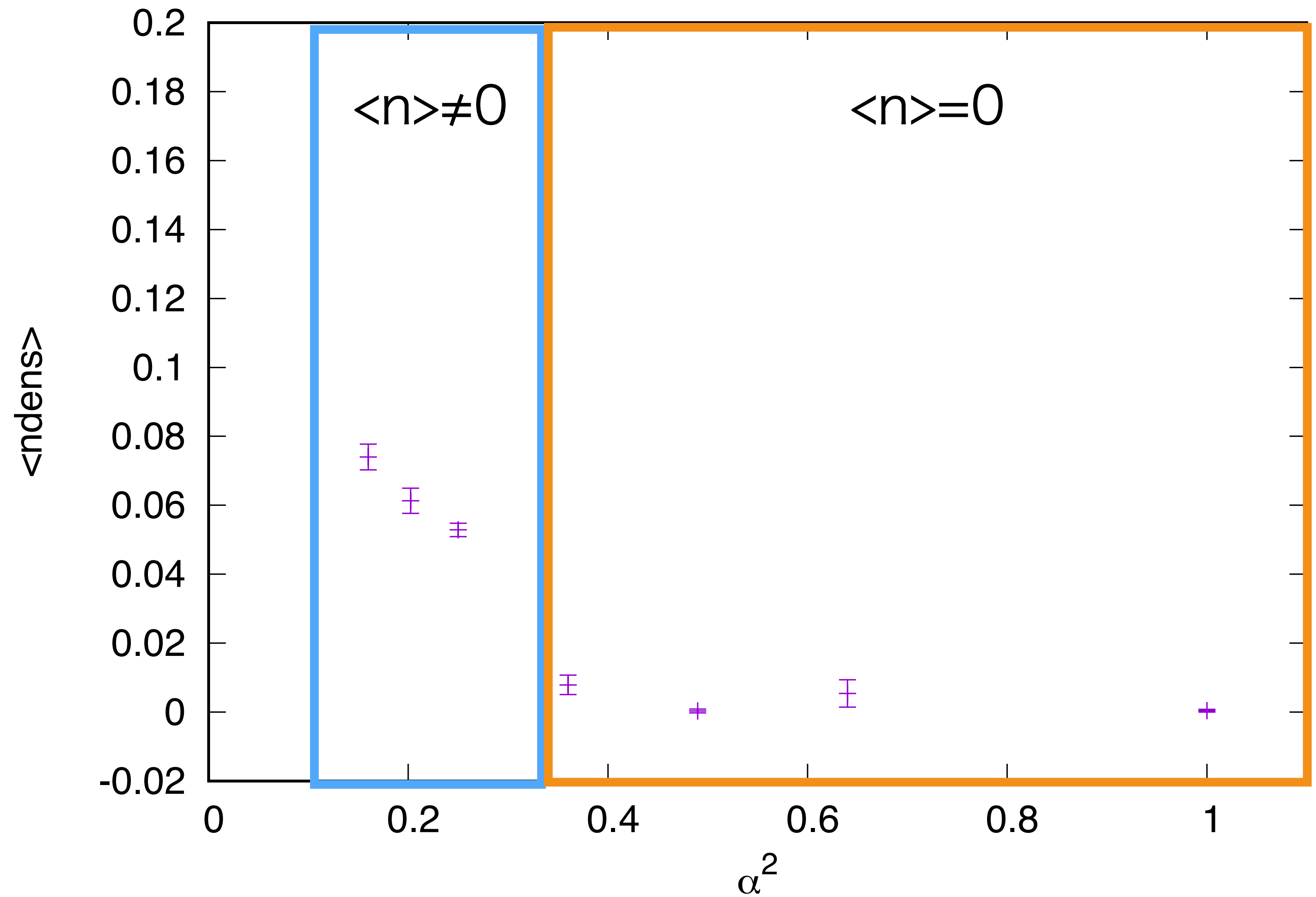

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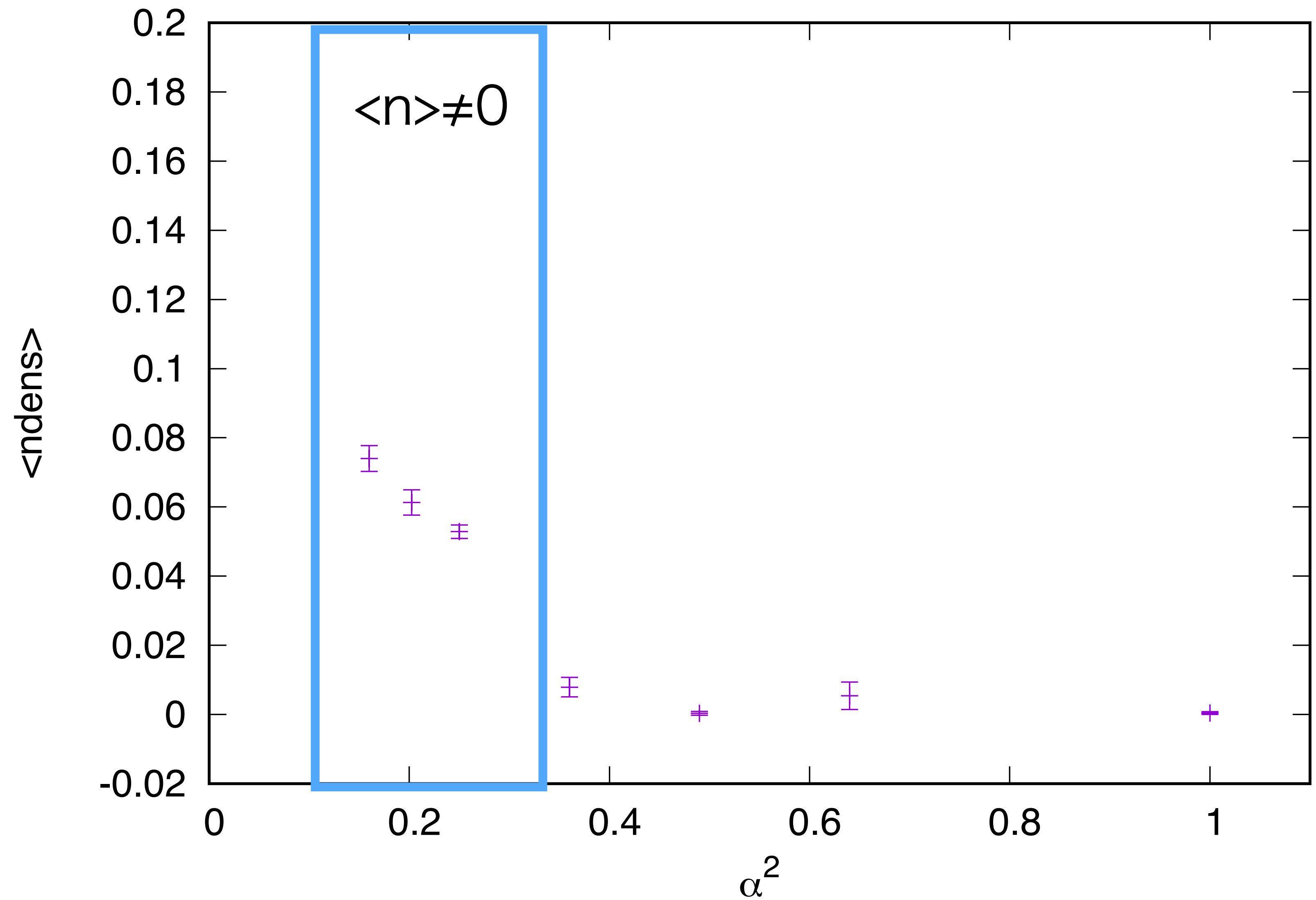
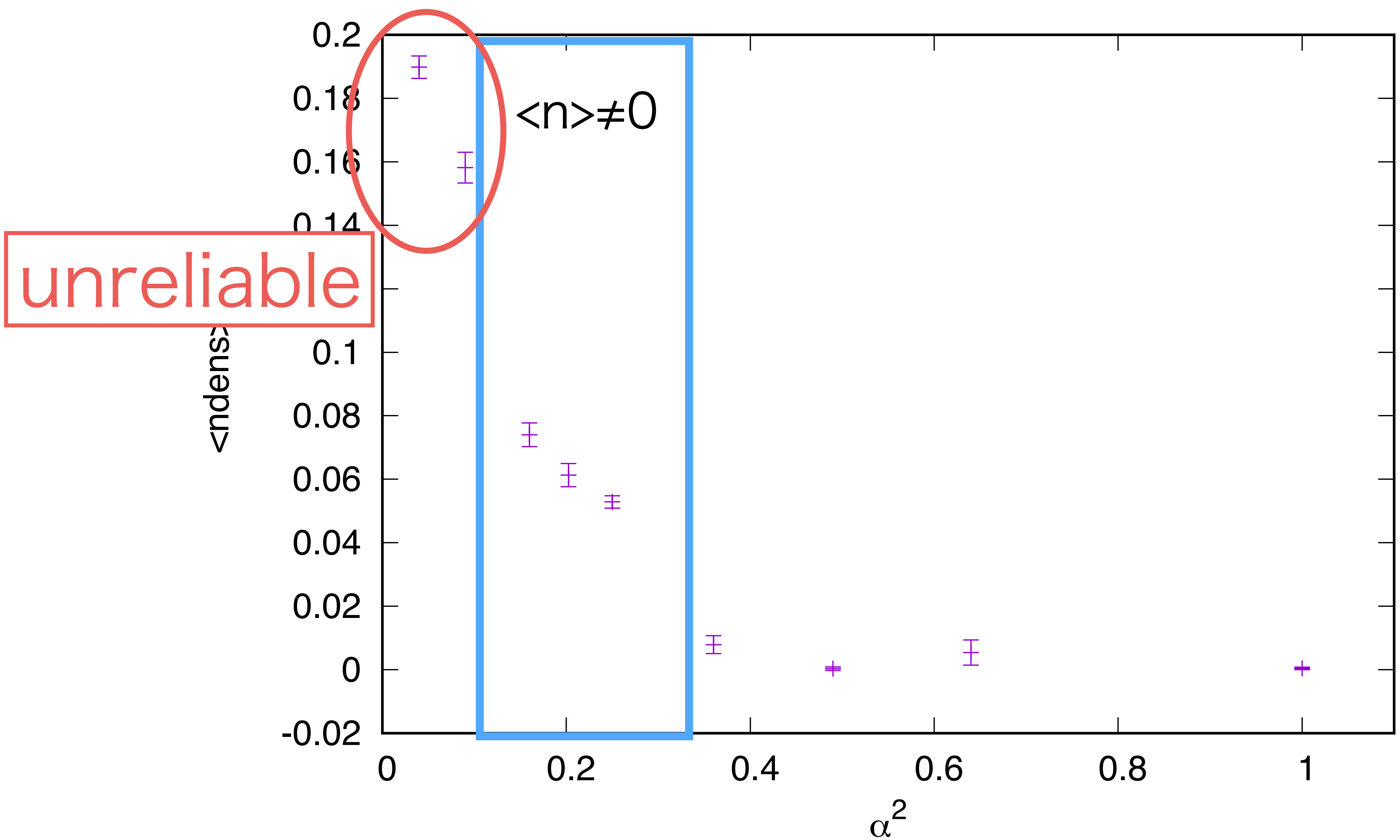
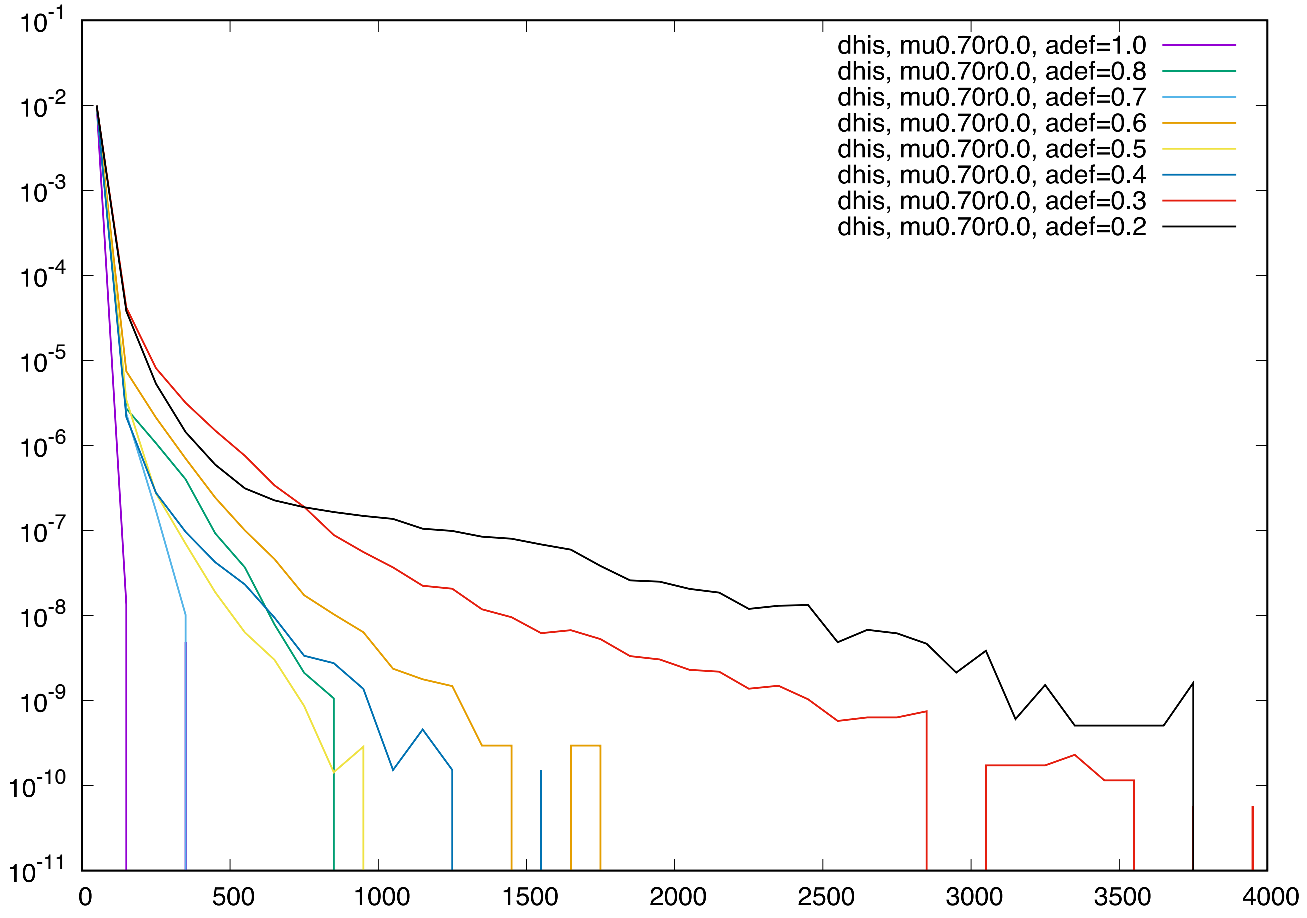


"$\langle n \rangle$ vs α^2" at μ=0.7

(4^3x8 lattice)



Prob. dist. (semilog) of the magnitude of the drift (μ=0.7)



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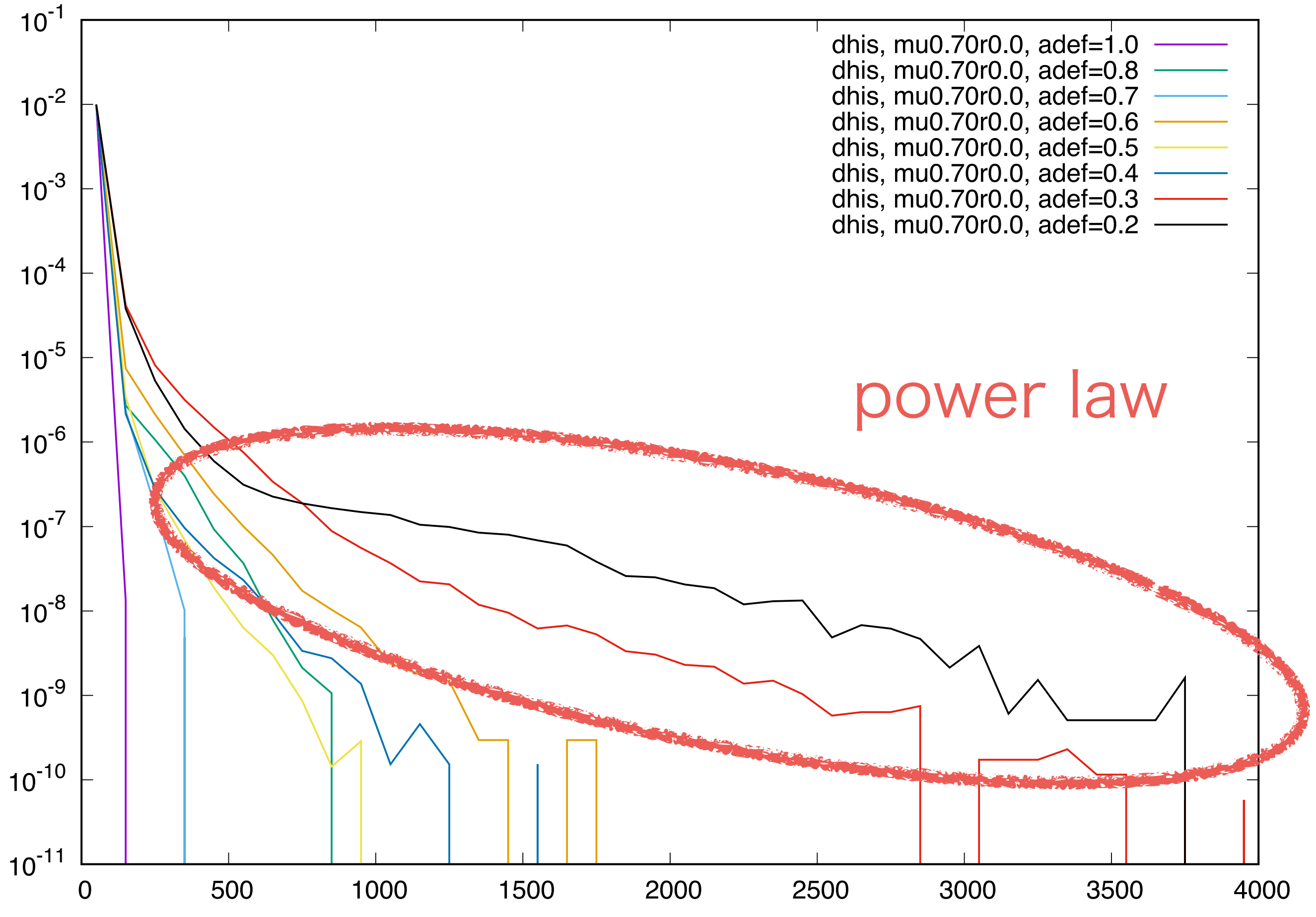

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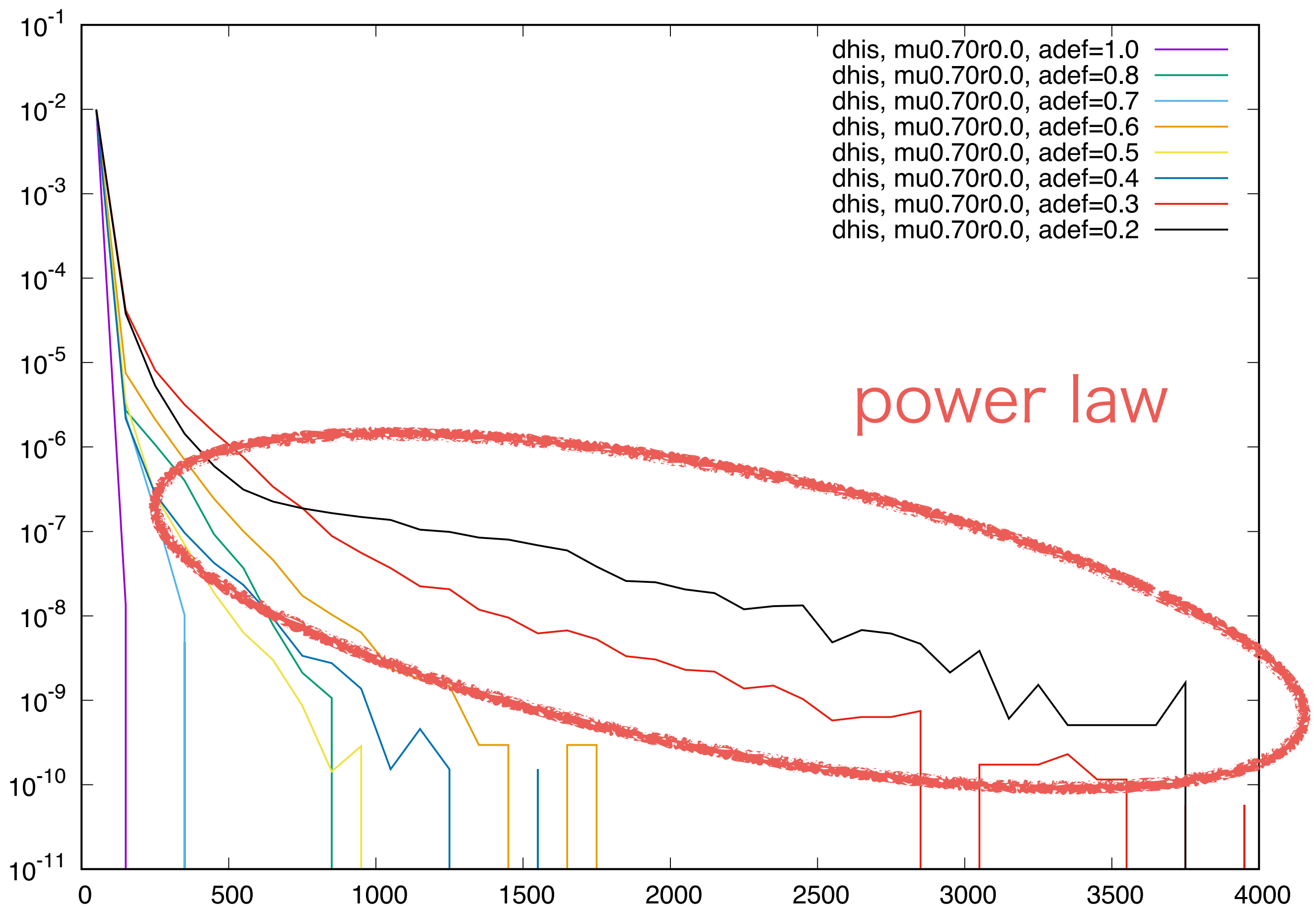
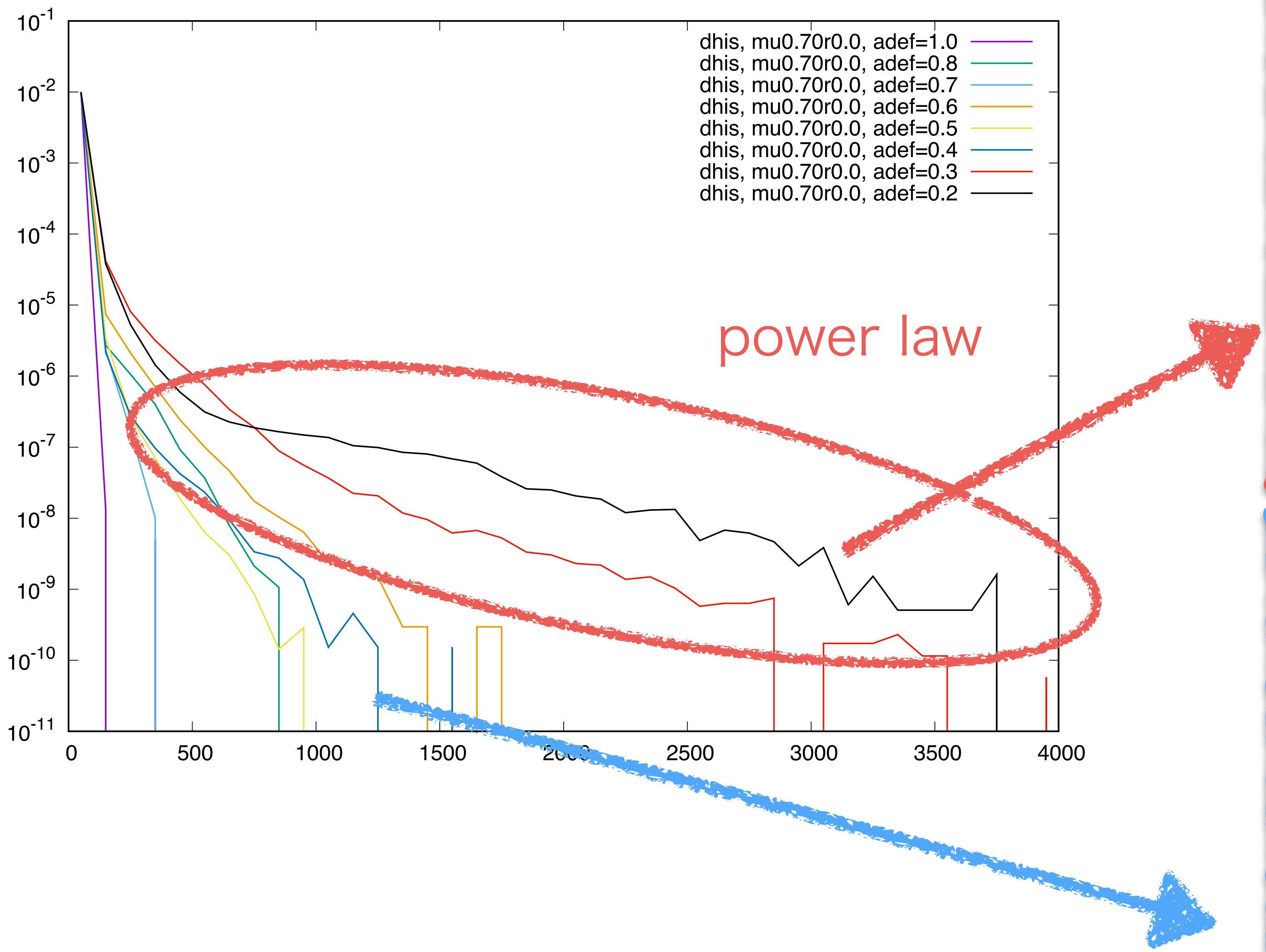


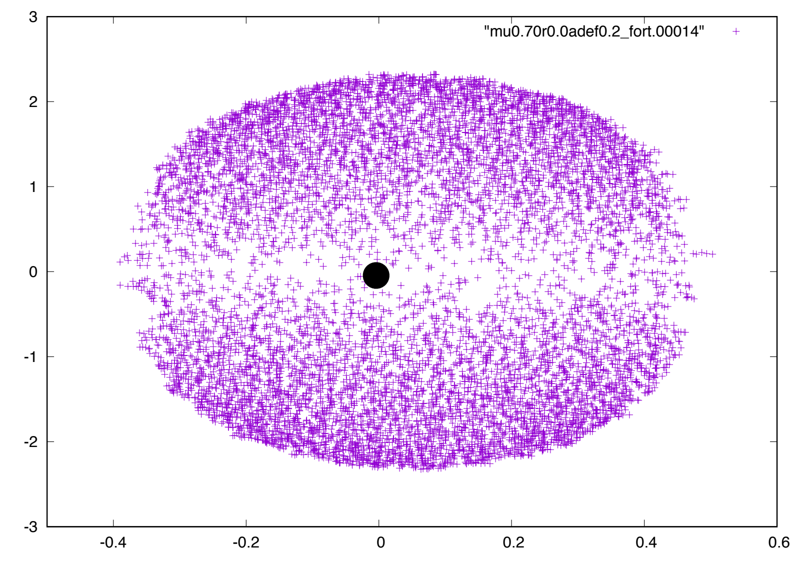
A drastic change of the dist. can be observed as α is varied.
→ indicates a transition from the exp. fall-off to the power law

Practically, the fall-off behavior can be judged from its α-dependence

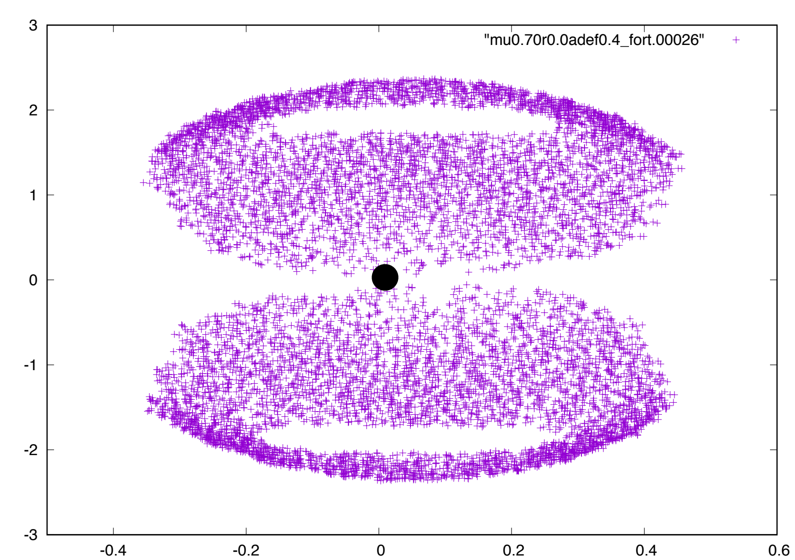
Eigenvalue distribution of M=D+m
(μ=0.7)



α=0.2

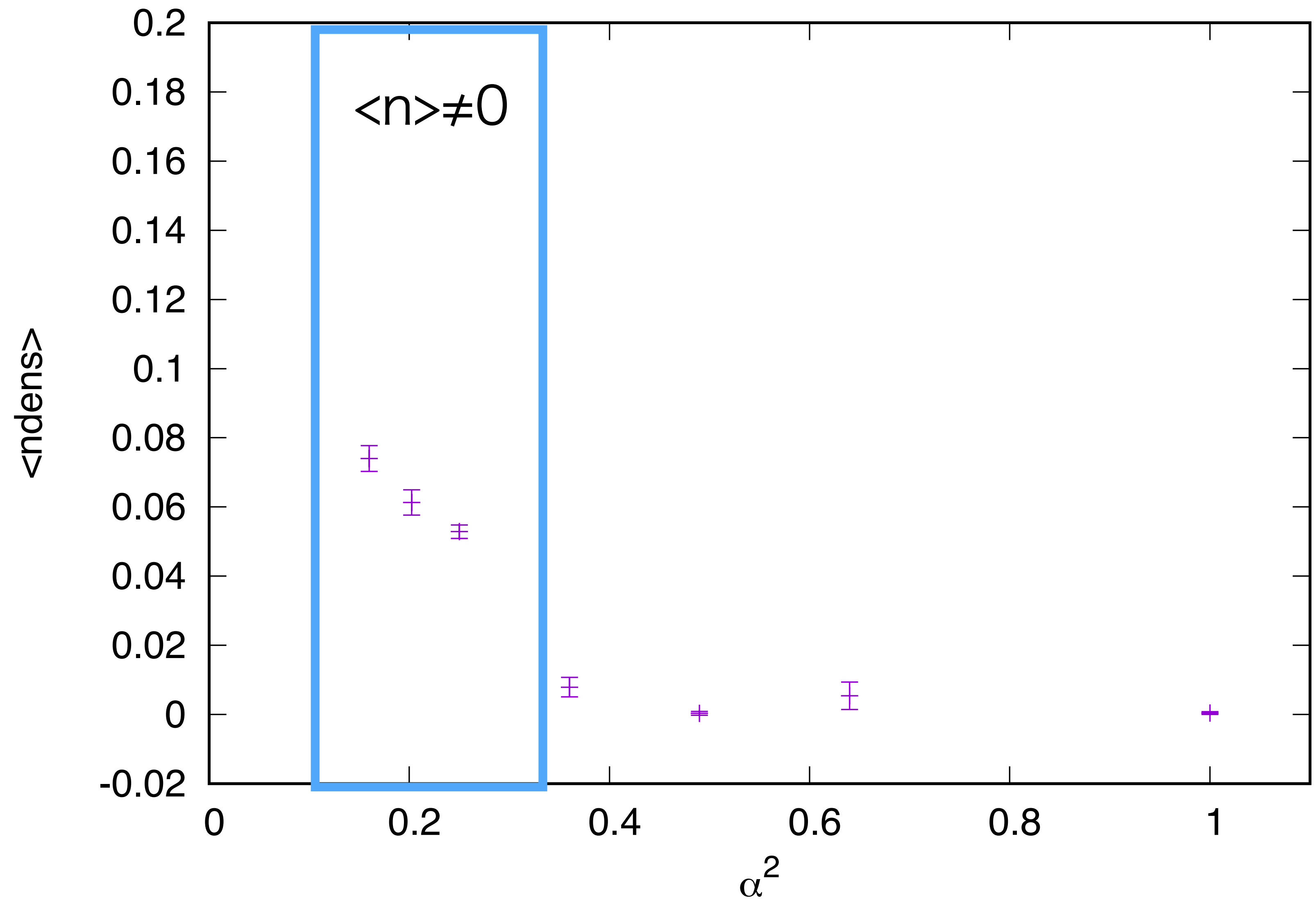


α=0.4



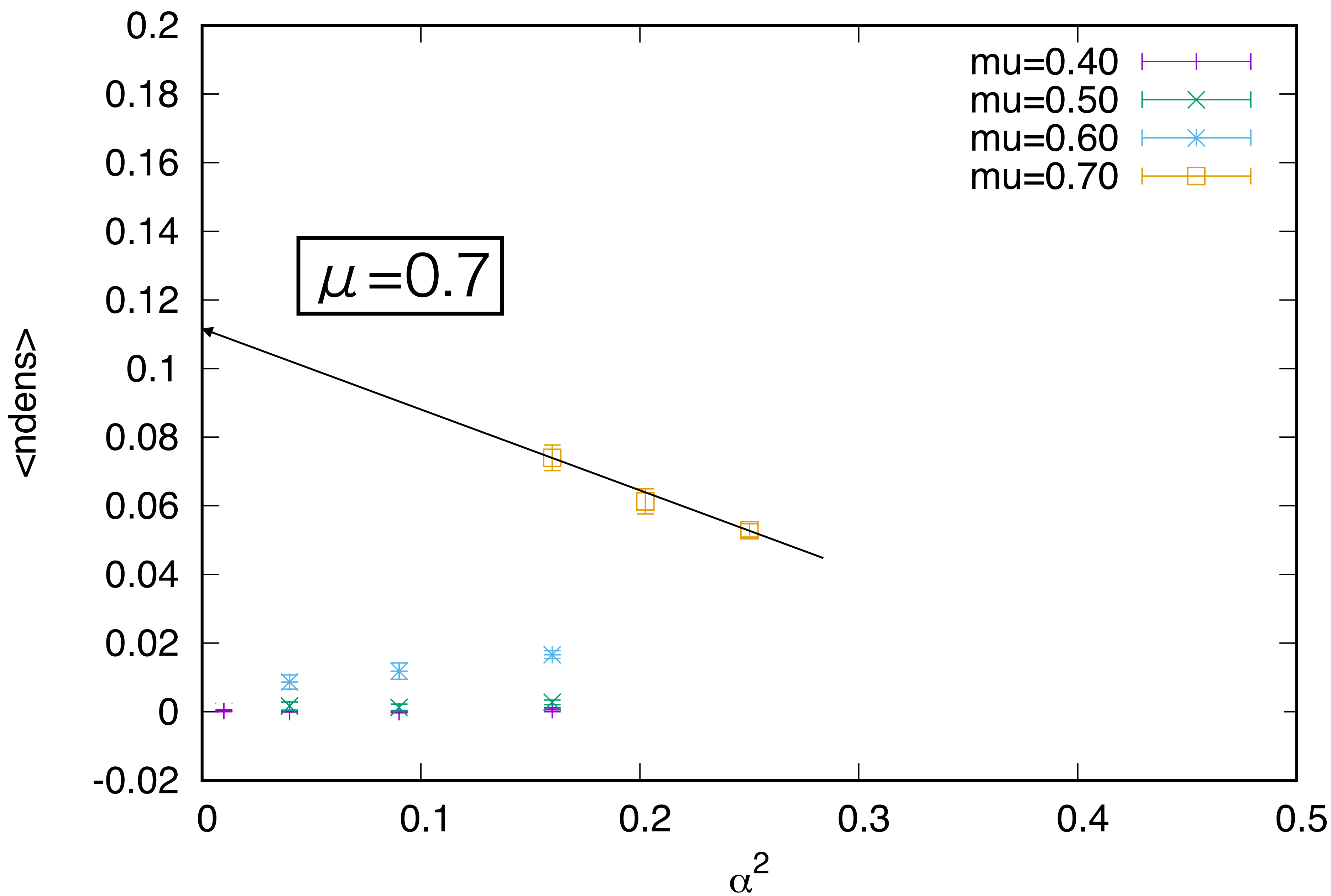
“ <n> vs α^2 ” at μ=0.7

(4^3x8 lattice)



“ <n> vs α^2 ” at μ=0.4, 0.5, 0.6, 0.7

(4^3x8 lattice)



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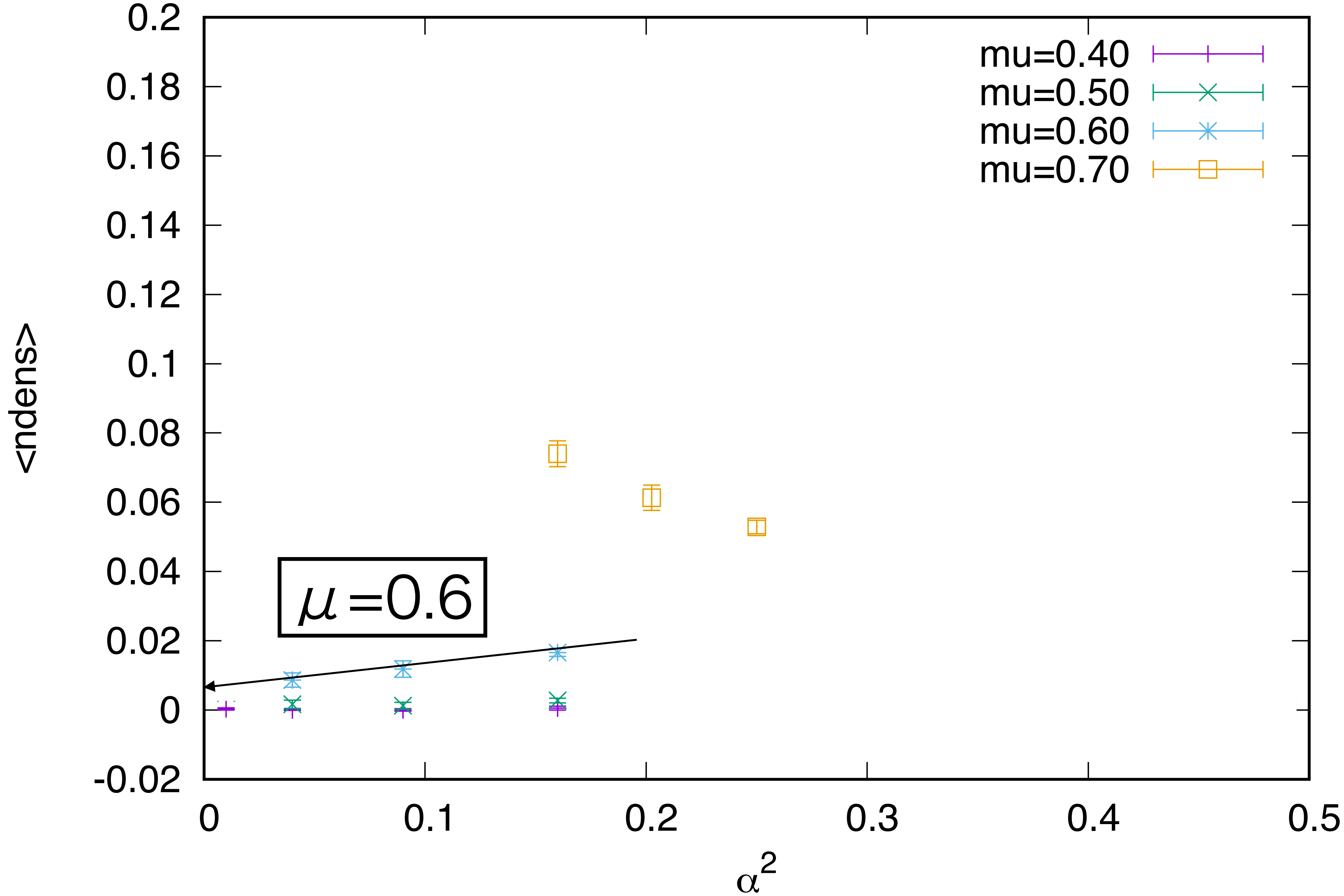

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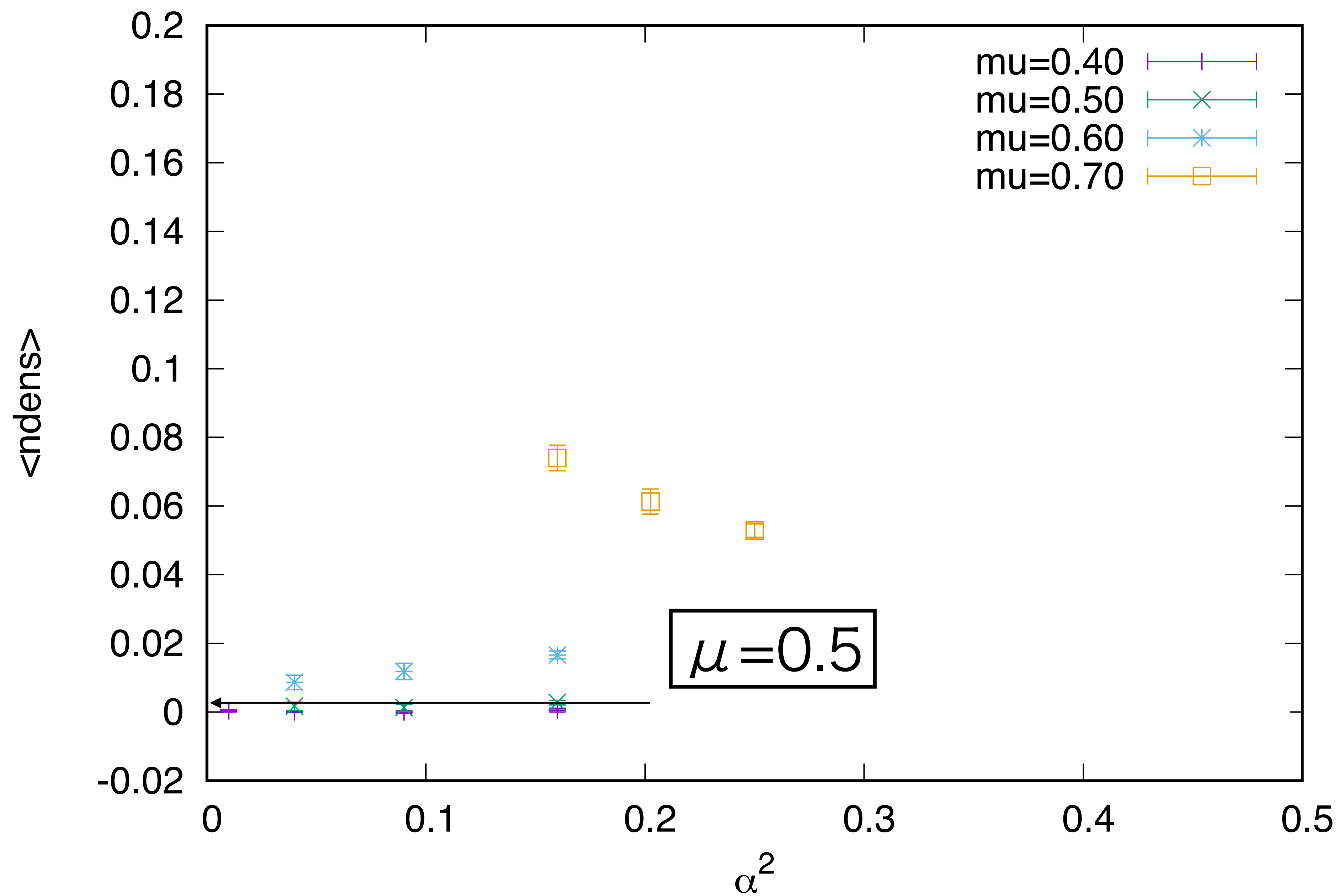

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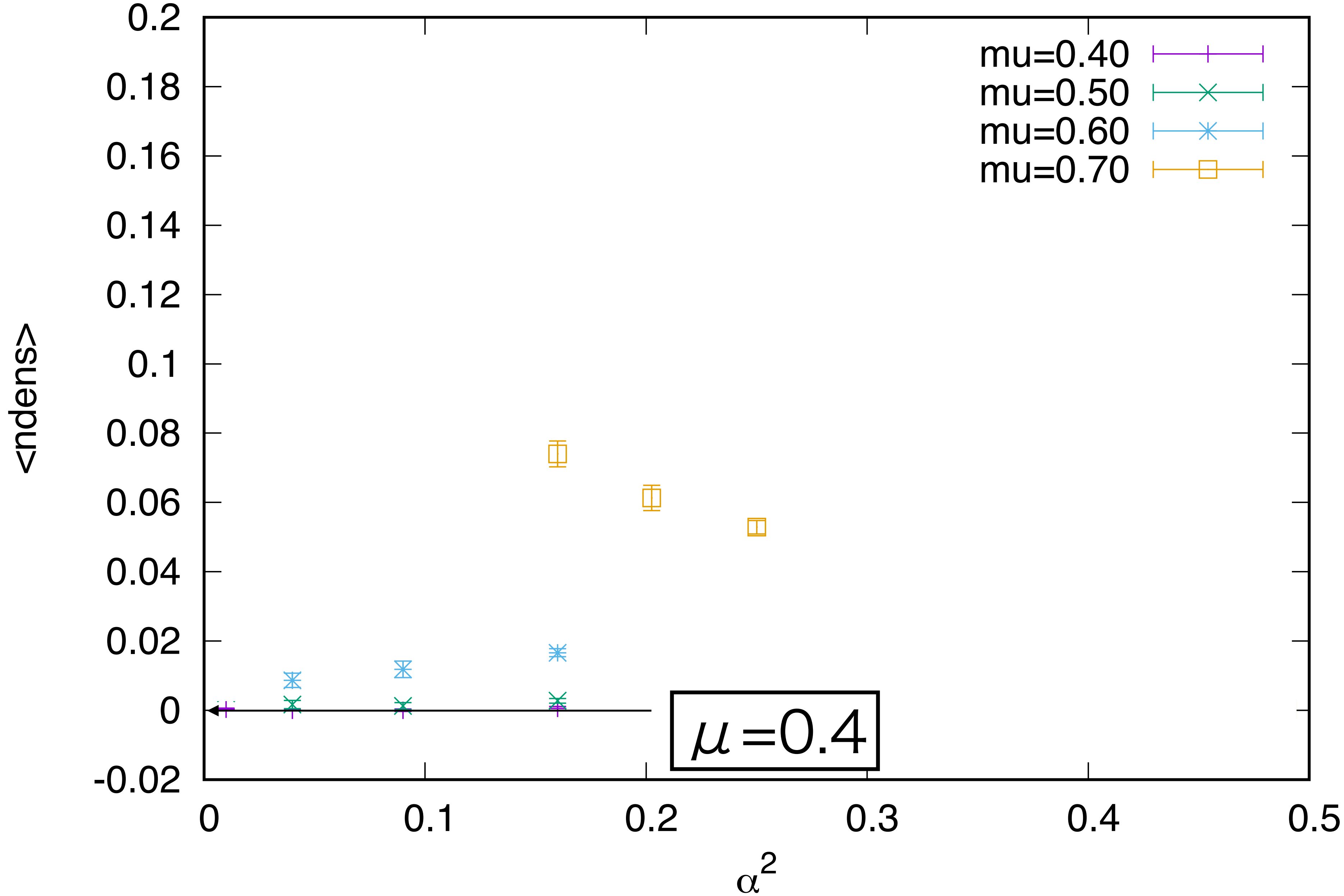
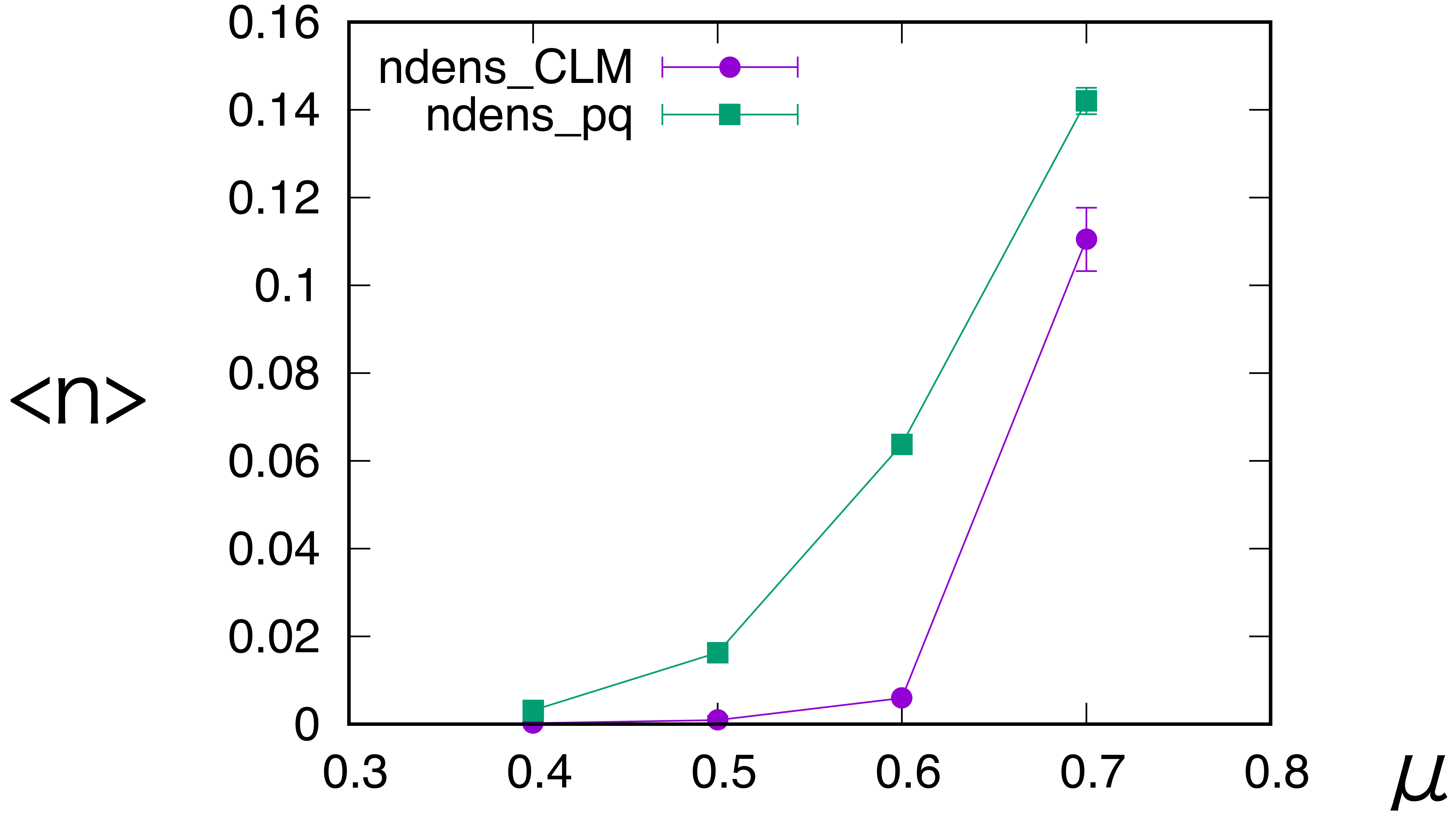
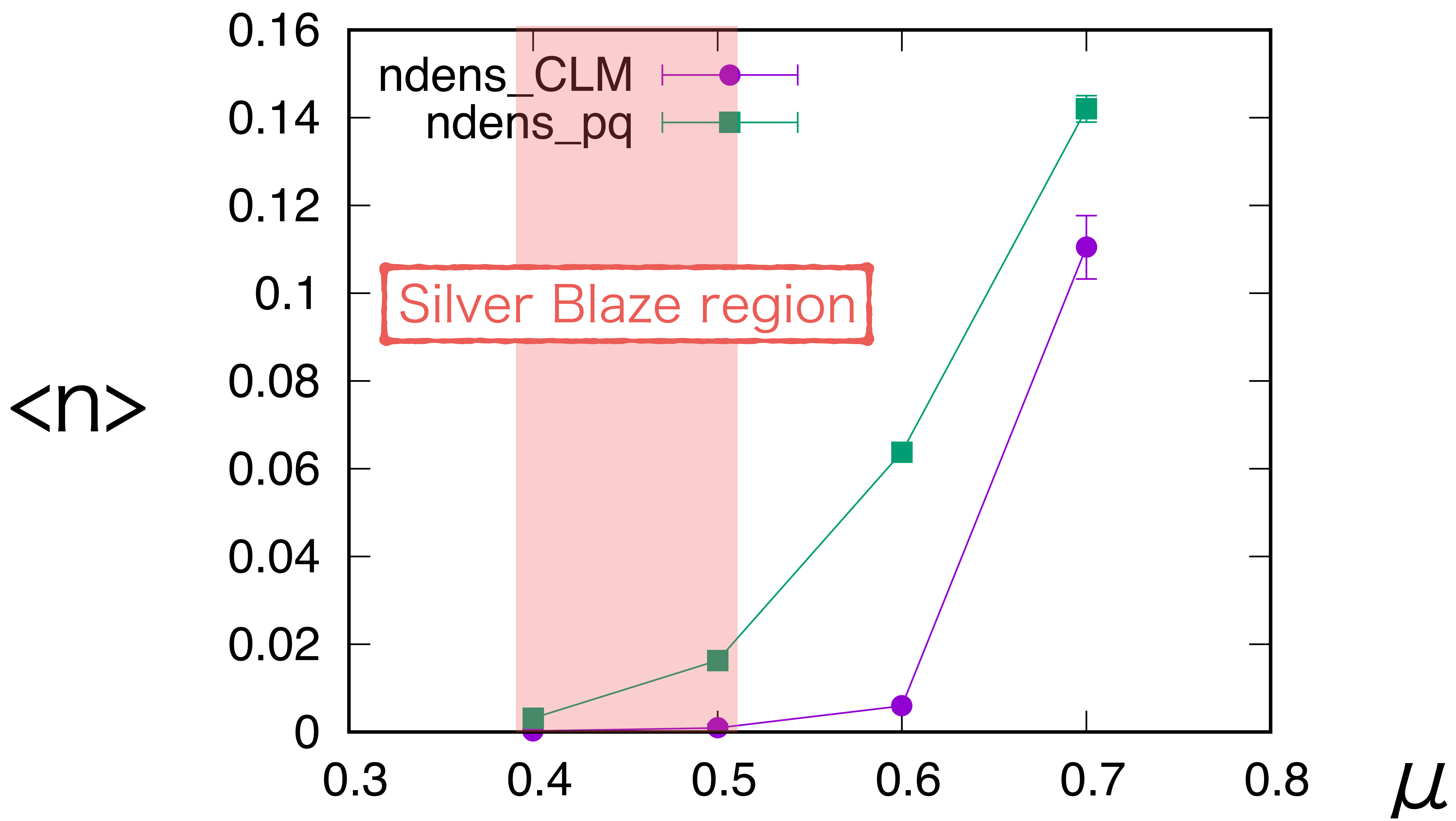


<n> vs μ (4^3x8)



<n>
μ
ndens_CLM
ndens_pq
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.3
0.4
0.5
0.6
0.7
0.8

<n> vs μ (4^3x8)



<n>
0.16
0.14
0.12
0.1
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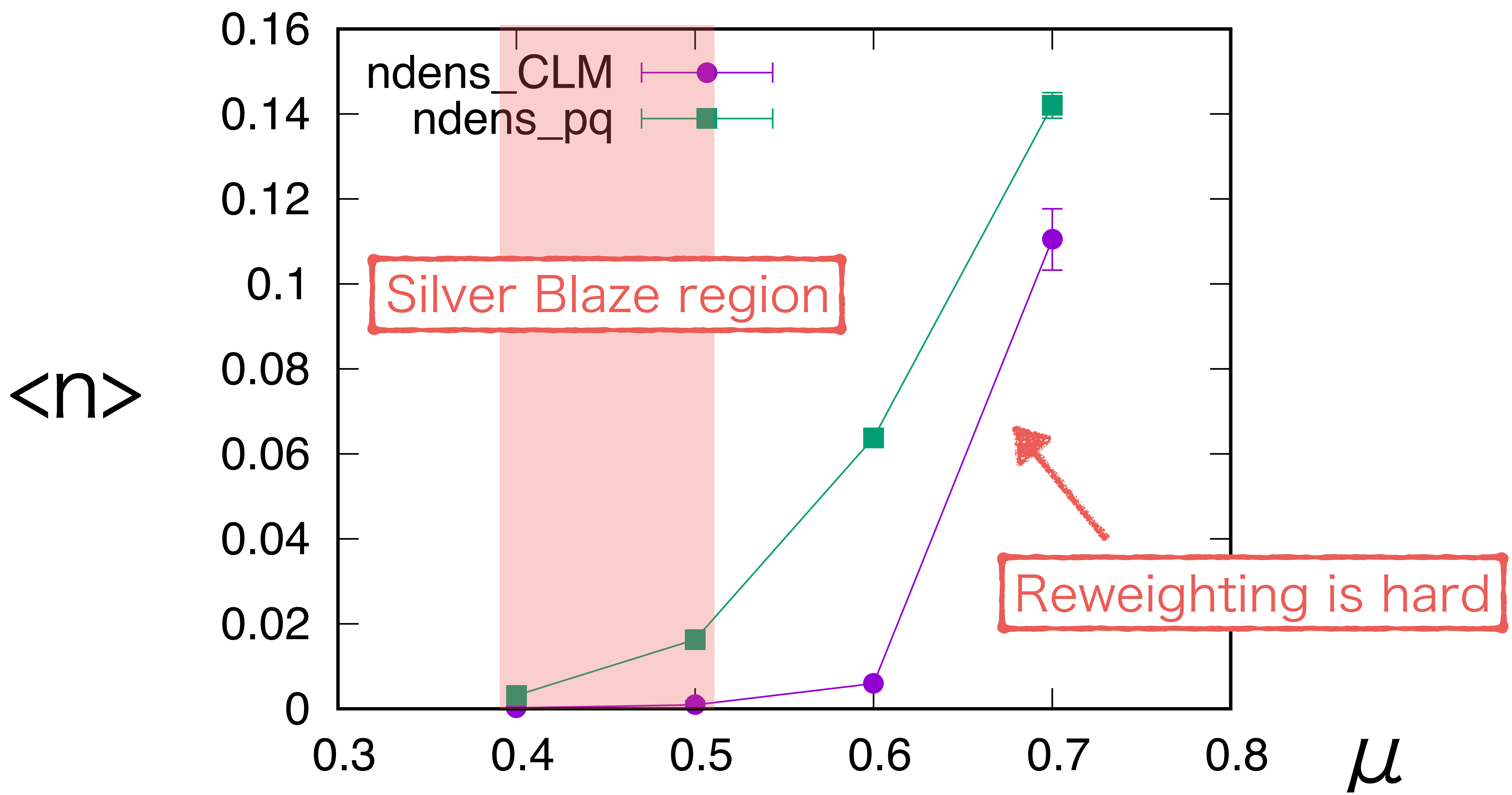
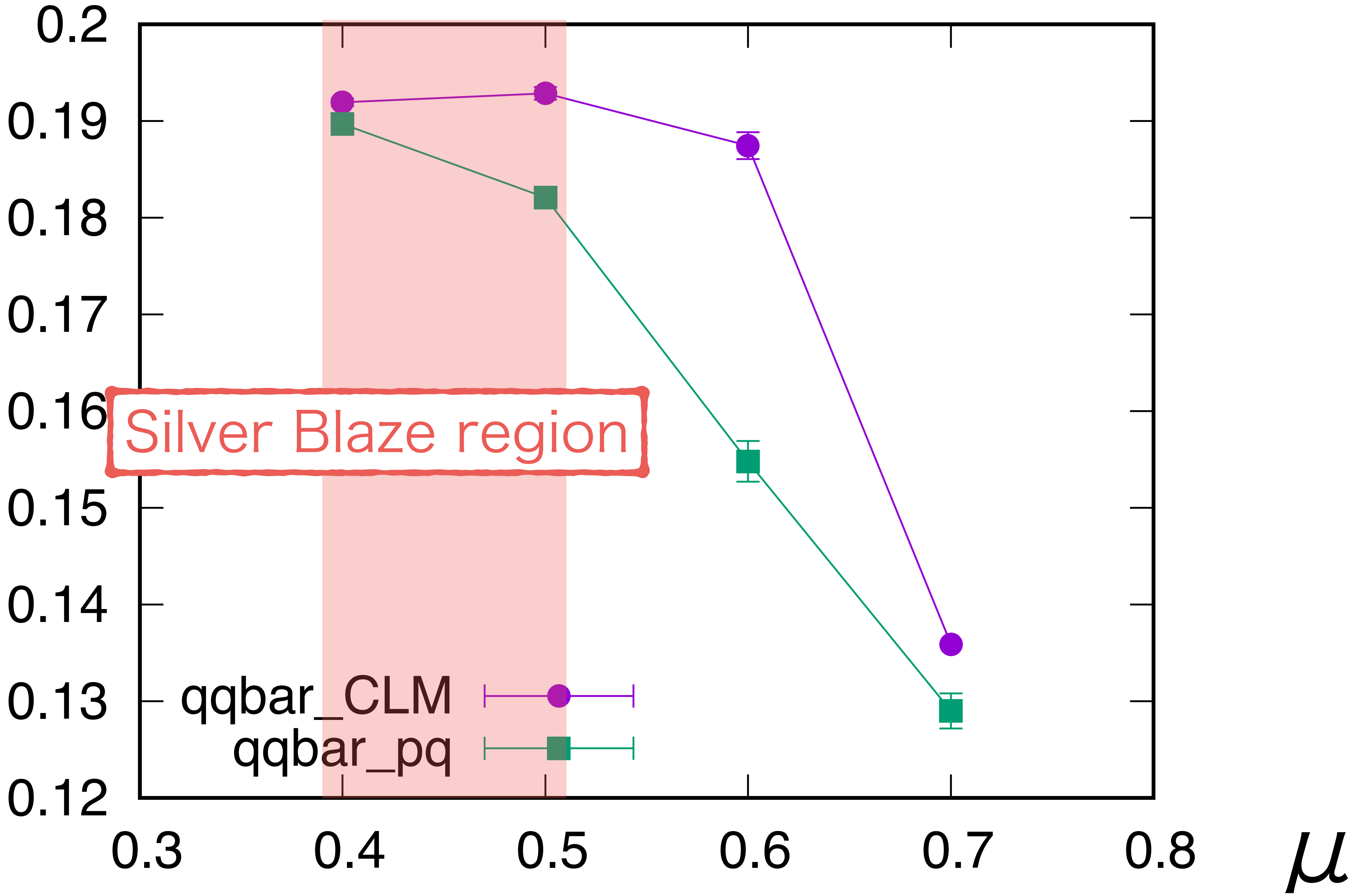
0.06
0.04
0.02
0
0.3
0.4
0.5
0.6
0.7
0.8
μ
ndens_CLM
ndens_pq
Silver Blaze region

<n> vs μ $(4^3 x8)$



0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.3
0.4
0.5
0.6
0.7
0.8
<n>
μ
ndens_CLM
ndens_pq
Silver Blaze region
Reweighting is hard

$<\Sigma>$ vs μ $(4^3 x8)$



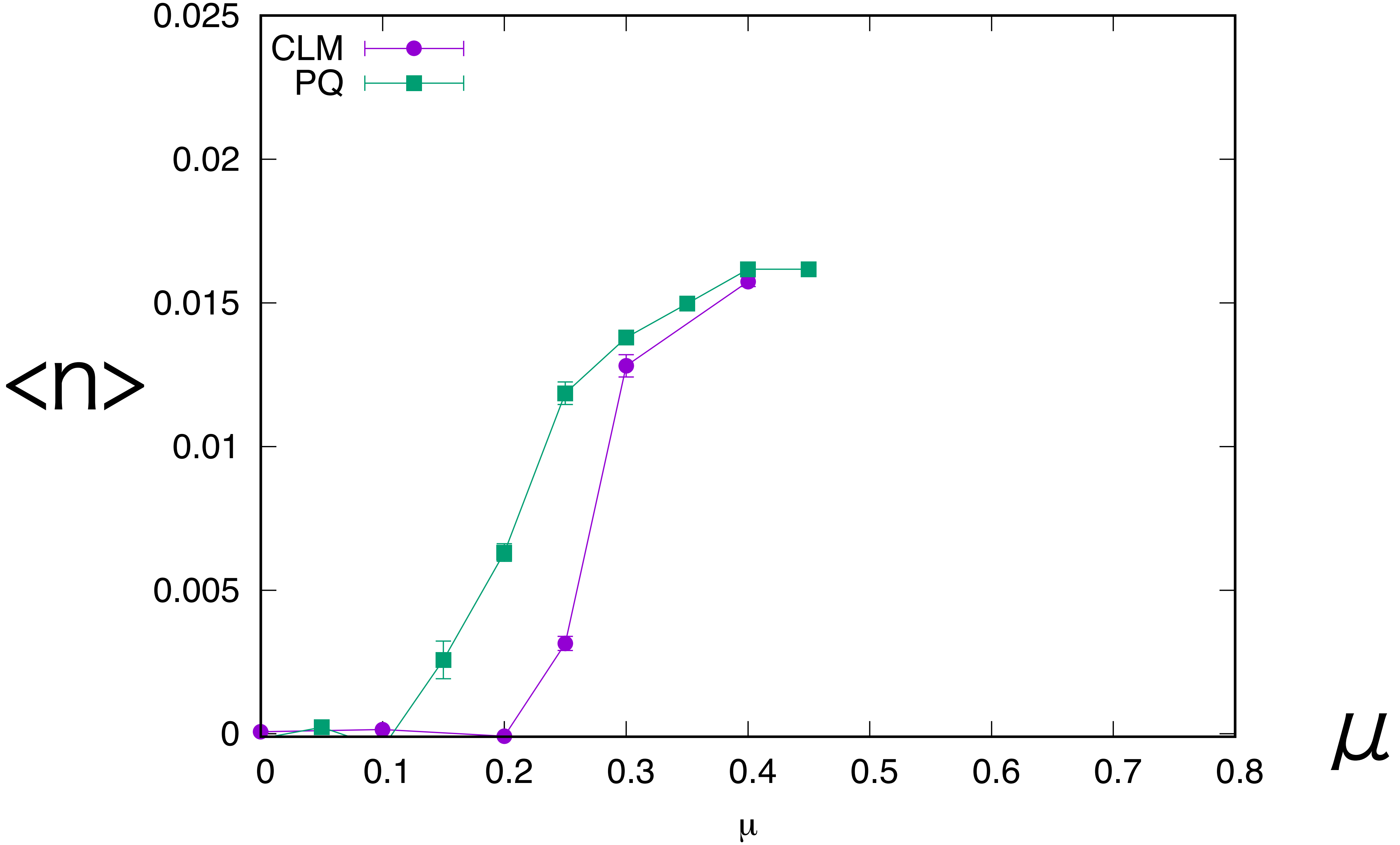
Preliminary results for $N_x=N_y=N_z=8$, $N_t=16$

parallel computation employed

[Ito, Matsufuru, Moritake, Nishimura, SS, Tsuchiya, Tsutsui]

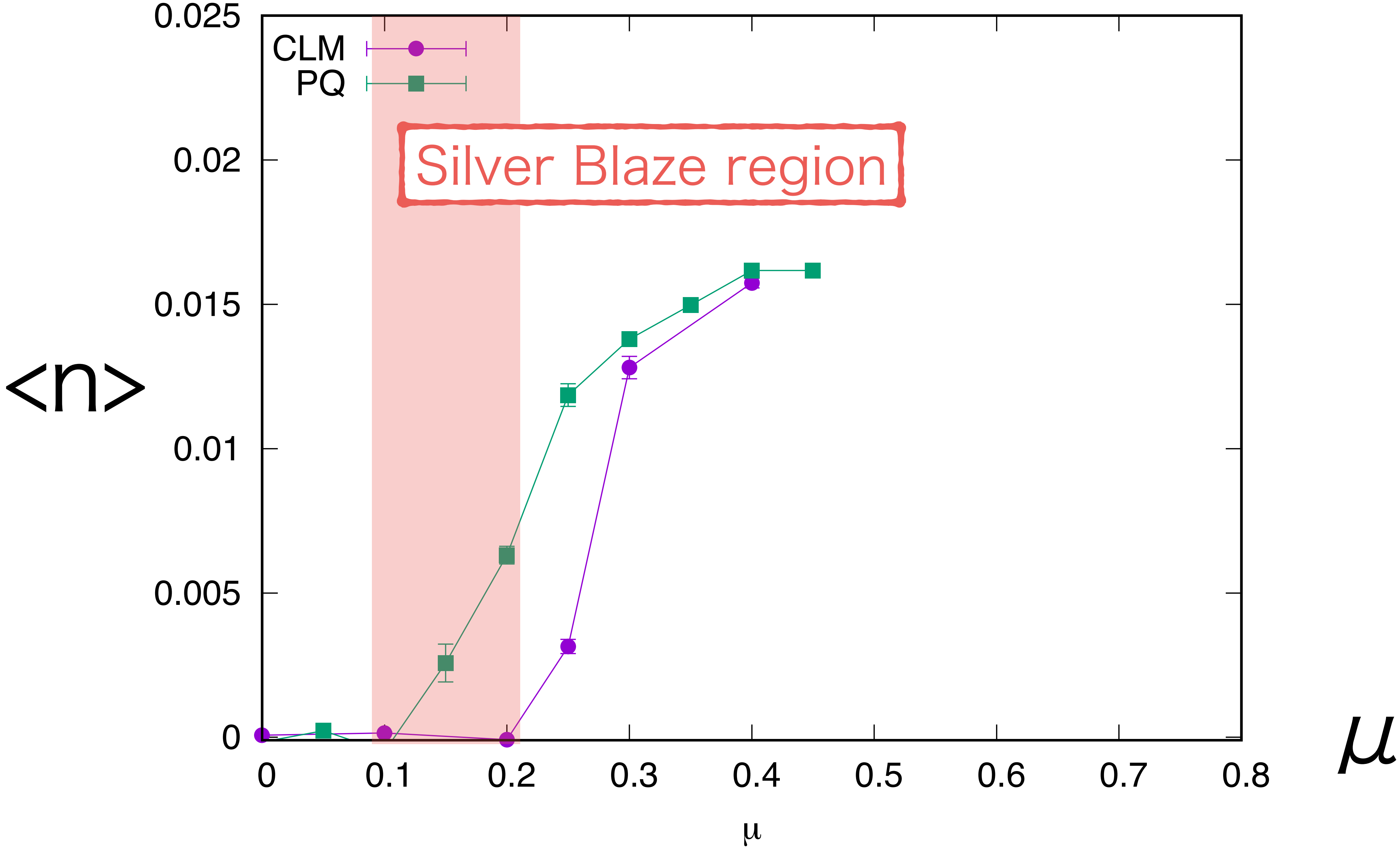
<n> vs μ (8^3x16)

preliminary



<n> vs μ (8^3x16)

preliminary



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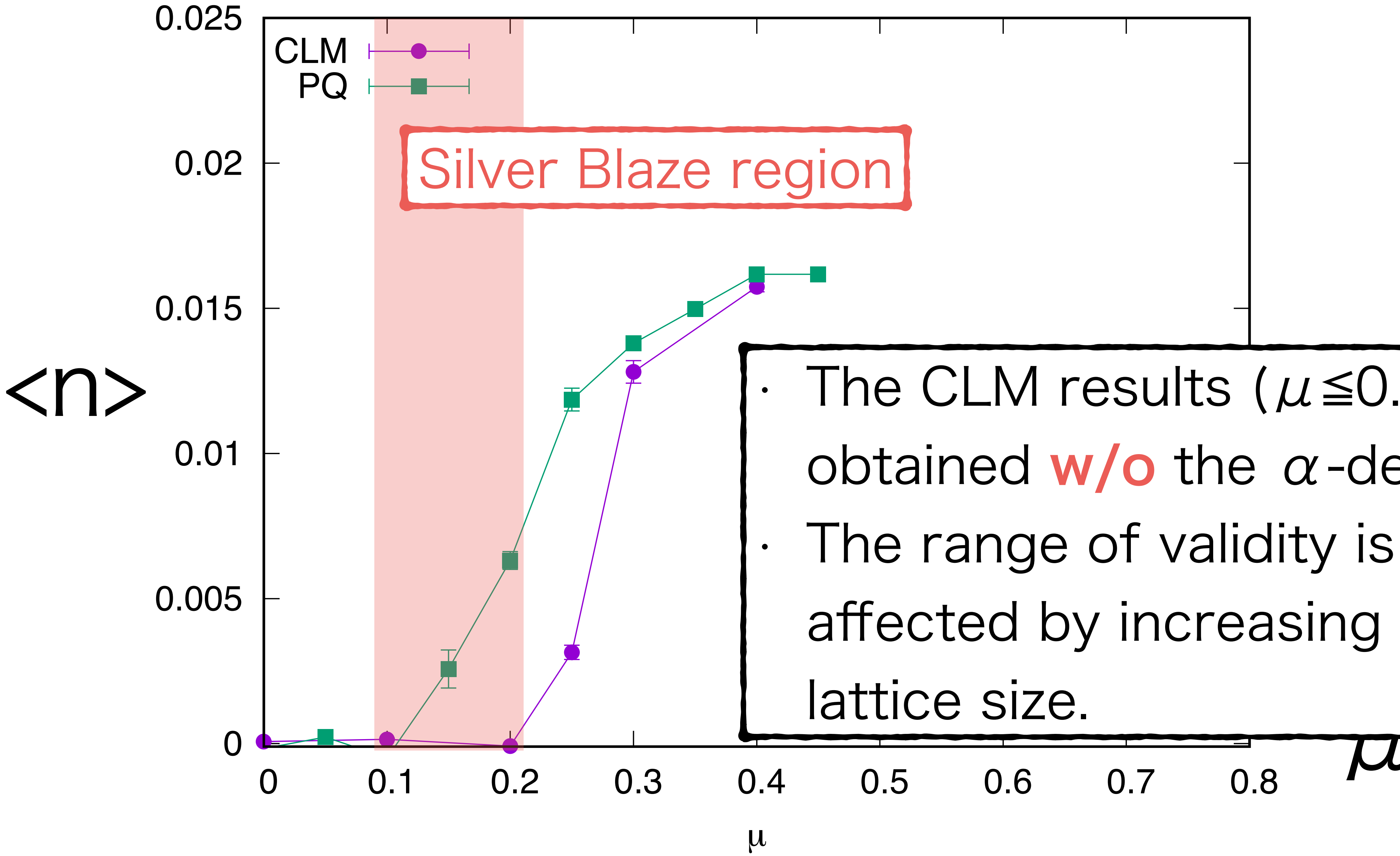
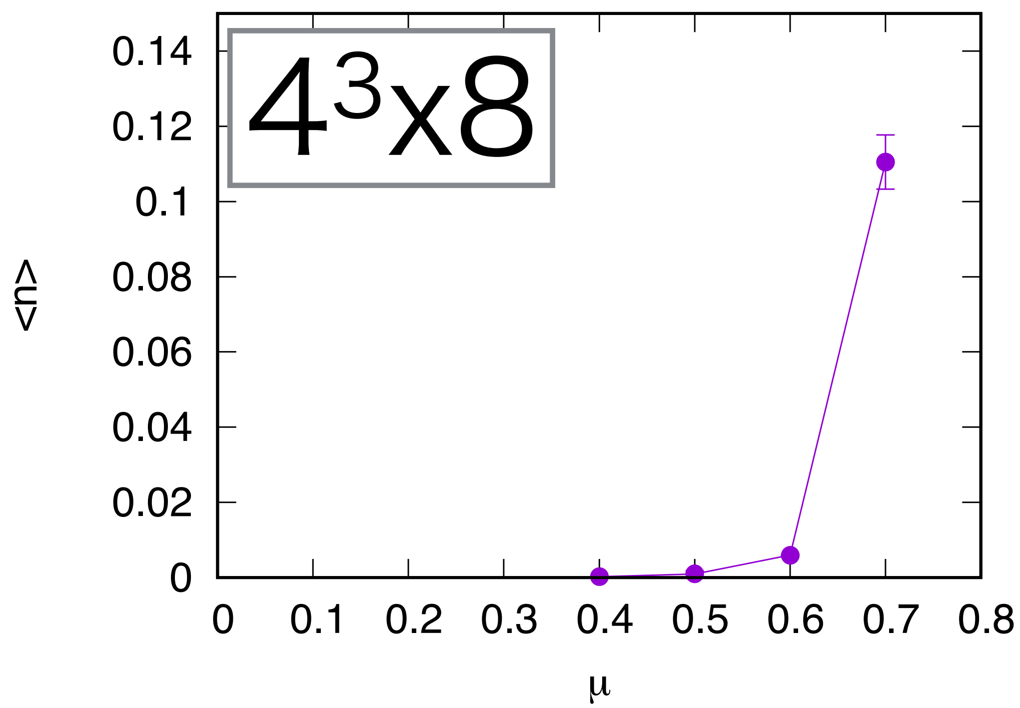
preliminary

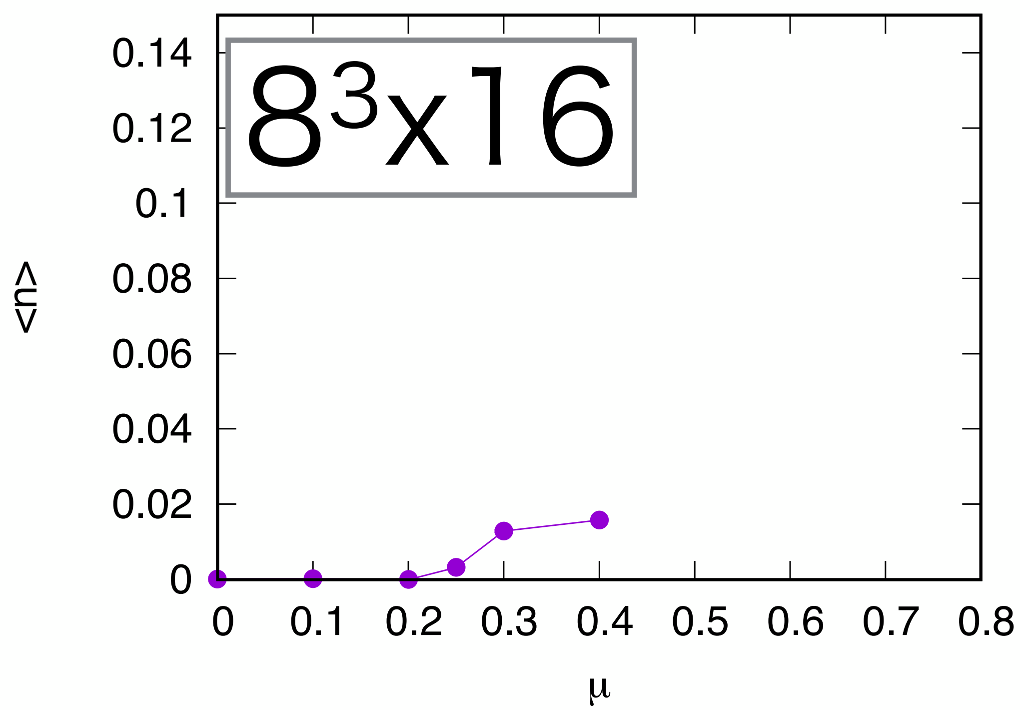


- The CLM results ($\mu \leqq 0.4$) are obtained **w/o** the α-def.
- The range of validity is not affected by increasing the lattice size.

Comparison of 8^3x16 with 4^3x8

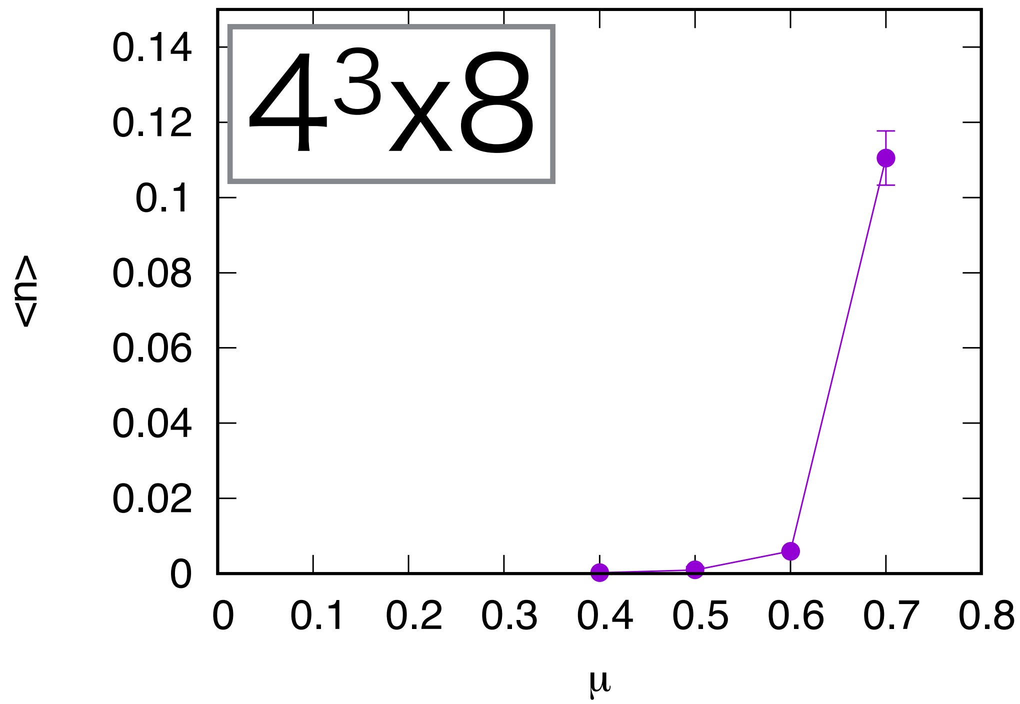
preliminary

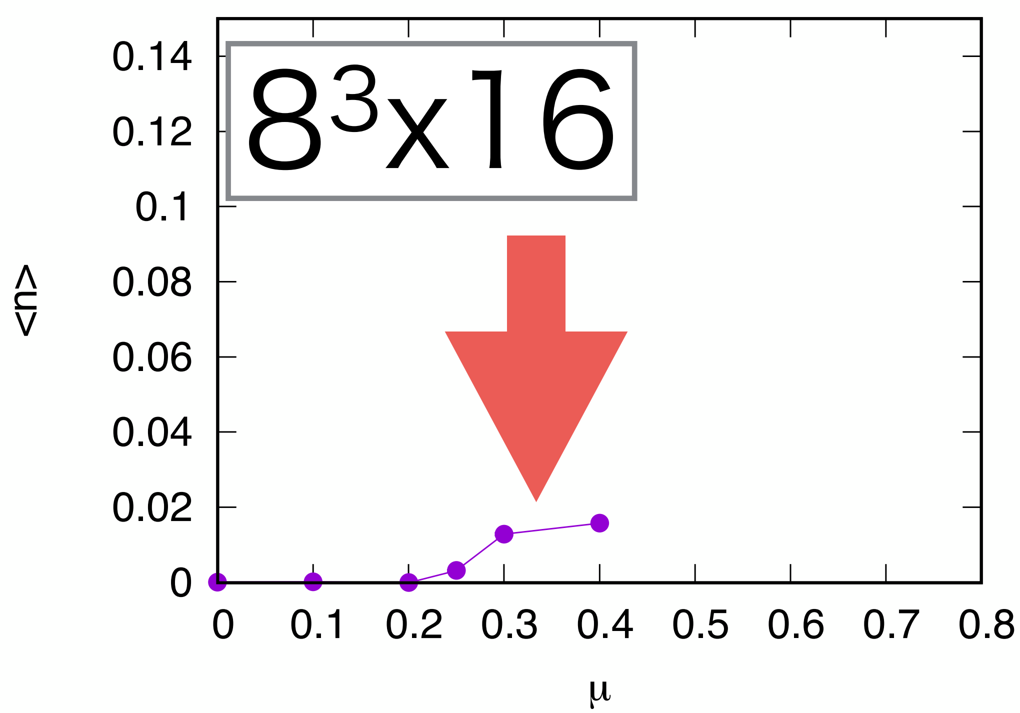




- 4^3x8 → 8^3x16
  V → 8xV
  T → T/2
- The onset of <n> for 8^3x16 is earlier than that for 4^3x8
  ← finite volume effect
- In 8^3x16, a saturation-like behavior is observed.
  - the nuclear matter phase?
  - the second phase transition to quark matter exists?

Comparison of 8^3x16 with 4^3x8





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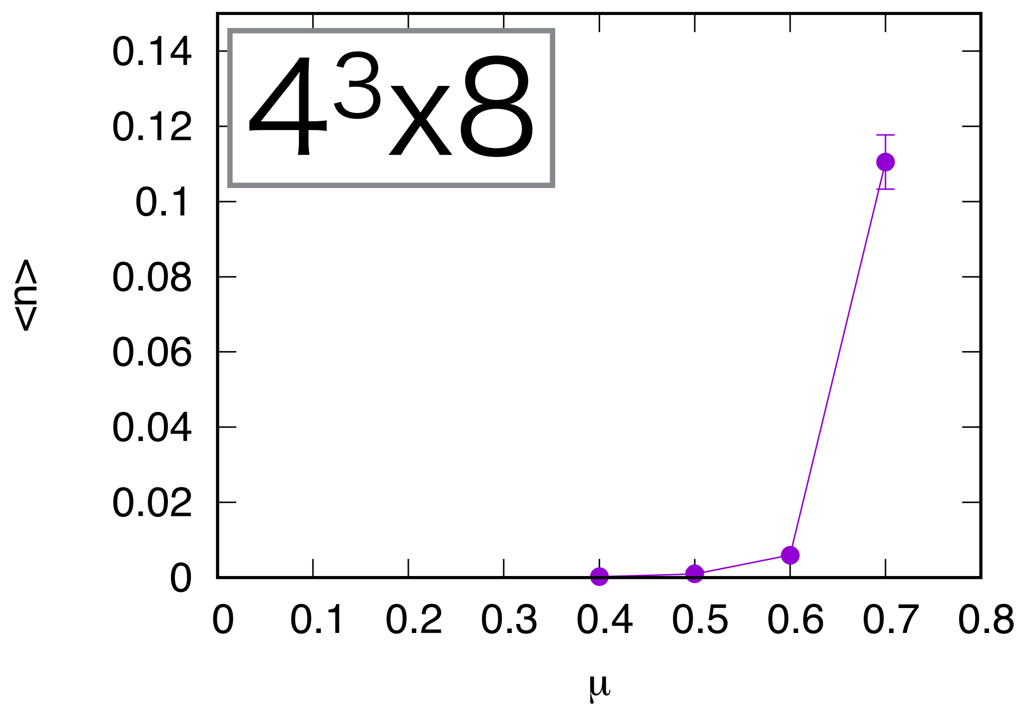

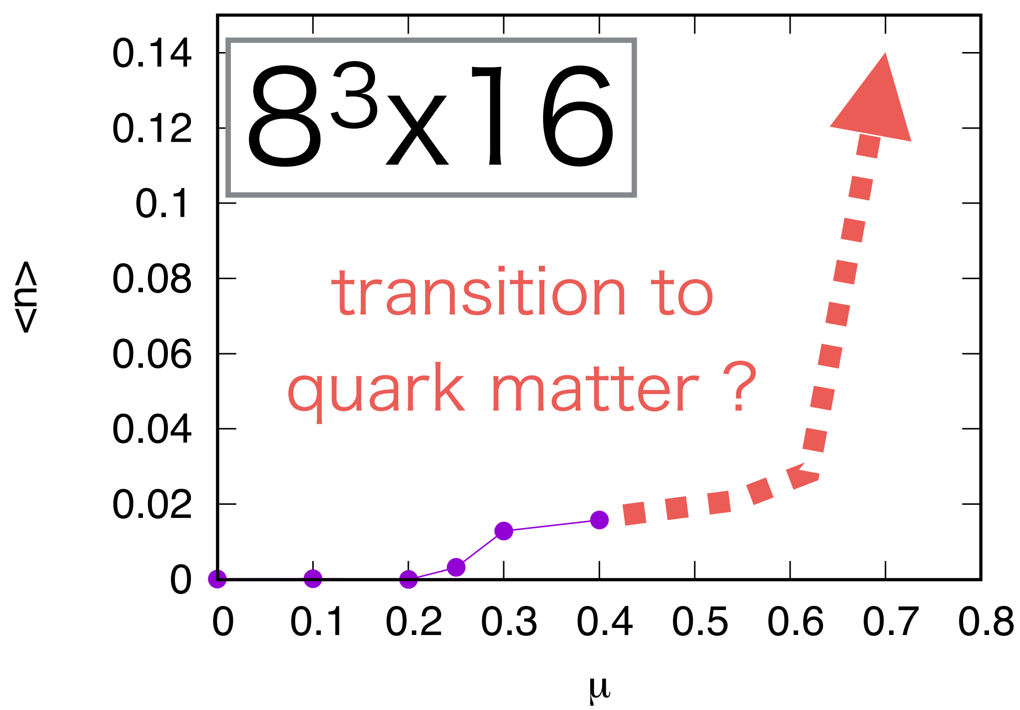

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  - the second phase transition to quark matter exists?

Summary

- We study QCD at finite density and low temperature by using the CLM with
  - ✓ Gauge cooling
  - ✓ Deformation of the Dirac operator
  - ✓ Reliability check of the obtained results by the prob. dist. of the drift term

- We have found a clear difference between the results obtained by the CLM and the PQ simulation
  - We have succeeded in observing the Silver Blaze phenomenon and the phase transition from $\langle n \rangle = 0$ to $\langle n \rangle \neq 0$.
  - The CLM works well even in the region where the reweighting-based methods are extremely hard.
  - The range of validity is unaffected much by increasing lattice size.

Future work

- Simulation of 8^3x16 lattice for larger μ by making use of the α-deformation of the Dirac operator.
    - Can we observe the second phase transition to quark matter?