ββ *decay matrix elements from lattice QCD*

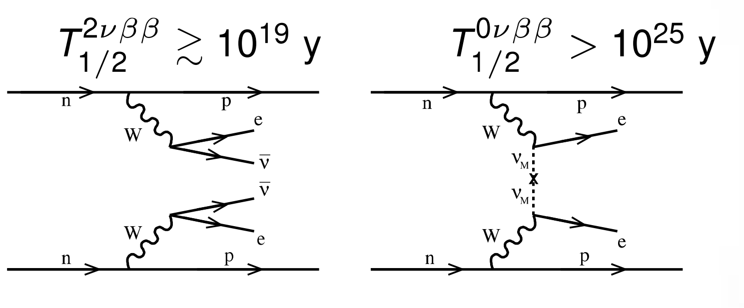
arXiv:1701.03456, 1702.02929

William Detmold, MIT

Lattice 2017, June 23rd 2017

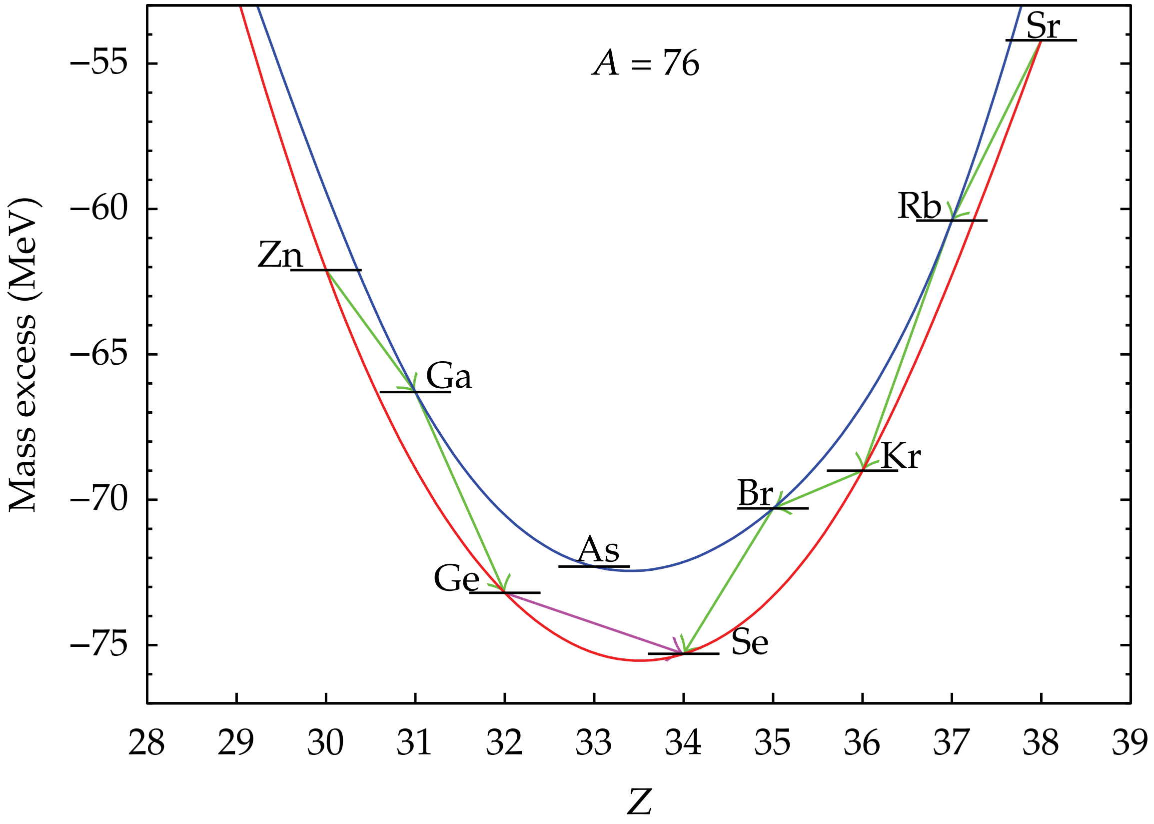
ββ decay

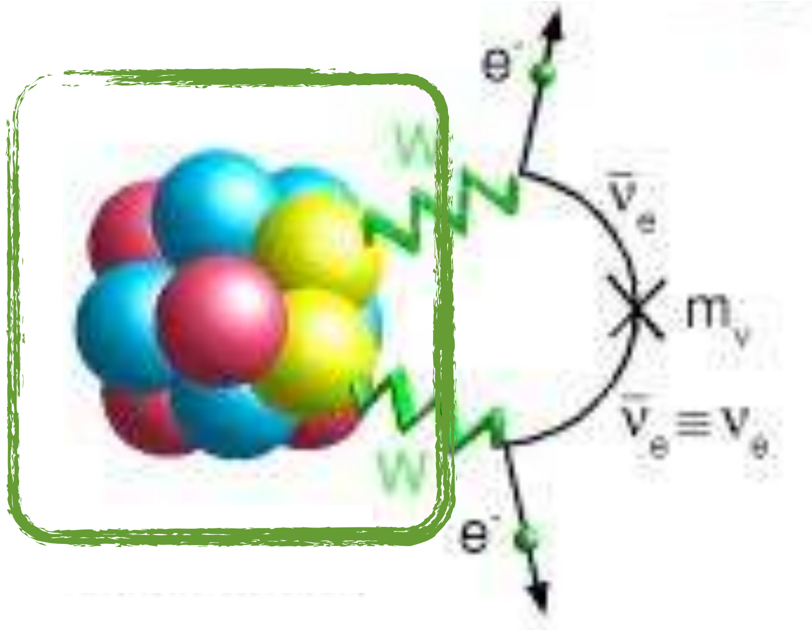
- Certain nuclei allow observable **ββ** decay



- 2**νββ**: rarest decays measured
- If neutrinos are massive Majorana fermions 0**νββ** decay is possible
- Half-life depends critically on the nuclear matrix elements of two weak currents!

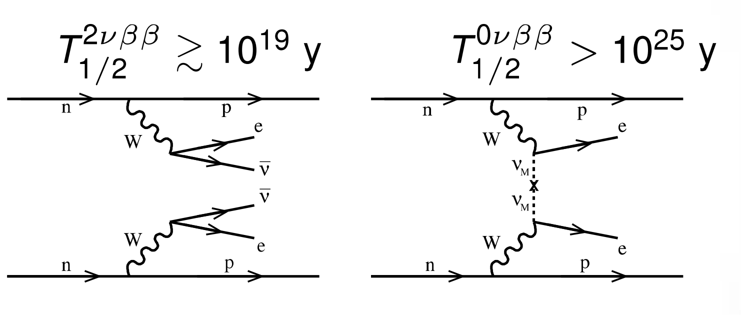
$$\left(T_{1/2}^{0\nu\beta\beta}(0^+ \rightarrow 0^+)\right)^{-1} = G_{01}\left|M^{0\nu\beta\beta}\right|^2\left(\frac{\langle m_\nu\rangle}{m_e}\right)^2$$





ββ decay

- Certain nuclei allow observable ββ decay

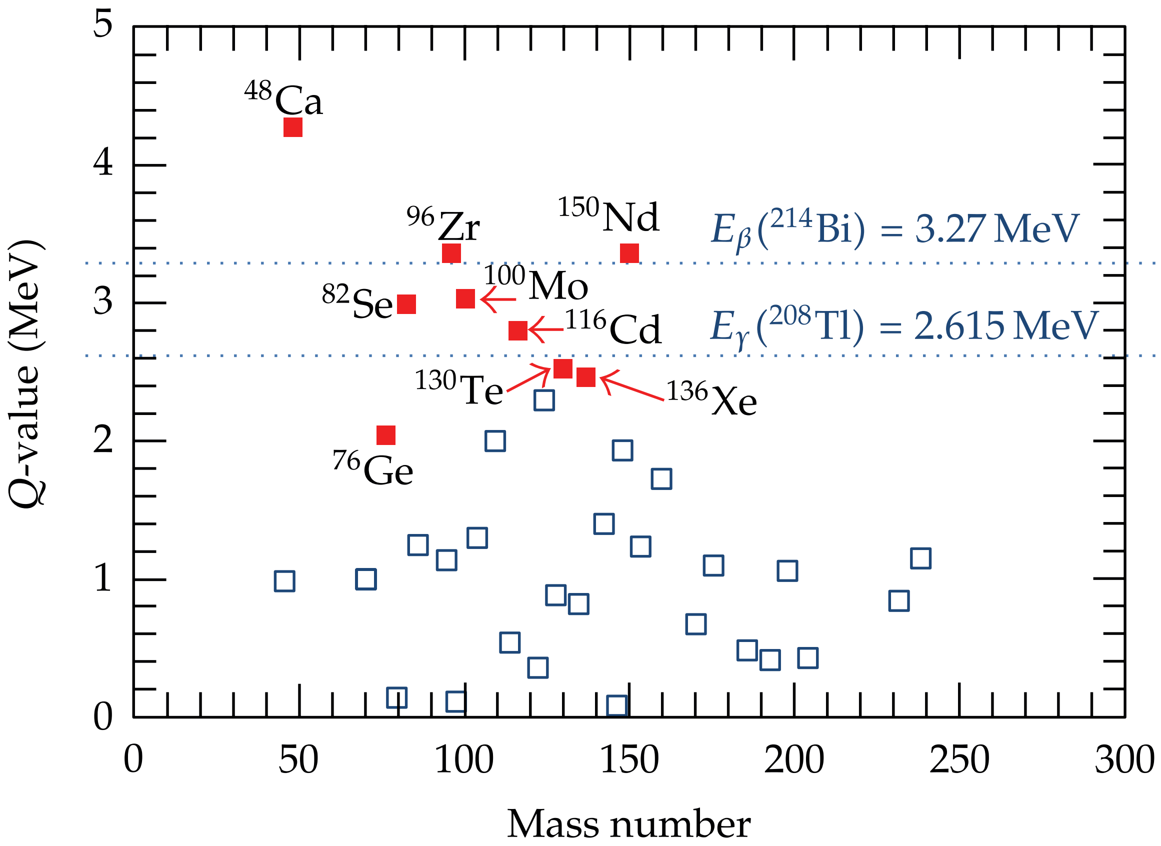


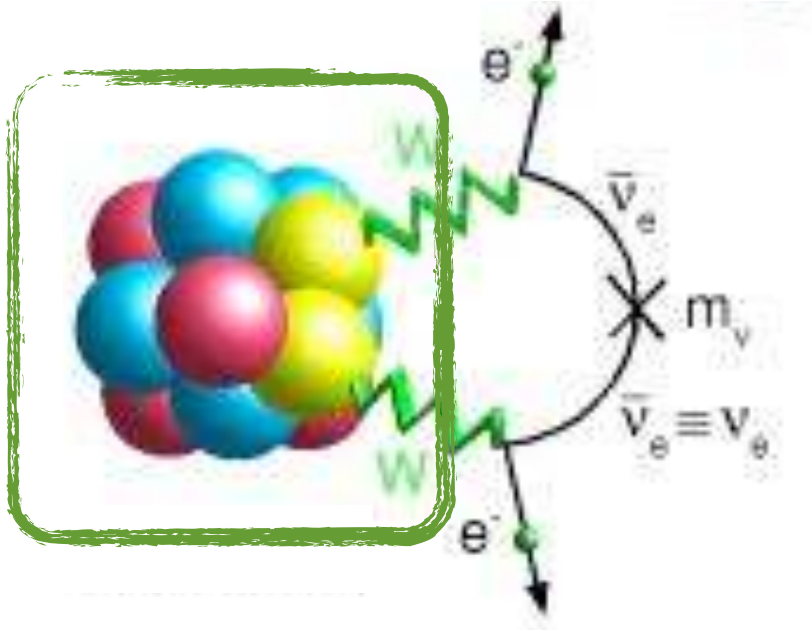
$T_{1/2}^{2\nu\beta\beta} \gtrsim 10^{19}$ y

$T_{1/2}^{0\nu\beta\beta} > 10^{25}$ y

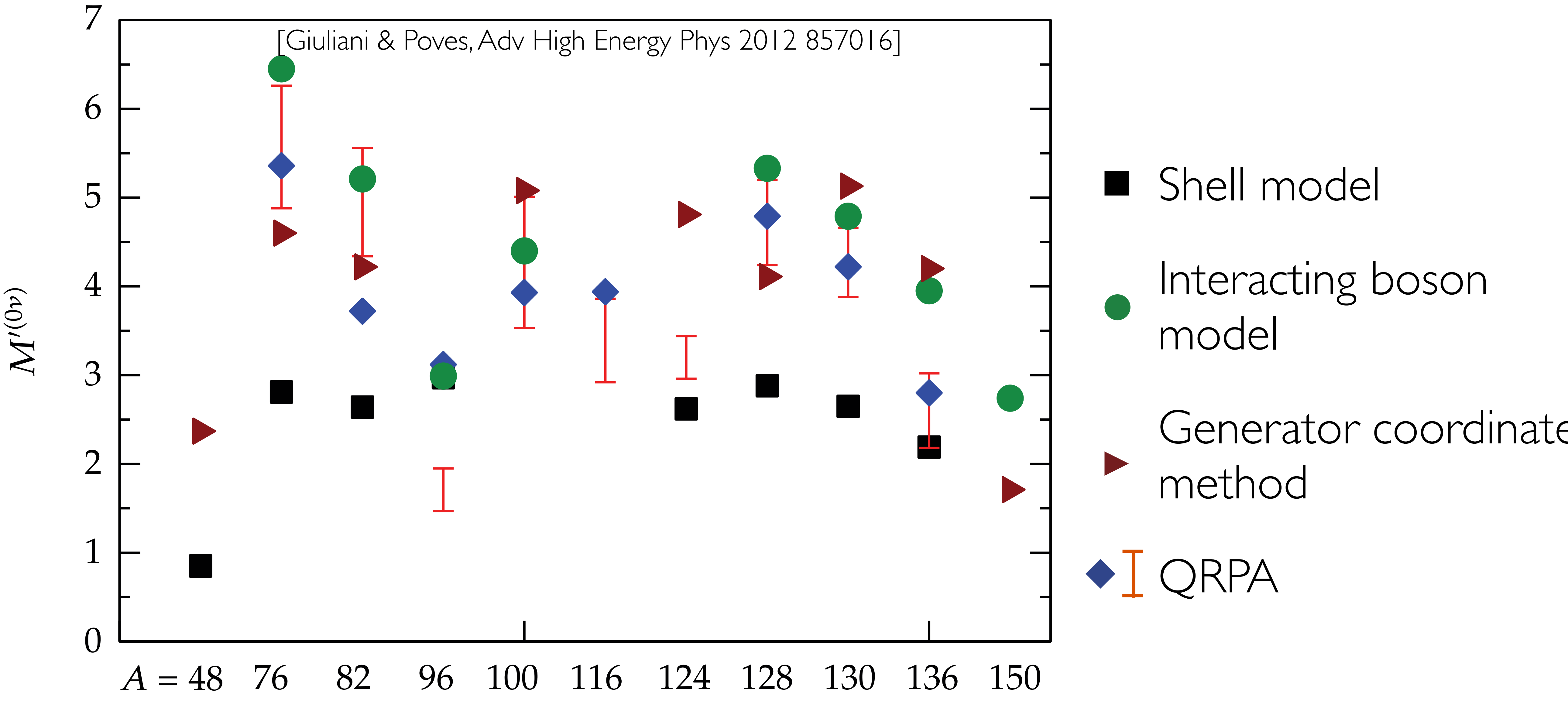
- 2νββ: rarest decays measured
- If neutrinos are massive Majorana fermions 0νββ decay is possible
- Half-life depends critically on the nuclear matrix elements of two weak currents!

$$\left(T_{1/2}^{0\nu\beta\beta}(0^+ \to 0^+)\right)^{-1} = G_{01}\left|M^{0\nu\beta\beta}\right|^2\left(\frac{\langle m_\nu\rangle}{m_e}\right)^2$$





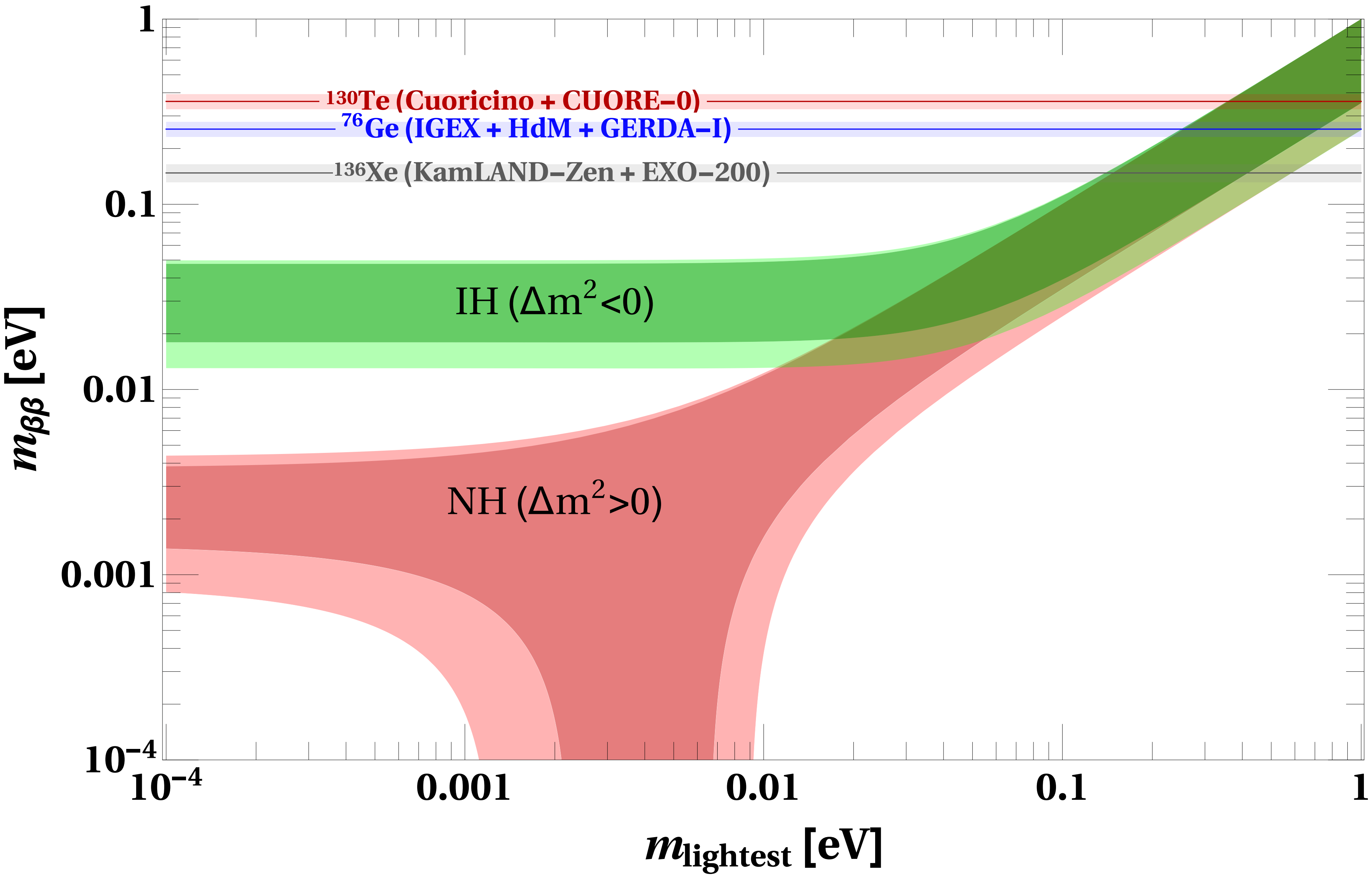
0νββ decay nuclear matrix elements


[Giuliani & Poves, Adv High Energy Phys 2012 857016]

- Large spread of calculations
- Is the spread representative of the true uncertainty?

0νββ decay

- Sensitivity of experiments depends on size of matrix elements



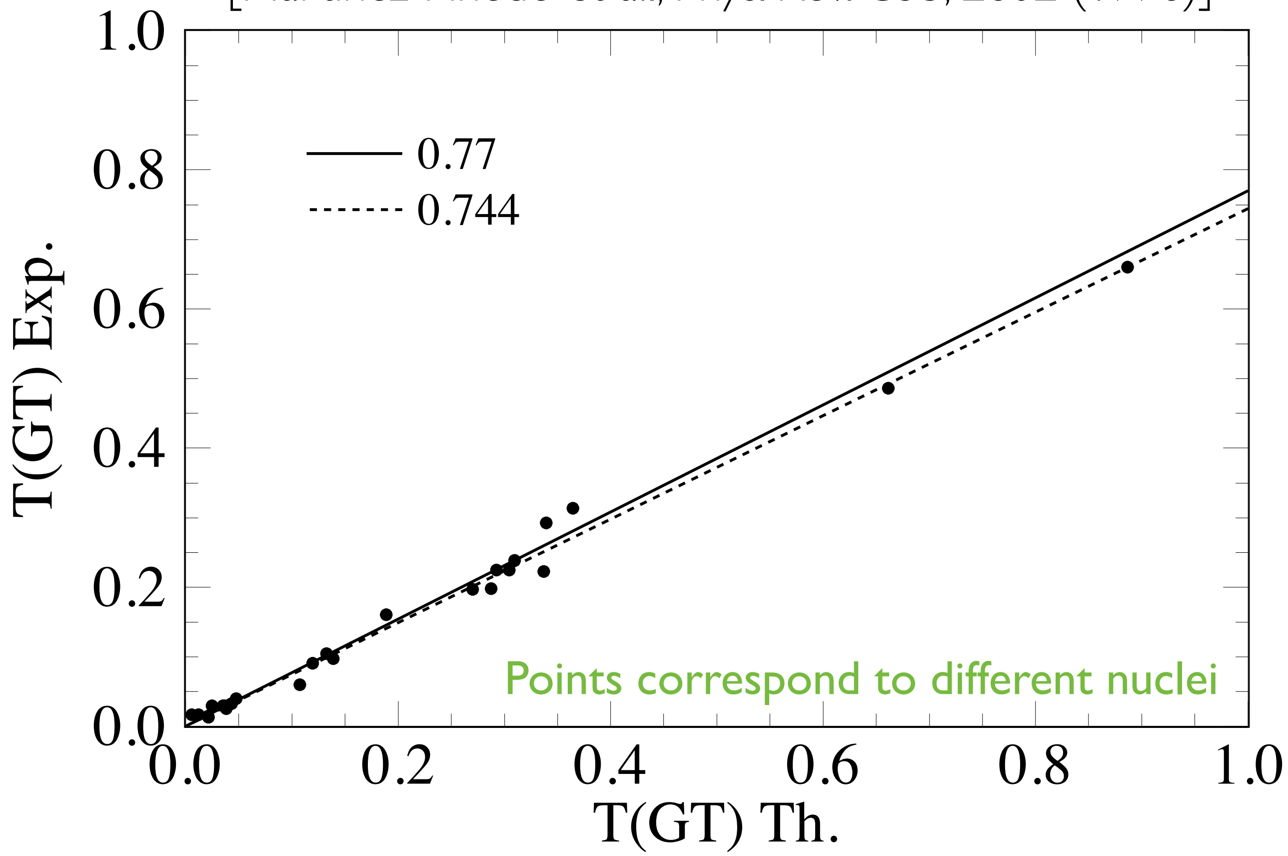
- Overestimating nuclear matrix elements: $$$$ mistake

Nuclear uncertainties

- How well do we know nuclear matrix elements?

😢 Stark example of problems: Gamow-Teller transitions in nuclei

  - Well measured for large range of nuclei (30<A<60)
  - Many nuclear structure calcs (QRPA, shell-model,…) – spectrum well described
  - Matrix elements systematically off by 20–30%
  - “Correct” by “quenching” axial charge in nuclei ...



[Martinez-Pinedo et al., Phys. Rev. C53, 2602 (1996)]

Points correspond to different nuclei

$$T(GT) \sim \sqrt{\sum_f \langle \boldsymbol{\sigma} \cdot \boldsymbol{\tau} \rangle_{i \to f}}$$

$$\langle \boldsymbol{\sigma\tau} \rangle = \frac{\langle f || \sum_k \boldsymbol{\sigma}^k \boldsymbol{t}_{\pm}^k || i \rangle}{\sqrt{2J_i + 1}}$$

Unphysical nuclei

- Case study QCD with unphysical quark masses
  - m_π~800 MeV, m_N~1,600 MeV
  - m_π~450 MeV, m_N~1,200 MeV

1. Spectrum of light nuclei (A<5)
   [PRD **87** (2013), 034506]

2. Nuclear structure: magnetic moments, polarisabilities (A<5)
   [PRL **113**, 252001 (2014),PRD 92, 114502 (2015)]

3. Nuclear reactions: $np \to d\gamma$
   [PRL **115**, 132001 (2015)]

4. Gamow-Teller transitions: $pp \to de\nu$, $g_A(^3H)$ [arXiv:1610.04545]

5. Double β decay: $pp \to nn$
   [1701.03456,1702.02929]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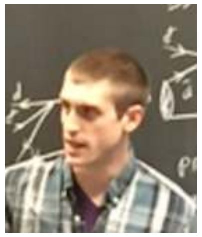
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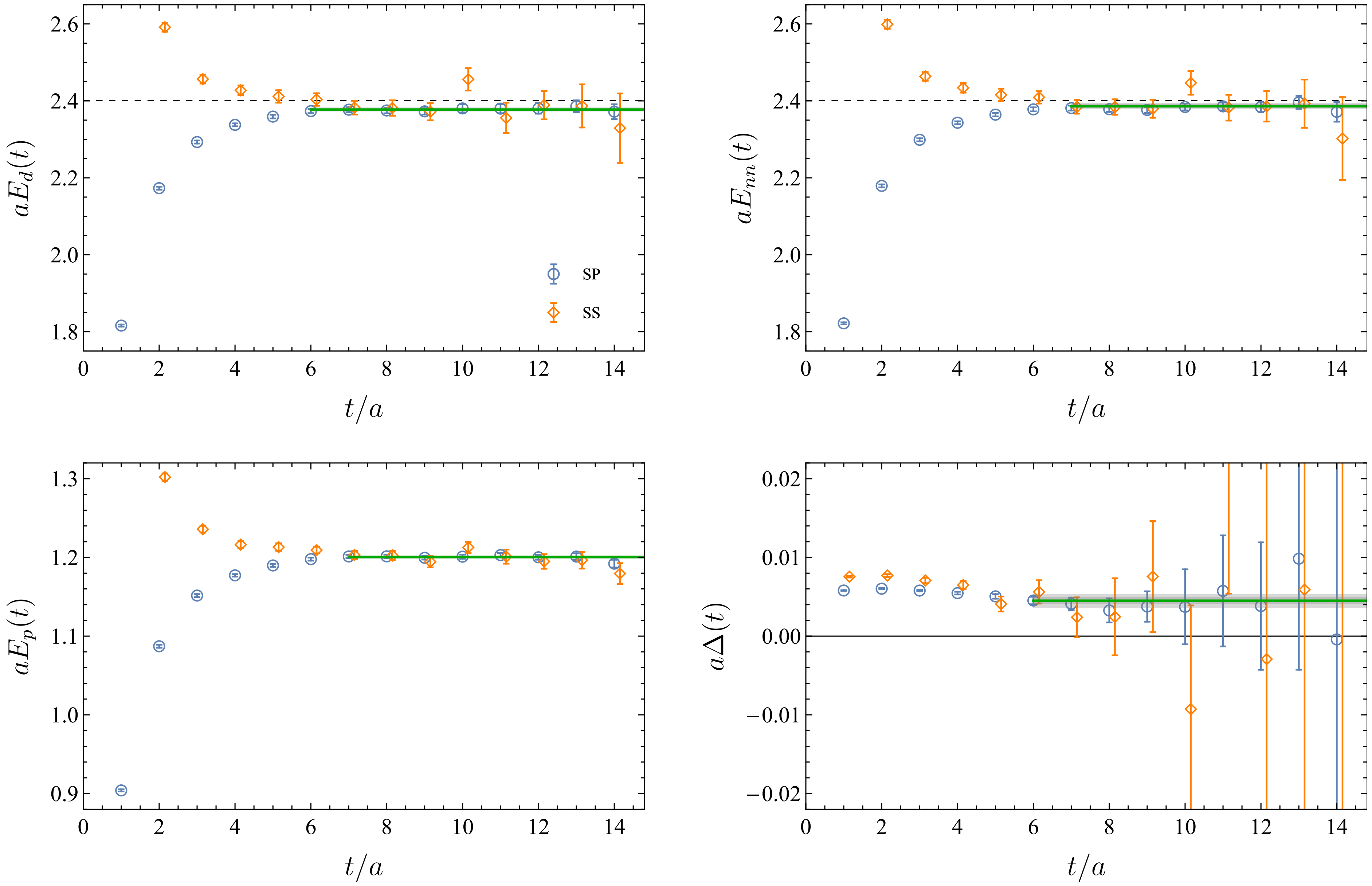
See also: Amy Nicholson Lattice 2016

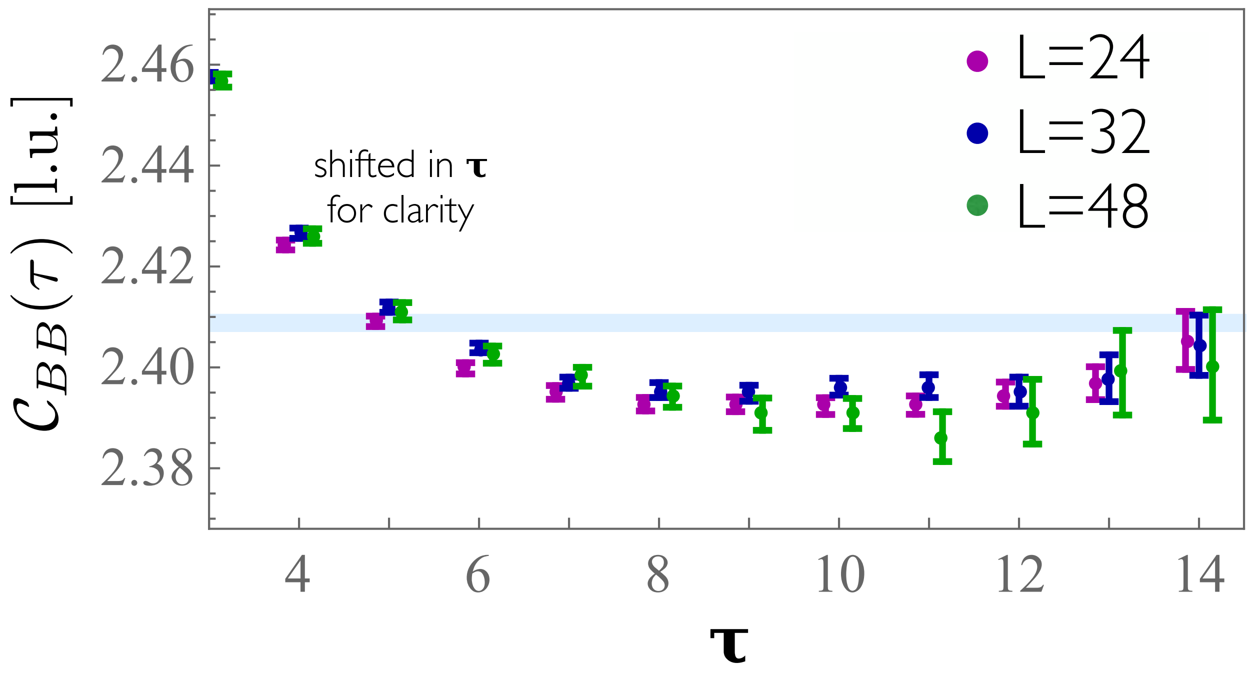
sanity check I

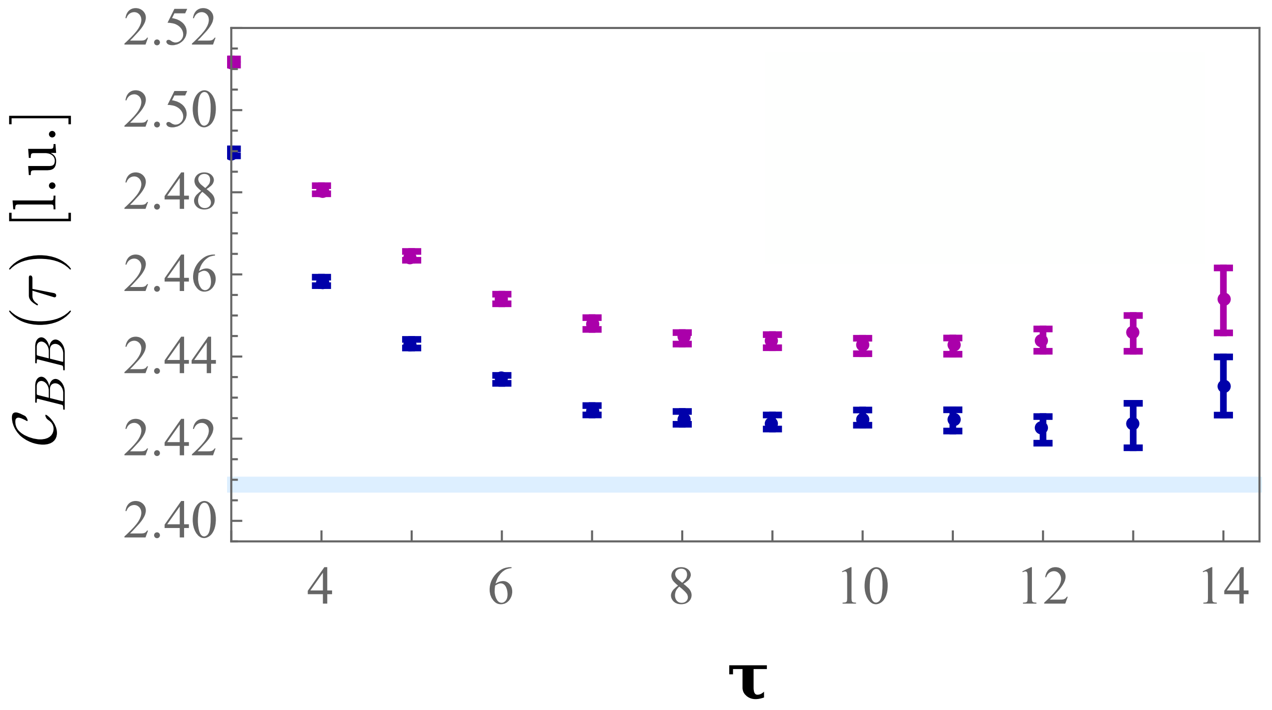
- Check for existence of *nn* and deuteron bound states



sanity check 2

- Potential for fake plateaus?
  - Two or more scattering states combine with relative signs to give flat behaviour
- Study at 3 volumes with same source structure
- Negative shifted state
  - Correlators fully consistent at L=24, 32, 48
- Excited state
  - Scales as 1/L^3 consistent with scattering state





Axial Background Field

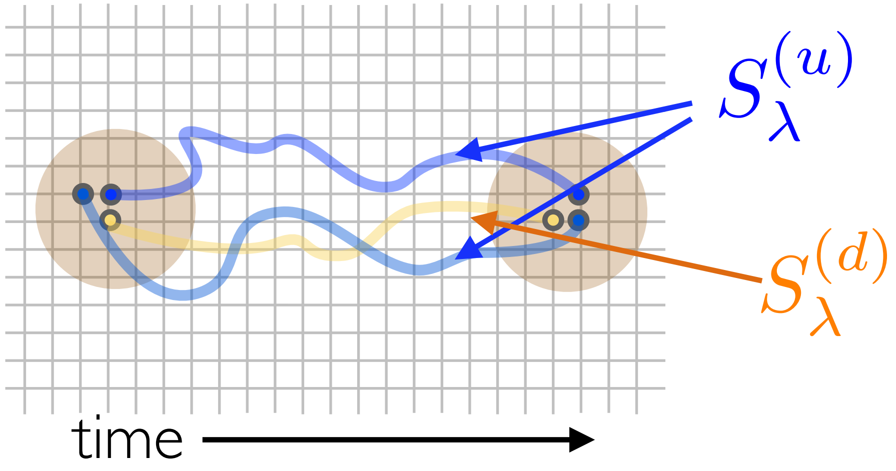
- Construct correlation functions from propagators modified in axial field

- Compound propagators

constant external field

$$S_\lambda^{(q)}(x,y) = S^{(q)}(x,y) + \boxed{\lambda_q} \int dz\, S^{(q)}(x,z)\,\gamma_3\gamma_5\, S^{(q)}(z,y)$$

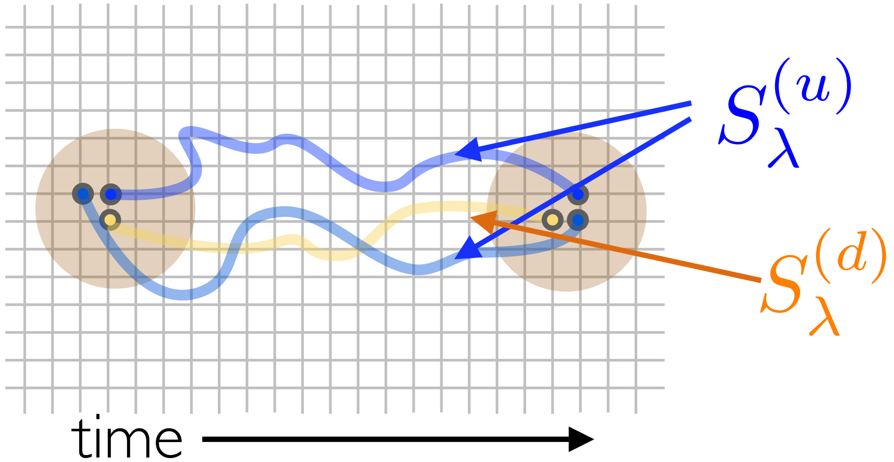
$$C_{\lambda_u;\lambda_d}(t) =$$



- Correlator becomes finite-order polynomial in field strength

Axial Background Field

$C_{\lambda_u;\lambda_d}(t) =$



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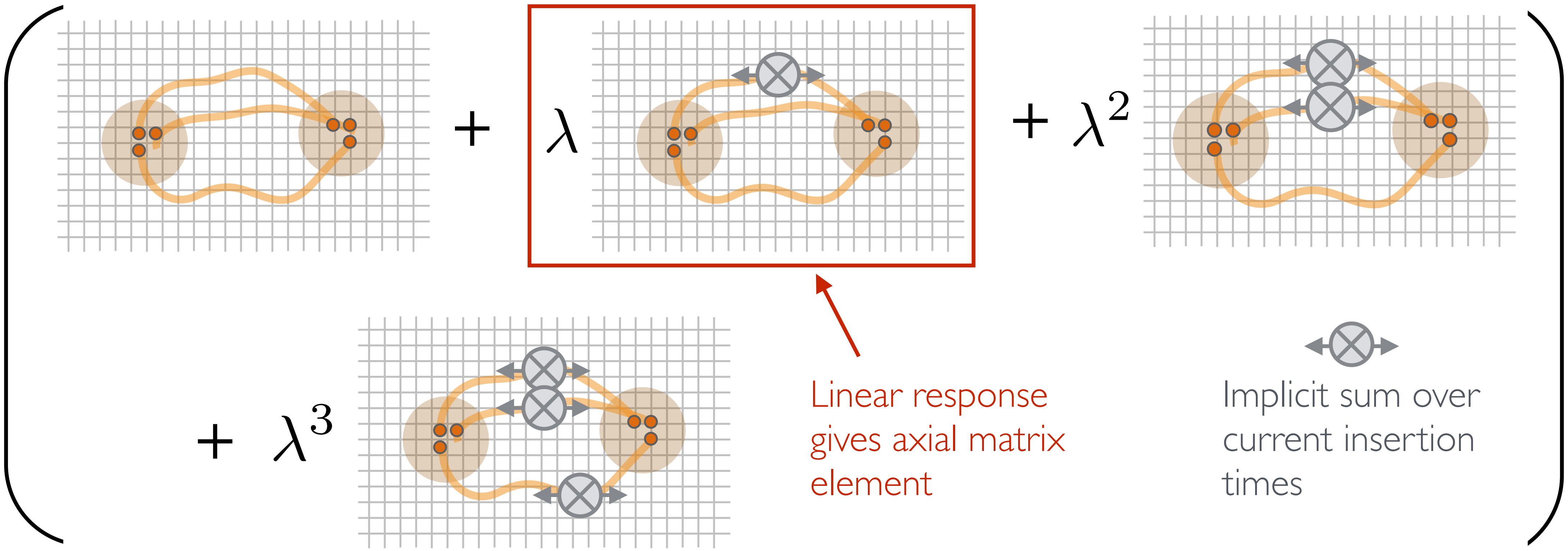
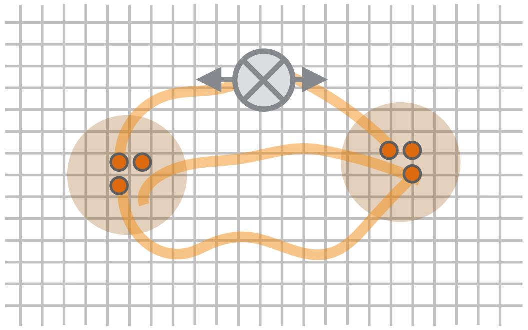


Linear response gives axial matrix element

Implicit sum over current insertion times

Axial Background Field

$$C_{\lambda_u;\lambda_d}(t)\Big|_{\mathcal{O}(\lambda)} =$$



Implicit sum over current insertion times

Example: determination of the proton axial charge

Excited states

Irrelevant constants

$$\begin{aligned} C_{\lambda_u;\lambda_d}(t)\Big|_{\mathcal{O}(\lambda)} &= \sum_{\tau=0}^{t} \langle 0|\chi^\dagger(t) J(\tau)\chi(0)|0\rangle \\ &= \ldots \\ &= Z_0 e^{-M_p t}\left[C + t\,\langle p|A_3^{(u)}(0)|p\rangle + \mathcal{O}(e^{-\delta t})\right] \end{aligned}$$

Matrix element

Time difference isolates matrix element part

$$\left(C_{\lambda_u;\lambda_d}(t+1) - C_{\lambda_u;\lambda_d}(t)\right)\Big|_{\mathcal{O}(\lambda)} = Z_0 e^{-M_p t}\langle p|A_3^{(u)}(0)|p\rangle + \mathcal{O}(e^{-\delta t})$$

Proton axial charge

- Extract matrix element through linear response of correlators to the background field
- Form ratios to cancel leading time-dependence

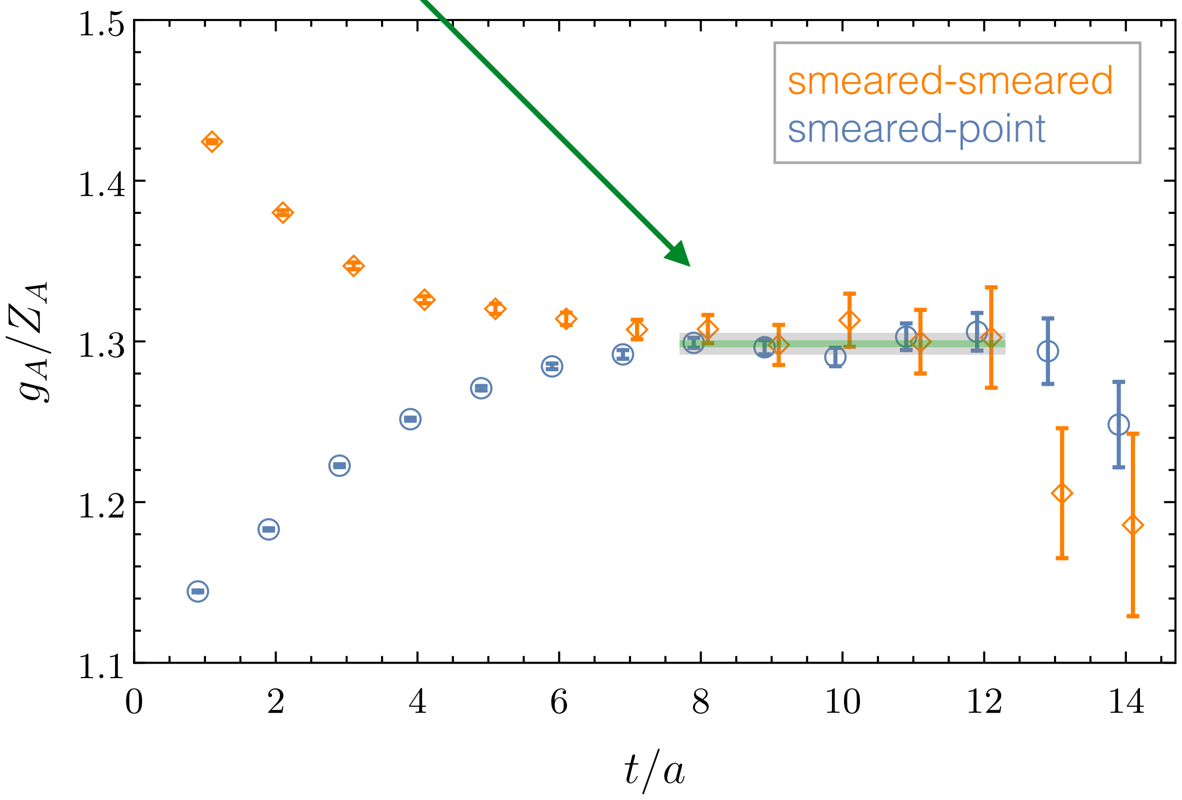
$$R_p(t) = \frac{\left(C^{(p)}_{\lambda_u;\lambda_d=0}(t) - C^{(p)}_{\lambda_u=0;\lambda_d}(t)\right)\Big|_{\mathcal{O}(\lambda)}}{C^{(p)}_{\lambda_u=0;\lambda_d=0}(t)}$$

At late times:

$$R_p(t+1) - R_p(t) \xrightarrow{t\to\infty} \frac{g_A}{Z_A}$$

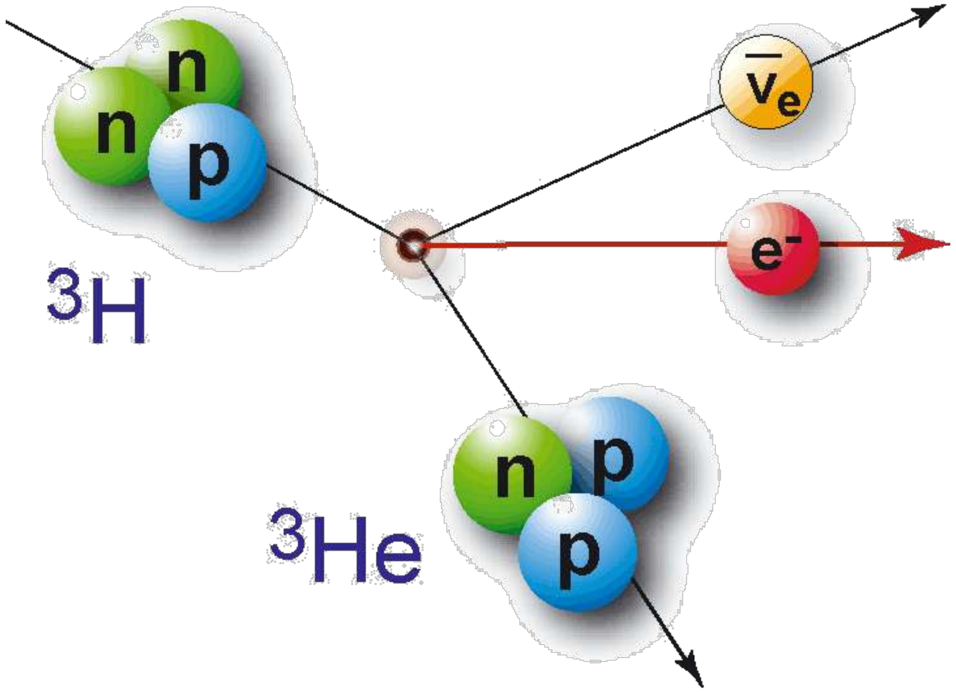
- Matrix element revealed through "effective matrix elt. plot"

constant fit to plateau region



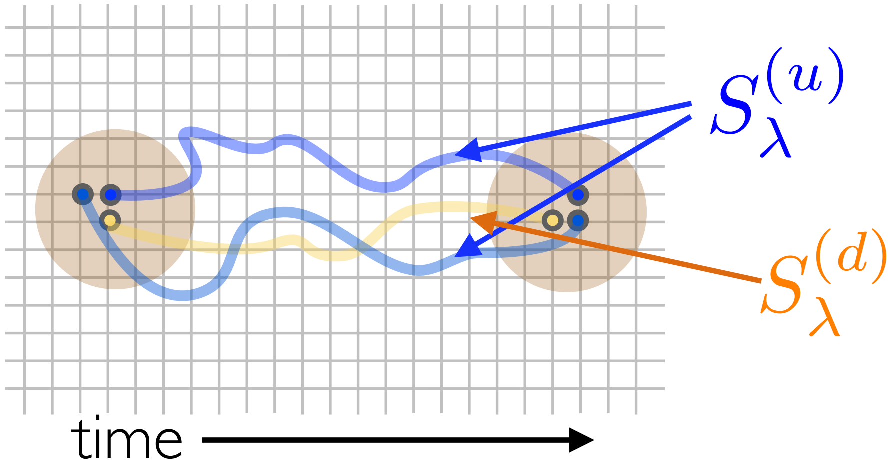
Gamow-Teller matrix elements

- Axial coupling to NN system
  - $pp \rightarrow de^{+}\nu$ fusion
  - Muon capture: MuSun @ PSI
  - $d\,\nu \rightarrow n\,n\,e+$: SNO
- Tritium half-life
  - Understand multi-body contributions to $\langle \mathbf{GT} \rangle$: better predictions for decay rates of larger nuclei
- **See Z Davoudi plenary on Saturday**

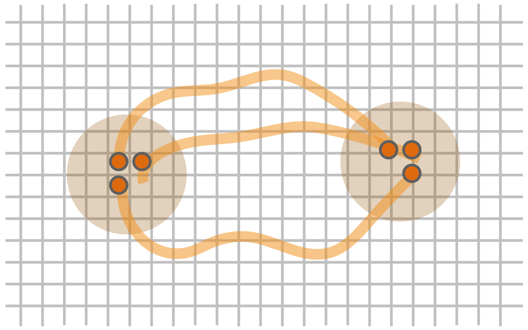


Second order weak interactions

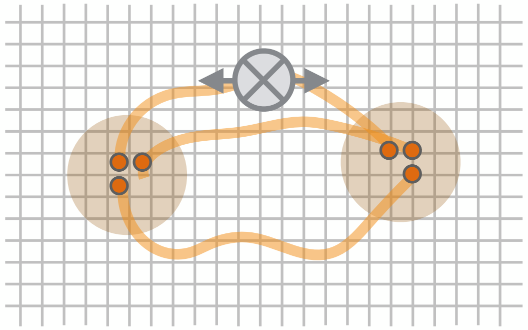
$C_{\lambda_u;\lambda_d}(t) =$



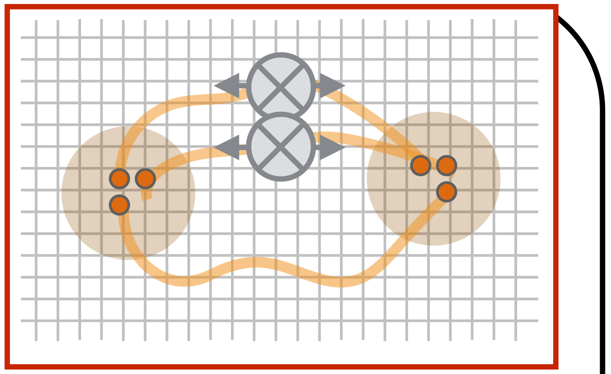
$C_{\lambda_u;\lambda_d}(t) = \Bigg($



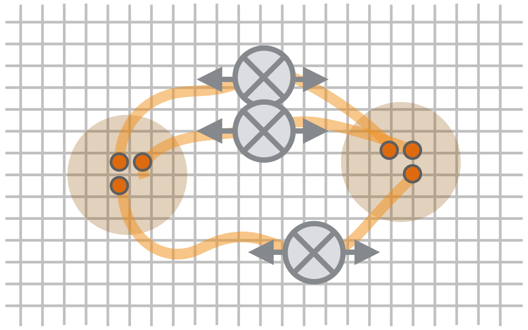
$+ \ \lambda$



$+ \ \lambda^2$



$+ \ \lambda^3$

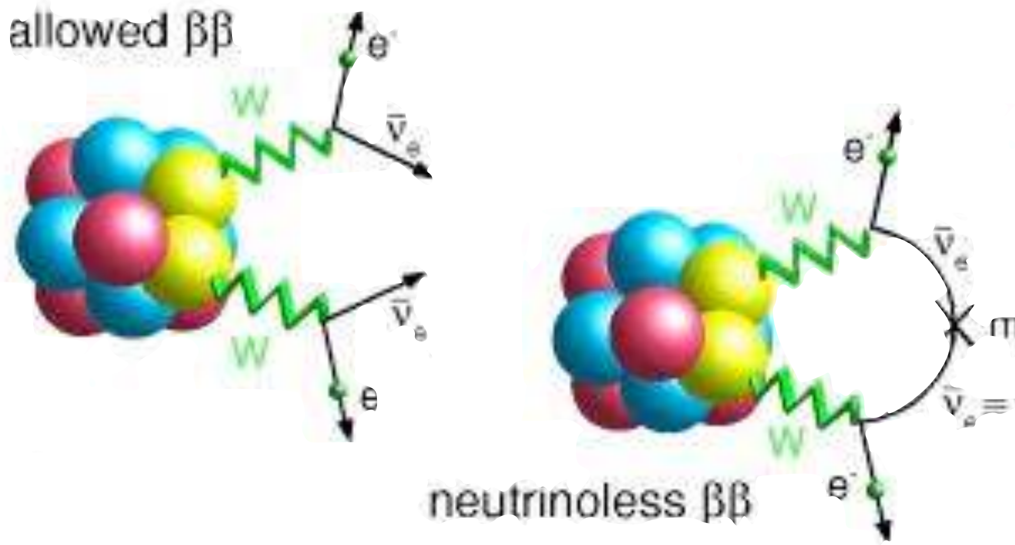


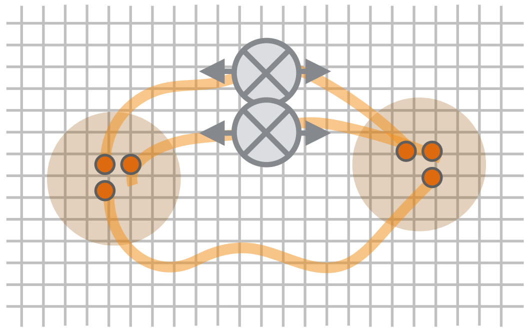
$\Bigg)$

Second order response

⊗ Implicit sum over current insertion times

Second order weak interactions





- Double beta decay
  - nn→pp transition matrix element

$$M^{2\nu}_{GT} = 6 \int d^4x d^4y \langle pp| T\left[J_3^+(x) J_3^+(y)\right] |nn\rangle$$

Somewhat similar to RBC **Δ**M$_K$ and rare kaon decays second-order weak (**Xu Feng plenary**)

  - Second order responses gives

$$C_{nn\to pp}(t) = \sum_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} \sum_{t_1=0}^{t} \sum_{t_2=0}^{t} \langle 0|\chi_{pp}(\boldsymbol{x},t) T\left\{J_3^+(\boldsymbol{y},t_1) J_3^+(\boldsymbol{z},t_2)\right\} \chi^\dagger_{nn}(0)|0\rangle$$

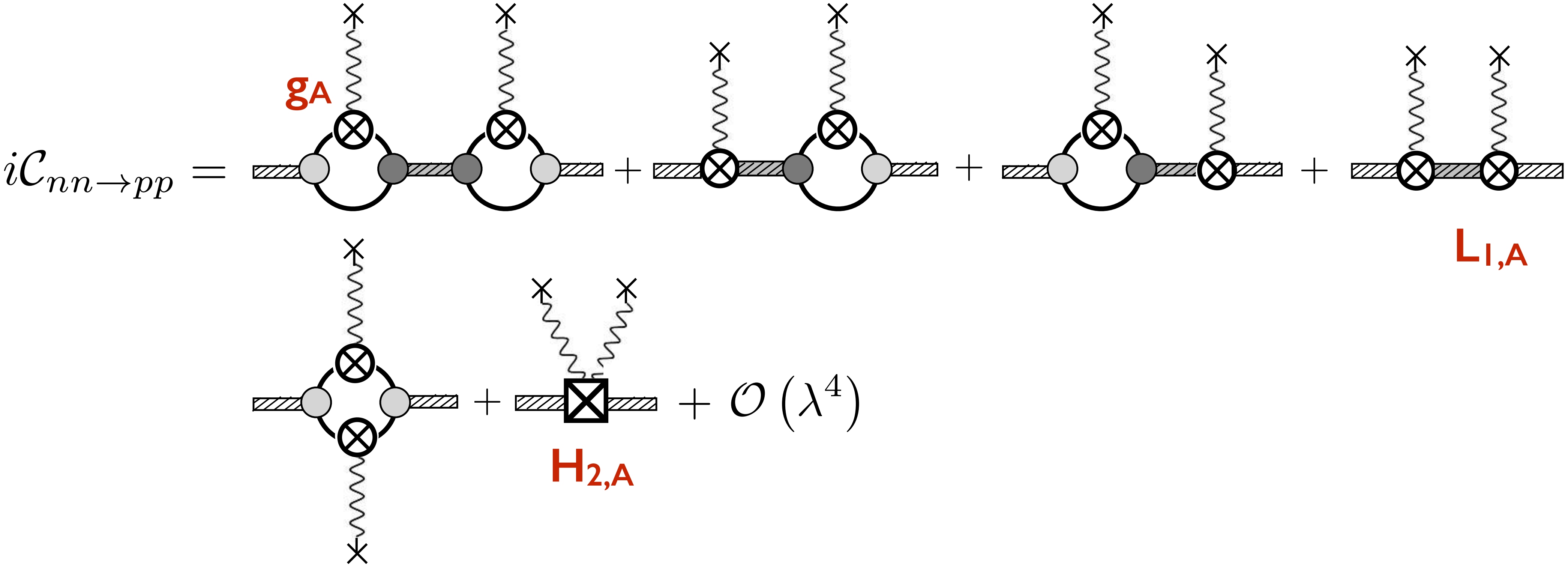
$$= \frac{2}{a^2} \sum_{\mathfrak{n},\mathfrak{m},\mathfrak{l}'} Z_{\mathfrak{n}} Z^\dagger_{\mathfrak{m}} e^{-E_{\mathfrak{n}} t} \frac{\langle \mathfrak{n}|\tilde{J}_3^+|\mathfrak{l}'\rangle\langle \mathfrak{l}'|\tilde{J}_3^+|\mathfrak{m}\rangle}{E_{\mathfrak{l}'} - E_{\mathfrak{m}}} \left(\frac{e^{-(E_{\mathfrak{l}'} - E_{\mathfrak{n}})t} - 1}{E_{\mathfrak{l}'} - E_{\mathfrak{n}}} + \frac{e^{(E_{\mathfrak{n}} - E_{\mathfrak{m}})t} - 1}{E_{\mathfrak{n}} - E_{\mathfrak{m}}} \right)$$

  - Both insertion times integrated

Second order weak interactions

- Contributions in effective field theory are

$$M^{2\nu}_{GT} = 6 \int d^4x d^4y \langle pp| T\left[J^+_3(x) J^+_3(y)\right] |nn\rangle$$



- One body (**gA**), two body single-weak current (**L1A**), two-body second order weak (**H2A**)

- **H2A** ~ ***isotensor axial polarisability*** ignored in all phenomenological analyses (unconstrained except by DBD)

Second order weak interactions

- Expand the summations over different state contributions

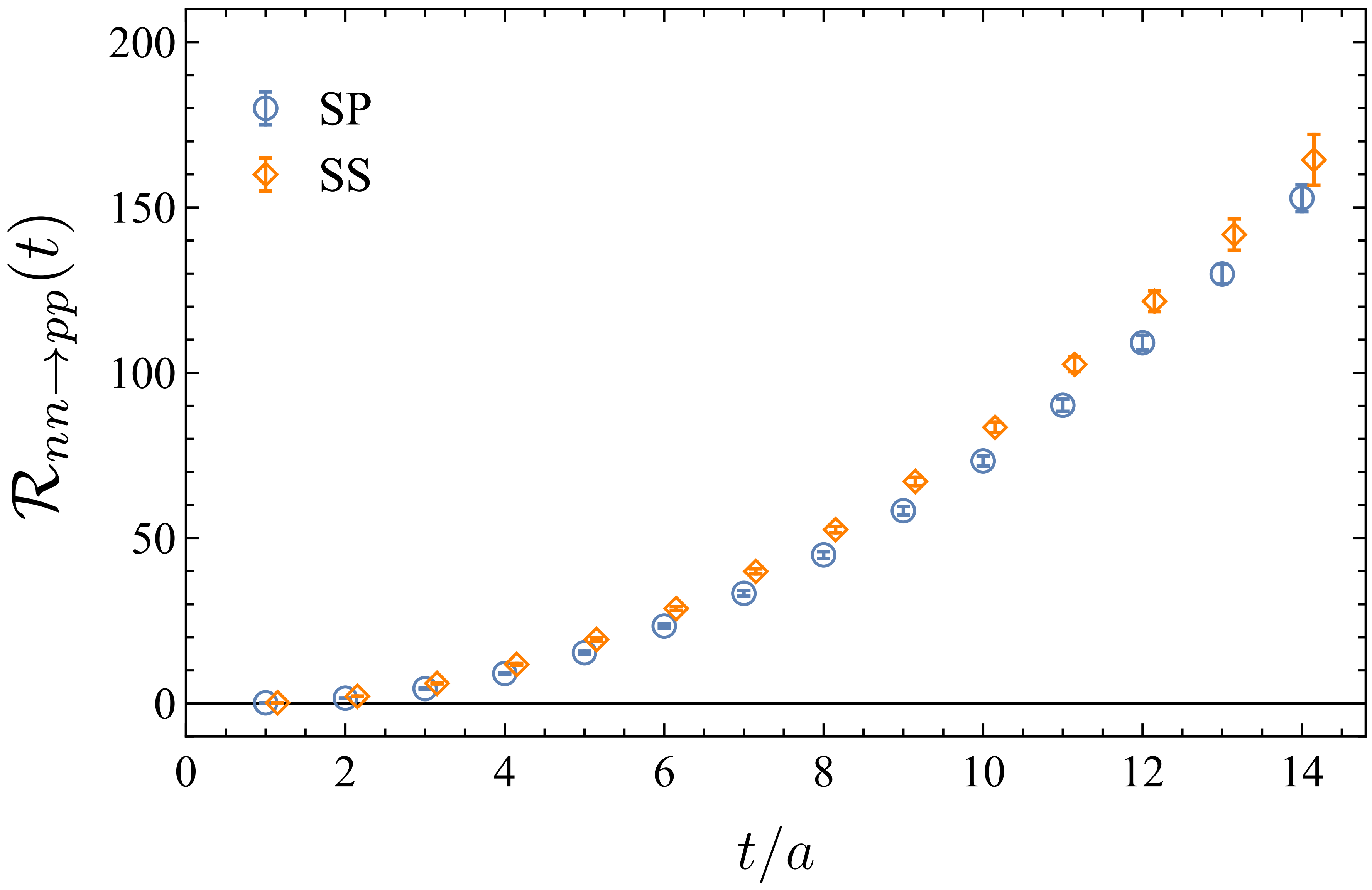
$$\begin{aligned}
a^2 C_{nn\to pp}(t) = 2 Z_{pp} Z^\dagger_{nn} e^{-E_{nn}t} \Bigg\{ & \left[\frac{e^{\Delta t}-1}{\Delta^2} - \frac{t}{\Delta}\right] \langle pp|\tilde{J}_3^+|d\rangle\langle d|\tilde{J}_3^+|nn\rangle \\
& + \sum_{\mathfrak{l}'\neq d} \left[\frac{t}{\delta_{\mathfrak{l}'}} - \frac{1}{\delta^2_{\mathfrak{l}'}}\right] \langle pp|\tilde{J}_3^+|\mathfrak{l}'\rangle\langle \mathfrak{l}'|\tilde{J}_3^+|nn\rangle \\
& + \sum_{\mathfrak{n}\neq nn,pp} \left[\frac{e^{\Delta t}}{\Delta(\Delta+\delta_{\mathfrak{n}})} - \frac{1}{\Delta\delta_{\mathfrak{n}}}\right] \left(\frac{Z_{\mathfrak{n}}}{Z_{pp}}\langle \mathfrak{n}|\tilde{J}_3^+|d\rangle\langle d|\tilde{J}_3^+|nn\rangle + \frac{Z^\dagger_{\mathfrak{n}}}{Z^\dagger_{nn}}\langle pp|\tilde{J}_3^+|d\rangle\langle d|\tilde{J}_3^+|\mathfrak{n}\rangle\right) \\
& + \sum_{\mathfrak{n}\neq nn,pp}\ \sum_{\mathfrak{l}'\neq d} \frac{1}{\delta_{\mathfrak{l}'}\delta_{\mathfrak{n}}} \left(\frac{Z_{\mathfrak{n}}}{Z_{pp}}\langle \mathfrak{n}|\tilde{J}_3^+|\mathfrak{l}'\rangle\langle \mathfrak{l}'|\tilde{J}_3^+|nn\rangle + \frac{Z^\dagger_{\mathfrak{n}}}{Z^\dagger_{nn}}\langle pp|\tilde{J}_3^+|\mathfrak{l}'\rangle\langle \mathfrak{l}'|\tilde{J}_3^+|\mathfrak{n}\rangle\right) \\
& + \sum_{\mathfrak{n},\mathfrak{m}\neq nn,pp} \frac{e^{\Delta t}}{(\Delta+\delta_{\mathfrak{n}})(\Delta+\delta_{\mathfrak{m}})} \frac{Z_{\mathfrak{n}}}{Z_{pp}} \frac{Z^\dagger_{\mathfrak{m}}}{Z^\dagger_{nn}} \langle \mathfrak{n}|\tilde{J}_3^+|d\rangle\langle d|\tilde{J}_3^+|\mathfrak{m}\rangle + \mathcal{O}(e^{-\delta t}, e^{-\delta' t}) \Bigg\}.
\end{aligned}$$

- Take ratio to two-point correlate

$$\begin{aligned}
\mathcal{R}_{nn\to pp}(t) = \frac{C_{nn\to pp}(t)}{2C^{(nn)}_{0;0}(t)} = \left[-t + \frac{e^{\Delta t}-1}{\Delta}\right] \frac{\langle pp|\tilde{J}_3^+|d\rangle\langle d|\tilde{J}_3^+|nn\rangle}{\Delta} + t \sum_{\mathfrak{l}'\neq d} \frac{\langle pp|\tilde{J}_3^+|\mathfrak{l}'\rangle\langle \mathfrak{l}'|\tilde{J}_3^+|nn\rangle}{\delta_{\mathfrak{l}'}} \\
+ C + D\, e^{\Delta t} + \mathcal{O}(e^{-\delta t}, e^{-\delta' t})
\end{aligned}$$

C,D irrelevant constants

Second order weak interactions



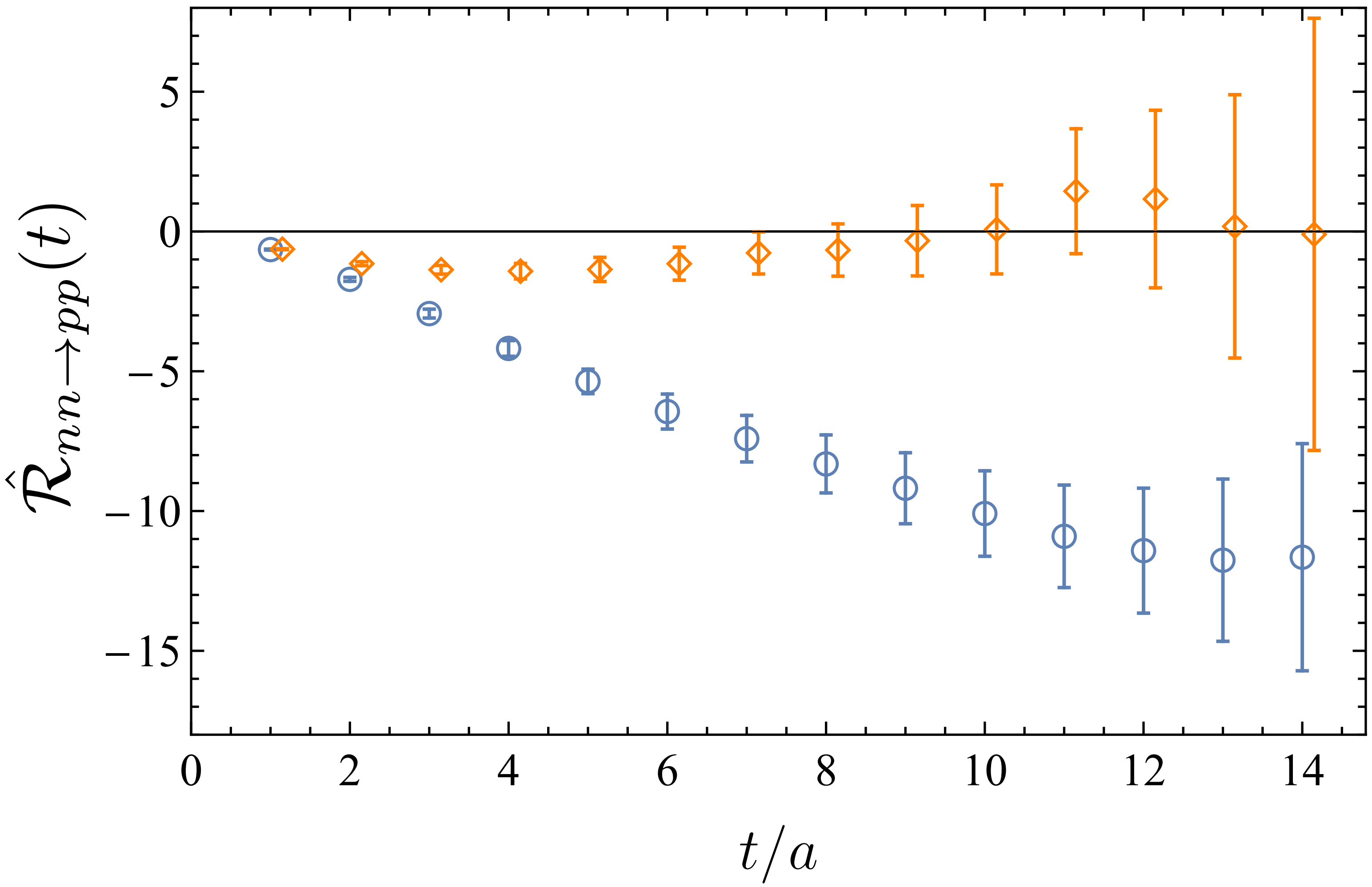
Second order weak interactions

- Correlated subtraction of deuteron pole term

$$\hat{\mathcal{R}}_{nn\to pp}(t) = \mathcal{R}_{nn\to pp}(t) - \frac{|\langle pp|\tilde{J}_3^+|d\rangle|^2}{a\Delta}\left[-\frac{t}{a} + \frac{e^{\Delta t}-1}{a\Delta}\right]$$

$$= \frac{t}{a}\sum_{\mathfrak{l}'\neq d}\frac{\langle pp|\tilde{J}_3^+|\mathfrak{l}'\rangle\langle\mathfrak{l}'|\tilde{J}_3^+|nn\rangle}{a\delta_{\mathfrak{l}'}} + c + d\;e^{\Delta t}.$$

c,d irrelevant constants



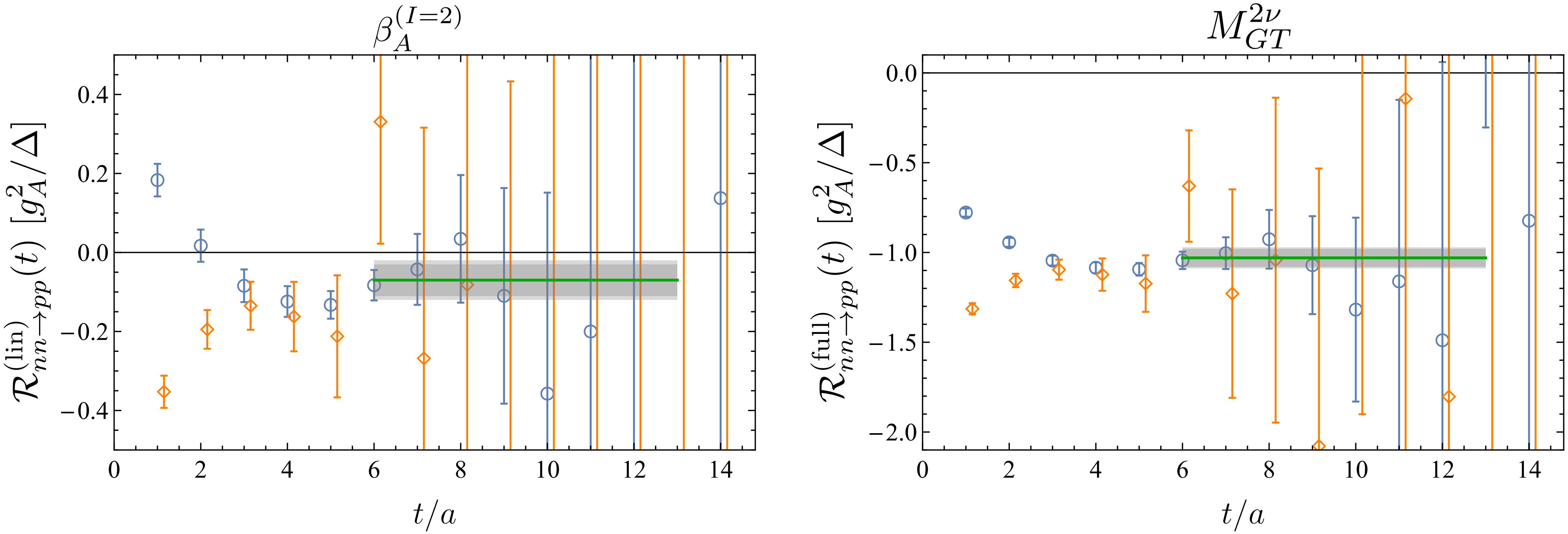
Second order weak interactions

- Take combinations to pull out isotensor axial polarisability (two body contribution)

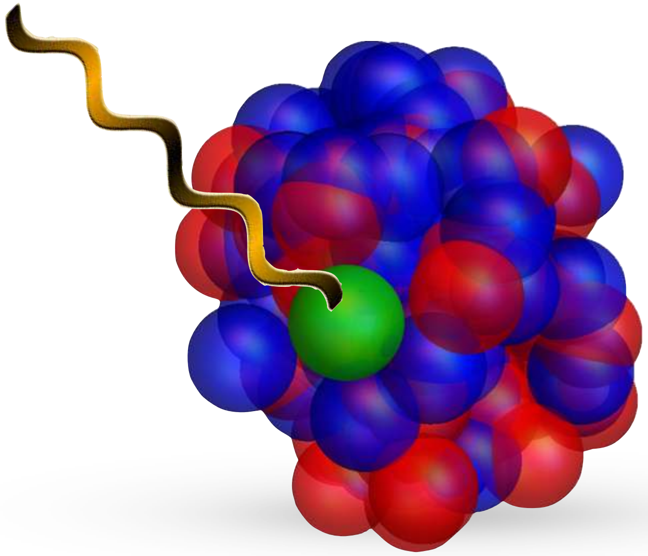
$$\mathcal{R}^{(\mathrm{lin})}_{nn\to pp}(t) = \frac{(e^{a\Delta}+1)\hat{\mathcal{R}}_{nn\to pp}(t+a) - \hat{\mathcal{R}}_{nn\to pp}(t+2a) - e^{a\Delta}\hat{\mathcal{R}}_{nn\to pp}(t)}{e^{a\Delta}-1} \xrightarrow{t\to\infty} \frac{1}{aZ_A^2}\frac{\beta_A^{(2)}}{6}$$

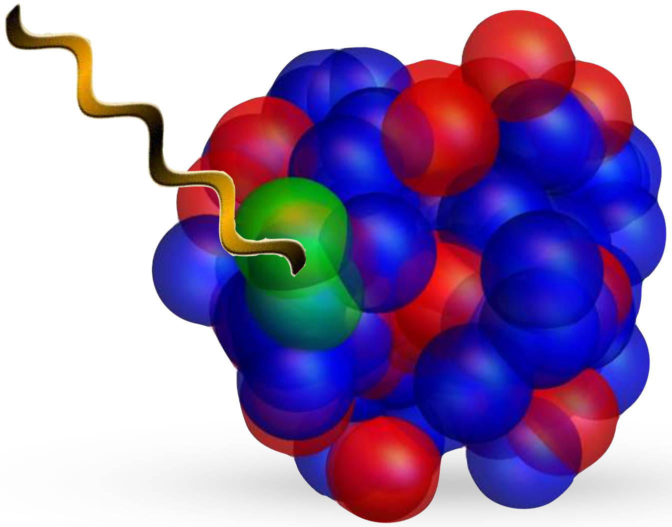
- Add back in deuteron pole

$$\mathcal{R}^{(\mathrm{full})}_{nn\to pp}(t) = \mathcal{R}^{(\mathrm{lin})}_{nn\to pp}(t) - \frac{|\langle pp|\tilde{J}_3^+|d\rangle|^2}{a\Delta} \xrightarrow{t\to\infty} \frac{1}{aZ_A^2}\frac{M_{GT}^{2\nu}}{6}$$



Larger nuclei





- What about larger (phenomenologically-relevant) nuclei?
- Nuclear effective field theory:
  - 1-body currents are dominant
  - 2-body currents are sub-leading *but non-negligible*
- Determine one body contributions from single nucleon
- Determine few-body contributions from A=2,3,4... (this calculation)
- Match EFT and many body methods to LQCD to make predictions for larger nuclei

fin

Acknowledgements

NPLQCD

USQCD

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ASCR Leadership Computing Challenge ALCC

NSF